Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/76-4.1.3.1-a+b-sin-^m-c+d-sin-^n-A+B-sin-

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [358]. This is test number [76].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (358)	0.00 (0)
Mathematica	97.21 (348)	2.79 (10)
Maple	81.01 (290)	18.99 (68)
Giac	79.89 (286)	20.11 (72)
Fricas	76.82 (275)	23.18 (83)
Mupad	49.72 (178)	50.28 (180)
Maxima	37.15 (133)	62.85 (225)
Sympy	28.49 (102)	71.51 (256)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

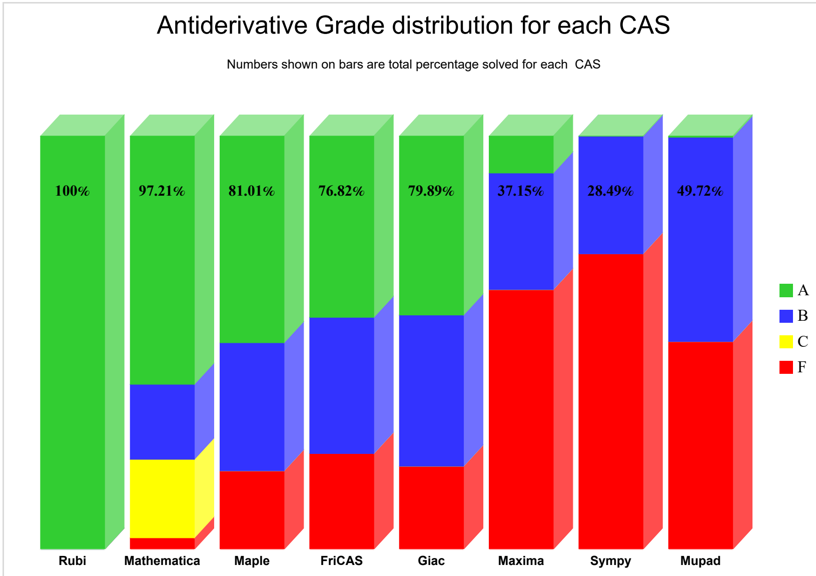
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

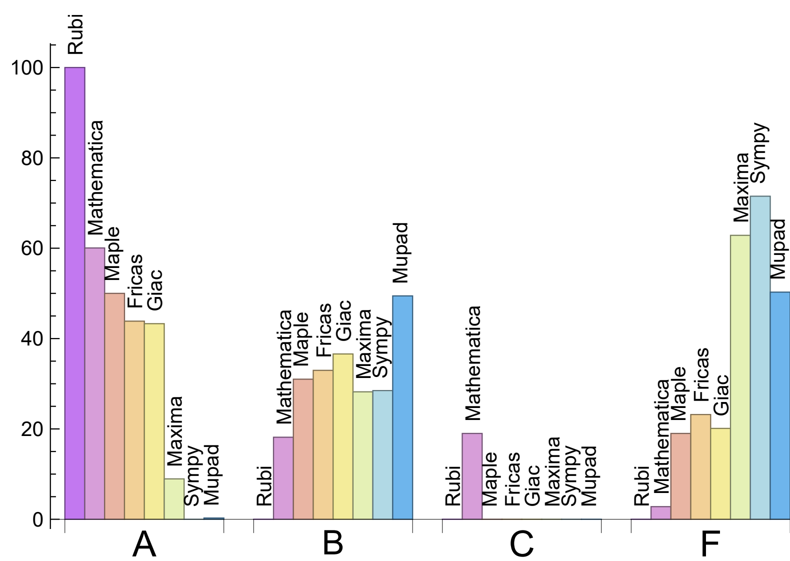
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	60.06	18.16	18.99	2.79
Maple	50.00	31.01	0.00	18.99
Fricas	43.85	32.96	0.00	23.18
Giac	43.30	36.59	0.00	20.11
Maxima	8.94	28.21	0.00	62.85
Mupad	N/A	49.44	0.00	50.28
Sympy	0.00	28.49	0.00	71.51

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	80.00 %	20.00 %	0.00 %
Maple	68	100.00 %	0.00 %	0.00 %
Fricas	83	100.00 %	0.00 %	0.00 %
Giac	72	80.56 %	12.50 %	6.94 %
Maxima	225	86.67 %	4.44 %	8.89 %
Sympy	256	39.84 %	42.97 %	17.19 %
Mupad	180	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

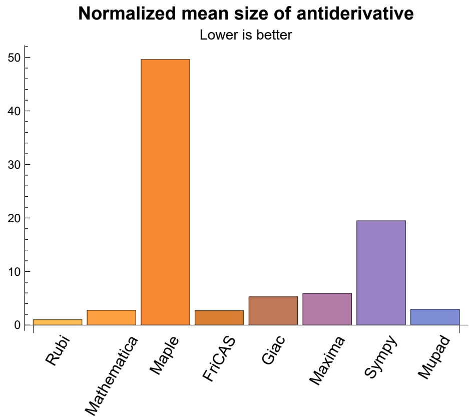
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

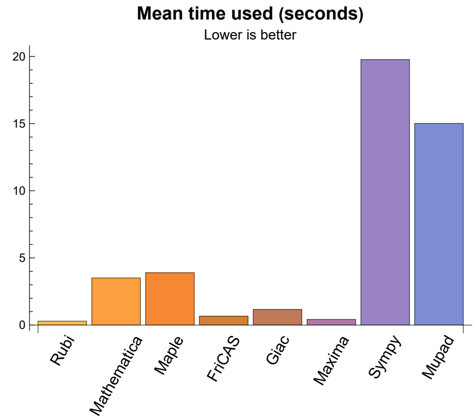
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.29	179.38	1.00	156.00	1.00
Mathematica	3.51	478.82	2.76	221.00	1.42
Maple	3.90	36143.47	49.58	210.50	1.48
Maxima	0.41	845.23	5.91	544.00	4.42
Fricas	0.66	544.42	2.68	245.00	1.87
Sympy	19.76	2842.08	19.47	1616.50	14.42
Giac	1.16	464.67	5.28	284.00	1.78
Mupad	15.01	473.44	2.94	298.50	2.09

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{358}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {7, 8, 11, 12, 195, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 215, 216, 217, 299, 304, 305, 306, 313, 320, 326, 327, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

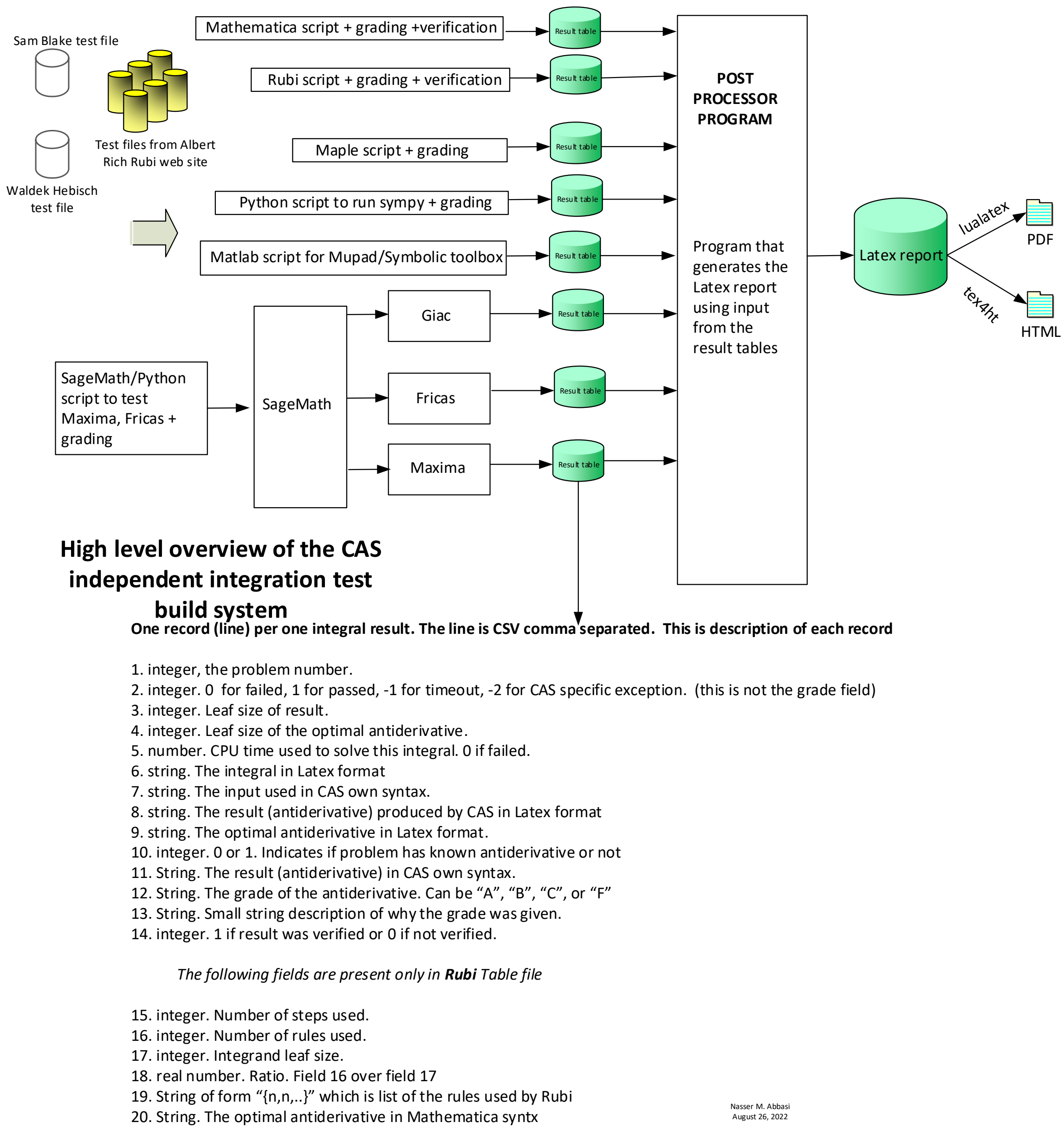
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 10, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 99, 101, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 206, 207, 208, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 235, 238, 239, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 298, 299, 301, 302, 303, 305, 306, 332, 334, 336, 337, 341, 350, 351, 352, 358 }

B grade: { 8, 11, 12, 14, 21, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 89, 90, 98, 100, 170, 171, 172, 232, 233, 234, 236, 237, 240, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 297, 304, 335, 339, 340, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

C grade: { 3, 9, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 195, 198, 199, 200, 201, 202, 205, 209, 210, 214, 215, 216, 217, 248, 249, 250, 290, 291, 292, 300, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338 }

F grade: { 13, 196, 197, 328, 329, 330, 331, 333, 348, 349 }

2.1.3 Maple

A grade: { 16, 18, 20, 21, 22, 23, 24, 25, 31, 32, 33, 35, 36, 37, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 137, 139, 140, 141, 142, 147, 149, 150, 151, 152, 160, 161, 162, 163, 177, 185, 191, 194, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 352, 358 }

B grade: { 17, 19, 26, 27, 28, 29, 30, 34, 38, 39, 40, 41, 42, 48, 49, 66, 67, 76, 77, 78, 79, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 131, 135, 136, 138, 143, 144, 145, 146, 148, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 250, 257, 260, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 353, 354, 355, 356, 357 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

2.1.4 Maxima

A grade: { 17, 18, 20, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261, 358 }

B grade: { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351, 358 }

B grade: { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { }

B grade: { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 218, 219, 220, 221, 224, 225, 226, 227, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 251, 252, 253, 254, 258, 259, 260, 261, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 222, 223, 228, 229, 230, 231, 232, 233, 234, 240, 241, 242, 243, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

2.1.7 Giac

A grade: { 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 77, 81, 82, 83, 84, 89, 112, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 147, 148, 151, 152, 153, 155, 156, 159, 163, 164, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 224, 225, 226, 227, 228, 230, 231, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 268, 269, 274, 275, 276, 277, 281, 282, 287, 288, 289, 290, 291, 294, 295, 296, 301, 303, 308, 309, 358 }

B grade: { 14, 15, 16, 21, 34, 35, 36, 37, 48, 49, 50, 51, 55, 61, 67, 68, 69, 76, 78, 79, 80, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 140, 141, 146, 149, 150, 157, 158, 160, 161, 162, 165, 170, 171, 178, 184, 222, 223, 229, 232, 234, 250, 256, 257, 262, 263, 264, 265, 267, 270, 271, 272, 273, 278, 279, 280, 283, 284, 285, 286, 292, 293, 297, 298, 299, 300, 302, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 154, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 316, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

2.1.8 Mupad

A grade: { 358 }

B grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 111, 118, 119, 125, 126, 127, 131, 132, 133, 134, 138, 139, 140, 141, 142, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 191, 205, 206, 207, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 350, 351, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 129, 130, 135, 136, 137, 143, 144, 145, 146, 153, 154, 155, 156, 157, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306,

$307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328,$
$329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349,$
$353, 354, 355, 356, 357 \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	248	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	1.615	0.828	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	204	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	1.056	1.424	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	392	0	0	0	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	3.384	1.203	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	157	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.598	0.335	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	212	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	0.875	1.699	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	260	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.589	2.915	1.872	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	596	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	29.705	0.227	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	478	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	26.739	0.213	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	409	0	0	0	0	0	-1
N.S.	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	44.824	0.241	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	250	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.259	24.220	0.286	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	523	0	0	0	0	0	-1
N.S.	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.452	55.883	0.286	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	5918	0	0	0	0	0	-1
N.S.	1	1.00	26.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	20.990	0.235	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	9.808	0.268	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	41	0	9496	61
N.S.	1	1.00	2.89	0.00	0.00	1.11	0.00	256.65	1.65
time (sec)	N/A	0.084	4.146	0.460	0.000	0.612	0.000	56.537	12.982

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	41	0	5502	38
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	157.20	1.09
time (sec)	N/A	0.063	0.269	0.367	0.000	0.377	0.000	26.689	12.848

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	212	0	823	0	371	2500
N.S.	1	1.00	0.96	1.39	0.00	5.38	0.00	2.42	16.34
time (sec)	N/A	0.273	0.595	0.415	0.000	0.445	0.000	0.460	17.182

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	131	342	363	129	853	184	454
N.S.	1	1.00	0.72	1.88	1.99	0.71	4.69	1.01	2.49
time (sec)	N/A	0.208	0.644	0.260	0.280	0.387	0.565	0.471	14.833

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	95	208	216	107	486	145	389
N.S.	1	1.00	0.67	1.46	1.52	0.75	3.42	1.02	2.74
time (sec)	N/A	0.168	0.572	0.179	0.283	0.376	0.365	0.417	13.800

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	105	74	185	193	86	396	114	345
N.S.	1	1.08	0.76	1.91	1.99	0.89	4.08	1.18	3.56
time (sec)	N/A	0.112	0.422	0.154	0.356	0.363	0.238	0.413	13.381

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	74	79	46	138	58	122
N.S.	1	1.00	0.98	1.51	1.61	0.94	2.82	1.18	2.49
time (sec)	N/A	0.047	0.112	0.094	0.272	0.369	0.133	0.409	14.329

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	125	67	287	122	828	124	111
N.S.	1	1.00	2.23	1.20	5.12	2.18	14.79	2.21	1.98
time (sec)	N/A	0.113	0.633	0.227	0.528	0.390	1.209	0.406	12.636

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	160	87	494	170	700	92	132
N.S.	1	1.00	2.22	1.21	6.86	2.36	9.72	1.28	1.83
time (sec)	N/A	0.154	0.429	0.273	0.515	0.374	2.438	0.436	12.506

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	147	115	801	195	1035	139	172
N.S.	1	1.00	1.41	1.11	7.70	1.88	9.95	1.34	1.65
time (sec)	N/A	0.164	0.499	0.345	0.302	0.363	5.360	0.454	12.981

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	174	159	1176	267	1831	187	228
N.S.	1	1.00	1.23	1.12	8.28	1.88	12.89	1.32	1.61
time (sec)	N/A	0.195	0.587	0.403	0.326	0.356	10.560	0.436	13.223

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	200	203	1553	325	3232	267	310
N.S.	1	1.00	1.14	1.15	8.82	1.85	18.36	1.52	1.76
time (sec)	N/A	0.215	0.583	0.507	0.405	0.361	20.035	0.463	13.345

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	219	569	615	165	1586	278	661
N.S.	1	1.00	0.96	2.48	2.69	0.72	6.93	1.21	2.89
time (sec)	N/A	0.247	1.242	0.356	0.308	0.378	1.171	0.492	15.132

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	163	463	495	141	1210	244	553
N.S.	1	1.00	0.86	2.45	2.62	0.75	6.40	1.29	2.93
time (sec)	N/A	0.205	0.985	0.269	0.293	0.389	0.811	0.467	14.888

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	137	365	387	119	910	208	542
N.S.	1	1.00	0.93	2.48	2.63	0.81	6.19	1.41	3.69
time (sec)	N/A	0.146	0.676	0.210	0.297	0.366	0.566	0.470	14.154

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	166	176	79	372	118	238
N.S.	1	1.00	0.61	1.87	1.98	0.89	4.18	1.33	2.67
time (sec)	N/A	0.097	0.103	0.128	0.277	0.368	0.334	0.456	14.205

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	186	193	81	396	111	339
N.S.	1	1.00	0.68	1.90	1.97	0.83	4.04	1.13	3.46
time (sec)	N/A	0.106	0.578	0.129	0.273	0.374	0.237	0.395	13.705

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	191	123	678	185	2365	163	244
N.S.	1	1.00	1.63	1.05	5.79	1.58	20.21	1.39	2.09
time (sec)	N/A	0.196	0.836	0.264	0.511	0.371	2.378	0.508	14.701

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	238	107	909	247	2474	135	246
N.S.	1	1.00	2.18	0.98	8.34	2.27	22.70	1.24	2.26
time (sec)	N/A	0.193	0.418	0.322	0.530	0.373	4.866	0.435	14.150

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	278	126	1237	289	1647	159	233
N.S.	1	1.00	2.48	1.12	11.04	2.58	14.71	1.42	2.08
time (sec)	N/A	0.185	0.475	0.398	0.526	0.370	9.578	0.485	14.679

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	191	161	1709	279	2008	229	269
N.S.	1	1.00	2.55	2.15	22.79	3.72	26.77	3.05	3.59
time (sec)	N/A	0.151	0.646	0.459	0.365	0.350	18.235	0.448	13.351

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	261	205	2273	355	3262	301	331
N.S.	1	1.00	2.27	1.78	19.77	3.09	28.37	2.62	2.88
time (sec)	N/A	0.192	0.779	0.565	0.363	0.366	33.150	0.500	13.358

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	285	249	2838	431	4816	373	423
N.S.	1	1.00	1.83	1.60	18.19	2.76	30.87	2.39	2.71
time (sec)	N/A	0.232	1.002	0.648	0.398	0.394	56.210	0.484	13.544

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	313	293	3402	503	6669	445	500
N.S.	1	1.00	1.59	1.49	17.27	2.55	33.85	2.26	2.54
time (sec)	N/A	0.275	2.317	0.545	0.425	0.369	93.867	0.509	14.053

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	255	651	713	189	1948	347	812
N.S.	1	1.00	0.96	2.46	2.69	0.71	7.35	1.31	3.06
time (sec)	N/A	0.265	2.623	0.492	0.303	0.400	2.132	0.493	14.869

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	232	611	665	165	1753	301	705
N.S.	1	1.00	1.05	2.75	3.00	0.74	7.90	1.36	3.18
time (sec)	N/A	0.218	1.609	0.434	0.300	0.410	1.576	0.480	14.929

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	209	568	615	143	1579	273	661
N.S.	1	1.00	1.15	3.14	3.40	0.79	8.72	1.51	3.65
time (sec)	N/A	0.166	1.204	0.348	0.301	0.393	1.167	0.483	14.782

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	64	263	284	97	682	162	325
N.S.	1	1.00	0.55	2.25	2.43	0.83	5.83	1.38	2.78
time (sec)	N/A	0.107	0.152	0.201	0.294	0.380	0.720	0.492	14.288

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	364	387	111	910	204	536
N.S.	1	1.00	0.96	2.64	2.80	0.80	6.59	1.48	3.88
time (sec)	N/A	0.140	0.670	0.213	0.287	0.381	0.567	0.532	14.166

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	95	208	216	105	486	145	390
N.S.	1	1.00	0.68	1.49	1.54	0.75	3.47	1.04	2.79
time (sec)	N/A	0.153	0.560	0.167	0.279	0.361	0.359	0.574	13.664

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	223	152	1241	228	4255	234	323
N.S.	1	1.00	1.43	0.97	7.96	1.46	27.28	1.50	2.07
time (sec)	N/A	0.210	0.919	0.287	0.517	0.373	4.619	0.522	14.120

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	280	166	1504	298	4665	233	341
N.S.	1	1.00	1.72	1.02	9.23	1.83	28.62	1.43	2.09
time (sec)	N/A	0.236	0.612	0.333	0.549	0.365	8.977	0.557	14.005

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	316	157	1831	351	4665	226	336
N.S.	1	1.00	2.07	1.03	11.97	2.29	30.49	1.48	2.20
time (sec)	N/A	0.240	0.734	0.444	0.612	0.370	17.182	0.583	14.002

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	356	177	2304	379	2951	213	316
N.S.	1	1.00	2.36	1.17	15.26	2.51	19.54	1.41	2.09
time (sec)	N/A	0.229	0.807	0.520	0.566	0.342	30.300	0.663	15.978

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	283	205	2941	351	3262	301	346
N.S.	1	1.00	3.68	2.66	38.19	4.56	42.36	3.91	4.49
time (sec)	N/A	0.157	1.635	0.602	0.380	0.532	51.282	0.651	13.392

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	313	249	3694	429	4816	373	408
N.S.	1	1.00	2.65	2.11	31.31	3.64	40.81	3.16	3.46
time (sec)	N/A	0.189	1.841	0.437	0.490	0.587	85.306	0.676	13.544

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	339	293	4446	503	6669	445	500
N.S.	1	1.00	2.17	1.88	28.50	3.22	42.75	2.85	3.21
time (sec)	N/A	0.240	3.345	0.542	0.472	0.591	135.346	0.680	13.794

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	365	337	5197	573	8821	517	577
N.S.	1	1.00	1.85	1.71	26.38	2.91	44.78	2.62	2.93
time (sec)	N/A	0.279	4.584	0.623	0.538	0.389	211.770	0.669	13.963

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	274	209	1962	273	6690	343	397
N.S.	1	1.00	1.44	1.10	10.33	1.44	35.21	1.81	2.09
time (sec)	N/A	0.253	1.539	0.321	0.525	0.366	8.623	0.568	14.778

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	220	152	1222	228	4255	235	319
N.S.	1	1.00	1.40	0.97	7.78	1.45	27.10	1.50	2.03
time (sec)	N/A	0.222	0.913	0.289	0.579	0.358	4.656	0.525	14.012

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	188	119	662	187	2365	164	241
N.S.	1	1.00	1.59	1.01	5.61	1.58	20.04	1.39	2.04
time (sec)	N/A	0.192	0.882	0.261	0.526	0.372	2.451	0.511	14.577

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	127	67	278	123	828	122	110
N.S.	1	1.00	2.23	1.18	4.88	2.16	14.53	2.14	1.93
time (sec)	N/A	0.110	0.402	0.224	0.493	0.377	1.253	0.499	12.941

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	57	37	30	83	41	39
N.S.	1	1.00	1.00	1.63	1.06	0.86	2.37	1.17	1.11
time (sec)	N/A	0.099	0.024	0.168	0.286	0.334	0.801	0.511	12.554

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	108	93	288	78	578	102	118
N.S.	1	1.00	1.71	1.48	4.57	1.24	9.17	1.62	1.87
time (sec)	N/A	0.139	0.409	0.232	0.302	0.345	2.698	0.486	12.323

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	157	145	459	114	1236	177	178
N.S.	1	1.00	1.54	1.42	4.50	1.12	12.12	1.74	1.75
time (sec)	N/A	0.184	0.586	0.322	0.297	0.346	5.573	0.554	12.469

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	240	189	673	150	2468	237	239
N.S.	1	1.00	1.69	1.33	4.74	1.06	17.38	1.67	1.68
time (sec)	N/A	0.222	0.756	0.358	0.304	0.363	11.821	0.521	13.233

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	354	252	3244	386	10608	412	500
N.S.	1	1.00	1.48	1.05	13.52	1.61	44.20	1.72	2.08
time (sec)	N/A	0.299	1.318	0.447	0.575	0.389	28.336	0.537	14.825

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	311	193	2276	336	7337	369	414
N.S.	1	1.00	1.73	1.07	12.64	1.87	40.76	2.05	2.30
time (sec)	N/A	0.254	0.842	0.407	0.552	0.364	16.258	0.534	14.952

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	274	162	1496	303	4665	233	336
N.S.	1	1.00	1.69	1.00	9.23	1.87	28.80	1.44	2.07
time (sec)	N/A	0.234	0.589	0.375	0.521	0.384	9.145	0.530	14.341

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	234	107	903	252	2474	136	242
N.S.	1	1.00	2.17	0.99	8.36	2.33	22.91	1.26	2.24
time (sec)	N/A	0.188	0.402	0.325	0.596	0.365	4.956	0.539	14.527

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	156	86	490	174	702	92	133
N.S.	1	1.00	2.17	1.19	6.81	2.42	9.75	1.28	1.85
time (sec)	N/A	0.144	0.397	0.266	0.565	0.400	2.506	0.482	12.826

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	110	97	287	74	578	102	117
N.S.	1	1.00	1.77	1.56	4.63	1.19	9.32	1.65	1.89
time (sec)	N/A	0.142	0.352	0.227	0.292	0.356	2.670	0.447	12.286

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	145	50	44	469	87	82
N.S.	1	1.00	0.85	2.34	0.81	0.71	7.56	1.40	1.32
time (sec)	N/A	0.098	0.090	0.205	0.285	0.362	2.420	0.460	12.383

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	237	183	705	123	2674	235	183
N.S.	1	1.00	2.55	1.97	7.58	1.32	28.75	2.53	1.97
time (sec)	N/A	0.152	0.696	0.336	0.304	0.362	11.352	0.496	12.450

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	285	233	905	160	4228	295	197
N.S.	1	1.00	2.11	1.73	6.70	1.19	31.32	2.19	1.46
time (sec)	N/A	0.187	0.660	0.450	0.336	0.353	22.302	0.469	12.749

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	329	277	1082	198	5868	355	337
N.S.	1	1.00	1.88	1.58	6.18	1.13	33.53	2.03	1.93
time (sec)	N/A	0.229	0.758	0.542	0.326	0.363	45.129	0.512	12.883

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	388	240	3572	447	10608	376	501
N.S.	1	1.00	1.60	0.99	14.70	1.84	43.65	1.55	2.06
time (sec)	N/A	0.296	1.777	0.513	0.691	0.376	48.373	0.514	14.754

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	348	201	2604	408	7337	305	419
N.S.	1	1.00	1.73	1.00	12.96	2.03	36.50	1.52	2.08
time (sec)	N/A	0.274	1.058	0.441	0.561	0.381	30.036	0.456	14.711

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	308	157	1825	352	4665	226	333
N.S.	1	1.00	2.01	1.03	11.93	2.30	30.49	1.48	2.18
time (sec)	N/A	0.228	0.721	0.410	0.550	0.365	17.511	0.473	14.365

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	272	127	1232	291	1647	159	230
N.S.	1	1.00	2.47	1.15	11.20	2.65	14.97	1.45	2.09
time (sec)	N/A	0.183	0.456	0.362	0.524	0.557	9.637	0.474	15.081

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	139	115	797	203	1035	138	172
N.S.	1	1.00	1.35	1.12	7.74	1.97	10.05	1.34	1.67
time (sec)	N/A	0.154	0.563	0.328	0.296	0.668	5.433	0.516	13.026

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	156	145	459	113	1236	175	178
N.S.	1	1.00	1.53	1.42	4.50	1.11	12.12	1.72	1.75
time (sec)	N/A	0.178	0.565	0.339	0.306	0.338	5.526	0.440	12.429

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	237	185	704	115	2674	235	183
N.S.	1	1.00	2.63	2.06	7.82	1.28	29.71	2.61	2.03
time (sec)	N/A	0.144	0.680	0.336	0.301	0.357	11.269	0.489	12.469

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	227	64	60	1098	134	126
N.S.	1	1.00	0.77	2.70	0.76	0.71	13.07	1.60	1.50
time (sec)	N/A	0.107	0.143	0.319	0.285	0.370	7.449	0.442	14.421

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	325	271	1105	159	6135	355	217
N.S.	1	1.00	2.69	2.24	9.13	1.31	50.70	2.93	1.79
time (sec)	N/A	0.159	0.764	0.497	0.332	0.363	44.453	0.497	13.158

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	373	321	1303	196	8396	415	231
N.S.	1	1.00	2.30	1.98	8.04	1.21	51.83	2.56	1.43
time (sec)	N/A	0.199	0.910	0.628	0.344	0.379	78.502	0.531	13.139

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	401	365	1505	234	11011	475	474
N.S.	1	1.00	1.96	1.78	7.34	1.14	53.71	2.32	2.31
time (sec)	N/A	0.241	2.246	0.450	0.368	0.377	136.236	0.485	14.522

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	149	119	0	302	0	312	-1
N.S.	1	1.00	0.75	0.60	0.00	1.53	0.00	1.58	-0.01
time (sec)	N/A	0.335	1.878	6.213	0.000	0.356	0.000	0.697	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	103	0	256	0	276	-1
N.S.	1	1.00	0.78	0.66	0.00	1.63	0.00	1.76	-0.01
time (sec)	N/A	0.278	0.979	5.451	0.000	0.371	0.000	0.700	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	81	0	195	0	205	-1
N.S.	1	1.00	0.90	0.70	0.00	1.68	0.00	1.77	-0.01
time (sec)	N/A	0.209	0.662	5.247	0.000	0.362	0.000	0.583	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	87	63	0	139	0	125	-1
N.S.	1	1.00	1.19	0.86	0.00	1.90	0.00	1.71	-0.01
time (sec)	N/A	0.168	0.281	5.184	0.000	0.348	0.000	0.543	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	166	159	0	274	0	314	-1
N.S.	1	1.00	1.36	1.30	0.00	2.25	0.00	2.57	-0.01
time (sec)	N/A	0.228	0.884	8.148	0.000	0.378	0.000	0.466	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	157	227	0	342	0	409	-1
N.S.	1	1.00	1.37	1.97	0.00	2.97	0.00	3.56	-0.01
time (sec)	N/A	0.222	1.128	6.365	0.000	0.373	0.000	0.514	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	199	268	0	422	0	440	-1
N.S.	1	1.00	1.58	2.13	0.00	3.35	0.00	3.49	-0.01
time (sec)	N/A	0.233	1.526	8.109	0.000	0.383	0.000	0.571	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	217	352	0	524	0	659	-1
N.S.	1	1.00	1.33	2.16	0.00	3.21	0.00	4.04	-0.01
time (sec)	N/A	0.264	2.249	10.470	0.000	0.371	0.000	0.614	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	1355	121	0	375	0	392	-1
N.S.	1	1.00	6.45	0.58	0.00	1.79	0.00	1.87	-0.00
time (sec)	N/A	0.374	6.451	6.194	0.000	0.385	0.000	0.782	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	1173	105	0	328	0	357	-1
N.S.	1	1.00	7.02	0.63	0.00	1.96	0.00	2.14	-0.01
time (sec)	N/A	0.294	6.357	5.744	0.000	0.356	0.000	0.661	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	83	0	241	0	251	-1
N.S.	1	1.00	0.88	0.69	0.00	2.01	0.00	2.09	-0.01
time (sec)	N/A	0.251	2.831	6.938	0.000	0.371	0.000	0.661	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	204	0	211	-1
N.S.	1	1.00	1.10	0.80	0.00	2.52	0.00	2.60	-0.01
time (sec)	N/A	0.209	0.382	4.940	0.000	0.357	0.000	0.580	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	175	197	0	332	0	502	-1
N.S.	1	1.00	1.09	1.22	0.00	2.06	0.00	3.12	-0.01
time (sec)	N/A	0.303	0.796	6.417	0.000	0.374	0.000	0.508	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	355	282	0	411	0	592	-1
N.S.	1	1.00	2.02	1.60	0.00	2.34	0.00	3.36	-0.01
time (sec)	N/A	0.317	0.623	6.725	0.000	0.382	0.000	0.586	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	344	386	0	479	0	630	-1
N.S.	1	1.00	1.97	2.21	0.00	2.74	0.00	3.60	-0.01
time (sec)	N/A	0.320	0.798	10.194	0.000	0.493	0.000	0.591	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	342	354	0	555	0	689	-1
N.S.	1	1.00	1.95	2.02	0.00	3.17	0.00	3.94	-0.01
time (sec)	N/A	0.326	1.219	9.523	0.000	0.593	0.000	0.630	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	357	440	0	694	0	781	-1
N.S.	1	1.00	1.61	1.98	0.00	3.13	0.00	3.52	-0.00
time (sec)	N/A	0.359	1.672	9.960	0.000	0.867	0.000	0.638	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	1569	121	0	424	0	481	-1
N.S.	1	1.00	7.47	0.58	0.00	2.02	0.00	2.29	-0.00
time (sec)	N/A	0.368	6.544	6.131	0.000	0.366	0.000	0.728	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	143	105	0	351	0	391	-1
N.S.	1	1.00	0.89	0.65	0.00	2.18	0.00	2.43	-0.01
time (sec)	N/A	0.317	5.923	6.063	0.000	0.369	0.000	0.751	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1157	83	0	302	0	310	-1
N.S.	1	1.00	9.33	0.67	0.00	2.44	0.00	2.50	-0.01
time (sec)	N/A	0.271	6.346	6.022	0.000	0.356	0.000	0.681	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	245	0	267	-1
N.S.	1	1.00	1.10	0.80	0.00	3.02	0.00	3.30	-0.01
time (sec)	N/A	0.209	0.660	5.572	0.000	0.373	0.000	0.669	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	193	233	0	377	0	677	-1
N.S.	1	1.00	0.96	1.16	0.00	1.88	0.00	3.38	-0.00
time (sec)	N/A	0.365	0.893	8.616	0.000	0.375	0.000	0.522	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	444	354	0	458	0	764	-1
N.S.	1	1.00	2.04	1.62	0.00	2.10	0.00	3.50	-0.00
time (sec)	N/A	0.387	1.108	7.244	0.000	0.373	0.000	0.597	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	434	434	0	537	0	815	-1
N.S.	1	1.00	1.93	1.93	0.00	2.39	0.00	3.62	-0.00
time (sec)	N/A	0.379	1.487	9.435	0.000	0.377	0.000	0.631	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	422	524	0	590	0	753	-1
N.S.	1	1.00	1.94	2.41	0.00	2.72	0.00	3.47	-0.00
time (sec)	N/A	0.376	2.084	9.839	0.000	0.375	0.000	0.665	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	355	432	0	673	0	974	-1
N.S.	1	1.00	1.64	1.99	0.00	3.10	0.00	4.49	-0.00
time (sec)	N/A	0.368	2.951	9.527	0.000	0.388	0.000	0.686	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	409	526	0	806	0	1037	-1
N.S.	1	1.00	1.54	1.98	0.00	3.03	0.00	3.90	-0.00
time (sec)	N/A	0.393	4.666	10.559	0.000	0.393	0.000	0.762	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	157	111	518	121	0	821	-1
N.S.	1	1.00	0.78	0.56	2.59	0.60	0.00	4.10	-0.00
time (sec)	N/A	0.263	3.507	4.507	0.573	0.343	0.000	0.740	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	134	95	418	100	0	605	-1
N.S.	1	1.00	0.84	0.60	2.63	0.63	0.00	3.81	-0.01
time (sec)	N/A	0.235	1.124	4.343	0.540	0.353	0.000	0.677	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	73	318	71	0	373	-1
N.S.	1	1.00	0.96	0.62	2.69	0.60	0.00	3.16	-0.01
time (sec)	N/A	0.212	0.417	4.128	0.512	0.337	0.000	0.594	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	188	47	0	185	128
N.S.	1	1.00	0.60	0.73	2.58	0.64	0.00	2.53	1.75
time (sec)	N/A	0.184	0.162	3.783	0.570	0.350	0.000	0.496	13.150

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	140	130	0	176	0	154	-1
N.S.	1	1.00	1.54	1.43	0.00	1.93	0.00	1.69	-0.01
time (sec)	N/A	0.183	0.319	6.872	0.000	0.348	0.000	0.506	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	284	225	0	250	0	426	-1
N.S.	1	1.00	2.09	1.65	0.00	1.84	0.00	3.13	-0.01
time (sec)	N/A	0.225	0.379	5.898	0.000	0.362	0.000	0.511	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	404	350	0	304	0	528	-1
N.S.	1	1.00	2.24	1.94	0.00	1.69	0.00	2.93	-0.01
time (sec)	N/A	0.283	0.581	8.977	0.000	0.370	0.000	0.628	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	826	162	0	1034	-1
N.S.	1	1.00	0.73	0.59	3.41	0.67	0.00	4.27	-0.00
time (sec)	N/A	0.440	4.677	5.532	0.558	0.362	0.000	0.820	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	159	121	726	141	0	818	-1
N.S.	1	1.00	0.79	0.60	3.61	0.70	0.00	4.07	-0.00
time (sec)	N/A	0.368	1.894	7.248	0.612	0.372	0.000	0.754	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	130	105	625	117	0	309	-1
N.S.	1	1.00	0.84	0.68	4.06	0.76	0.00	2.01	-0.01
time (sec)	N/A	0.322	0.801	5.917	0.542	0.357	0.000	0.621	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	113	81	522	86	0	299	492
N.S.	1	1.00	0.98	0.70	4.54	0.75	0.00	2.60	4.28
time (sec)	N/A	0.268	0.477	6.464	0.540	0.349	0.000	0.620	17.178

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	87	63	371	65	0	236	137
N.S.	1	1.00	1.12	0.81	4.76	0.83	0.00	3.03	1.76
time (sec)	N/A	0.201	0.219	5.181	0.530	0.342	0.000	0.606	17.508

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	168	0	236	0	271	-1
N.S.	1	1.00	1.30	1.24	0.00	1.75	0.00	2.01	-0.01
time (sec)	N/A	0.236	0.365	7.085	0.000	0.378	0.000	0.530	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	300	258	0	222	0	523	-1
N.S.	1	1.00	1.71	1.47	0.00	1.27	0.00	2.99	-0.01
time (sec)	N/A	0.259	0.582	6.742	0.000	0.374	0.000	0.578	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	430	426	0	300	0	787	-1
N.S.	1	1.00	1.91	1.89	0.00	1.33	0.00	3.50	-0.00
time (sec)	N/A	0.326	0.963	8.992	0.000	0.372	0.000	0.672	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	1025	176	0	417	-1
N.S.	1	1.00	0.73	0.59	4.24	0.73	0.00	1.72	-0.00
time (sec)	N/A	0.430	2.878	5.455	0.547	0.380	0.000	0.708	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	158	121	926	157	0	818	-1
N.S.	1	1.00	0.76	0.58	4.43	0.75	0.00	3.91	-0.00
time (sec)	N/A	0.376	1.802	5.490	0.559	0.370	0.000	0.806	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	132	105	825	133	0	474	904
N.S.	1	1.00	0.82	0.66	5.16	0.83	0.00	2.96	5.65
time (sec)	N/A	0.327	0.826	5.533	0.537	0.362	0.000	0.661	22.785

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	113	83	719	102	0	395	683
N.S.	1	1.00	0.93	0.69	5.94	0.84	0.00	3.26	5.64
time (sec)	N/A	0.272	0.495	5.589	0.538	0.346	0.000	0.589	19.157

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	89	65	547	83	0	390	479
N.S.	1	1.00	1.05	0.76	6.44	0.98	0.00	4.59	5.64
time (sec)	N/A	0.205	0.230	5.923	0.580	0.355	0.000	0.543	17.535

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	204	200	0	284	0	472	-1
N.S.	1	1.00	1.17	1.15	0.00	1.63	0.00	2.71	-0.01
time (sec)	N/A	0.294	0.545	7.180	0.000	0.381	0.000	0.567	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	357	308	0	298	0	682	-1
N.S.	1	1.00	1.59	1.38	0.00	1.33	0.00	3.04	-0.00
time (sec)	N/A	0.325	0.948	6.859	0.000	0.385	0.000	0.625	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	479	410	0	256	0	946	-1
N.S.	1	1.00	1.86	1.59	0.00	0.99	0.00	3.67	-0.00
time (sec)	N/A	0.378	1.510	8.254	0.000	0.398	0.000	0.695	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	174	0	148	0	159	173
N.S.	1	1.00	1.26	1.85	0.00	1.57	0.00	1.69	1.84
time (sec)	N/A	0.214	0.707	22.542	0.000	0.380	0.000	0.514	16.166

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	102	129	0	128	0	159	149
N.S.	1	1.00	1.09	1.37	0.00	1.36	0.00	1.69	1.59
time (sec)	N/A	0.218	0.572	0.384	0.000	0.360	0.000	0.571	14.959

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	91	0	98	0	153	122
N.S.	1	1.00	0.89	0.97	0.00	1.04	0.00	1.63	1.30
time (sec)	N/A	0.221	0.385	0.378	0.000	0.370	0.000	0.550	1.787

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	57	0	66	0	148	75
N.S.	1	1.00	0.68	0.62	0.00	0.72	0.00	1.61	0.82
time (sec)	N/A	0.209	0.122	0.367	0.000	0.360	0.000	0.508	0.938

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	125	394	187	0	0	148	-1
N.S.	1	1.00	1.25	3.94	1.87	0.00	0.00	1.48	-0.01
time (sec)	N/A	0.221	0.810	0.339	0.510	0.000	0.000	0.494	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	118	403	0	0	0	118	-1
N.S.	1	1.00	1.19	4.07	0.00	0.00	0.00	1.19	-0.01
time (sec)	N/A	0.240	0.674	0.346	0.000	0.000	0.000	0.475	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	101	137	0	94	0	118	-1
N.S.	1	1.00	1.10	1.49	0.00	1.02	0.00	1.28	-0.01
time (sec)	N/A	0.233	0.370	0.313	0.000	0.386	0.000	0.480	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	103	205	0	114	0	118	153
N.S.	1	1.00	1.10	2.18	0.00	1.21	0.00	1.26	1.63
time (sec)	N/A	0.234	0.426	0.324	0.000	0.374	0.000	0.526	17.626

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	205	185	0	156	0	262	323
N.S.	1	1.00	1.40	1.27	0.00	1.07	0.00	1.79	2.21
time (sec)	N/A	0.239	1.015	0.411	0.000	0.386	0.000	0.581	17.532

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	172	147	0	133	0	362	174
N.S.	1	1.00	1.18	1.01	0.00	0.91	0.00	2.48	1.19
time (sec)	N/A	0.239	1.099	0.385	0.000	0.400	0.000	0.583	16.567

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	96	86	0	89	0	252	103
N.S.	1	1.00	0.72	0.64	0.00	0.66	0.00	1.88	0.77
time (sec)	N/A	0.229	0.513	0.348	0.000	0.379	0.000	0.526	1.842

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	91	0	93	0	153	122
N.S.	1	1.00	0.84	0.95	0.00	0.97	0.00	1.59	1.27
time (sec)	N/A	0.222	0.401	0.377	0.000	0.372	0.000	0.512	14.292

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	494	0	0	0	256	-1
N.S.	1	1.00	0.94	3.41	0.00	0.00	0.00	1.77	-0.01
time (sec)	N/A	0.252	0.480	0.382	0.000	0.000	0.000	0.562	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	210	748	394	0	0	236	-1
N.S.	1	1.00	1.33	4.73	2.49	0.00	0.00	1.49	-0.01
time (sec)	N/A	0.256	0.605	0.352	0.525	0.000	0.000	0.516	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	198	594	0	0	0	215	-1
N.S.	1	1.00	1.33	3.99	0.00	0.00	0.00	1.44	-0.01
time (sec)	N/A	0.252	0.656	0.346	0.000	0.000	0.000	0.537	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	125	223	0	134	0	193	-1
N.S.	1	1.00	1.30	2.32	0.00	1.40	0.00	2.01	-0.01
time (sec)	N/A	0.184	0.689	0.354	0.000	0.391	0.000	0.498	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	123	217	0	144	0	193	245
N.S.	1	1.00	0.84	1.49	0.00	0.99	0.00	1.32	1.68
time (sec)	N/A	0.255	0.893	0.325	0.000	0.414	0.000	0.525	18.994

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	126	339	0	166	0	193	279
N.S.	1	1.00	0.82	2.20	0.00	1.08	0.00	1.25	1.81
time (sec)	N/A	0.252	1.267	0.361	0.000	0.405	0.000	0.504	20.227

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	223	203	0	168	0	480	383
N.S.	1	1.00	1.13	1.03	0.00	0.85	0.00	2.42	1.93
time (sec)	N/A	0.312	1.774	0.451	0.000	0.439	0.000	0.637	18.173

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	113	114	0	124	0	376	131
N.S.	1	1.00	0.63	0.63	0.00	0.69	0.00	2.09	0.73
time (sec)	N/A	0.307	0.523	0.376	0.000	0.396	0.000	0.567	16.003

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	165	147	0	126	0	262	174
N.S.	1	1.00	1.16	1.04	0.00	0.89	0.00	1.85	1.23
time (sec)	N/A	0.237	1.191	0.375	0.000	0.419	0.000	0.541	16.324

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	102	131	0	124	0	159	149
N.S.	1	1.00	1.06	1.36	0.00	1.29	0.00	1.66	1.55
time (sec)	N/A	0.219	0.569	0.388	0.000	0.408	0.000	0.513	2.701

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	177	590	0	0	0	318	-1
N.S.	1	1.00	0.92	3.06	0.00	0.00	0.00	1.65	-0.01
time (sec)	N/A	0.315	1.012	0.378	0.000	0.000	0.000	0.594	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	231	845	0	0	0	0	-1
N.S.	1	1.00	1.10	4.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	1.145	0.313	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	207	1092	546	0	0	311	-1
N.S.	1	1.00	0.98	5.15	2.58	0.00	0.00	1.47	-0.00
time (sec)	N/A	0.330	0.769	0.309	0.700	0.000	0.000	0.562	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	204	832	0	0	0	289	-1
N.S.	1	1.00	1.04	4.24	0.00	0.00	0.00	1.47	-0.01
time (sec)	N/A	0.333	0.794	0.335	0.000	0.000	0.000	0.546	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	145	309	0	176	0	282	-1
N.S.	1	1.00	1.51	3.22	0.00	1.83	0.00	2.94	-0.01
time (sec)	N/A	0.189	1.912	0.323	0.000	0.378	0.000	0.554	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	368	0	194	0	282	341
N.S.	1	1.00	1.00	2.52	0.00	1.33	0.00	1.93	2.34
time (sec)	N/A	0.255	2.662	0.392	0.000	0.383	0.000	0.573	20.756

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	144	423	0	209	0	282	357
N.S.	1	1.00	0.73	2.16	0.00	1.07	0.00	1.44	1.82
time (sec)	N/A	0.326	3.589	0.429	0.000	0.408	0.000	0.532	20.715

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	269	259	0	192	0	584	482
N.S.	1	1.00	1.08	1.04	0.00	0.77	0.00	2.34	1.93
time (sec)	N/A	0.394	4.563	0.427	0.000	0.430	0.000	0.658	20.031

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	135	142	0	142	0	480	384
N.S.	1	1.00	0.60	0.63	0.00	0.63	0.00	2.12	1.70
time (sec)	N/A	0.375	1.061	0.431	0.000	0.417	0.000	0.610	17.653

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	232	203	0	158	0	376	383
N.S.	1	1.00	1.21	1.06	0.00	0.82	0.00	1.96	1.99
time (sec)	N/A	0.317	1.392	0.492	0.000	0.444	0.000	0.580	18.428

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	212	187	0	150	0	262	321
N.S.	1	1.00	1.49	1.32	0.00	1.06	0.00	1.85	2.26
time (sec)	N/A	0.244	1.257	0.438	0.000	0.400	0.000	0.546	18.169

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	121	174	0	147	0	159	173
N.S.	1	1.00	1.26	1.81	0.00	1.53	0.00	1.66	1.80
time (sec)	N/A	0.223	0.680	0.382	0.000	0.385	0.000	0.525	16.679

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	183	671	0	0	0	494	-1
N.S.	1	1.00	0.77	2.81	0.00	0.00	0.00	2.07	-0.00
time (sec)	N/A	0.391	1.847	0.325	0.000	0.000	0.000	0.652	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	292	927	0	0	0	411	-1
N.S.	1	1.00	1.08	3.42	0.00	0.00	0.00	1.52	-0.00
time (sec)	N/A	0.407	2.319	0.309	0.000	0.000	0.000	0.620	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	251	1189	0	0	0	433	-1
N.S.	1	1.00	0.95	4.52	0.00	0.00	0.00	1.65	-0.00
time (sec)	N/A	0.402	1.691	0.309	0.000	0.000	0.000	0.594	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	244	1455	809	0	0	376	-1
N.S.	1	1.00	0.92	5.51	3.06	0.00	0.00	1.42	-0.00
time (sec)	N/A	0.417	1.985	0.309	0.550	0.000	0.000	0.582	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	238	1019	0	0	0	354	-1
N.S.	1	1.00	0.96	4.13	0.00	0.00	0.00	1.43	-0.00
time (sec)	N/A	0.409	1.793	0.385	0.000	0.000	0.000	0.632	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	434	389	0	212	0	359	-1
N.S.	1	1.00	4.52	4.05	0.00	2.21	0.00	3.74	-0.01
time (sec)	N/A	0.184	6.606	0.369	0.000	0.411	0.000	0.572	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	442	393	0	228	0	359	406
N.S.	1	1.00	3.03	2.69	0.00	1.56	0.00	2.46	2.78
time (sec)	N/A	0.248	6.665	0.409	0.000	0.431	0.000	0.550	22.848

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	442	505	0	249	0	359	827
N.S.	1	1.00	2.19	2.50	0.00	1.23	0.00	1.78	4.09
time (sec)	N/A	0.334	6.736	0.472	0.000	0.450	0.000	0.567	25.256

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	436	560	0	259	0	359	841
N.S.	1	1.00	1.77	2.28	0.00	1.05	0.00	1.46	3.42
time (sec)	N/A	0.400	6.706	0.542	0.000	0.470	0.000	0.546	28.344

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	185	601	0	0	0	317	-1
N.S.	1	1.00	0.94	3.05	0.00	0.00	0.00	1.61	-0.01
time (sec)	N/A	0.316	0.881	0.383	0.000	0.000	0.000	0.591	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	507	0	0	0	260	-1
N.S.	1	1.00	1.00	3.47	0.00	0.00	0.00	1.78	-0.01
time (sec)	N/A	0.248	0.472	0.329	0.000	0.000	0.000	0.553	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	126	403	188	0	0	150	-1
N.S.	1	1.00	1.31	4.20	1.96	0.00	0.00	1.56	-0.01
time (sec)	N/A	0.220	0.790	0.314	0.523	0.000	0.000	0.497	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	167	0	0	0	150	-1
N.S.	1	1.00	0.86	1.48	0.00	0.00	0.00	1.33	-0.01
time (sec)	N/A	0.247	0.233	0.276	0.000	0.000	0.000	0.478	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	191	308	0	363	0	227	-1
N.S.	1	1.00	1.85	2.99	0.00	3.52	0.00	2.20	-0.01
time (sec)	N/A	0.178	0.381	0.318	0.000	0.443	0.000	0.529	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	222	471	0	456	0	263	-1
N.S.	1	1.00	1.45	3.08	0.00	2.98	0.00	1.72	-0.01
time (sec)	N/A	0.247	0.417	0.339	0.000	0.439	0.000	0.548	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	271	948	0	0	0	413	-1
N.S.	1	1.00	1.00	3.50	0.00	0.00	0.00	1.52	-0.00
time (sec)	N/A	0.395	2.319	0.325	0.000	0.000	0.000	0.599	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	212	865	0	0	0	266	-1
N.S.	1	1.00	1.01	4.12	0.00	0.00	0.00	1.27	-0.00
time (sec)	N/A	0.329	1.070	0.316	0.000	0.000	0.000	0.500	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	190	774	395	0	0	237	-1
N.S.	1	1.00	1.19	4.87	2.48	0.00	0.00	1.49	-0.01
time (sec)	N/A	0.260	0.612	0.320	0.530	0.000	0.000	0.526	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	157	413	0	0	0	120	-1
N.S.	1	1.00	1.57	4.13	0.00	0.00	0.00	1.20	-0.01
time (sec)	N/A	0.247	0.813	0.299	0.000	0.000	0.000	0.525	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	186	307	0	355	0	222	-1
N.S.	1	1.00	1.81	2.98	0.00	3.45	0.00	2.16	-0.01
time (sec)	N/A	0.179	0.385	0.301	0.000	0.444	0.000	0.497	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	178	131	0	292	0	239	-1
N.S.	1	1.00	1.19	0.87	0.00	1.95	0.00	1.59	-0.01
time (sec)	N/A	0.253	0.481	0.303	0.000	0.472	0.000	0.519	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	306	435	0	467	0	323	-1
N.S.	1	1.00	1.41	2.00	0.00	2.15	0.00	1.49	-0.00
time (sec)	N/A	0.335	0.649	0.290	0.000	0.454	0.000	0.515	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	286	1301	0	0	0	475	-1
N.S.	1	1.00	0.89	4.03	0.00	0.00	0.00	1.47	-0.00
time (sec)	N/A	0.461	4.096	0.322	0.000	0.000	0.000	0.599	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	243	1219	0	0	0	434	-1
N.S.	1	1.00	0.92	4.63	0.00	0.00	0.00	1.65	-0.00
time (sec)	N/A	0.395	1.699	0.306	0.000	0.000	0.000	0.612	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	199	1120	544	0	0	312	-1
N.S.	1	1.00	0.94	5.31	2.58	0.00	0.00	1.48	-0.00
time (sec)	N/A	0.325	0.768	0.305	0.527	0.000	0.000	0.535	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	179	611	0	0	0	181	-1
N.S.	1	1.00	1.20	4.10	0.00	0.00	0.00	1.21	-0.01
time (sec)	N/A	0.257	0.662	0.308	0.000	0.000	0.000	0.522	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	135	0	92	0	117	156
N.S.	1	1.00	1.05	1.44	0.00	0.98	0.00	1.24	1.66
time (sec)	N/A	0.220	0.357	0.292	0.000	0.383	0.000	0.485	14.861

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	214	471	0	440	0	260	-1
N.S.	1	1.00	1.42	3.12	0.00	2.91	0.00	1.72	-0.01
time (sec)	N/A	0.242	0.453	0.316	0.000	0.435	0.000	0.505	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	305	435	0	449	0	318	-1
N.S.	1	1.00	1.47	2.09	0.00	2.16	0.00	1.53	-0.00
time (sec)	N/A	0.331	0.688	0.295	0.000	0.466	0.000	0.542	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	246	152	0	328	0	286	-1
N.S.	1	1.00	1.00	0.62	0.00	1.34	0.00	1.17	-0.00
time (sec)	N/A	0.394	0.641	0.342	0.000	0.470	0.000	0.557	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	1931	0	0	0	0	0	-1
N.S.	1	1.00	11.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	12.413	0.323	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	180.011	0.849	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	180.014	1.237	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	462	0	0	0	0	0	-1
N.S.	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	3.661	1.312	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.279	0.665	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	6999	0	0	0	0	0	-1
N.S.	1	1.00	56.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	25.728	0.411	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	9240	0	0	0	0	0	-1
N.S.	1	1.00	62.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	6.591	1.816	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	12302	0	0	0	0	0	-1
N.S.	1	1.00	83.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	6.825	2.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	5.594	0.375	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	1.667	0.375	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	667	0	763	574	0	0	749
N.S.	1	1.00	2.43	0.00	2.77	2.09	0.00	0.00	2.72
time (sec)	N/A	0.339	6.594	0.326	0.575	0.431	0.000	0.000	21.060

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	192	174	0	528	323	0	0	480
N.S.	1	1.16	1.05	0.00	3.18	1.95	0.00	0.00	2.89
time (sec)	N/A	0.239	1.178	0.302	0.547	0.401	0.000	0.000	19.203

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	345	173	0	0	105
N.S.	1	1.00	1.12	0.00	3.32	1.66	0.00	0.00	1.01
time (sec)	N/A	0.187	0.322	0.303	0.542	0.375	0.000	0.000	1.442

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	1.559	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	3178	0	0	0	0	0	-1
N.S.	1	1.00	23.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	6.565	0.316	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	8147	0	0	0	0	0	-1
N.S.	1	1.00	60.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	6.699	0.319	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	353	0	0	212	0	0	368
N.S.	1	1.00	1.32	0.00	0.00	0.79	0.00	0.00	1.38
time (sec)	N/A	0.298	17.912	1.371	0.000	0.405	0.000	0.000	22.233

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	269	0	0	143	0	0	239
N.S.	1	1.00	1.41	0.00	0.00	0.75	0.00	0.00	1.25
time (sec)	N/A	0.216	14.861	1.326	0.000	0.414	0.000	0.000	15.484

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	211	0	0	94	0	0	134
N.S.	1	1.00	1.85	0.00	0.00	0.82	0.00	0.00	1.18
time (sec)	N/A	0.151	11.734	0.586	0.000	0.411	0.000	0.000	14.158

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	675	0	0	0	0	0	-1
N.S.	1	1.00	4.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	11.557	0.446	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	1767	0	0	0	0	0	-1
N.S.	1	1.00	11.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	18.865	0.408	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	3601	0	0	0	0	0	-1
N.S.	1	1.00	21.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	89.504	0.509	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	5163	0	0	0	0	0	-1
N.S.	1	1.00	29.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	61.822	1.393	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	63	0	0	84	898	0	64
N.S.	1	1.00	1.85	0.00	0.00	2.47	26.41	0.00	1.88
time (sec)	N/A	0.176	0.381	1.087	0.000	0.439	80.426	0.000	14.447

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	67	0	0	83	898	0	61
N.S.	1	1.00	1.97	0.00	0.00	2.44	26.41	0.00	1.79
time (sec)	N/A	0.153	0.783	1.068	0.000	0.388	80.810	0.000	14.457

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	84	898	0	61
N.S.	1	1.00	2.00	0.00	0.00	2.55	27.21	0.00	1.85
time (sec)	N/A	0.157	1.149	1.331	0.000	0.388	80.044	0.000	14.378

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	0	0	83	898	0	64
N.S.	1	1.00	1.74	0.00	0.00	2.37	25.66	0.00	1.83
time (sec)	N/A	0.155	0.396	1.089	0.000	0.374	81.098	0.000	14.363

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	39	0	9672	36
N.S.	1	1.00	1.00	0.00	0.00	1.08	0.00	268.67	1.00
time (sec)	N/A	0.089	0.341	0.418	0.000	0.374	0.000	45.402	13.542

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	40	0	9673	37
N.S.	1	1.00	1.00	0.00	0.00	1.08	0.00	261.43	1.00
time (sec)	N/A	0.086	0.324	0.410	0.000	0.398	0.000	45.271	13.497

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	87	158	157	105	440	131	300
N.S.	1	1.00	0.62	1.13	1.12	0.75	3.14	0.94	2.14
time (sec)	N/A	0.132	0.112	0.241	0.280	0.369	0.664	0.468	15.304

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	77	136	138	91	359	113	256
N.S.	1	1.00	0.64	1.12	1.14	0.75	2.97	0.93	2.12
time (sec)	N/A	0.123	0.084	0.171	0.275	0.509	0.455	0.551	15.286

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	55	117	112	77	267	77	292
N.S.	1	1.00	0.57	1.22	1.17	0.80	2.78	0.80	3.04
time (sec)	N/A	0.083	0.330	0.143	0.288	0.386	0.303	0.448	15.030

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	86	63	196	77	250
N.S.	1	1.00	0.66	1.09	1.05	0.77	2.39	0.94	3.05
time (sec)	N/A	0.073	0.245	0.118	0.287	0.360	0.199	0.448	15.188

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	74	88	85	92	0	107	212
N.S.	1	1.00	0.97	1.16	1.12	1.21	0.00	1.41	2.79
time (sec)	N/A	0.071	0.116	0.233	0.269	0.415	0.000	0.424	13.227

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	80	83	111	0	153	226
N.S.	1	1.00	0.97	1.01	1.05	1.41	0.00	1.94	2.86
time (sec)	N/A	0.128	0.156	0.207	0.294	0.417	0.000	0.442	13.194

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	142	78	90	152	0	137	220
N.S.	1	1.00	1.82	1.00	1.15	1.95	0.00	1.76	2.82
time (sec)	N/A	0.086	0.035	0.234	0.272	0.404	0.000	0.462	13.500

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	141	101	117	175	0	150	245
N.S.	1	1.00	1.81	1.29	1.50	2.24	0.00	1.92	3.14
time (sec)	N/A	0.090	0.360	0.217	0.292	0.383	0.000	0.438	13.316

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	210	114	145	166	0	174	244
N.S.	1	1.00	2.44	1.33	1.69	1.93	0.00	2.02	2.84
time (sec)	N/A	0.104	0.050	0.245	0.283	0.502	0.000	0.604	13.116

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	268	140	175	201	0	174	244
N.S.	1	1.00	2.55	1.33	1.67	1.91	0.00	1.66	2.32
time (sec)	N/A	0.172	0.053	0.222	0.287	0.366	0.000	0.471	13.155

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	306	163	207	240	0	242	340
N.S.	1	1.00	2.35	1.25	1.59	1.85	0.00	1.86	2.62
time (sec)	N/A	0.138	0.060	0.257	0.283	0.421	0.000	0.448	13.365

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	254	154	715	248	3614	156	326
N.S.	1	1.00	1.97	1.19	5.54	1.92	28.02	1.21	2.53
time (sec)	N/A	0.149	0.655	0.395	0.516	0.380	27.944	0.489	17.076

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	228	115	543	225	2290	113	261
N.S.	1	1.00	2.21	1.12	5.27	2.18	22.23	1.10	2.53
time (sec)	N/A	0.134	0.544	0.349	0.509	0.379	16.284	0.453	16.929

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	189	96	392	204	1268	93	178
N.S.	1	1.00	2.12	1.08	4.40	2.29	14.25	1.04	2.00
time (sec)	N/A	0.125	0.602	0.303	0.487	0.363	9.086	0.509	14.924

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	107	71	348	156	461	63	110
N.S.	1	1.00	1.30	0.87	4.24	1.90	5.62	0.77	1.34
time (sec)	N/A	0.106	0.325	0.279	0.313	0.380	5.045	0.430	13.448

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	92	86	387	154	573	79	134
N.S.	1	1.00	1.59	1.48	6.67	2.66	9.88	1.36	2.31
time (sec)	N/A	0.076	0.168	0.247	0.299	0.375	2.807	0.441	13.309

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	313	95	433	310	0	99	199
N.S.	1	1.00	3.19	0.97	4.42	3.16	0.00	1.01	2.03
time (sec)	N/A	0.120	0.716	0.431	0.298	0.385	0.000	0.461	14.771

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	167	120	519	406	0	146	210
N.S.	1	1.00	1.48	1.06	4.59	3.59	0.00	1.29	1.86
time (sec)	N/A	0.285	1.965	0.457	0.285	0.422	0.000	0.505	15.774

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	245	148	622	498	0	180	288
N.S.	1	1.00	1.78	1.07	4.51	3.61	0.00	1.30	2.09
time (sec)	N/A	0.164	2.483	0.533	0.309	0.402	0.000	0.532	15.741

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	348	174	706	594	0	213	314
N.S.	1	1.00	2.27	1.14	4.61	3.88	0.00	1.39	2.05
time (sec)	N/A	0.171	6.185	0.520	0.300	0.392	0.000	0.495	15.280

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	267	422	438	245	996	314	830
N.S.	1	1.00	0.82	1.29	1.34	0.75	3.05	0.96	2.54
time (sec)	N/A	0.405	1.399	0.276	0.291	0.435	0.423	0.743	15.576

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	185	274	285	165	571	198	547
N.S.	1	1.00	0.87	1.29	1.34	0.77	2.68	0.93	2.57
time (sec)	N/A	0.250	1.070	0.186	0.285	0.415	0.276	0.528	15.413

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	104	147	155	88	277	101	134
N.S.	1	1.00	0.94	1.32	1.40	0.79	2.50	0.91	1.21
time (sec)	N/A	0.113	0.297	0.145	0.289	0.377	0.148	0.501	13.314

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	62	46	94	48	100
N.S.	1	1.00	0.94	1.23	1.29	0.96	1.96	1.00	2.08
time (sec)	N/A	0.016	0.078	0.122	0.282	0.373	0.085	0.468	13.263

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	196	120	0	303	5508	141	2500
N.S.	1	1.00	2.00	1.22	0.00	3.09	56.20	1.44	25.51
time (sec)	N/A	0.196	0.452	0.299	0.000	0.412	172.698	0.557	16.580

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	217	174	0	672	0	204	2500
N.S.	1	1.00	1.75	1.40	0.00	5.42	0.00	1.65	20.16
time (sec)	N/A	0.227	0.929	0.430	0.000	0.459	0.000	0.677	20.316

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	345	424	0	990	0	594	554
N.S.	1	1.00	1.96	2.41	0.00	5.62	0.00	3.38	3.15
time (sec)	N/A	0.274	2.273	0.648	0.000	0.422	0.000	0.590	15.589

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	437	745	778	371	1865	474	1291
N.S.	1	1.00	0.94	1.61	1.68	0.80	4.02	1.02	2.78
time (sec)	N/A	0.650	2.068	0.365	0.314	0.433	0.669	0.807	15.987

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	296	496	514	251	1129	311	765
N.S.	1	1.00	0.88	1.48	1.53	0.75	3.36	0.93	2.28
time (sec)	N/A	0.484	1.021	0.266	0.288	0.420	0.442	0.623	15.705

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	278	289	149	571	172	492
N.S.	1	1.00	0.96	1.67	1.74	0.90	3.44	1.04	2.96
time (sec)	N/A	0.184	0.523	0.185	0.293	0.391	0.271	0.644	14.603

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	117	123	74	199	88	91
N.S.	1	1.00	1.13	1.24	1.31	0.79	2.12	0.94	0.97
time (sec)	N/A	0.042	0.224	0.128	0.279	0.373	0.146	0.610	13.161

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	177	235	0	467	0	314	2500
N.S.	1	1.00	1.04	1.37	0.00	2.73	0.00	1.84	14.62
time (sec)	N/A	0.351	0.437	0.358	0.000	0.480	0.000	0.587	20.069

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	192	251	0	750	0	498	2500
N.S.	1	1.00	0.97	1.27	0.00	3.79	0.00	2.52	12.63
time (sec)	N/A	0.403	0.686	0.516	0.000	0.429	0.000	0.795	21.578

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	226	431	0	1508	0	703	2500
N.S.	1	1.00	1.05	2.00	0.00	7.01	0.00	3.27	11.63
time (sec)	N/A	0.428	0.938	0.699	0.000	0.457	0.000	0.726	22.497

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	528	1077	1136	440	2878	566	1395
N.S.	1	1.00	0.87	1.78	1.88	0.73	4.76	0.94	2.31
time (sec)	N/A	0.984	3.118	0.496	0.320	0.439	0.958	0.825	16.176

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	355	725	758	306	1804	380	976
N.S.	1	1.00	0.77	1.57	1.64	0.66	3.90	0.82	2.11
time (sec)	N/A	0.760	1.607	0.362	0.304	0.422	0.647	0.652	15.626

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	414	430	184	960	217	550
N.S.	1	1.00	0.78	2.06	2.14	0.92	4.78	1.08	2.74
time (sec)	N/A	0.231	0.793	0.264	0.323	0.404	0.411	0.756	14.623

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	117	120	178	185	98	371	116	330
N.S.	1	0.92	0.94	1.40	1.46	0.77	2.92	0.91	2.60
time (sec)	N/A	0.072	0.349	0.177	0.333	0.376	0.236	0.567	14.256

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	233	378	0	644	0	617	2500
N.S.	1	1.00	0.95	1.54	0.00	2.62	0.00	2.51	10.16
time (sec)	N/A	0.619	0.672	0.416	0.000	0.498	0.000	0.604	21.761

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	244	406	0	1048	0	588	2500
N.S.	1	1.00	0.86	1.43	0.00	3.70	0.00	2.08	8.83
time (sec)	N/A	0.645	1.035	0.601	0.000	0.465	0.000	0.531	23.883

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	830	586	0	1697	0	986	2500
N.S.	1	1.00	2.72	1.92	0.00	5.56	0.00	3.23	8.20
time (sec)	N/A	0.651	2.141	0.882	0.000	0.470	0.000	0.536	25.413

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	788	337	1226	480	14644	479	839
N.S.	1	1.00	3.58	1.53	5.57	2.18	66.56	2.18	3.81
time (sec)	N/A	0.240	0.900	0.301	0.588	0.469	4.986	0.541	14.054

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	141	200	193	660	311	5763	222	297
N.S.	1	0.99	1.40	1.35	4.62	2.17	40.30	1.55	2.08
time (sec)	N/A	0.138	0.337	0.246	0.534	0.384	2.554	0.495	16.666

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	126	81	278	160	1307	160	122
N.S.	1	1.00	1.88	1.21	4.15	2.39	19.51	2.39	1.82
time (sec)	N/A	0.137	0.339	0.224	0.505	0.368	1.250	0.548	13.681

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	42	84	70	109	40	35
N.S.	1	1.00	2.26	1.20	2.40	2.00	3.11	1.14	1.00
time (sec)	N/A	0.034	0.122	0.125	0.506	0.368	0.624	0.431	12.920

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	148	94	0	616	0	113	154
N.S.	1	1.00	1.47	0.93	0.00	6.10	0.00	1.12	1.52
time (sec)	N/A	0.120	0.250	0.487	0.000	0.397	0.000	0.470	13.322

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	209	197	0	1571	0	443	437
N.S.	1	1.00	1.15	1.09	0.00	8.68	0.00	2.45	2.41
time (sec)	N/A	0.236	0.867	0.526	0.000	0.423	0.000	0.483	15.531

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	313	482	0	3348	0	753	1076
N.S.	1	1.00	1.11	1.70	0.00	11.83	0.00	2.66	3.80
time (sec)	N/A	0.379	1.161	1.141	0.000	0.463	0.000	0.548	17.439

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	547	340	1500	596	14612	494	663
N.S.	1	1.00	2.40	1.49	6.58	2.61	64.09	2.17	2.91
time (sec)	N/A	0.353	2.393	0.343	0.559	0.401	9.255	0.469	16.346

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	338	193	901	385	5358	277	365
N.S.	1	1.00	2.56	1.46	6.83	2.92	40.59	2.10	2.77
time (sec)	N/A	0.337	1.142	0.317	0.521	0.401	4.914	0.530	16.038

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	180	110	492	216	1062	141	94
N.S.	1	1.00	2.12	1.29	5.79	2.54	12.49	1.66	1.11
time (sec)	N/A	0.138	0.245	0.284	0.511	0.399	2.430	0.508	13.738

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	70	232	125	372	68	97
N.S.	1	1.00	0.66	1.08	3.57	1.92	5.72	1.05	1.49
time (sec)	N/A	0.035	0.046	0.198	0.309	0.357	1.286	0.480	13.388

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	229	159	0	1318	0	259	302
N.S.	1	1.00	1.51	1.05	0.00	8.67	0.00	1.70	1.99
time (sec)	N/A	0.284	0.438	0.612	0.000	0.412	0.000	0.567	14.783

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	313	263	0	3168	0	425	844
N.S.	1	1.00	1.14	0.96	0.00	11.52	0.00	1.55	3.07
time (sec)	N/A	0.459	1.941	0.667	0.000	0.467	0.000	0.519	16.763

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	1257	547	0	5054	0	944	1686
N.S.	1	1.00	3.26	1.42	0.00	13.09	0.00	2.45	4.37
time (sec)	N/A	0.654	5.772	1.426	0.000	0.520	0.000	0.530	17.693

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	366	349	1828	663	11456	596	593
N.S.	1	1.00	1.63	1.55	8.12	2.95	50.92	2.65	2.64
time (sec)	N/A	0.548	1.996	0.454	0.667	0.411	17.202	0.497	15.695

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	514	241	1230	444	3468	382	286
N.S.	1	1.00	3.13	1.47	7.50	2.71	21.15	2.33	1.74
time (sec)	N/A	0.311	0.618	0.408	0.547	0.572	9.641	0.482	16.544

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	176	151	797	283	1819	223	245
N.S.	1	1.00	1.39	1.19	6.28	2.23	14.32	1.76	1.93
time (sec)	N/A	0.154	0.491	0.355	0.310	0.791	5.365	0.513	14.259

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	114	421	202	1015	130	150
N.S.	1	1.00	0.62	1.12	4.13	1.98	9.95	1.27	1.47
time (sec)	N/A	0.053	0.068	0.236	0.320	0.386	2.962	0.467	13.804

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	502	252	0	2337	0	578	591
N.S.	1	1.00	2.19	1.10	0.00	10.21	0.00	2.52	2.58
time (sec)	N/A	0.492	0.856	0.769	0.000	0.456	0.000	0.576	17.184

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	1253	355	0	4543	0	772	1349
N.S.	1	1.00	3.29	0.93	0.00	11.92	0.00	2.03	3.54
time (sec)	N/A	0.742	6.286	0.885	0.000	0.501	0.000	0.541	17.669

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	548	639	0	7352	0	1272	2387
N.S.	1	1.00	1.08	1.26	0.00	14.47	0.00	2.50	4.70
time (sec)	N/A	0.988	3.303	1.542	0.000	0.572	0.000	0.636	19.882

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	305	242	0	480	0	579	-1
N.S.	1	1.00	1.19	0.95	0.00	1.88	0.00	2.26	-0.00
time (sec)	N/A	0.310	0.826	5.871	0.000	0.403	0.000	0.744	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	176	161	0	317	0	367	-1
N.S.	1	1.00	0.92	0.84	0.00	1.65	0.00	1.91	-0.01
time (sec)	N/A	0.226	0.501	6.509	0.000	0.382	0.000	0.585	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	117	102	0	184	0	197	-1
N.S.	1	1.00	0.99	0.86	0.00	1.56	0.00	1.67	-0.01
time (sec)	N/A	0.168	0.251	6.129	0.000	0.394	0.000	0.553	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	92	0	90	-1
N.S.	1	1.00	1.32	0.94	0.00	1.48	0.00	1.45	-0.02
time (sec)	N/A	0.039	0.091	5.500	0.000	0.395	0.000	0.470	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	685	0	130	-1
N.S.	1	1.00	9.03	1.39	0.00	6.85	0.00	1.30	-0.01
time (sec)	N/A	0.163	6.343	8.023	0.000	0.871	0.000	0.488	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	1054	0	221	-1
N.S.	1	1.00	7.15	2.17	0.00	8.37	0.00	1.75	-0.01
time (sec)	N/A	0.184	6.185	11.341	0.000	1.013	0.000	0.508	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	967	628	0	1804	0	443	-1
N.S.	1	1.00	5.04	3.27	0.00	9.40	0.00	2.31	-0.01
time (sec)	N/A	0.251	7.220	12.290	0.000	1.486	0.000	0.583	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	390	312	0	652	0	793	-1
N.S.	1	1.00	1.04	0.83	0.00	1.74	0.00	2.12	-0.00
time (sec)	N/A	0.612	2.941	6.075	0.000	0.416	0.000	0.815	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	267	207	0	443	0	523	-1
N.S.	1	1.00	0.91	0.70	0.00	1.51	0.00	1.78	-0.00
time (sec)	N/A	0.474	1.494	5.356	0.000	0.405	0.000	0.667	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	144	150	0	268	0	301	-1
N.S.	1	1.00	0.87	0.91	0.00	1.62	0.00	1.82	-0.01
time (sec)	N/A	0.217	0.745	5.815	0.000	0.385	0.000	0.615	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	146	0	147	-1
N.S.	1	1.00	1.00	0.76	0.00	1.45	0.00	1.46	-0.01
time (sec)	N/A	0.058	0.279	5.489	0.000	0.364	0.000	0.517	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	356	291	0	918	0	287	-1
N.S.	1	1.00	2.33	1.90	0.00	6.00	0.00	1.88	-0.01
time (sec)	N/A	0.340	2.445	8.832	0.000	1.023	0.000	0.611	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	381	590	0	1474	0	381	-1
N.S.	1	1.00	1.99	3.09	0.00	7.72	0.00	1.99	-0.01
time (sec)	N/A	0.373	3.463	11.039	0.000	1.177	0.000	0.591	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	416	895	0	2262	0	655	-1
N.S.	1	1.00	1.88	4.05	0.00	10.24	0.00	2.96	-0.00
time (sec)	N/A	0.405	3.777	14.549	0.000	1.681	0.000	0.605	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1565	374	0	880	0	1061	-1
N.S.	1	1.00	2.93	0.70	0.00	1.65	0.00	1.99	-0.00
time (sec)	N/A	0.812	6.604	6.021	0.000	0.612	0.000	1.009	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	328	257	0	608	0	717	-1
N.S.	1	1.00	0.76	0.60	0.00	1.42	0.00	1.67	-0.00
time (sec)	N/A	0.729	5.956	5.725	0.000	0.528	0.000	0.749	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	202	152	0	374	0	425	-1
N.S.	1	1.00	0.95	0.72	0.00	1.76	0.00	2.00	-0.00
time (sec)	N/A	0.247	2.753	6.578	0.000	0.402	0.000	0.646	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	202	0	213	-1
N.S.	1	1.00	0.86	0.72	0.00	1.46	0.00	1.54	-0.01
time (sec)	N/A	0.077	0.999	5.737	0.000	0.378	0.000	0.515	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	450	543	0	1356	0	548	-1
N.S.	1	1.00	2.06	2.49	0.00	6.22	0.00	2.51	-0.00
time (sec)	N/A	0.603	4.296	11.739	0.000	1.893	0.000	0.582	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	460	932	0	2096	0	625	-1
N.S.	1	1.00	1.74	3.52	0.00	7.91	0.00	2.36	-0.00
time (sec)	N/A	0.636	4.467	13.429	0.000	1.672	0.000	0.645	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	504	1587	0	3104	0	938	-1
N.S.	1	1.00	1.64	5.15	0.00	10.08	0.00	3.05	-0.00
time (sec)	N/A	0.666	5.672	15.551	0.000	2.441	0.000	0.723	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	375	610	0	653	0	573	-1
N.S.	1	1.00	1.32	2.15	0.00	2.30	0.00	2.02	-0.00
time (sec)	N/A	0.659	0.609	8.720	0.000	0.438	0.000	0.673	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	246	396	0	470	0	375	-1
N.S.	1	1.00	1.23	1.98	0.00	2.35	0.00	1.88	-0.00
time (sec)	N/A	0.392	0.378	7.365	0.000	0.425	0.000	0.602	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	232	0	323	0	229	-1
N.S.	1	1.00	1.04	1.78	0.00	2.48	0.00	1.76	-0.01
time (sec)	N/A	0.180	0.345	7.423	0.000	0.533	0.000	0.532	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	228	0	149	151
N.S.	1	1.00	1.34	1.62	0.00	2.89	0.00	1.89	1.91
time (sec)	N/A	0.047	0.163	7.876	0.000	0.435	0.000	0.449	1.056

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	238	199	0	786	0	261	-1
N.S.	1	1.00	1.75	1.46	0.00	5.78	0.00	1.92	-0.01
time (sec)	N/A	0.197	2.262	9.329	0.000	1.037	0.000	0.559	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	374	899	0	2231	0	510	-1
N.S.	1	1.00	1.81	4.34	0.00	10.78	0.00	2.46	-0.00
time (sec)	N/A	0.428	5.012	12.520	0.000	2.863	0.000	0.616	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	490	2275	0	4268	0	909	-1
N.S.	1	1.00	1.59	7.36	0.00	13.81	0.00	2.94	-0.00
time (sec)	N/A	0.726	8.508	17.833	0.000	4.768	0.000	0.735	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	684	1031	0	812	0	673	-1
N.S.	1	1.00	2.42	3.64	0.00	2.87	0.00	2.38	-0.00
time (sec)	N/A	0.670	0.735	9.930	0.000	0.475	0.000	0.624	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	357	694	0	610	0	478	-1
N.S.	1	1.00	1.76	3.42	0.00	3.00	0.00	2.35	-0.00
time (sec)	N/A	0.391	0.512	7.077	0.000	0.456	0.000	0.607	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	246	389	0	431	0	0	-1
N.S.	1	1.00	1.85	2.92	0.00	3.24	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.320	6.894	0.000	0.417	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	315	0	192	-1
N.S.	1	1.00	1.72	2.02	0.00	3.62	0.00	2.21	-0.01
time (sec)	N/A	0.051	0.147	4.798	0.000	0.420	0.000	0.490	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	419	624	0	1633	0	488	-1
N.S.	1	1.00	2.24	3.34	0.00	8.73	0.00	2.61	-0.01
time (sec)	N/A	0.399	2.218	9.638	0.000	2.066	0.000	0.602	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	542	2049	0	3491	0	898	-1
N.S.	1	1.00	1.86	7.02	0.00	11.96	0.00	3.08	-0.00
time (sec)	N/A	0.714	6.696	14.527	0.000	4.602	0.000	0.725	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	1395	4707	0	5968	0	1309	-1
N.S.	1	1.00	3.47	11.71	0.00	14.85	0.00	3.26	-0.00
time (sec)	N/A	1.070	11.245	21.270	0.000	8.879	0.000	0.943	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	523	1438	0	1012	0	778	-1
N.S.	1	1.00	1.70	4.67	0.00	3.29	0.00	2.53	-0.00
time (sec)	N/A	0.717	1.760	12.454	0.000	0.430	0.000	0.796	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	544	982	0	774	0	550	-1
N.S.	1	1.00	2.48	4.48	0.00	3.53	0.00	2.51	-0.00
time (sec)	N/A	0.394	0.782	11.490	0.000	0.409	0.000	0.643	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	267	448	0	564	0	378	-1
N.S.	1	1.00	1.77	2.97	0.00	3.74	0.00	2.50	-0.01
time (sec)	N/A	0.196	0.514	9.062	0.000	0.398	0.000	0.542	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	420	0	246	-1
N.S.	1	1.00	1.80	2.21	0.00	3.33	0.00	1.95	-0.01
time (sec)	N/A	0.070	0.267	9.954	0.000	0.413	0.000	0.503	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	550	1418	0	2665	0	787	-1
N.S.	1	1.00	2.11	5.43	0.00	10.21	0.00	3.02	-0.00
time (sec)	N/A	0.672	3.896	16.362	0.000	4.036	0.000	0.640	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	1318	4092	0	5255	0	1138	-1
N.S.	1	1.00	3.34	10.36	0.00	13.30	0.00	2.88	-0.00
time (sec)	N/A	1.065	10.533	21.731	0.000	8.623	0.000	0.905	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	2103	7322	0	8675	0	0	-1
N.S.	1	1.00	4.05	14.11	0.00	16.71	0.00	0.00	-0.00
time (sec)	N/A	1.464	13.045	32.625	0.000	27.881	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	27.855	0.889	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	6.572	0.725	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	6.185	0.388	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.236	30.614	2.039	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	245	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.623	33.119	0.303	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	5.899	0.267	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	244	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	12.270	0.302	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	603	0	0	0	0	0	-1
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	55.981	0.310	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	300	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	10.421	0.815	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	198	212	0	0	0	0	0	-1
N.S.	1	0.99	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	4.253	0.712	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	1.250	0.006	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	473	0	0	0	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	5.014	0.449	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	651	0	0	0	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	1.633	2.254	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	651	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.907	1.729	2.158	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	3281	0	0	0	0	0	-1
N.S.	1	1.00	11.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	7.430	0.283	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	17.127	0.283	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	4.014	0.323	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	672	0	0	0	0	0	-1
N.S.	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	4.536	0.293	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	682	0	0	0	0	0	-1
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	4.200	0.323	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	573	0	0	0	0	0	-1
N.S.	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	4.524	0.520	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	12.747	0.350	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	5.879	0.497	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	61	0	0	98
N.S.	1	1.00	1.00	0.00	0.00	1.56	0.00	0.00	2.51
time (sec)	N/A	0.117	0.481	1.526	0.000	0.456	0.000	0.000	14.774

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	62	0	0	99
N.S.	1	1.00	1.00	0.00	0.00	1.55	0.00	0.00	2.48
time (sec)	N/A	0.117	0.528	1.517	0.000	0.460	0.000	0.000	14.621

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	188	374	0	1327	0	776	2500
N.S.	1	1.00	0.94	1.88	0.00	6.67	0.00	3.90	12.56
time (sec)	N/A	0.396	1.101	0.724	0.000	0.529	0.000	0.464	27.619

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	840	840	2042	6059287	0	0	0	0	-1
N.S.	1	1.00	2.43	7213.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.139	6.538	98.622	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	1901	3148347	0	0	0	0	-1
N.S.	1	1.00	3.02	4997.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	29.540	59.498	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	1949	99490	0	0	0	0	-1
N.S.	1	1.00	4.67	238.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	6.413	20.678	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	2266	198633	0	0	0	0	-1
N.S.	1	1.00	4.17	365.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.932	7.014	27.275	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	2837	867629	0	0	0	0	-1
N.S.	1	1.00	3.31	1011.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.761	7.852	50.631	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	16.633	0.290	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [350] had the largest ratio of [55]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	33	0.152
2	A	6	5	1.00	33	0.152
3	A	5	4	1.00	31	0.129
4	A	4	3	1.00	33	0.091
5	A	5	3	1.00	33	0.091
6	A	6	3	1.00	33	0.091
7	A	6	5	1.00	35	0.143
8	A	5	5	1.00	35	0.143
9	A	4	4	1.00	35	0.114
10	A	9	9	1.00	35	0.257
11	A	10	10	1.00	35	0.286
12	A	9	5	1.00	33	0.152
13	A	4	3	1.00	34	0.088
14	A	1	1	1.00	43	0.023
15	A	1	1	1.00	37	0.027
16	A	6	6	1.00	31	0.194
17	A	7	6	1.00	34	0.176
18	A	6	6	1.00	34	0.176
19	A	5	5	1.08	34	0.147
20	A	4	4	1.00	32	0.125
21	A	4	3	1.00	34	0.088
22	A	4	4	1.00	34	0.118
23	A	4	4	1.00	34	0.118
24	A	5	5	1.00	34	0.147
25	A	6	5	1.00	34	0.147

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	6	1.00	36	0.167
27	A	7	6	1.00	36	0.167
28	A	6	5	1.00	36	0.139
29	A	5	4	1.00	36	0.111
30	A	5	5	1.00	34	0.147
31	A	5	5	1.00	36	0.139
32	A	5	5	1.00	36	0.139
33	A	5	4	1.00	36	0.111
34	A	3	3	1.00	36	0.083
35	A	4	4	1.00	36	0.111
36	A	5	4	1.00	36	0.111
37	A	6	4	1.00	36	0.111
38	A	9	6	1.00	36	0.167
39	A	8	6	1.00	36	0.167
40	A	7	5	1.00	36	0.139
41	A	6	4	1.00	36	0.111
42	A	6	5	1.00	36	0.139
43	A	6	6	1.00	34	0.176
44	A	6	6	1.00	36	0.167
45	A	6	6	1.00	36	0.167
46	A	6	5	1.00	36	0.139
47	A	6	4	1.00	36	0.111
48	A	3	3	1.00	36	0.083
49	A	4	4	1.00	36	0.111
50	A	5	4	1.00	36	0.111
51	A	6	4	1.00	36	0.111
52	A	7	7	1.00	36	0.194
53	A	6	6	1.00	36	0.167
54	A	5	5	1.00	36	0.139
55	A	4	3	1.00	34	0.088
56	A	4	4	1.00	36	0.111
57	A	4	4	1.00	36	0.111
58	A	5	5	1.00	36	0.139
59	A	6	5	1.00	36	0.139
60	A	8	7	1.00	36	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	36	0.167
62	A	6	6	1.00	36	0.167
63	A	5	5	1.00	36	0.139
64	A	4	4	1.00	34	0.118
65	A	4	4	1.00	36	0.111
66	A	4	3	1.00	36	0.083
67	A	4	3	1.00	36	0.083
68	A	5	4	1.00	36	0.111
69	A	6	4	1.00	36	0.111
70	A	8	6	1.00	36	0.167
71	A	7	6	1.00	36	0.167
72	A	6	5	1.00	36	0.139
73	A	5	4	1.00	36	0.111
74	A	4	4	1.00	34	0.118
75	A	5	5	1.00	36	0.139
76	A	4	3	1.00	36	0.083
77	A	4	3	1.00	36	0.083
78	A	4	3	1.00	36	0.083
79	A	5	4	1.00	36	0.111
80	A	6	4	1.00	36	0.111
81	A	6	4	1.00	36	0.111
82	A	5	4	1.00	36	0.111
83	A	4	4	1.00	36	0.111
84	A	3	3	1.00	36	0.083
85	A	5	5	1.00	36	0.139
86	A	5	5	1.00	36	0.139
87	A	5	5	1.00	36	0.139
88	A	6	6	1.00	36	0.167
89	A	6	4	1.00	38	0.105
90	A	5	4	1.00	38	0.105
91	A	4	4	1.00	38	0.105
92	A	3	3	1.00	38	0.079
93	A	6	5	1.00	38	0.132
94	A	6	5	1.00	38	0.132
95	A	6	6	1.00	38	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	5	1.00	38	0.132
97	A	7	6	1.00	38	0.158
98	A	6	4	1.00	38	0.105
99	A	5	4	1.00	38	0.105
100	A	4	4	1.00	38	0.105
101	A	3	3	1.00	38	0.079
102	A	7	5	1.00	38	0.132
103	A	7	5	1.00	38	0.132
104	A	7	6	1.00	38	0.158
105	A	7	6	1.00	38	0.158
106	A	7	5	1.00	38	0.132
107	A	8	6	1.00	38	0.158
108	A	6	4	1.00	38	0.105
109	A	5	4	1.00	38	0.105
110	A	4	4	1.00	38	0.105
111	A	3	3	1.00	38	0.079
112	A	4	4	1.00	38	0.105
113	A	5	5	1.00	38	0.132
114	A	6	6	1.00	38	0.158
115	A	7	4	1.00	38	0.105
116	A	6	4	1.00	38	0.105
117	A	5	4	1.00	38	0.105
118	A	4	4	1.00	38	0.105
119	A	3	3	1.00	38	0.079
120	A	5	5	1.00	38	0.132
121	A	6	6	1.00	38	0.158
122	A	7	7	1.00	38	0.184
123	A	7	4	1.00	38	0.105
124	A	6	4	1.00	38	0.105
125	A	5	4	1.00	38	0.105
126	A	4	4	1.00	38	0.105
127	A	3	3	1.00	38	0.079
128	A	6	5	1.00	38	0.132
129	A	7	7	1.00	38	0.184
130	A	8	7	1.00	38	0.184

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	2	1.00	40	0.050
132	A	3	2	1.00	40	0.050
133	A	3	2	1.00	40	0.050
134	A	3	2	1.00	40	0.050
135	A	5	5	1.00	40	0.125
136	A	5	5	1.00	40	0.125
137	A	3	2	1.00	40	0.050
138	A	3	2	1.00	40	0.050
139	A	3	3	1.00	40	0.075
140	A	3	3	1.00	40	0.075
141	A	3	3	1.00	40	0.075
142	A	3	2	1.00	40	0.050
143	A	5	5	1.00	40	0.125
144	A	5	5	1.00	40	0.125
145	A	5	5	1.00	40	0.125
146	A	2	2	1.00	40	0.050
147	A	3	3	1.00	40	0.075
148	A	3	3	1.00	40	0.075
149	A	4	3	1.00	40	0.075
150	A	4	3	1.00	40	0.075
151	A	3	3	1.00	40	0.075
152	A	3	2	1.00	40	0.050
153	A	6	5	1.00	40	0.125
154	A	6	5	1.00	40	0.125
155	A	6	6	1.00	40	0.150
156	A	6	5	1.00	40	0.125
157	A	2	2	1.00	40	0.050
158	A	3	3	1.00	40	0.075
159	A	4	3	1.00	40	0.075
160	A	5	3	1.00	40	0.075
161	A	5	3	1.00	40	0.075
162	A	4	3	1.00	40	0.075
163	A	3	3	1.00	40	0.075
164	A	3	2	1.00	40	0.050
165	A	7	5	1.00	40	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	7	5	1.00	40	0.125
167	A	7	6	1.00	40	0.150
168	A	7	6	1.00	40	0.150
169	A	7	5	1.00	40	0.125
170	A	2	2	1.00	40	0.050
171	A	3	3	1.00	40	0.075
172	A	4	3	1.00	40	0.075
173	A	5	3	1.00	40	0.075
174	A	6	5	1.00	40	0.125
175	A	5	5	1.00	40	0.125
176	A	5	5	1.00	40	0.125
177	A	7	4	1.00	40	0.100
178	A	3	3	1.00	40	0.075
179	A	4	4	1.00	40	0.100
180	A	7	5	1.00	40	0.125
181	A	6	5	1.00	40	0.125
182	A	5	5	1.00	40	0.125
183	A	5	5	1.00	40	0.125
184	A	3	3	1.00	40	0.075
185	A	4	4	1.00	40	0.100
186	A	5	4	1.00	40	0.100
187	A	8	6	1.00	40	0.150
188	A	7	6	1.00	40	0.150
189	A	6	6	1.00	40	0.150
190	A	5	5	1.00	40	0.125
191	A	3	2	1.00	40	0.050
192	A	4	4	1.00	40	0.100
193	A	5	4	1.00	40	0.100
194	A	6	4	1.00	40	0.100
195	A	5	5	1.00	36	0.139
196	A	5	5	1.00	36	0.139
197	A	5	5	1.00	36	0.139
198	A	5	5	1.00	34	0.147
199	A	3	3	1.00	23	0.130
200	A	5	5	1.00	36	0.139

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	5	1.00	36	0.139
202	A	5	5	1.00	36	0.139
203	A	4	4	1.00	38	0.105
204	A	4	4	1.00	38	0.105
205	A	4	3	1.00	38	0.079
206	A	3	3	1.16	38	0.079
207	A	3	2	1.00	38	0.053
208	A	4	4	1.00	38	0.105
209	A	4	4	1.00	38	0.105
210	A	4	4	1.00	38	0.105
211	A	4	3	1.00	40	0.075
212	A	3	3	1.00	40	0.075
213	A	2	2	1.00	40	0.050
214	A	5	5	1.00	40	0.125
215	A	5	5	1.00	38	0.132
216	A	5	5	1.00	40	0.125
217	A	5	5	1.00	40	0.125
218	A	2	2	1.00	46	0.043
219	A	2	2	1.00	45	0.044
220	A	2	2	1.00	44	0.045
221	A	2	2	1.00	43	0.047
222	A	1	1	1.00	47	0.021
223	A	1	1	1.00	46	0.022
224	A	13	4	1.00	32	0.125
225	A	12	4	1.00	32	0.125
226	A	10	5	1.00	30	0.167
227	A	5	5	1.00	24	0.208
228	A	7	5	1.00	30	0.167
229	A	9	7	1.00	32	0.219
230	A	7	6	1.00	32	0.188
231	A	7	4	1.00	32	0.125
232	A	10	5	1.00	32	0.156
233	A	12	6	1.00	32	0.188
234	A	12	4	1.00	32	0.125
235	A	11	6	1.00	32	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	9	4	1.00	32	0.125
237	A	8	3	1.00	32	0.094
238	A	8	3	1.00	30	0.100
239	A	3	3	1.00	24	0.125
240	A	9	4	1.00	30	0.133
241	A	15	9	1.00	32	0.281
242	A	13	7	1.00	32	0.219
243	A	15	7	1.00	32	0.219
244	A	5	4	1.00	33	0.121
245	A	4	4	1.00	33	0.121
246	A	3	3	1.00	31	0.097
247	A	1	1	1.00	21	0.048
248	A	6	6	1.00	33	0.182
249	A	6	6	1.00	33	0.182
250	A	7	7	1.00	33	0.212
251	A	6	5	1.00	35	0.143
252	A	5	5	1.00	35	0.143
253	A	4	4	1.00	33	0.121
254	A	2	2	1.00	23	0.087
255	A	7	7	1.00	35	0.200
256	A	7	7	1.00	35	0.200
257	A	7	7	1.00	35	0.200
258	A	7	5	1.00	35	0.143
259	A	6	5	1.00	35	0.143
260	A	10	8	1.00	33	0.242
261	A	8	6	0.92	23	0.261
262	A	8	7	1.00	35	0.200
263	A	8	8	1.00	35	0.229
264	A	8	7	1.00	35	0.200
265	A	3	3	1.00	35	0.086
266	A	2	2	0.99	35	0.057
267	A	4	4	1.00	33	0.121
268	A	2	2	1.00	23	0.087
269	A	5	5	1.00	35	0.143
270	A	6	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	7	6	1.00	35	0.171
272	A	3	2	1.00	35	0.057
273	A	5	5	1.00	35	0.143
274	A	4	4	1.00	33	0.121
275	A	2	2	1.00	23	0.087
276	A	6	5	1.00	35	0.143
277	A	7	6	1.00	35	0.171
278	A	8	6	1.00	35	0.171
279	A	6	5	1.00	35	0.143
280	A	5	5	1.00	35	0.143
281	A	4	4	1.00	33	0.121
282	A	3	3	1.00	23	0.130
283	A	7	5	1.00	35	0.143
284	A	8	6	1.00	35	0.171
285	A	9	6	1.00	35	0.171
286	A	5	5	1.00	37	0.135
287	A	4	4	1.00	37	0.108
288	A	4	4	1.00	35	0.114
289	A	2	2	1.00	25	0.080
290	A	3	3	1.00	37	0.081
291	A	3	3	1.00	37	0.081
292	A	4	4	1.00	37	0.108
293	A	6	6	1.00	37	0.162
294	A	5	5	1.00	37	0.135
295	A	5	5	1.00	35	0.143
296	A	3	3	1.00	25	0.120
297	A	4	4	1.00	37	0.108
298	A	4	4	1.00	37	0.108
299	A	4	4	1.00	37	0.108
300	A	7	6	1.00	37	0.162
301	A	6	5	1.00	37	0.135
302	A	6	5	1.00	35	0.143
303	A	4	3	1.00	25	0.120
304	A	5	4	1.00	37	0.108
305	A	5	5	1.00	37	0.135

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	5	4	1.00	37	0.108
307	A	7	6	1.00	37	0.162
308	A	6	6	1.00	37	0.162
309	A	5	5	1.00	35	0.143
310	A	3	3	1.00	25	0.120
311	A	5	5	1.00	37	0.135
312	A	6	6	1.00	37	0.162
313	A	7	6	1.00	37	0.162
314	A	7	7	1.00	37	0.189
315	A	6	6	1.00	37	0.162
316	A	5	5	1.00	35	0.143
317	A	3	3	1.00	25	0.120
318	A	6	6	1.00	37	0.162
319	A	7	7	1.00	37	0.189
320	A	8	7	1.00	37	0.189
321	A	7	6	1.00	37	0.162
322	A	6	6	1.00	37	0.162
323	A	5	5	1.00	35	0.143
324	A	4	4	1.00	25	0.160
325	A	7	6	1.00	37	0.162
326	A	8	7	1.00	37	0.189
327	A	9	7	1.00	37	0.189
328	A	7	4	1.00	35	0.114
329	A	8	6	1.00	33	0.182
330	A	7	5	1.00	35	0.143
331	A	7	4	1.00	35	0.114
332	A	11	7	1.00	37	0.189
333	A	4	4	1.00	37	0.108
334	A	7	7	1.00	37	0.189
335	A	7	4	1.00	37	0.108
336	A	6	6	1.00	35	0.171
337	A	5	5	0.99	33	0.152
338	A	3	3	1.00	23	0.130
339	A	6	6	1.00	35	0.171
340	A	7	7	1.00	35	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	8	7	1.00	35	0.200
342	A	9	5	1.00	37	0.135
343	A	9	5	1.00	37	0.135
344	A	9	5	1.00	37	0.135
345	A	9	5	1.00	37	0.135
346	A	9	5	1.00	35	0.143
347	A	7	6	1.00	39	0.154
348	A	4	4	1.00	36	0.111
349	A	4	4	1.00	40	0.100
350	A	1	1	1.00	55	0.018
351	A	1	1	1.00	51	0.020
352	A	6	6	1.00	35	0.171
353	A	7	7	1.00	39	0.180
354	A	5	5	1.00	39	0.128
355	A	3	3	1.00	39	0.077
356	A	4	4	1.00	39	0.103
357	A	5	5	1.00	39	0.128
358	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=373

$$-\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)}+\frac{a^3(B(15+19n+4n^2)+A(2$$

```
[Out] -a^3*(B*(2*n^2+14*n+27)+A*(2*n^2+15*n+28))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/
d/f/(4+n)/(n^2+5*n+6)-a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^
2/d/f/(4+n)-(A*(4+n)+B*(6+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^3+a^3*sin(
f*x+e))/d/f/(3+n)/(4+n)+a^3*(B*(4*n^2+19*n+15)+A*(4*n^2+21*n+20))*cos(f*x+e
)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)
/d/f/(4+n)/(n^2+3*n+2)/(cos(f*x+e)^2)^(1/2)+a^3*(B*(9+4*n)+A*(11+4*n))*cos(
f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n
)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3055, 3047, 3102, 2827, 2722}

$$\frac{a^3(A(4n+11)+B(4n+9))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3(A(4n^2+21n+20)+B(4n^2+19n+15))\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)(n+2)(n+4)\sqrt{\cos^2(e+fx)}}-\frac{a^3(A(2n^2+15n+28)+B(2n^2+14n+27))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)(n+3)(n+4)}-\frac{(A(n+4)+B(n+6))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(d\sin(e+fx))^{n+1}}{df(n+3)(n+4)}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^3*(B*(27 + 14*n + 2*n^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[
e + f*x])^(1 + n))/(d*f*(2 + n)*(3 + n)*(4 + n))) + (a^3*(B*(15 + 19*n + 4*
n^2) + A*(20 + 21*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)
*(4 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e +
f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e
+ f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2]) - (a*B*Cos[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*(4 + n)) - ((A
*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^3 + a^3*Sin[
e + f*x]))/(d*f*(3 + n)*(4 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx &= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\,\mathrm{si}}{df(4+n)} \\
&= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\,\mathrm{si}}{df(4+n)} \\
&= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\,\mathrm{si}}{df(4+n)} \\
&= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n}{df(2+n)(3+n} \\
&= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n}{df(2+n)(3+n} \\
&= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n}{df(2+n)(3+n}
\end{aligned}
$$

Mathematica [A]
time = 1.61, size = 248, normalized size = 0.66

$$
\frac{a^3\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(3A+B)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)}{2+n}+\sin(e+fx)\left(\frac{3(A+B)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(e+fx)\right)}{3+n}+\sin(e+fx)\left(\frac{(A+3B)\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(e+fx)\right)}{4+n}+\frac{B\,{}_2F_1\left(\frac{1}{2},\frac{5+n}{2};\frac{7+n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{5+n}\right)\right)\right)\right)}{f\sqrt{\cos^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((3*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4 + n)
/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (5 + n)
/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n))))))/(f*Sqrt[Cos[e + f
*x]^2])
```

Maple [F]
time = 0.83, size = 0, normalized size = 0.00

$$
\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{3}\left(A+B\sin\left(fx+e\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*sin(
f*x + e))^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin (e + f x))^n (A + B \sin (e + f x)) (a + a \sin (e + f x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3, x)
```

3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=277

$$-\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)}+\frac{a^2(2B(1+n)+A(3+2n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\right.}{df(1+n)(2+n)\sqrt{}}$$

```
[Out] -a^2*(A*(3+n)+B*(4+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)/(3+n)-B*co
s(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^2+a^2*sin(f*x+e))/d/f/(3+n)+a^2*(2*B*(1+n)
+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)
*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a^2*(2*A*(3+n)+B
*(5+2*n))*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*si
n(f*x+e))^(2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3055, 3047, 3102, 2827, 2722}

$$\frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^2(A(2n+3)+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)(n+2)\sqrt{\cos^2(e+fx)}}-\frac{a^2(A(n+3)+B(n+4))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)(n+3)}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*
(2 + n)*(3 + n))) + (a^2*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeom
etric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n
))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) + B*(5 +
2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x
]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2])
 - (B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\right.}{df(3+n)}$$

$$= -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\right.}{df(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin}{df(2+n)(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin}{df(2+n)(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin}{df(2+n)(3+n)}$$

Mathematica [A]
time = 1.06, size = 204, normalized size = 0.74

$$\frac{a^2\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(2A+B)\,_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)}{2+n}+\sin(e+fx)\left(\frac{(A+2B)\,_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(e+fx)\right)}{3+n}+\frac{B\,_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{4+n}\right)\right)\right)}{f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((2*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*(((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Sin[e +
 f*x]^2]*Sin[e + f*x])/(4 + n)))))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [F]
time = 1.42, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^2 (A+B\sin(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2, x)
```

3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=191

$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)}+\frac{a(B(1+n)+A(2+n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)+a*(B*(1+n)+A*(2+n))*cos(f*x+
e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n
)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a*(A+B)*cos(f*x+e)*hypergeom([1/2, 1
+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(cos(f*x+e
)^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3047, 3102, 2827, 2722}

$$\frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{df(n+1)(n+2)\sqrt{\cos^2(e+fx)}}-\frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + (a*(B*(1 + n
) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)
/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*Sqrt[Cos[e + f
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (d\sin(e+fx))^n\left(aA+(aA+aB)\sin(e+\right.$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{\int(d\,\mathrm{si}}{}$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{(a(A}{}$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{a(B(1}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.38, size = 392, normalized size = 2.05

$$-\frac{2^{-2-n}ae^{ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n\left(\frac{2(A+B)e^{-i(e+f(1+n)x)}\,{}_2F_1\left(\frac{1}{2}(-1-n),-n;\frac{1-n}{2};e^{2i(e+fx)}\right)}{1+n}-\frac{2(A+B)e^{i(e-f(-1+n)x)}\,{}_2F_1\left(\frac{1-n}{2},-n;\frac{3-n}{2};e^{2i(e+fx)}\right)}{-1+n}+i\left(\frac{Be^{-i(2e+f(2+n)x)}\,{}_2F_1\left(-1-\frac{n}{2},-n;-\frac{n}{2};e^{2i(e+fx)}\right)}{2+n}+\frac{e^{-ifnx}\left(Be^{2i(e+fx)}n\,{}_2F_1\left(1-\frac{n}{2},-n;2-\frac{n}{2};e^{2i(e+fx)}\right)-2(2A+B)(-2+n)\,{}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};e^{2i(e+fx)}\right)\right)}{(-2+n)n}\right)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(-2 - n)*a*E^(I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*
x)))^n*((2*(A + B)*Hypergeometric2F1[(-1 - n)/2, -n, (1 - n)/2, E^((2*I)*(e
 + f*x))])/(E^(I*(e + f*(1 + n)*x))*(1 + n)) - (2*(A + B)*E^(I*(e - f*(-1 +
 n)*x))*Hypergeometric2F1[(1 - n)/2, -n, (3 - n)/2, E^((2*I)*(e + f*x))])/(
-1 + n) + I*((B*Hypergeometric2F1[-1 - n/2, -n, -1/2*n, E^((2*I)*(e + f*x))
])/(E^(I*(2*e + f*(2 + n)*x))*(2 + n)) + (B*E^((2*I)*(e + f*x))*n*Hypergeom
etric2F1[1 - n/2, -n, 2 - n/2, E^((2*I)*(e + f*x))] - 2*(2*A + B)*(-2 + n)*
Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^((2*I)*(e + f*x))])/(E^(I*f*n*x)*(
```

```
-2 + n)*n)))*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x]))/((1 - E^((2*I)*(e + f*x
)))^n*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[e + f*x]^n))
```

Maple [**F**]
time = 1.20, size = 0, normalized size = 0.00

$$\int (d \sin (fx+e))^n (a+a\sin (fx+e)) (A+B\sin (fx+e)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e))^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\sin(e+fx))^n (A+B\sin(e+fx)) (a+a\sin(e+fx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)), x)
```

3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=202

$$\frac{(B-An+Bn)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{adf(1+n)\sqrt{\cos^2(e+fx)}}+\frac{(A-B)(1+n)\cos(e+fx)}{ad^2}$$

```
[Out] (A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))+(-A*n+B*n+B)*cos
(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))
^(1+n)/a/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*(1+n)*cos(f*x+e)*hypergeom([1
/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a/d^2/f/(2+n)/(co
s(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3057, 2827, 2722}

$$\frac{(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{ad^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{(-An+Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{adf(n+1)\sqrt{\cos^2(e+fx)}}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(a*d^2*f*(2 + n)*Sqrt[Cos[e
 + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*S
in[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{\int(d\sin(e+fx))^n(ad$$
$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{((A-B)(1+n))\int(}{a}$$
$$= \frac{(B-An+Bn)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin}{adf(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A]
time = 0.60, size = 157, normalized size = 0.78

$$\frac{\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{(B-An+Bn)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)}{(1+n)\sqrt{\cos^2(e+fx)}}+\frac{(A-B)(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{(2+n)\sqrt{\cos^2(e+fx)}}+\frac{A-B}{1+\sin(e+fx)}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(((B - A*n + B*n)*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*Sqrt[Cos[e + f*
x]^2]) + ((A - B)*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[
e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[Cos[e + f*x]^2]) + (A - B)/(1 + Sin
[e + f*x])))/(a*f)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e))^n(A+B\sin(fx+e))}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{a+a\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)), x)
```

3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=279

$$-\frac{n(A-2An+2B(1+n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}+\frac{(1+n)(B+2A(1$$

```
[Out] 1/3*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+sin(f*x+
e))+1/3*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^2-1/3*n*
(A-2*A*n+2*B*(1+n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f
*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+1/3*(1+n)*
(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e
)^2)*(d*sin(f*x+e))^(2+n)/a^2/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3057, 2827, 2722}

$$\frac{(n+1)(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{3a^2d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n(-2An+A+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{3a^2df(n+1)\sqrt{\cos^2(e+fx)}}+\frac{(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3a^2df(\sin(e+fx)+1)}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a^2*d*f*(1 + n)
*Sqrt[Cos[e + f*x]^2]) + ((1 + n)*(B + 2*A*(1 - n) + 2*B*n)*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])
^(2 + n))/(3*a^2*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((B + 2*A*(1 - n) +
2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + Sin[e + f*x])
) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f
*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2} + \frac{\int \frac{(d\sin(e+fx))^n(ad(2A-}{}}{}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A}{}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A}{}$$

$$= -\frac{n(A-2An+2B(1+n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(\right.}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A]
time = 0.87, size = 212, normalized size = 0.76

$$\frac{\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(-\frac{n(A-2An+2B(1+n))\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)}{(1+n)\sqrt{\cos^2(e+fx)}}+\frac{(1+n)(B-2A(-1+n)+2Bn)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{(2+n)\sqrt{\cos^2(e+fx)}}+\frac{A-B}{(1+\sin(e+fx))^2}+\frac{(-A+B)n}{1+\sin(e+fx)}+\frac{2A+B-An+Bn}{1+\sin(e+fx)}\right)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(-((n*(A - 2*A*n + 2*B*(1 + n
))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*S
qrt[Cos[e + f*x]^2])) + ((1 + n)*(B - 2*A*(-1 + n) + 2*B*n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[C
os[e + f*x]^2]) + (A - B)/(1 + Sin[e + f*x])^2 + ((-A + B)*n)/(1 + Sin[e +
f*x]) + (2*A + B - A*n + B*n)/(1 + Sin[e + f*x])))/(3*a^2*f)
```

Maple [F]
time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e))^n\,(A+B\sin(fx+e))}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2, x)
```

3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=362

$$-\frac{n(B(3-n-4n^2)+A(2-9n+4n^2))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}+\frac{(1-$$

```
[Out] 1/5*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^3+1/15*(A*(5
-2*n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a/d/f/(a+a*sin(f*x+e))^2+1/15*
(1-n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a^3+a^3*s
in(f*x+e))-1/15*n*(B*(-4*n^2-n+3)+A*(4*n^2-9*n+2))*cos(f*x+e)*hypergeom([1/
2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^3/d/f/(1+n)/
(cos(f*x+e)^2)^(1/2)+1/15*(1-n)*(1+n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*hyp
ergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^3/d^2/
f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3057, 2827, 2722}

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{15a^3d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n(A(4n^2-9n+2)+B(-4n^2-n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{15a^3df(n+1)\sqrt{\cos^2(e+fx)}}+\frac{(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{15df\left(a^3\sin(e+fx)+a^3\right)}+\frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{n+1}}{15adf(a\sin(e+fx)+a)^2}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))
/(a^3*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((1 - n)*(1 + n)*(7*A + 3*B - 4*A
*n + 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e
 + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(15*a^3*d^2*f*(2 + n)*Sqrt[Cos[e + f*x
]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e
 + f*x])^3) + ((A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))
/(15*a*d*f*(a + a*Sin[e + f*x])^2) + ((1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*C
os[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*d*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{\int \frac{(d\sin(e+fx))^n(ad(4A-\cdots}{\cdots}}{\cdots}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cdots}{15adf(\cdots}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cdots}{15adf(\cdots}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cdots}{15adf(\cdots}$$

$$= -\frac{n(B(3-n-4n^2)+A(2-9n+4n^2))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\cdots\right.}{15a^3df(1+n)\sqrt{\cos^2(e+\cdots}}$$

Mathematica [A]
time = 2.91, size = 260, normalized size = 0.72

$$\frac{(d\sin(e+fx))^n\left(\frac{2\cos(e+fx)\sin(e+fx)\left(n\left(A(-2+9n-4n^2)+B(-3+n+4n^2)\right){}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)+\frac{(-1+n)(1+n)^2(A(-7+4n)-B(3+4n))\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{2+n}\right)}{(1+n)\sqrt{\cos^2(e+fx)}}+\frac{3(A-B)\sin(2(e+fx))}{(1+\sin(e+fx))^3}+\frac{(A(5-2n)+2Bn)\sin(2(e+fx))}{(1+\sin(e+fx))^2}+\frac{(-1+n)(A(-7+4n)-B(3+4n))\sin(2(e+fx))}{1+\sin(e+fx)}\right)}{30a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] ((d*Sin[e + f*x])^n*((2*Cos[e + f*x]*Sin[e + f*x]*(n*(A*(-2 + 9*n - 4*n^2)
+ B*(-3 + n + 4*n^2))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e +
```

```
f*x]^2] + ((-1 + n)*(1 + n)^2*(A*(-7 + 4*n) - B*(3 + 4*n))*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(2 + n)))/((1 +
n)*Sqrt[Cos[e + f*x]^2]) + (3*(A - B)*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])^
3 + ((A*(5 - 2*n) + 2*B*n)*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])^2 + ((-1 +
n)*(A*(-7 + 4*n) - B*(3 + 4*n))*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])))/(30*
a^3*f)
```

Maple [F]
time = 1.87, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x+e\right)\right)^{n} \left(A+B \sin \left(f x+e\right)\right)}{\left(a+a \sin \left(f x+e\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 - 4
*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3, x)
```

3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=336

$$-\frac{2a^3(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(\right.}{f(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(3/2)/d/f/(7+2*n)-2
*a^3*(2*B*(16*n^3+104*n^2+203*n+115)+A*(32*n^3+224*n^2+478*n+301))*cos(f*x+
e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*
n)/(7+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a^3*(2*B*(4*n^2+23*n+35)
+A*(8*n^2+50*n+77))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(5+2*n)/(7+
2*n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(5+n)+A*(7+2*n))*cos(f*x+e)*(d*sin(f
*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)/(7+2*n)
```

Rubi [A]

time = 0.61, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3060, 2855, 69, 67}

$$-\frac{2a^3(A(32n^3+224n^2+478n+301)+2B(16n^3+104n^2+203n+115))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}}-\frac{2a^3(A(8n^2+50n+77)+2B(4n^2+23n+35))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n+7)+2B(n+5))\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(d\sin(e+fx))^{n+1}}{df(2n+5)(2n+7)}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^3*(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 +
32*n^3))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*
Sin[e + f*x])^n)/(f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a
*Sin[e + f*x]]) - (2*a^3*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*
Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*
(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n)*(7 + 2*n)
) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2)
)/(d*f*(7 + 2*n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+}{df(7+2n)}$$

$$= -\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx}{df(5+2n}$$

$$= -\frac{2a^3(2B(35+23n+4n^2)+A(77+50n}{df(3+2n)(5+2n)(7+2}$$

$$= -\frac{2a^3(2B(35+23n+4n^2)+A(77+50n}{df(3+2n)(5+2n)(7+2}$$

$$= -\frac{2a^3(2B(35+23n+4n^2)+A(77+50n}{df(3+2n)(5+2n)(7+2}$$

$$= -\frac{2a^3(2B(115+203n+104n^2+16n^3)+}{}$$

Mathematica [A]
time = 29.71, size = 596, normalized size = 1.77

$$\frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{5/2}\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}\right)^n\left(1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^n\left(\frac{A\,{}_2F_1\left(\frac{1+n}{2},\frac{9}{2}+n;\frac{3+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{1+n}+\frac{A\,{}_2F_1\left(4+\frac{n}{2},\frac{9}{2}+n;5+\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan^7\left(\frac{1}{2}(e+fx)\right)}{8+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{(5A+2B)\,{}_2F_1\left(\frac{2+n}{2},\frac{9}{2}+n;\frac{4+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{2+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{(11A+10B)\,{}_2F_1\left(\frac{3+n}{2},\frac{9}{2}+n;\frac{5+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{3+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{5(3A+4B)\,{}_2F_1\left(\frac{4+n}{2},\frac{9}{2}+n;\frac{6+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{4+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{5(3A+4B)\,{}_2F_1\left(\frac{9}{2}+n,\frac{5+n}{2};\frac{7+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{5+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{(11A+10B)\,{}_2F_1\left(\frac{9}{2}+n,\frac{6+n}{2};\frac{8+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{6+n}+\frac{(5A+2B)\,{}_2F_1\left(\frac{9}{2}+n,\frac{7+n}{2};\frac{9+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{7+n}\right)\right)\right)\right)\right)\right)}{f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((5*A +
 2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2]
)/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 + n)/2, 9
/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((5*(3*
A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e + f*x)/2]
^2])/(4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1[9/2 + n,
(5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + Tan[(e + f*x)/2]*(((1
1*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -Tan[(e + f*x)
/2]^2])/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 + n)/2, (9 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(7 + n))))))))/(f*Sqrt[Sec[(e +
 f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]^n)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \left(a+a \sin \left(f x+e\right)\right)^{\frac{5}{2}} \left(A+B \sin \left(f x+e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e
))^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin(e + f x))^n (A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

3.8 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=229

$$-\frac{2a^2(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d}{f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a^2*(2*B*(4*n^2+13*n+9)+A*(8*n^2+30*n+25))*cos(f*x+e)*hypergeom([1/2, -n
],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*n)/(sin(f*x+e)^n)/(a+
a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(3+n)+A*(5+2*n))*cos(f*x+e)*(d*sin(f*x+e))^(
1+n)/d/f/(3+2*n)/(5+2*n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(d*sin(f*x
+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A]
time = 0.34, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3060, 2855, 69, 67}

$$-\frac{2a^2(A(8n^2+30n+25)+2B(4n^2+13n+9))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^2*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2))*Cos[e + f*x]*Hyper
geometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*
n)*(5 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(3 + n)
 + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 +
2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a}}{df(5+2n)}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+f}{df(3+2n)(5+2n)\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+f}{df(3+2n)(5+2n)\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+f}{df(3+2n)(5+2n)\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{2a^2(2B(9+13n+4n^2)+A(25+30n}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 478 vs. 2(229) = 458.
time = 26.74, size = 478, normalized size = 2.09

$$\frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{3/2}\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}\right)^n\left(1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^n\left(\frac{A\,{}_2F_1\left(\frac{1+n}{2},\frac{7}{2}+n;\frac{3+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{1+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{(3A+2B)\,{}_2F_1\left(\frac{2+n}{2},\frac{7}{2}+n;\frac{4+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{2+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{2(2A+3B)\,{}_2F_1\left(\frac{3+n}{2},\frac{7}{2}+n;\frac{5+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{3+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{2(2A+3B)\,{}_2F_1\left(\frac{7}{2}+n,\frac{4+n}{2};\frac{6+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{4+n}+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{(3A+2B)\,{}_2F_1\left(\frac{7}{2}+n,\frac{5+n}{2};\frac{7+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{5+n}+\frac{A\,{}_2F_1\left(\frac{7}{2}+n,\frac{6+n}{2};\frac{8+n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{6+n}\right)\right)\right)\right)\right)}{f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[
(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e + f*x
)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2, -Tan[
(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2
F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2])/(4 + n) + Tan[(e +
f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n)/2, -Tan
[(e + f*x)/2]^2])/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)/2, (8 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(6 + n)))))))/(f*Sqrt[Sec[(e +
f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]^n)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^
n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^{\frac{3}{2}}(d\sin(e+fx))^n(A+B\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(d*sin(e + f*x))**n*(A + B*sin(e + f
*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^
n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin(e + f x))^n (A + B \sin(e + f x)) (a + a \sin(e + f x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

3.9 $\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=137

$$-\frac{2a(2B(1+n)+A(3+2n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f(3+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a}{d}$$

```
[Out] -2*a*(2*B*(1+n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e
))*(d*sin(f*x+e))^n/f/(3+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*c
os(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3060, 2855, 69, 67}

$$-\frac{2a(A(2n+3)+2B(n+1))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3
/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[
a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(
3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
```

```
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx &= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\Big(\\ &= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{\Big(}{}\\ &= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{\Big(}{}\\ &= -\frac{2a\left(A+\frac{2B(1+n)}{3+2n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n\right.}{f\sqrt{a}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 44.82, size = 409, normalized size = 2.99

$$-\frac{(1+i)2^{-2-n}e^{-\frac{3ie}{2}+ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n\left(\frac{2Be^{-\frac{1}{2}if(3+2n)x}\,{}_2F_1\left(\frac{1}{4}(-3-2n),-n;\frac{1}{4}(1-2n);e^{2i(e+fx)}\right)}{f(3+2n)}+2e^{ie}\left(-\frac{i(2A+B)e^{-\frac{1}{2}if(1+2n)x}\,{}_2F_1\left(\frac{1}{4}(-1-2n),-n;\frac{1}{4}(3-2n);e^{2i(e+fx)}\right)}{f+2fn}+\frac{e^{\frac{1}{2}i(2e+f(1-2n)x)}\left(-\left((2A+B)(-3+2n)\,{}_2F_1\left(\frac{1}{4}(1-2n),-n;\frac{1}{4}(5-2n);e^{2i(e+fx)}\right)\right)+iBe^{i(e+fx)}(-1+2n)\,{}_2F_1\left(\frac{1}{4}(3-2n),-n;\frac{1}{4}(7-2n);e^{2i(e+fx)}\right)\right)}{f(-3+2n)(-1+2n)}\right)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{a(1+\sin(e+fx))}}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),
x]
```

```
[Out] ((-1 - I)*2^(-2 - n)*E^(((-3*I)/2)*e + I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e +
f*x))))/E^(I*(e + f*x)))^n*((2*B*Hypergeometric2F1[(-3 - 2*n)/4, -n, (1 - 2
*n)/4, E^((2*I)*(e + f*x))])/(E^((I/2)*f*(3 + 2*n)*x)*f*(3 + 2*n)) + 2*E^(I
*e)*(((-I)*(2*A + B)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2
```

```
*I)*(e + f*x))])/(E^((I/2)*f*(1 + 2*n)*x)*(f + 2*f*n)) + (E^((I/2)*(2*e + f
*(1 - 2*n)*x))*(-((2*A + B)*(-3 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (
5 - 2*n)/4, E^((2*I)*(e + f*x))]) + I*B*E^(I*(e + f*x))*(-1 + 2*n)*Hypergeo
metric2F1[(3 - 2*n)/4, -n, (7 - 2*n)/4, E^((2*I)*(e + f*x))]))/(f*(-3 + 2*n
)*(-1 + 2*n))))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])])/((1 - E^((2*
I)*(e + f*x)))^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]^n)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x +e \right)\right)^{n} \sqrt{a +a \sin \left(f x +e \right)}\, \left(A +B \sin \left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sin{\left(e + f x \right)} + 1\right)} \left(d \sin{\left(e + f x \right)}\right)^{n} \left(A + B \sin{\left(e + f x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{(A-B)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}-2I$$

```
[Out] -(A-B)*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d
*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*B*cos(f*x+e)*hyper
geom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*s
in(f*x+e))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3066, 2866, 2865, 2864, 129, 440, 2855, 69, 67}

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2
]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x
]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*
(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
```

```
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = (A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int(d\sin(e+fx))^n\sqrt{a}}{a}$$

$$= \frac{\left((A-B)\sqrt{1+\sin(e+fx)}\right)\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} + \frac{(aB}{}$$

$$= \frac{\left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)c}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 24.22, size = 250, normalized size = 1.64

$$\frac{\cos(e+fx)\sin^n(e+fx)(d\sin(e+fx))^n\left(-\sin^2(e+fx)\right)^{-n}\sqrt{a(1+\sin(e+fx))}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4(A-B)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(-\sin(e+fx))^n\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}-(A+B)(1+2n)F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\left(1-\frac{1}{1+\sin(e+fx)}\right)^n\right)}{4af(1+2n)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 +
 Sin[e + f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e
+ f*x])] - (A + B)*(1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2,
1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n)
)/(4*a*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e
+ f*x])^(-1))^n)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (fx+e))^{n} (A+B \sin (fx+e))}{\sqrt{a+a \sin (fx+e)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sin (e+fx))^{n} (A+B \sin (e+fx))}{\sqrt{a (\sin (e+fx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)
), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\sin(e+fx))^n\,(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2), x
)
```

3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}}-\frac{(A-4An+B(3+4n))F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(}{4af\sqrt{a+a\sin(e+}}$$

```
[Out] 1/2*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A
-4*A*n+B*(3+4*n))*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-1/2*(A-
B)*(1+2*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e)
)^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3057, 3066, 2866, 2865, 2864, 129, 440, 2855, 69, 67}

$$-\frac{(-4An+A+B(4n+3))\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{4af\sqrt{a\sin(e+fx)+a}}-\frac{(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{2af\sqrt{a\sin(e+fx)+a}}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x])
^(3/2)) - ((A - 4*A*n + B*(3 + 4*n))*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e +
f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(4*a*f*Sin[e +
 f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*(1 + 2*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e
 + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(d\sin(e+fx))^n(ad(A+}{\sqrt{}}}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{((A-B)(1+2n))}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\Big(\big(-\frac{1}{2}a^2(A-B)d(}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\Big(\big(-\frac{1}{2}a^2(A-B)d(}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\,\mathrm{co}}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\,\mathrm{co}}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A+3B-4An+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 523 vs. 2(226) = 452.
time = 55.88, size = 523, normalized size = 2.31

$$\frac{\sec(e+fx)(d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(aF_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\,(-\sin(e+fx))^{-n}(1+\sin(e+fx))-\frac{4\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(-2a(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(1+\sin(e+fx))\right)}{-1+4n^2}\right)+A\left(a^2F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sqrt{2-2\sin(e+fx)}\,(-\sin(e+fx))^{-n}(1+\sin(e+fx))^2-\frac{4a\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}(1+\sin(e+fx))\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(2a(1+2n)F_1\left(\frac{1}{2}-n;-\frac{1}{2},-n;\frac{3}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{1}{1+\sin(e+fx)}\right)(1+\sin(e+fx))\right)}{-1+4n^2}\right)\right)}{8a^3f\sqrt{a(1+\sin(e+fx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1/
2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*
(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Si
n[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Si
n[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/
2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e +
 f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)) + A*((a^2*AppellF1
[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e +
f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Sin[e + f*x
])/(1 + Sin[e + f*x])]*(1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n,
-1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^
(-1))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e))^n\,(A+B\sin(fx+e))}{(a+a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/
(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(e+fx))^n (A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(
3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2), x
)
```

3.12 $\int (d \sin(e+fx))^n (a+a\sin(e+fx))^m (A+B\sin(e+fx))\,dx$

Optimal. Leaf size=221

$$-\frac{2^{\frac{3}{2}+m}BF_1\left(\frac{1}{2};-n,-\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))$$
$$f$$

```
[Out] -2^(3/2+m)*B*AppellF1(1/2,-n,-1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(sin
(f*x+e)^n)-2^(1/2+m)*(A-B)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*s
in(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x
+e))^m/f/(sin(f*x+e)^n)
```

Rubi [A]
time = 0.32, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3066, 2866, 2865, 2864, 138}

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f}-\frac{B2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,-m-\frac{1}{2};\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Si
n[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*Appell
F1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f
*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/(f*Sin[e + f*x]^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = (A-B)\int (d\sin(e+fx))^n(a+a\sin(e+$$

$$= \left((A-B)(1+\sin(e+fx))^{-m}(a+a\sin(e$$

$$= \left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1$$

$$= -\frac{\left((A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\,s\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+m}BF_1\left(\frac{1}{2};-n,-\frac{1}{2}-m;\frac{3}{2};1-\sin(e-\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5918 vs. $2(221) = 442$.
time = 20.99, size = 5918, normalized size = 26.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x +e \right)\right)^{n} \left(a +a \sin \left(f x +e \right)\right)^{m} \left(A +B \sin \left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{m} \left(d \sin{\left(e + f x \right)}\right)^{n} \left(A + B \sin{\left(e + f x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n*(A + B*sin(e + f*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.13 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=114

$$\frac{F_1\left(1+n;-\frac{1}{2},\frac{1}{2}-m;2+n;\sin(e+fx),-\sin(e+fx)\right)\sec(e+fx)(d\sin(e+fx))^{1+n}(1+\sin(e+fx))^{\frac{1}{2}-m}(}{df(1+n)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1+n,1/2-m,-1/2,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(d*sin(f*x+e
))^(1+n)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m/d/f/(1+
n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3087, 140, 138}

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},\frac{1}{2}-m;n+2;\sin(e+fx),-\sin(e+fx)\right)}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e +
 f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e +
f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
```

```
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx = \frac{\Big(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin\ldots}}{\ldots}$$

$$= \frac{\Big(\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin\ldots}}{\ldots}$$

$$= \frac{\Big(\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a-a\sin\ldots}{\ldots}$$

$$= \frac{F_1\left(1+n;-\frac{1}{2},\frac{1}{2}-m;2+n;\sin(e+fx),-\ldots\right.}{\ldots}$$

Mathematica [F]
time = 9.81, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
[Out] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m, x
]
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\int\left(-(d\sin(e+fx))^n\left(a\sin(e+fx)+a\right)^m\right)dx+\int(d\sin(e+fx))^n\left(a\sin(e+fx)+a\right)^m\sin(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m,x)
```

```
[Out] -a*(Integral(-(d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m, x) + Integral((d
*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(a-a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)),x)
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)), x)
```

3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(c+dx))^{-2-n}}{d}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))^(-2-n)/d
```

Rubi [A]
time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{3053\}$

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c +
 d*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(37) = 74.
time = 4.15, size = 107, normalized size = 2.89

$$-\frac{2^n\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{4}(c+dx)\right)\left(-\sin\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{3}{4}(c+dx)\right)\right)\right)^n(1+\cos(c+dx)-\sin(c+dx))(a(1+\sin(c+dx)))^{-2-n}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*S
in[c + d*x]),x]
```

```
[Out] -((2^n*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/4]*(-Sin[(c + d*x)/4] + Sin[(3*(c + d*x))/4]))^n*(1 + Cos[c + d*x] - Sin[
c + d*x])*(a*(1 + Sin[c + d*x]))^(-2 - n))/d)
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int (\sin^n (dx+c)) (a+a\sin (dx+c))^{-2-n} (-1-n-(-2-n)\sin (dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="maxima")
```

```
[Out] integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin(
d*x + c)^n, x)
```

Fricas [A]
time = 0.61, size = 41, normalized size = 1.11

$$-\frac{(a\sin (dx+c)+a)^{-n-2}\sin (dx+c)^n\cos (dx+c)\sin (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x
)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 9496 vs. $2(37) = 74$.
time = 56.54, size = 9496, normalized size = 256.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="giac")
```

```
[Out] -8*(cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) +
4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) -
 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8
*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*e^(-n*log(
sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2
*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*
x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4
*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*ta
n(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*t
an(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)
^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*
x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 +
 1)) + n*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*
log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x +
c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8
*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
 + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2
)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 +
```

```
 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)
^2 + 1)))*tan(1/4*pi*n*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1
/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 +
8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(4*a*tan(1/2
*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) +
1/2*pi*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*ta
n(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x
+ 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*n - pi*floor(d*x/pi + c/pi
+ 1/2) + 1/2*pi*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2
 + 4*a*tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^3 - 2*e^(-n*log(sqrt(2
)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1
/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d
*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d
*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
+ c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan
(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2
*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2
*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) +
 n*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(sq
rt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*t
an(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1
/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(
d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2
*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/
2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2
 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1
)))*sin(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d...
```

Mupad [B]
time = 12.98, size = 61, normalized size = 1.65

$$-\frac{\sin\left(c+d\,x\right)^n\,\sin\left(2\,c+2\,d\,x\right)}{a^2\,d\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^n\,\left(2\,{\sin\left(c+d\,x\right)}^2+4\,\sin\left(c+d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(c + d*x)^n*(n - sin(c + d*x)*(n + 2) + 1))/(a + a*sin(c + d*x))^(
n + 2),x)
```

```
[Out] -(sin(c + d*x)^n*sin(2*c + 2*d*x))/(a^2*d*(a*(sin(c + d*x) + 1))^n*(4*sin(c
 + d*x) + 2*sin(c + d*x)^2 + 2))
```

3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$

Optimal. Leaf size=35

$$-\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+dx))^m}{d}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(-1-m)*(a+a*sin(d*x+c))^m/d
```

Rubi [A]
time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {3053}

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),x
]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a}{d}$$

Mathematica [A]
time = 0.27, size = 35, normalized size = 1.00

$$-\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a(1+\sin(c+dx)))^m}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d
*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a*(1 + Sin[c + d*x]))^m)/d)
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(\sin^{-2-m}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{m}\left(1+m-m\sin\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m
 - 2), x)
```

Fricas [A]
time = 0.38, size = 41, normalized size = 1.17

$$-\frac{\left(a\sin\left(dx+c\right)+a\right)^{m}\sin\left(dx+c\right)^{-m-2}\cos\left(dx+c\right)\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2)*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5502 vs. $2(35) = 70$.
time = 26.69, size = 5502, normalized size = 157.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="giac")
```

```
[Out] -8*(cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2
*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)
*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(
d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)
^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*
c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(
a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x +
1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*ta
n(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2
*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d
*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/
2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m +
 pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^3 - 2*e^(m*log(sqrt(
2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
 + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
```

```
2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d
*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 +
2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))
- m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4
*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*sin(2*pi*m*floor(
-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2
*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x
 + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*tan(-1/2*pi + 1/4*pi
*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/
2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/
2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi +
 c/pi + 1/2))*tan(1/2*d*x + 1/2*c)^3 - cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/
8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^
2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*ta
n(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1
/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2
+ 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*lo...
```

Mupad [B]
time = 12.85, size = 38, normalized size = 1.09

$$-\frac{\sin\left(2\,c+2\,d\,x\right)\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^m}{2\,d\,{\sin\left(c+d\,x\right)}^{m+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(c + d*x))^m*(m - m*sin(c + d*x) + 1))/sin(c + d*x)^(m + 2),
x)
```

```
[Out] -(sin(2*c + 2*d*x)*(a*(sin(c + d*x) + 1))^m)/(2*d*sin(c + d*x)^(m + 2))
```

3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=153

$$\frac{(Ab-2aB)x}{b^3}-\frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f}-\frac{B\cos(e+fx)}{b^2f}+\frac{a^2(Ab-aB}{b^2\left(a^2-b^2\right)f(a}$$

```
[Out] (A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((b+a*tan(1
/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f-B*cos(f*x+e)/b^2/f+a^
2*(A*b-B*a)*cos(f*x+e)/b^2/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A]
time = 0.27, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3067, 3102, 2814, 2739, 632, 210}

$$\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{x(Ab-2aB)}{b^3}-\frac{B\cos(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f) - (
B*Cos[e + f*x])/(b^2*f) + (a^2*(A*b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} + \frac{\int \frac{ab(Ab-aB)+(a^2-b^2)(Ab-aB)\sin(e+f}{a+b\sin(e+fx}}{b^2(a^2-b}$$

$$= -\frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))} + \frac{\int \frac{ab^2(Ab-aB)}{}}{}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2(a^2-b^2)f(a+b\sin(e+fx))}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\tan^{-1}\left(\frac{b+a\tan(}{\sqrt{a^2}}\right.}{b^3(a^2-b^2)^{3/2}f}$$

Mathematica [A]
time = 0.60, size = 147, normalized size = 0.96

$$\frac{(Ab-2aB)(e+fx)+\frac{2a(-a^2Ab+2Ab^3+2a^3B-3ab^2B)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-bB\cos(e+fx)+\frac{a^2b(Ab-aB)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*B
*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Si
n[e + f*x])))/(b^3*f)
```

Maple [A]
time = 0.42, size = 212, normalized size = 1.39

method	result
derivativedivides	$-\frac{\frac{2a\left(\frac{-\frac{b^2(Ab-aB)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-aB)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(A a^2b-2A b^3-2B a^3+3Ba b^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^3}+\frac{-\frac{2Bb}{1+\tan^2\left(\frac{fx}{2}+\right.}}{}}{f}$

default	$-\frac{2a\left(\frac{-\frac{b^2(Ab-aB)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-aB)}{a^2-b^2}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(A\,a^2b-2A\,b^3-2B\,a^3+3Ba\,b^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^3}+\frac{-\frac{2Bb}{1+\tan^2\left(\frac{fx}{2} \cdots\right.}\cdots}{f}$
risch	$\frac{xA}{b^2}-\frac{2xaB}{b^3}-\frac{B\,\mathrm{e}^{i(fx+e)}}{2b^2f}-\frac{B\,\mathrm{e}^{-i(fx+e)}}{2b^2f}+\frac{2ia^2(-Ab+aB)\left(ib+a\,\mathrm{e}^{i(fx+e)}\right)}{b^3(a^2-b^2)f\left(-i\mathrm{e}^{2i(fx+e)}b+ib+2a\,\mathrm{e}^{i(fx+e)}\right)}+\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{\cdots}}{\cdots}\right.}{\sqrt{-a^2+\cdots}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-2*a/b^3*((-b^2*(A*b-B*a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)-b*a*(A*b-B*a)/(
a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+(A*a^2*b-2*A*b^
3-2*B*a^3+3*B*a*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b
)/(a^2-b^2)^(1/2)))+2/b^3*(-B*b/(1+tan(1/2*f*x+1/2*e)^2)+(A*b-2*B*a)*arctan
(tan(1/2*f*x+1/2*e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(156) = 312.
time = 0.45, size = 823, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a
*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A
```

```
*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
 e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 -
2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 +
 A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^
3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6
)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) +
 (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A
*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 +
2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e
))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x +
e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x
+ e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*
b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((
a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f
)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(156) = 312.
time = 0.46, size = 371, normalized size = 2.42

$$\frac{\frac{2\,(2\,Ba^4-Aa^3b-3\,Ba^2b^2+2\,Aab^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{a^2-b^2}}-\frac{2\left(Ba^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Aab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Aa^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Ba^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Aab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Bb^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ba^3-Aa^2b-Bab^2\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\right)(a^2b^2-b^4)}-\frac{(2\,Ba-Ab)(fx+e)}{b^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^
2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*b^2
*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(1/2*
f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*x + 1
/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) + 2*B*a
^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*
e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(a^2*b^2
- b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f
```

Mupad [B]
time = 17.18, size = 2500, normalized size = 16.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(A + B*sin(e + f*x)))/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(A*a^2*b - 2*B*a^3 + B*a*b^2))/(b^2*(a^2 - b^2)) - (2*tan(e/2 + (f*x)/2
)^3*(B*a^2 - A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(2*B*b^2 - 3*B
*a^2 + A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(A*a^2*b - 2*B*a^3
 + B*a*b^2))/(b^2*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*tan(e/
2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) + (log
(tan(e/2 + (f*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*f) - (log(tan(e/2 + (f*x)/
2) - 1i)*(A*b*1i - B*a*2i))/(b^3*f) - (a*atan(((a*(-(a + b)^3*(a - b)^3)^(1
/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2
*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/
(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*
a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 +
28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^
6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b
^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^
6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(
b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^
9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 +
7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^
3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^
4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b
^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b
^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((3
2*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^
4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 -
2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8
 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*
a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 +
 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(
1/2)*((32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/
(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^
4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2
*a^2*b^8 + a^4*b^6) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a
^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b
^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
 a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*
```

a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(4*B^3*a^8 + 2*A^3*a^3*b^5 - A^3*a^5*b^3 - 6*B^3*a^6*b^2 - 8*A*B^2*a^7*b + 13*A*B^2*a^5*b^3 - 9*A^2*B*a^4*b^4 + 5*A^2*B*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (64*tan(e/2 + (f*x)/2)*(16*B^3*a^9 - 4*A^3*a^2*b^7 + 6*A^3*a^4*b^5 - 2*A^3*a^6*b^3 + 24*B^3*a^5*b^4 - 40*B^3*a^7*b^2 - 24*A*B^2*a^8*b - 40*A*B^2*a^4*b^5 + 64*A*B^2*a^6*b^3 + 22*A^2*B*a^3*b^6 - 34*A^2*B*a^5*b^4 + 12*A^2*B*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A*...

3.17 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=182

$$\frac{7}{16}a(2A-B)c^4x+\frac{7a(2A-B)c^4\cos^3(e+fx)}{24f}+\frac{7a(2A-B)c^4\cos(e+fx)\sin(e+fx)}{16f}-\frac{aBc\cos^3(e+fx)(c}{6f}$$

```
[Out] 7/16*a*(2*A-B)*c^4*x+7/24*a*(2*A-B)*c^4*cos(f*x+e)^3/f+7/16*a*(2*A-B)*c^4*c
os(f*x+e)*sin(f*x+e)/f-1/6*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^3/f+1/10*a*(
2*A-B)*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/40*a*(2*A-B)*cos(f*x+e)^3*(c
^4-c^4*sin(f*x+e))/f
```

Rubi [A]
time = 0.21, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\frac{7ac^4(2A-B)\cos^3(e+fx)}{24f}+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f}+\frac{7ac^4(2A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{7}{16}ac^4x(2A-B)+\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f}-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a*(2*A - B)*c^4*x)/16 + (7*a*(2*A - B)*c^4*Cos[e + f*x]^3)/(24*f) + (7*a
*(2*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a*B*c*Cos[e + f*x]^3*(c
 - c*Sin[e + f*x])^3)/(6*f) + (a*(2*A - B)*Cos[e + f*x]^3*(c^2 - c^2*Sin[e
+ f*x])^2)/(10*f) + (7*a*(2*A - B)*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))
/(40*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx)$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}$$

$$= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} - \frac{aBc\cos^3}{}$$

$$= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} + \frac{7a(2A-}{}$$

$$= \frac{7}{16}a(2A-B)c^4x + \frac{7a(2A-B)c^4\cos^3(}{24f}$$

Mathematica [A]
time = 0.64, size = 131, normalized size = 0.72

$$\frac{ac^4(840Afx-420Bfx+120(7A-5B)\cos(e+fx)+20(13A-7B)\cos(3(e+fx))-12A\cos(5(e+fx))+36B\cos(5(e+fx))+240A\sin(2(e+fx))+15B\sin(2(e+fx))-90A\sin(4(e+fx))+105B\sin(4(e+fx))-5B\sin(6(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,
x]
```

```
[Out] (a*c^4*(840*A*f*x - 420*B*f*x + 120*(7*A - 5*B)*Cos[e + f*x] + 20*(13*A - 7
*B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] + 36*B*Cos[5*(e + f*x)] + 240*
A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] - 90*A*Sin[4*(e + f*x)] + 105*B*
Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(170) = 340$.
time = 0.26, size = 342, normalized size = 1.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(-3*A*c^4*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-2/3*A*c^4*a*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^4*a*(-1/2*cos(f*x+e)*sin(f
*x+e)+1/2*f*x+1/2*e)+3*A*c^4*a*cos(f*x+e)+A*c^4*a*(f*x+e)+3/5*B*c^4*a*(8/3+
sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*B*c^4*a*(-1/4*(sin(f*x+e)^3+3/2
```

```
*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*B*c^4*a*(2+sin(f*x+e)^2)*cos(f*x
+e)-3*B*c^4*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*c^4*a*cos(f*x+e)
-1/5*A*c^4*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*c^4*a*(-1/6*(
sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [**A**]
time = 0.28, size = 363, normalized size = 1.99

$$-\frac{64\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))Aac^4-640\,(\cos(fx+e)^3-3\,\cos(fx+e))Aac^4+90\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Aac^4-480\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aac^4-960\,(fx+e)Aac^4-192\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))Bac^4-640\,(\cos(fx+e)^3-3\,\cos(fx+e))Bac^4-5\,(4\,\sin(2\,fx+2\,e)^3+60\,fx+60\,e+9\,\sin(4\,fx+4\,e)-48\,\sin(2\,fx+2\,e))Bac^4-60\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Bac^4+720\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac^4-2880\,Aac^4\cos(fx+e)+960\,Bac^4\cos(fx+e)}{960\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a*c^4
 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - sin(2*f*x +
 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a*c^4 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*
B*a*c^4 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2
*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^4 - 2880*
A*a*c^4*cos(f*x + e) + 960*B*a*c^4*cos(f*x + e))/f
```

Fricas [**A**]
time = 0.39, size = 129, normalized size = 0.71

$$-\frac{48\,(A-3\,B)ac^4\cos(fx+e)^5-320\,(A-B)ac^4\cos(fx+e)^3-105\,(2\,A-B)ac^4fx+5\,(8\,Bac^4\cos(fx+e)^5+2\,(18\,A-25\,B)ac^4\cos(fx+e)^3-21\,(2\,A-B)ac^4\cos(fx+e))\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] -1/240*(48*(A - 3*B)*a*c^4*cos(f*x + e)^5 - 320*(A - B)*a*c^4*cos(f*x + e)^
3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*cos(f*x + e)^5 + 2*(18*A - 25*B)
*a*c^4*cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(163) = 326$.
time = 0.57, size = 853, normalized size = 4.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)**
4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4*co
s(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos(e +
f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin(e + f
*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*cos(e +
 f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f*x)**6/1
6 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*x*sin(e +
 f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a*c**4*
x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)**2/2 + 5*B*
a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 - 3*B*a*c**4*x
*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 3*B*
a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)**3*cos(e +
f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)
/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a*c**4*sin(e + f*
x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 8*B
*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)**3/(3*f) - B*a*c**4
*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c
)**4, True))
```

Giac [**A**]
time = 0.47, size = 184, normalized size = 1.01

$$-\frac{Bac^4\sin(6fx+6e)}{192f}+\frac{7}{16}(2Aac^4-Bac^4)x-\frac{(Aac^4-3Bac^4)\cos(5fx+5e)}{80f}+\frac{(13Aac^4-7Bac^4)\cos(3fx+3e)}{48f}+\frac{(7Aac^4-5Bac^4)\cos(fx+e)}{8f}-\frac{(6Aac^4-7Bac^4)\sin(4fx+4e)}{64f}+\frac{(16Aac^4+Bac^4)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/192*B*a*c^4*sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A*
a*c^4 - 3*B*a*c^4)*cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*cos(3
*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*cos(f*x + e)/f - 1/64*(6*A*a*c^
4 - 7*B*a*c^4)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*sin(2*f*x +
 2*e)/f
```

Mupad [**B**]
time = 14.83, size = 454, normalized size = 2.49

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{Aac^4}{4}+\frac{7Bac^4}{8}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(6Aac^4-2Bac^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(12Aac^4-4Bac^4)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}\left(\frac{Aac^4}{4}+\frac{7Bac^4}{8}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(22Aac^4-18Bac^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(\frac{13Aac^4}{2}-\frac{37Bac^4}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(\frac{13Aac^4}{2}-\frac{37Bac^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{38Aac^4}{5}-\frac{34Bac^4}{5}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(\frac{68Aac^4}{3}-\frac{44Bac^4}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{27Aac^4}{4}-\frac{73Bac^4}{24}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(\frac{27Aac^4}{4}-\frac{73Bac^4}{24}\right)+\frac{34Aac^4}{15}-\frac{22Bac^4}{15}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+20\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}+\frac{7ac^4\,\mathrm{atan}\left(\frac{7ac^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2A-B)}{8\left(\frac{7Aac^4}{4}-\frac{7Bac^4}{8}\right)}\right)(2A-B)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^10*(
6*A*a*c^4 - 2*B*a*c^4) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^4 - 4*B*a*c^4) - ta
n(e/2 + (f*x)/2)^11*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^8*(2
2*A*a*c^4 - 18*B*a*c^4) + tan(e/2 + (f*x)/2)^5*((13*A*a*c^4)/2 - (37*B*a*c^
4)/4) - tan(e/2 + (f*x)/2)^7*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) + tan(e/2 +
(f*x)/2)^2*((38*A*a*c^4)/5 - (34*B*a*c^4)/5) + tan(e/2 + (f*x)/2)^6*((68*A*
a*c^4)/3 - (44*B*a*c^4)/3) + tan(e/2 + (f*x)/2)^3*((27*A*a*c^4)/4 - (73*B*a
*c^4)/24) - tan(e/2 + (f*x)/2)^9*((27*A*a*c^4)/4 - (73*B*a*c^4)/24) + (34*A
*a*c^4)/15 - (22*B*a*c^4)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*
x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (
f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (7*a*c^4*atan((7*a*c^4*tan(e/2 +
 (f*x)/2)*(2*A - B))/(8*((7*A*a*c^4)/4 - (7*B*a*c^4)/8)))*(2*A - B))/(8*f)
```

3.18 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=142

$$\frac{1}{8}a(5A-2B)c^3x+\frac{a(5A-2B)c^3\cos^3(e+fx)}{12f}+\frac{a(5A-2B)c^3\cos(e+fx)\sin(e+fx)}{8f}-\frac{aBc\cos^3(e+fx)(c}{5f}$$

```
[Out] 1/8*a*(5*A-2*B)*c^3*x+1/12*a*(5*A-2*B)*c^3*cos(f*x+e)^3/f+1/8*a*(5*A-2*B)*c
^3*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^2/f+1/20
*a*(5*A-2*B)*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A]
time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\frac{ac^3(5A-2B)\cos^3(e+fx)}{12f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}+\frac{ac^3(5A-2B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ac^3x(5A-2B)-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*(5*A - 2*B)*c^3*x)/8 + (a*(5*A - 2*B)*c^3*Cos[e + f*x]^3)/(12*f) + (a*(5
*A - 2*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(c -
 c*Sin[e + f*x])^2)/(5*f) + (a*(5*A - 2*B)*Cos[e + f*x]^3*(c^3 - c^3*Sin[e
+ f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(\\ &= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} + \\ &= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} + \\ &= \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} - \frac{aBc\cos^3(e \\ &= \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} + \frac{a(5A-2B \\ &= \frac{1}{8}a(5A-2B)c^3x + \frac{a(5A-2B)c^3\cos^3(e}{12f}\end{aligned}$$

Mathematica [A]

time = 0.57, size = 95, normalized size = 0.67

$$\frac{ac^3(60(4A-3B)\cos(e+fx)+10(8A-5B)\cos(3(e+fx))+6B\cos(5(e+fx))+15(4(5A-2B)fx+8A\sin(2(e+fx))-(A-2B)\sin(4(e+fx))))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,
x]
```

```
[Out] (a*c^3*(60*(4*A - 3*B)*Cos[e + f*x] + 10*(8*A - 5*B)*Cos[3*(e + f*x)] + 6*B
*Cos[5*(e + f*x)] + 15*(4*(5*A - 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A - 2*B
)*Sin[4*(e + f*x)])))/(480*f)
```

Maple [A]
time = 0.18, size = 208, normalized size = 1.46

method	result
risch	$\frac{5a\,c^3xA}{8}-\frac{a\,c^3xB}{4}+\frac{a\,c^3\cos(fx+e)A}{2f}-\frac{3a\,c^3\cos(fx+e)B}{8f}+\frac{Ba\,c^3\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)A\,c^3a}{32f}+\sin($
derivativedivides	$\frac{-\frac{2A\,c^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A\,c^3a\cos(fx+e)+A\,c^3a(fx+e)+2B\,c^3a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3}{\ldots}\right.}{}$
default	$\frac{-\frac{2A\,c^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A\,c^3a\cos(fx+e)+A\,c^3a(fx+e)+2B\,c^3a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3}{\ldots}\right.}{}$
norman	$\frac{(\frac{5}{8}A\,c^3a-\frac{1}{4}B\,c^3a)x+(\frac{5}{8}A\,c^3a-\frac{1}{4}B\,c^3a)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,c^3a-\frac{5}{2}B\,c^3a)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,c^3a-\frac{5}{2}B\,c^3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(-2/3*A*c^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^3*a*cos(f*x+e)+A*c^3*a*
(f*x+e)+2*B*c^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e)-2*B*c^3*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*c^3*a*cos(f*x+e
)-A*c^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/5
*B*c^3*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A]
time = 0.28, size = 216, normalized size = 1.52

$$\frac{320\left(\cos(fx+e)^3-3\cos(fx+e)\right)Aac^3-15\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)Aac^3+480(fx+e)Aac^3+32\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)Bac^3+30\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)Bac^3-240\left(2fx+2e-\sin(2fx+2e)\right)Bac^3+960Aac^3\cos(fx+e)-480Bac^3\cos(fx+e)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 + 32
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*x +
2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*co
s(f*x + e))/f
```

Fricas [A]
time = 0.38, size = 107, normalized size = 0.75

$$\frac{24\,Bac^3\cos\left(fx+e\right)^5+80\,(A-B)ac^3\cos\left(fx+e\right)^3+15\,(5\,A-2\,B)ac^3fx-15\,\left(2\,(A-2\,B)ac^3\cos\left(fx+e\right)^3-(5\,A-2\,B)ac^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5*
A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a*c
^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(129) = 258$.
time = 0.36, size = 486, normalized size = 3.42

$$\begin{cases} -\frac{3Aac^3x\sin^4(e+fx)}{8}-\frac{3Aac^3x\sin^2(e+fx)\cos^2(e+fx)}{4}-\frac{3Aac^3x\cos^4(e+fx)}{8}+Aac^3x+\frac{5Aac^3\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{2Aac^3\sin^2(e+fx)\cos(e+fx)}{f}+\frac{3Aac^3\sin(e+fx)\cos^3(e+fx)}{8f}-\frac{4Aac^3\cos^3(e+fx)}{3f}+\frac{2Aac^3\cos(e+fx)}{f}+\frac{3Bac^3x\sin^4(e+fx)}{4}+\frac{3Bac^3x\sin^2(e+fx)\cos^2(e+fx)}{2}-Bac^3x\sin^2(e+fx)+\frac{3Bac^3x\cos^4(e+fx)}{4}-Bac^3x\cos^2(e+fx)+\frac{Bac^3\sin^4(e+fx)\cos(e+fx)}{f}-\frac{5Bac^3\sin^3(e+fx)\cos(e+fx)}{4f}+\frac{4Bac^3\sin^2(e+fx)\cos^3(e+fx)}{3f}-\frac{3Bac^3\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{Bac^3\sin(e+fx)\cos(e+fx)}{f}+\frac{8Bac^3\cos^5(e+fx)}{15f}-\frac{Bac^3\cos(e+fx)}{f} & \text{for}\ f\neq 0 \\ x(A+B\sin(e))(a\sin(e)+a)(-c\sin(e)+c)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e + f
*x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**4/4 +
 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e + f*x)**
2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*a*c**3*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**3*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))
```

Giac [A]
time = 0.42, size = 145, normalized size = 1.02

$$\frac{Bac^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{Aac^3\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(5\,Aac^3-2\,Bac^3\right)x+\frac{\left(8\,Aac^3-5\,Bac^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}+\frac{\left(4\,Aac^3-3\,Bac^3\right)\cos\left(fx+e\right)}{8\,f}-\frac{\left(Aac^3-2\,Bac^3\right)\sin\left(4\,fx+4\,e\right)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f + 1
/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*sin(
4*f*x + 4*e)/f
```

Mupad [B]
time = 13.80, size = 389, normalized size = 2.74

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aac^3}{4}+\frac{Bac^3}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(4Aac^3-2Bac^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{7Aac^3}{2}-3Bac^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(\frac{7Aac^3}{2}-3Bac^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(\frac{3Aac^3}{4}+\frac{Bac^3}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(8Aac^3-8Bac^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{8Aac^3}{3}-\frac{8Bac^3}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{16Aac^3}{3}-\frac{4Bac^3}{3}\right)+\frac{4Aac^3}{3}-\frac{14Bac^3}{15}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}+\frac{ac^3\,\mathrm{atan}\left(\frac{ac^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(5A-2B)}{4\left(\frac{5Aac^3}{4}-\frac{Bac^3}{2}\right)}\right)(5A-2B)}{4f}-\frac{ac^3(5A-2B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^8*(4
*A*a*c^3 - 2*B*a*c^3) + tan(e/2 + (f*x)/2)^3*((7*A*a*c^3)/2 - 3*B*a*c^3) -
tan(e/2 + (f*x)/2)^7*((7*A*a*c^3)/2 - 3*B*a*c^3) - tan(e/2 + (f*x)/2)^9*((3
*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 - 8*B*a*c^3) +
 tan(e/2 + (f*x)/2)^2*((8*A*a*c^3)/3 - (8*B*a*c^3)/3) + tan(e/2 + (f*x)/2)^
4*((16*A*a*c^3)/3 - (4*B*a*c^3)/3) + (4*A*a*c^3)/3 - (14*B*a*c^3)/15)/(f*(5
*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 +
 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) + (a*c^3*atan((a*c^3*
tan(e/2 + (f*x)/2)*(5*A - 2*B))/(4*((5*A*a*c^3)/4 - (B*a*c^3)/2)))*(5*A - 2
*B))/(4*f) - (a*c^3*(5*A - 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.19 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=97

$$\frac{1}{8}a(4A-B)c^2x+\frac{a(A-B)c^2\cos^3(e+fx)}{3f}+\frac{a(4A-B)c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{aBc^2\cos^3(e+fx)\sin(e+$$

```
[Out] 1/8*a*(4*A-B)*c^2*x+1/3*a*(A-B)*c^2*cos(f*x+e)^3/f+1/8*a*(4*A-B)*c^2*cos(f*
x+e)*sin(f*x+e)/f+1/4*a*B*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = $\{3046, 2939, 2748, 2715, 8\}$

$$\frac{ac^2(4A-B)\cos^3(e+fx)}{12f}+\frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ac^2x(4A-B)-\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A - B)*c^2*x)/8 + (a*(4*A - B)*c^2*Cos[e + f*x]^3)/(12*f) + (a*(4*A -
 B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*Cos[e + f*x]^3*(c^2 - c^2*S
in[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))$$

$$= -\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}$$

$$= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} - \frac{aB\cos^3(e}{}$$

$$= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} + \frac{a(4A-B}{}$$

$$= \frac{1}{8}a(4A-B)c^2x + \frac{a(4A-B)c^2\cos^3(e-}{12f}$$

Mathematica [A]
time = 0.42, size = 74, normalized size = 0.76

$$\frac{ac^2(24(A-B)\cos(e+fx)+8(A-B)\cos(3(e+fx))+3(16Afx-4Bfx+8A\sin(2(e+fx))+B\sin(4(e+fx))))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (a*c^2*(24*(A - B)*Cos[e + f*x] + 8*(A - B)*Cos[3*(e + f*x)] + 3*(16*A*f*x
- 4*B*f*x + 8*A*Sin[2*(e + f*x)] + B*Sin[4*(e + f*x)])))/(96*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(89) = 178$.
time = 0.15, size = 185, normalized size = 1.91

method	result
risch	$\frac{a\,c^2 x A}{2} - \frac{a\,c^2 x B}{8} + \frac{a\,c^2 \cos(fx+e) A}{4f} - \frac{a\,c^2 \cos(fx+e) B}{4f} + \frac{B\,c^2 a \sin(4fx+4e)}{32f} + \frac{a\,c^2 \cos(3fx+3e) A}{12f} - \frac{a\,c^2 \cos(}{}$
derivativedivides	$\frac{-A\,c^2 a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\,c^2 a\cos(fx+e)+A\,c^2 a(fx+e)+\frac{B\,c^2 a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-B\,c^2 a\left(-\frac{\cos(f}{}\right.}{}$
default	$\frac{-A\,c^2 a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\,c^2 a\cos(fx+e)+A\,c^2 a(fx+e)+\frac{B\,c^2 a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-B\,c^2 a\left(-\frac{\cos(f}{}\right.}{}$
norman	$\frac{(\frac{1}{2}A\,c^2 a-\frac{1}{8}B\,c^2 a)x+(2A\,c^2 a-\frac{1}{2}B\,c^2 a)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2A\,c^2 a-\frac{1}{2}B\,c^2 a)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3A\,c^2 a-\frac{3}{4}B\,c^2 a)x\left(\mathrm{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(-A*c^2*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*cos(f*x+e)
+A*c^2*a*(f*x+e)+1/3*B*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)-B*c^2*a*(-1/2*cos(
f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*c^2*a*cos(f*x+e)-1/3*A*c^2*a*(2+sin(f*x+
e)^2)*cos(f*x+e)+B*c^2*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(94) = 188$.
time = 0.36, size = 193, normalized size = 1.99

$$\frac{32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aac^2-24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aac^2+96\left(fx+e\right)Aac^2-32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Bac^2+3\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\sin\left(2\,fx+2\,e\right)\right)Bac^2-24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Bac^2+96\,Aac^2\cos\left(fx+e\right)-96\,Bac^2\cos\left(fx+e\right)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin(
2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos(f
*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*cos(f*
x + e) - 96*B*a*c^2*cos(f*x + e))/f
```

Fricas [A]
time = 0.36, size = 86, normalized size = 0.89

$$\frac{8\,(A-B)ac^2\cos\left(fx+e\right)^3+3\,(4\,A-B)ac^2 fx+3\left(2\,Bac^2\cos\left(fx+e\right)^3+(4\,A-B)ac^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^2
*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(85) = 170$.
time = 0.24, size = 396, normalized size = 4.08

$$\begin{cases} - \frac{Aac^2x\sin^2{(e+fx)}}{2} - \frac{Aac^2x\cos^2{(e+fx)}}{2} + Aac^2x - \frac{Aac^2\sin^2{(e+fx)}\cos{(e+fx)}}{f} + \frac{Aac^2\sin{(e+fx)}\cos{(e+fx)}}{2f} - \frac{2Aac^2\cos^3{(e+fx)}}{3f} + \frac{Aac^2\cos{(e+fx)}}{f} + \frac{3Bac^2x\sin^4{(e+fx)}}{8} + \frac{3Bac^2x\sin^2{(e+fx)}\cos^2{(e+fx)}}{4} - \frac{Bac^2x\sin^2{(e+fx)}}{2} + \frac{3Bac^2x\cos^4{(e+fx)}}{8} - \frac{Bac^2x\cos^2{(e+fx)}}{2} - \frac{5Bac^2\sin^3{(e+fx)}\cos{(e+fx)}}{8f} + \frac{Bac^2\sin^2{(e+fx)}\cos{(e+fx)}}{f} - \frac{3Bac^2\sin{(e+fx)}\cos^3{(e+fx)}}{8f} + \frac{Bac^2\sin{(e+fx)}\cos{(e+fx)}}{2f} + \frac{2Bac^2\cos^3{(e+fx)}}{3f} - \frac{Bac^2\cos{(e+fx)}}{f} & \text{for}\ f \neq 0 \\ x(A + B\sin{(e)})\left(a\sin{(e)} + a\right)\left(-c\sin{(e)} + c\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 + A
*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*x)
*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e + f
*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x)**4/
8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e + f*x)*c
os(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a*c**
2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

Giac [A]
time = 0.41, size = 114, normalized size = 1.18

$$\frac{Bac^2\sin(4\,fx + 4\,e)}{32\,f} + \frac{Aac^2\sin(2\,fx + 2\,e)}{4\,f} + \frac{1}{8}\left(4\,Aac^2 - Bac^2\right)x + \frac{\left(Aac^2 - Bac^2\right)\cos(3\,fx + 3\,e)}{12\,f} + \frac{\left(Aac^2 - Bac^2\right)\cos(fx + e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*A
*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(A*
a*c^2 - B*a*c^2)*cos(f*x + e)/f
```

Mupad [B]
time = 13.38, size = 345, normalized size = 3.56

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a\,c^2+\frac{B\,a\,c^2}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,A\,a\,c^2-2\,B\,a\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(2\,A\,a\,c^2-2\,B\,a\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{2\,A\,a\,c^2}{3}-\frac{2\,B\,a\,c^2}{3}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(A\,a\,c^2+\frac{B\,a\,c^2}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(A\,a\,c^2-\frac{7\,B\,a\,c^2}{4}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(A\,a\,c^2-\frac{7\,B\,a\,c^2}{4}\right)+\frac{2\,A\,a\,c^2}{3}-\frac{2\,B\,a\,c^2}{3}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{a\,c^2\,\mathrm{atan}\left(\frac{a\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,A-B\right)}{4\,\left(A\,a\,c^2-\frac{B\,a\,c^2}{4}\right)}\right)\,\left(4\,A-B\right)}{4\,f}-\frac{a\,c^2\,\left(4\,A-B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^4*(2*A*a*c
^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2 +
(f*x)/2)^2*((2*A*a*c^2)/3 - (2*B*a*c^2)/3) - tan(e/2 + (f*x)/2)^7*(A*a*c^2
+ (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^3*(A*a*c^2 - (7*B*a*c^2)/4) - tan(e/2 +
 (f*x)/2)^5*(A*a*c^2 - (7*B*a*c^2)/4) + (2*A*a*c^2)/3 - (2*B*a*c^2)/3)/(f*(
4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 +
tan(e/2 + (f*x)/2)^8 + 1)) + (a*c^2*atan((a*c^2*tan(e/2 + (f*x)/2)*(4*A - B
))/(4*(A*a*c^2 - (B*a*c^2)/4)))*(4*A - B))/(4*f) - (a*c^2*(4*A - B)*(atan(t
an(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.20 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*A*c*x-1/3*a*B*c*cos(f*x+e)^3/f+1/2*a*A*c*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3046, 2748, 2715, 8}

$$\frac{aAc\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*A*c*x)/2 - (a*B*c*Cos[e + f*x]^3)/(3*f) + (a*A*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))\,d$$
$$= -\frac{aBc\cos^3(e+fx)}{3f} + (aAc)\int \cos^2(e+f$$
$$= -\frac{aBc\cos^3(e+fx)}{3f} + \frac{aAc\cos(e+fx)\sin}{2f}$$
$$= \frac{1}{2}aAcx - \frac{aBc\cos^3(e+fx)}{3f} + \frac{aAc\cos(e+}{}$$

Mathematica [A]
time = 0.11, size = 48, normalized size = 0.98

$$-\frac{ac(3B\cos(e+fx)+B\cos(3(e+fx))-3A(-2e+2fx+\sin(2(e+fx))))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
[Out] -1/12*(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Sin
[2*(e + f*x)])))/f
```

Maple [A]
time = 0.09, size = 74, normalized size = 1.51

method	result
risch	$\frac{aAcx}{2}-\frac{Bac\cos(fx+e)}{4f}-\frac{Bac\cos(3fx+3e)}{12f}+\frac{Aac\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{\frac{Bac\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
default	$\frac{\frac{Bac\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
norman	$\frac{\frac{Aac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2Bac}{3f}-\frac{2Bac\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{aAcx}{2}-\frac{Aac\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3aAcx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3aAcx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f*(1/3*B*a*c*(2+sin(f*x+e)^2)*cos(f*x+e)-A*a*c*(-1/2*cos(f*x+e)*sin(f*x+e
)+1/2*f*x+1/2*e)-B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

Maxima [A]
time = 0.27, size = 79, normalized size = 1.61

$$-\frac{3\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))Aac-12\,(fx+e)Aac+4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)Bac+12\,Bac\cos\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(c
os(f*x + e)^3 - 3*cos(f*x + e))*B*a*c + 12*B*a*c*cos(f*x + e))/f
```

Fricas [A]
time = 0.37, size = 46, normalized size = 0.94

$$-\frac{2\,Bac\cos\left(fx+e\right)^{3}-3\,Aacfx-3\,Aac\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x +
 e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(46) = 92$.
time = 0.13, size = 138, normalized size = 2.82

$$\begin{cases} -\frac{Aacx\sin^{2}\left(e+fx\right)}{2}-\frac{Aacx\cos^{2}\left(e+fx\right)}{2}+Aacx+\frac{Aac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{Bac\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{2Bac\cos^{3}\left(e+fx\right)}{3f}-\frac{Bac\cos\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x(A+B\sin\left(e\right))\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*x
 + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e + f*
```

```
x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))
```

Giac [A]
time = 0.41, size = 58, normalized size = 1.18

$$\frac{1}{2} Aacx - \frac{Bac\cos(3fx+3e)}{12f} - \frac{Bac\cos(fx+e)}{4f} + \frac{Aac\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1/
4*A*a*c*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.33, size = 122, normalized size = 2.49

$$\frac{A\,a\,c\,x}{2} - \frac{A\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5 + \left(\frac{a\,c\,(12\,B-9\,A\,(e+f\,x))}{6} + \frac{3\,A\,a\,c\,(e+f\,x)}{2}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - A\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) + \frac{a\,c\,(4\,B-3\,A\,(e+f\,x))}{6} + \frac{A\,a\,c\,(e+f\,x)}{2}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

```
[Out] (A*a*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((a*c*(12*B - 9*A*(e + f*x)))/6 + (3*A*
a*c*(e + f*x))/2) + (a*c*(4*B - 3*A*(e + f*x)))/6 - A*a*c*tan(e/2 + (f*x)/2
) + (A*a*c*(e + f*x))/2 + A*a*c*tan(e/2 + (f*x)/2)^5)/(f*(tan(e/2 + (f*x)/2
)^2 + 1)^3)
```

3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{a(A+2B)x}{c}+\frac{aB\cos(e+fx)}{cf}+\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}$$

```
[Out] -a*(A+2*B)*x/c+a*B*cos(f*x+e)/c/f+2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3046, 2936, 2718}

$$\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax(A+2B)}{c}+\frac{aB\cos(e+fx)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((a*(A + 2*B)*x)/c) + (a*B*Cos[e + f*x])/(c*f) + (2*a*(A + B)*Cos[e + f*x]
)/(f*(c - c*Sin[e + f*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} + \frac{a\int(-Ac-2Bc-Bc\sin(e+fx)}{c^2}$$
$$= -\frac{a(A+2B)x}{c} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} - \frac{(aB)\int \sin(e+}{c}$$
$$= -\frac{a(A+2B)x}{c} + \frac{aB\cos(e+fx)}{cf} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.
time = 0.63, size = 125, normalized size = 2.23

$$\frac{a\left(-((A+2B)x)+\frac{B\cos(e)\cos(fx)}{f}-\frac{B\sin(e)\sin(fx)}{f}+\frac{4(A+B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(1+\sin(e+fx))}{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a*(-((A + 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A
+ B)*Sin[(f*x)/2])/(f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])))*(1 + Sin[e + f*x]))/(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A]
time = 0.23, size = 67, normalized size = 1.20

method	result
derivativedivides	$\frac{2a\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{fc}$
default	$\frac{2a\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{fc}$
risch	$-\frac{axA}{c}-\frac{2axB}{c}+\frac{Ba\,e^{i(fx+e)}}{2cf}+\frac{Ba\,e^{-i(fx+e)}}{2cf}+\frac{4aA}{fc\left(e^{i(fx+e)}-i\right)}+\frac{4aB}{fc\left(e^{i(fx+e)}-i\right)}$
norman	$\frac{\frac{a(A+2B)x}{c}+\frac{a(A+2B)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4aA+4aB}{cf}-\frac{2aB\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2aB\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{(4aA+2aB)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-}{\left(1+\tan^2\left(\frac{fx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*a/c*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A+2*B)*arctan(tan(1/2*f*x+1/2*e))-(2*A
+2*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(60) = 120$.
time = 0.53, size = 287, normalized size = 5.12

$$-\frac{2\left(Ba\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+Aa\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)+Ba\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{Aa}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c -
 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(60) = 120$.
time = 0.39, size = 122, normalized size = 2.18

$$-\frac{(A+2B)afx - Ba\cos(fx+e)^2 - 2(A+B)a + ((A+2B)afx - (2A+3B)a)\cos(fx+e) - ((A+2B)afx - Ba\cos(fx+e) + 2(A+B)a)\sin(fx+e)}{cf\cos(fx+e) - cf\sin(fx+e) + cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x - (
2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A + B
)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(48) = 96$.
time = 1.21, size = 828, normalized size = 14.79

$$\begin{cases} -\frac{Aafx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{Aafx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{Aafx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{Aafx}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{4Aa\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{4Aa}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{2Bafx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{2Bafx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{2Bafx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{2Bafx}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{4Ba\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{2Ba\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{6Ba}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))(a\sin(e)+a)}{-c\sin(e)+c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(
e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)**2
/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
- c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 +
f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2)**3
- c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e/2 + f
*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
 c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 +
 f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*
f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2
+ c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f
*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*f*
x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
 - c*f) - 4*B*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2
+ f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*tan(e/2 + f*x/2)/(c*f*tan
(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) -
6*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*
x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c), T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(60) = 120.
time = 0.41, size = 124, normalized size = 2.21

$$-\frac{\frac{(Aa+2\,Ba)(fx+e)}{c}+\frac{2\left(2\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Aa+3\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(1
/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f*x
 + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f
```

Mupad [B]
time = 12.64, size = 111, normalized size = 1.98

$$\frac{\left(4\,A\,a+4\,B\,a\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,a+6\,B\,a}{f\,\left(-c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}-\frac{A\,a\,f\,x+2\,B\,a\,f\,x}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x)),x)
```

```
[Out] (4*A*a + 6*B*a + tan(e/2 + (f*x)/2)^2*(4*A*a + 4*B*a) - 2*B*a*tan(e/2 + (f*
x)/2))/(f*(c - c*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2 - c*tan(e/2 +
(f*x)/2)^3)) - (A*a*f*x + 2*B*a*f*x)/(c*f)
```

3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$\frac{aBx}{c^2}-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] a*B*x/c^2-1/3*a*(A+7*B)*cos(f*x+e)/c^2/f/(1-sin(f*x+e))+2/3*a*(A+B)*cos(f*x
+e)/f/(c-c*sin(f*x+e))^2
```

Rubi [A]
time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3046, 2936, 2814, 2727}

$$-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{aBx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/c^2 - (a*(A + 7*B)*Cos[e + f*x])/(3*c^2*f*(1 - Sin[e + f*x])) + (2*
a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{a\int \frac{-Ac-4Bc-3Bc\sin(e+fx)}{c-c\sin(e+fx)}\,dx}{3c^2} \\
&= \frac{aBx}{c^2}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}-\frac{(a(A+7B))\int \frac{1}{c-c\sin(e+f}}{3c} \\
&= \frac{aBx}{c^2}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}-\frac{a(A+7B)\cos(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.
time = 0.43, size = 160, normalized size = 2.22

$$
-\frac{a\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A+3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)+14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e+\frac{3fx}{2}\right)+24B\sin\left(\frac{fx}{2}\right)+9Bfx\sin\left(e+\frac{fx}{2}\right)+3Bfx\sin\left(e+\frac{3fx}{2}\right)\right)}{6c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] -1/6*(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e +
 (3*f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B
*Sin[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/(c^
2*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

Maple [**A**]
time = 0.27, size = 87, normalized size = 1.21

method	result
risch	$\frac{aBx}{c^2}-\frac{2\left(3Aa\,e^{2i(fx+e)}-12iBa\,e^{i(fx+e)}+9Ba\,e^{2i(fx+e)}-aA-7aB\right)}{3\left(e^{i(fx+e)}-i\right)^3 f\,c^2}$

derivativedivides	$\frac{2a\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}\right)}{f\,c^{2}}$
default	$\frac{2a\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}\right)}{f\,c^{2}}$
norman	$\frac{axB\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{2aA-10aB}{3cf}-\frac{axB}{c}-\frac{16aB\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8aB\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8aB\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf}-\frac{(2aA-2aB)\left(\tan^{6}\left(\frac{f}{2}\right.\right.}{cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a/c^2*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(4
*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(71) = 142$.
time = 0.51, size = 494, normalized size = 6.86

$$\frac{2\left(Ba\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-4}{c^{2}-\frac{3c^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{c^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{2}}\right)-\frac{Aa\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-2\right)}{c^{2}-\frac{3c^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{c^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{Aa\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{2}-\frac{3c^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{c^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{Ba\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{2}-\frac{3c^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{c^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a*(3
*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(71) = 142$.
time = 0.37, size = 170, normalized size = 2.36

$$-\frac{6\,Bafx-(3\,Bafx+(A+7\,B)a)\cos(fx+e)^{2}+2\,(A+B)a+(3\,Bafx+(A-5\,B)a)\cos(fx+e)-(6\,Bafx-2\,(A+B)a+(3\,Bafx-(A+7\,B)a)\cos(fx+e))\sin(fx+e)}{3\,(c^{2}f\cos(fx+e)^{2}-c^{2}f\cos(fx+e)-2\,c^{2}f+(c^{2}f\cos(fx+e)+2\,c^{2}f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a +
(3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*a*
f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*
f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(65) = 130$.
time = 2.44, size = 700, normalized size = 9.72

$$\begin{cases} -\frac{6Aa\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{2Aa}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} + \frac{3Bafx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{9Bafx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} + \frac{9Bafx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{3Bafx}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} + \frac{6Ba\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} - \frac{24Ba\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} + \frac{10Ba}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} & \text{for}\ f \neq 0 \\ \frac{x(A+B\sin(e))(a\sin(e)+a)}{(-c\sin(e)+c)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*A*a/(3*
c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/
2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 +
f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*B*a*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)
**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3*c**2*f*tan(e/2 +
 f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c
**2*f) + 6*B*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*B*a*tan(e
/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 +
9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 10*B*a/(3*c**2*f*tan(e/2 + f*x/2)**
3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), N
e(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))
```

Giac [A]
time = 0.44, size = 92, normalized size = 1.28

$$\frac{\frac{3\,(fx+e)Ba}{c^2} - \frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa-5\,Ba\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2*
f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2*f
*x + 1/2*e) - 1)^3))/f
```

Mupad [B]
time = 12.51, size = 132, normalized size = 1.83

$$\frac{B\,a\,x}{c^2}-\frac{\left(\frac{a\,\left(6\,A-6\,B+9\,B\,\left(e+f\,x\right)\right)}{3}-3\,B\,a\,\left(e+f\,x\right)\right)\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(\frac{a\,\left(24\,B-9\,B\,\left(e+f\,x\right)\right)}{3}+3\,B\,a\,\left(e+f\,x\right)\right)\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a\,\left(2\,A-10\,B+3\,B\,\left(e+f\,x\right)\right)}{3}-B\,a\,\left(e+f\,x\right)}{c^2\,f\,{\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (B*a*x)/c^2 - ((a*(2*A - 10*B + 3*B*(e + f*x)))/3 + tan(e/2 + (f*x)/2)^2*((
a*(6*A - 6*B + 9*B*(e + f*x)))/3 - 3*B*a*(e + f*x)) + tan(e/2 + (f*x)/2)*((
a*(24*B - 9*B*(e + f*x)))/3 + 3*B*a*(e + f*x)) - B*a*(e + f*x))/(c^2*f*(tan
(e/2 + (f*x)/2) - 1)^3)
```

3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3} dx$

Optimal. Leaf size=104

$$\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}-\frac{a(A+11B)c\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] 2/5*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^3-1/15*a*(A+11*B)*c*cos(f*x+e)/f/
(c^2-c^2*sin(f*x+e))^2-1/15*a*(A-4*B)*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3046, 2936, 2829, 2727}

$$-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}-\frac{ac(A+11B)\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) - (a*(A + 11*B)*c*C
os[e + f*x])/(15*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(A - 4*B)*Cos[e + f*x])
/(15*f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
```

```
b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} + \frac{a\int \frac{-Ac-6Bc-5Bc\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{5c^2}$$
$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2} - \frac{(a(A$$
$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2} - \frac{a(A}{15f(}$$

Mathematica [A]
time = 0.50, size = 147, normalized size = 1.41

$$\frac{a\left(15(A-B)\cos\left(e+\frac{fx}{2}\right)-5(A-B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)+25B\sin\left(\frac{fx}{2}\right)+15B\sin\left(2e+\frac{3fx}{2}\right)+A\sin\left(2e+\frac{5fx}{2}\right)-4B\sin\left(2e+\frac{5fx}{2}\right)\right)}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(f
*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*x
)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 0.34, size = 115, normalized size = 1.11

method	result

derivativedivides	$\frac{2a\left(-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}\right)}{f\,c^{3}}$
default	$\frac{2a\left(-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}\right)}{f\,c^{3}}$
risch	$\frac{-\frac{10Ba\,e^{2i(fx+e)}}{3}-2iBa\,e^{3i(fx+e)}+\frac{2iBa\,e^{i(fx+e)}}{3}+2Ba\,e^{4i(fx+e)}-\frac{2iAa\,e^{i(fx+e)}}{3}-\frac{2Aa\,e^{2i(fx+e)}}{3}-\frac{2aA}{15}+\frac{8aB}{15}+2iAa\,e^{3i}}{\left(e^{i(fx+e)}-i\right)^{5}f\,c^{3}}$
norman	$\frac{-\frac{8aA-2aB}{15cf}-\frac{2aA\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{10(aA-aB)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2(aA-aB)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(2aA-2aB)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}-\frac{2(11}{}}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}c^{2}\left(t\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a/c^3*(-1/4*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(6*A+2*B)/(tan(1/2
*f*x+1/2*e)-1)^2-1/5*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+1/2*
e)-1)-1/3*(14*A+10*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(107) = 214$.
time = 0.30, size = 801, normalized size = 7.70

$$-\frac{2\left(\frac{Aa\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-7\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{3Aa\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{3Ba\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{2Ba\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 3*A*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/
(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3
*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)
/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1
```

```
)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*a*(5*sin(f*x + e)/(cos
(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A]
time = 0.36, size = 195, normalized size = 1.88

$$-\frac{(A-4B)a\cos(fx+e)^3-(2A+7B)a\cos(fx+e)^2+3(A+B)a\cos(fx+e)+6(A+B)a+\left((A-4B)a\cos(fx+e)^2+3(A+B)a\cos(fx+e)+6(A+B)a\right)\sin(fx+e)}{15\left(c^3f\cos(fx+e)^3+3c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)-4c^3f-\left(c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)-4c^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A + B
)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*a*
cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*c
os(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*
c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(92) = 184$.
time = 5.36, size = 1035, normalized size = 9.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*
c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*ta
n(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan(e/
2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**
4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c*
*3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c**3*f*
tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 1
5*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a/(15*c**3*f
*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
 f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75
*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*t
```

```
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e
/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)*
*4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f*x/2)/(15*c**3*f*ta
n(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*
x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*
c**3*f) + 2*B*a/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e
) + a)/(-c*sin(e) + c)**3, True))
```

Giac [A]
time = 0.45, size = 139, normalized size = 1.34

$$-\frac{2 \left(15 \, A a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{4} - 15 \, A a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} + 15 \, B a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} + 25 \, A a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} + 5 \, B a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} - 5 \, A a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 5 \, B a \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 4 \, A a - B a\right)}{15 \, c^{3} f {\left(\tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) - 1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*a*tan(1/2*f*x + 1/2*e)^4 - 15*A*a*tan(1/2*f*x + 1/2*e)^3 + 15*B
*a*tan(1/2*f*x + 1/2*e)^3 + 25*A*a*tan(1/2*f*x + 1/2*e)^2 + 5*B*a*tan(1/2*f
*x + 1/2*e)^2 - 5*A*a*tan(1/2*f*x + 1/2*e) + 5*B*a*tan(1/2*f*x + 1/2*e) + 4
*A*a - B*a)/(c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

Mupad [B]
time = 12.98, size = 172, normalized size = 1.65

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{11\,A\,a\,\cos\left(e+f\,x\right)}{2}-\frac{B\,a}{4}-\frac{41\,A\,a}{4}+\frac{B\,a\,\cos\left(e+f\,x\right)}{2}+5\,A\,a\,\sin\left(e+f\,x\right)-5\,B\,a\,\sin\left(e+f\,x\right)+\frac{3\,A\,a\,\cos\left(2\,e+2\,f\,x\right)}{4}+\frac{3\,B\,a\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{5\,A\,a\,\sin\left(2\,e+2\,f\,x\right)}{4}+\frac{5\,B\,a\,\sin\left(2\,e+2\,f\,x\right)}{4}\right)}{15\,c^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi }{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi }{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi }{4}+\frac{5\,f\,x}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((11*A*a*cos(e + f*x))/2 - (B*a)/4 - (41*A*a)/4 + (B*
a*cos(e + f*x))/2 + 5*A*a*sin(e + f*x) - 5*B*a*sin(e + f*x) + (3*A*a*cos(2*
e + 2*f*x))/4 + (3*B*a*cos(2*e + 2*f*x))/4 - (5*A*a*sin(2*e + 2*f*x))/4 + (
5*B*a*sin(2*e + 2*f*x))/4))/(15*c^3*f*((5*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f
*x)/2))/4 - (5*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2
+ pi/4 + (5*f*x)/2))/4))
```

3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=142

$$\frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] 2/7*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^4-1/35*a*(A+15*B)*cos(f*x+e)/c/f/
(c-c*sin(f*x+e))^3-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^2-1/
105*a*(2*A-5*B)*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3046, 2936, 2829, 2729, 2727}

$$-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}+\frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) - (a*(A + 15*B)*Cos
[e + f*x])/(35*c*f*(c - c*Sin[e + f*x])^3) - (a*(2*A - 5*B)*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^
4 - c^4*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} + \frac{a\int \frac{-Ac-8Bc-7Bc\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{7c^2} \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{(a(2\cdots}{\cdots} \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2\cdots}{105\cdots} \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2\cdots}{105\cdots}
\end{aligned}$$

Mathematica [A]

time = 0.59, size = 174, normalized size = 1.23

$$\frac{a\left(35(4A-B)\cos\left(e+\frac{fx}{2}\right)-42A\cos\left(e+\frac{3fx}{2}\right)+2A\cos\left(3e+\frac{7fx}{2}\right)-5B\cos\left(3e+\frac{7fx}{2}\right)+70A\sin\left(\frac{fx}{2}\right)+140B\sin\left(\frac{fx}{2}\right)+105B\sin\left(2e+\frac{3fx}{2}\right)+14A\sin\left(2e+\frac{5fx}{2}\right)-35B\sin\left(2e+\frac{5fx}{2}\right)\right)}{420c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e +
 (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f*x
)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin[2*
e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^7)
```

Maple [A]
time = 0.40, size = 159, normalized size = 1.12

method	result
risch	$-\frac{2ia\left(140iA\,e^{4i(fx+e)}-35iB\,e^{4i(fx+e)}+105B\,e^{5i(fx+e)}-42iA\,e^{2i(fx+e)}-70A\,e^{3i(fx+e)}-140B\,e^{3i(fx+e)}+2iA-14A\,e^{i(\cdots}}{105f\,c^4\left(e^{i(fx+e)}-i\right)^7}$
derivativedivides	$\frac{2a\left(-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{28A+14B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}\right.}{f\,c^4}$
default	$\frac{2a\left(-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{28A+14B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}\right.}{f\,c^4}$
norman	$\frac{(4aA-2aB)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{46aA-10aB}{105cf}-\frac{2aA\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(16aA-10aB)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cf}+\frac{2(22aA-10aB)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a/c^4*(-1/7*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/4*(56*A+40*B)/(tan(1
/2*f*x+1/2*e)-1)^4-1/5*(68*A+60*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+
1/2*e)-1)-1/3*(28*A+14*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(8*A+2*B)/(tan(1/2*f
*x+1/2*e)-1)^2-1/6*(48*A+48*B)/(tan(1/2*f*x+1/2*e)-1)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(146) = 292$.
time = 0.33, size = 1176, normalized size = 8.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] 2/105*(A*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
```

```
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + B*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(
c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7) - 3*A*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos
(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*B*a*(14*sin(f*
x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7))/f
```

Fricas [A]
time = 0.36, size = 267, normalized size = 1.88

$$\frac{(2A-5B)a\cos(fx+e)^4+4(2A-5B)a\cos(fx+e)^3-3(3A+10B)a\cos(fx+e)^2+15(A+B)a\cos(fx+e)+30(A+B)a-\left((2A-5B)a\cos(fx+e)^3-3(2A-5B)a\cos(fx+e)^2-15(A+B)a\cos(fx+e)-30(A+B)a\right)\sin(fx+e)}{105\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+4c^4f\cos(fx+e)+8c^4f+\left(c^4f\cos(fx+e)^3+4c^4f\cos(fx+e)^2-4c^4f\cos(fx+e)-8c^4f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(3
*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a - ((
2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B)*a
*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f
*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f +
 (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*
c^4*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1831 vs. $2(124) = 248$.
time = 10.56, size = 1831, normalized size = 12.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*A*a*tan(e
/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 910*A*a*tan(e/2 + f*x/2)**4/(105*c**4*
f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/
2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x
/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 10
5*c**4*f) + 700*A*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 546*A*a*tan(e
/2 + f*x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 112*A*a*tan(e/2 + f*x/2)/(105*c**4*f*t
an(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
 f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)
**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c
**4*f) - 46*A*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/
2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c*
*4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 210*B*a*tan(e/2 + f*x/2)**5/(105*c**4
*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e
/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*
x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 1
05*c**4*f) + 70*B*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 280*B*a*tan(e
/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 70*B*a*tan(e/2 + f*x/2)/(105*c**4*f*ta
n(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)*
*3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c*
*4*f) + 10*B*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
```

```
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e)
+ a)/(-c*sin(e) + c)**4, True))
```

Giac [A]
time = 0.44, size = 187, normalized size = 1.32

$$-\frac{2\left(105\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+105\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-35\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-350\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+140\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+273\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-56\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+35\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+23\,Aa-5\,Ba\right)}{105\,c^{4}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -2/105*(105*A*a*tan(1/2*f*x + 1/2*e)^6 - 210*A*a*tan(1/2*f*x + 1/2*e)^5 + 1
05*B*a*tan(1/2*f*x + 1/2*e)^5 + 455*A*a*tan(1/2*f*x + 1/2*e)^4 - 35*B*a*tan
(1/2*f*x + 1/2*e)^4 - 350*A*a*tan(1/2*f*x + 1/2*e)^3 + 140*B*a*tan(1/2*f*x
+ 1/2*e)^3 + 273*A*a*tan(1/2*f*x + 1/2*e)^2 - 56*A*a*tan(1/2*f*x + 1/2*e) +
 35*B*a*tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

Mupad [B]
time = 13.22, size = 228, normalized size = 1.61

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{15\,B\,a}{4}-\frac{171\,A\,a}{2}+\frac{353\,A\,a\cos(e+f\,x)}{8}+\frac{5\,B\,a\cos(e+f\,x)}{4}+\frac{595\,A\,a\sin(e+f\,x)}{8}-35\,B\,a\sin\left(e+f\,x\right)+\frac{43\,A\,a\cos(2\,e+2\,f\,x)}{2}-\frac{25\,A\,a\cos(3\,e+3\,f\,x)}{8}-\frac{5\,B\,a\cos(2\,e+2\,f\,x)}{4}+\frac{5\,B\,a\cos(3\,e+3\,f\,x)}{4}-\frac{77\,A\,a\sin(2\,e+2\,f\,x)}{4}-\frac{21\,A\,a\sin(3\,e+3\,f\,x)}{8}+\frac{35\,B\,a\sin(2\,e+2\,f\,x)}{4}\right)}{105\,c^{4}\,f\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\frac{21\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{8}-\frac{7\,\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{8}+\frac{\sqrt{2}\,\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^4,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((15*B*a)/4 - (171*A*a)/2 + (353*A*a*cos(e + f*x))/8
 + (5*B*a*cos(e + f*x))/4 + (595*A*a*sin(e + f*x))/8 - 35*B*a*sin(e + f*x)
+ (43*A*a*cos(2*e + 2*f*x))/2 - (25*A*a*cos(3*e + 3*f*x))/8 - (5*B*a*cos(2*
e + 2*f*x))/4 + (5*B*a*cos(3*e + 3*f*x))/4 - (77*A*a*sin(2*e + 2*f*x))/4 -
(21*A*a*sin(3*e + 3*f*x))/8 + (35*B*a*sin(2*e + 2*f*x))/4))/(105*c^4*f*((35
*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3
*f*x)/2))/8 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos(
(7*e)/2 - pi/4 + (7*f*x)/2))/8))
```

3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=176

$$\frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5}-\frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4}-\frac{a(A-2B)c\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a(A-2B)c\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2}$$

```
[Out] 2/9*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^5-1/63*a*(A+19*B)*cos(f*x+e)/c/f/
(c-c*sin(f*x+e))^4-1/105*a*(A-2*B)*c*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^3-2/
315*a*(A-2*B)*c*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))^2-2/315*a*(A-2*B)*cos(f*x
+e)/f/(c^5-c^5*sin(f*x+e))
```

Rubi [A]
time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3046, 2936, 2829, 2729, 2727}

$$-\frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}-\frac{2ac(A-2B)\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{ac(A-2B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4}+\frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(9*f*(c - c*Sin[e + f*x])^5) - (a*(A + 19*B)*Cos
[e + f*x])/(63*c*f*(c - c*Sin[e + f*x])^4) - (a*(A - 2*B)*c*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a*(A - 2*B)*c*Cos[e + f*x])/(315*f*(
c^3 - c^3*Sin[e + f*x])^2) - (2*a*(A - 2*B)*Cos[e + f*x])/(315*f*(c^5 - c^5
*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} + \frac{a\int \frac{-Ac-10Bc-9Bc\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{9c^2}$$
$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{(a(A$$
$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(}{105c}$$
$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(}{105c}$$
$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(}{105c}$$

Mathematica [A]

time = 0.58, size = 200, normalized size = 1.14

$$\frac{a\left(315A\cos\left(e+\frac{fx}{2}\right)-42(2A+B)\cos\left(e+\frac{3fx}{2}\right)+9A\cos\left(3e+\frac{7fx}{2}\right)-18B\cos\left(3e+\frac{7fx}{2}\right)+189A\sin\left(\frac{fx}{2}\right)+252B\sin\left(\frac{fx}{2}\right)+210B\sin\left(2e+\frac{3fx}{2}\right)+36A\sin\left(2e+\frac{5fx}{2}\right)-72B\sin\left(2e+\frac{5fx}{2}\right)-A\sin\left(4e+\frac{9fx}{2}\right)+2B\sin\left(4e+\frac{9fx}{2}\right)\right)}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] (a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e
+ (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin[(
f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*Sin
[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))/(12
60*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

Maple [A]
time = 0.51, size = 203, normalized size = 1.15

method	result
risch	$-\frac{4\left(9iAa\,e^{i(fx+e)}-18iBa\,e^{i(fx+e)}-2aB+aA-36Aa\,e^{2i(fx+e)}+72Ba\,e^{2i(fx+e)}-84iAa\,e^{3i(fx+e)}-42iBa\,e^{3i(fx+e)}+315\right.}{315\left(e^{i(fx+e)}-i\right)^9 f\,c^5}$
derivativedivides	$\frac{2a\left(-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{248A+232B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{32A+32B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{236A+168B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^5}$
default	$\frac{2a\left(-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{248A+232B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{32A+32B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{236A+168B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^5}$
norman	$\frac{(34aA-10aB)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{116aA-22aB}{315cf}-\frac{2aA\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2(3aA-aB)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2(31aA-3aB)\left(\tan^{10}\left(\frac{fx}{2}+\right.\right.}{3cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a/c^5*(-1/2*(10*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/7*(248*A+232*B)/(tan(
1/2*f*x+1/2*e)-1)^7-1/9*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/4*(128*A+72*
B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(236*A+168*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(t
an(1/2*f*x+1/2*e)-1)-1/3*(46*A+18*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/6*(296*A+24
8*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/8*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)^8)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1553 vs. $2(181) = 362$.
time = 0.40, size = 1553, normalized size = 8.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="maxima")
```

```
[Out] -2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 5*A*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14
7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*
c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a*(45*sin(f*x + e)/(cos(f*x + e
) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x +
e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*
c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*B*a
*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^
5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [A]
time = 0.36, size = 325, normalized size = 1.85

$$-\frac{2\,(A-2\,B)a\cos\left(fx+e\right)^{5}-8\,(A-2\,B)a\cos\left(fx+e\right)^{4}-25\,(A-2\,B)a\cos\left(fx+e\right)^{3}+5\,(4\,A+13\,B)a\cos\left(fx+e\right)^{2}-35\,(A+B)a\cos\left(fx+e\right)-70\,(A+B)a+\left(2\,(A-2\,B)a\cos\left(fx+e\right)^{4}+10\,(A-2\,B)a\cos\left(fx+e\right)^{3}-15\,(A-2\,B)a\cos\left(fx+e\right)^{2}-35\,(A+B)a\cos\left(fx+e\right)-70\,(A+B)a\right)\sin\left(fx+e\right)}{315\,\left(c^{5}f\cos\left(fx+e\right)^{5}+5\,c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{3}-20\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f\cos\left(fx+e\right)+16\,c^{5}f-\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}-12\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f\cos\left(fx+e\right)+16\,c^{5}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="fricas")
```

```
[Out] -1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(A
 - 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*c
os(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B)*a
*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x + e)
 - 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)
^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e
) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*co
s(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3232 vs. $2(160) = 320$.
time = 20.04, size = 3232, normalized size = 18.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*A*a*tan
(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*
x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)*
*6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 +
26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*
c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*A*a*tan(e/2 + f*x/2)**6/(315*c
**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*
tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e
/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 +
f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2)
 - 315*c**5*f) + 6930*A*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)*
*9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 2
6460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*
c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*
f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 7686*A
*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f
*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)
**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 +
 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 4494*A*a*tan(e/2 + f*x/2)**3/
(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c
```

```
**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f
*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(
e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 +
f*x/2) - 315*c**5*f) - 2286*A*a*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f
*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)*
*7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 -
39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340
*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) +
414*A*a*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(
e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 +
 f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/
2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2
 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 116*A*a/(315*c**5*f*tan(e/2
 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x
/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**
5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 1
1340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f
) - 630*B*a*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5
*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*ta
n(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2
 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*
x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 630*B*a*tan(e/2 + f*
x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 +
 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 3969
0*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**
5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*ta
n(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan
(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 +
 f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/
2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3
 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c*
*5*f) + 882*B*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*
c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*
f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan
(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
+ f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1218*B*a*tan(e/2
 + f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) + 162*B*a*tan(e/2 + f*x/2)**2/(315*c**5*f
*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*...
```

Giac [A]

time = 0.46, size = 267, normalized size = 1.52

$$-\frac{2\left(315\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+315\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-315\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-3465\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+945\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3843\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-441\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2247\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+609\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+1143\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-81\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-207\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+99\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+58\,Aa-11\,Ba\right)}{315\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="giac")
```

```
[Out] -2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 + 3
15*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a*t
an(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/2*f
*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x + 1/2
*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)^3 +
1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207*A*a*t
an(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)/(c^5*f
*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B]
time = 13.34, size = 310, normalized size = 1.76

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{1357\,A\,a}{4}-\frac{461\,B\,a}{16}-\frac{635\,A\,a\,\cos(e+f\,x)}{4}+\frac{5\,B\,a\,\cos(e+f\,x)}{2}-\frac{1575\,A\,a\,\sin(e+f\,x)}{4}+\frac{945\,B\,a\,\sin(e+f\,x)}{8}-\frac{625\,A\,a\,\cos(2\,e+2\,f\,x)}{4}+\frac{121\,A\,a\,\cos(3\,e+3\,f\,x)}{4}+\frac{7\,A\,a\,\cos(4\,e+4\,f\,x)}{2}+\frac{95\,B\,a\,\cos(2\,e+2\,f\,x)}{4}-8\,B\,a\,\cos\left(3\,e+3\,f\,x\right)-\frac{7\,B\,a\,\cos(4\,e+4\,f\,x)}{16}+\frac{399\,A\,a\,\sin(2\,e+2\,f\,x)}{4}+\frac{141\,A\,a\,\sin(3\,e+3\,f\,x)}{4}-\frac{15\,A\,a\,\sin(4\,e+4\,f\,x)}{4}-\frac{231\,B\,a\,\sin(2\,e+2\,f\,x)}{8}-\frac{39\,B\,a\,\sin(3\,e+3\,f\,x)}{8}+\frac{15\,B\,a\,\sin(4\,e+4\,f\,x)}{16}\right)}{315\,c^5\,f\left(\frac{63\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\frac{21\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{9\,\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}+\frac{9\,\sqrt{2}\,\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{16}+\frac{\sqrt{2}\,\cos\left(\frac{9\,e}{2}+\frac{\pi}{4}+\frac{9\,f\,x}{2}\right)}{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((1357*A*a)/4 - (461*B*a)/16 - (635*A*a*cos(e + f*x))
/4 + (5*B*a*cos(e + f*x))/2 - (1575*A*a*sin(e + f*x))/4 + (945*B*a*sin(e +
f*x))/8 - (625*A*a*cos(2*e + 2*f*x))/4 + (121*A*a*cos(3*e + 3*f*x))/4 + (7*
A*a*cos(4*e + 4*f*x))/2 + (95*B*a*cos(2*e + 2*f*x))/4 - 8*B*a*cos(3*e + 3*f
*x) - (7*B*a*cos(4*e + 4*f*x))/16 + (399*A*a*sin(2*e + 2*f*x))/4 + (141*A*a
*sin(3*e + 3*f*x))/4 - (15*A*a*sin(4*e + 4*f*x))/4 - (231*B*a*sin(2*e + 2*f
*x))/8 - (39*B*a*sin(3*e + 3*f*x))/8 + (15*B*a*sin(4*e + 4*f*x))/16))/(315*
c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 -
 pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4 + (9*
2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2 + pi/4 +
 (9*f*x)/2))/16))
```

3.26 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=229

$$\frac{9}{128}a^2(8A-3B)c^5x+\frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f}+\frac{9a^2(8A-3B)c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{3a^2(8A-3B)}{}$$

```
[Out] 9/128*a^2*(8*A-3*B)*c^5*x+3/80*a^2*(8*A-3*B)*c^5*cos(f*x+e)^5/f+9/128*a^2*(
8*A-3*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+3/64*a^2*(8*A-3*B)*c^5*cos(f*x+e)^3*si
n(f*x+e)/f+1/56*a^2*(8*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^2/f-1/8*a^2
*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^3/f+3/112*a^2*(8*A-3*B)*cos(f*x+e)^5*(
c^5-c^5*sin(f*x+e))/f
```

Rubi [A]
time = 0.25, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\frac{3a^2c^5(8A-3B)\cos^5(e+fx)}{80f}+\frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f}+\frac{3a^2c^5(8A-3B)\sin(e+fx)\cos^3(e+fx)}{64f}+\frac{9a^2c^5(8A-3B)\sin(e+fx)\cos(e+fx)}{128f}+\frac{9}{128}a^2c^5x(8A-3B)+\frac{a^2c^3(8A-3B)\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f}-\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (9*a^2*(8*A - 3*B)*c^5*x)/128 + (3*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]^5)/(80*
f) + (9*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (3*a^2*(8*
A - 3*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64*f) + (a^2*(8*A - 3*B)*c^3*Cos
[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(56*f) - (a^2*B*c^2*Cos[e + f*x]^5*(c -
 c*Sin[e + f*x])^3)/(8*f) + (3*a^2*(8*A - 3*B)*Cos[e + f*x]^5*(c^5 - c^5*Si
n[e + f*x]))/(112*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+
$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx)}{8f}$$
$$= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(}{56f}$$
$$= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(}{56f}$$
$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{a^2(8A}{}$$
$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{3a^2(8}{}$$
$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{9a^2(8}{}$$
$$= \frac{9}{128}a^2(8A-3B)c^5x + \frac{3a^2(8A-3B)c}{80}$$

Mathematica [**A**]
time = 1.24, size = 219, normalized size = 0.96

$$\frac{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5(2520(8A-3B)(e+fx)+560(27A-17B)\cos(e+fx)+560(13A-7B)\cos(3(e+fx))+112(11A-B)\cos(5(e+fx))-80(A-3B)\cos(7(e+fx))+560(19A-3B)\sin(2(e+fx))-280(2A-7B)\sin(4(e+fx))-560(A-B)\sin(6(e+fx))-35B\sin(8(e+fx)))}{35840f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x)
+ 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 112*
(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A - 3
*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*Sin[6
*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(215) = 430$.
time = 0.36, size = 569, normalized size = 2.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^2*A*c^5*(f*x+e)-B*a^2*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*
sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+B*a^2*c^5*(8/3+sin(f*x+e)^4+4/3*sin
(f*x+e)^2)*cos(f*x+e)+5*B*a^2*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f
*x+e)+3/8*f*x+3/8*e)+3*a^2*A*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*
sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-3/7*B*a^2*c^5*(16/5+sin(f*x+e)^6+6/
5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+1/7*a^2*A*c^5*(16/5+sin(f*x+e)^
6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^5*(-1/8*(sin(f*x+e)
^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*
f*x+35/128*e)-B*a^2*c^5*cos(f*x+e)+3*a^2*A*c^5*cos(f*x+e)-3*B*a^2*c^5*(-1/2
*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c^5*(-1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-1/3*B*a^2*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)+1/5*a^2*A*c^5*(8/
3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-5*a^2*A*c^5*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-5/3*a^2*A*c^5*(2+sin(f*x+e)^2)*c
os(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(229) = 458$.
time = 0.31, size = 615, normalized size = 2.69

$$-\frac{3072\,(5\cos(fx+e)^7 - 21\cos(fx+e)^5 + 35\cos(fx+e)^3 - 35\cos(fx+e))Aa^2c^5 - 7168\,(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e))Aa^2c^5 - 179200\,(\cos(fx+e)^3 - 3\cos(fx+e))Aa^2c^5 - 1680\,(4\sin(2fx+2e)^3 + 60fx + 60e + 9\sin(4fx+4e) - 48\sin(2fx+2e))Aa^2c^5 + 16800\,(12fx + 12e + \sin(4fx+4e) - 8\sin(2fx+2e))Aa^2c^5 - 26880\,(2fx + 2e - \sin(2fx+2e))Aa^2c^5 - 107520\,(fx+e)Aa^2c^5 - 9216\,(5\cos(fx+e)^7 - 21\cos(fx+e)^5 + 35\cos(fx+e)^3 - 35\cos(fx+e))Ba^2c^5 - 35840\,(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e))Ba^2c^5 - 35840\,(\cos(fx+e)^3 - 3\cos(fx+e))Ba^2c^5 + 35\,(128\sin(2fx+2e)^3 + 840fx + 840e + 3\sin(8fx+8e) + 168\sin(4fx+4e) - 768\sin(2fx+2e))Ba^2c^5 + 560\,(4\sin(2fx+2e)^3 + 60fx + 60e + 9\sin(4fx+4e) - 48\sin(2fx+2e))Ba^2c^5 - 16800\,(12fx + 12e + \sin(4fx+4e) - 8\sin(2fx+2e))Ba^2c^5 + 80640\,(2fx + 2e - \sin(2fx+2e))Ba^2c^5 - 322560\,Aa^2c^5\cos(fx+e) + 107520\,Ba^2c^5\cos(fx+e)}{107520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2
*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*
sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*
c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f*x + e)
^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^2*
c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*s
in(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^
5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x + e))/f
```

Fricas [A]
time = 0.38, size = 165, normalized size = 0.72

$$-\frac{640\,(A-3\,B)a^2c^5\cos(fx+e)^7 - 3584\,(A-B)a^2c^5\cos(fx+e)^5 - 315\,(8\,A-3\,B)a^2c^5fx + 35\,(16\,Ba^2c^5\cos(fx+e)^7 + 8\,(8\,A-11\,B)a^2c^5\cos(fx+e)^5 - 6\,(8\,A-3\,B)a^2c^5\cos(fx+e)^3 - 9\,(8\,A-3\,B)a^2c^5\cos(fx+e))\sin(fx+e)}{4480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f*
x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7 +
8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x + e)^
3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(218) = 436$.

time = 1.17, size = 1586, normalized size = 6.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*x*co
s(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*cos(e
 + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f -
 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*sin(e +
 f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5
*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**5*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*A*a**2*c**
5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)
**5/(16*f) + 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*c**
5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*x)**7/(35*f) +
 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e + f*x)**3/(3*f
) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e + f*x)**8/128 - 3
5*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a**2*c**5*x*sin(e
+ f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 15*B*
a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a**2*c**5*x*sin(e + f
*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 15*B*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 15*B*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 - 3*B*a**2*c**5*x*sin(e + f*x)**2/2 - 35*B*a**2*c**5*x*
cos(e + f*x)**8/128 - 5*B*a**2*c**5*x*cos(e + f*x)**6/16 + 15*B*a**2*c**5*x
*cos(e + f*x)**4/8 - 3*B*a**2*c**5*x*cos(e + f*x)**2/2 + 93*B*a**2*c**5*sin
(e + f*x)**7*cos(e + f*x)/(128*f) - 3*B*a**2*c**5*sin(e + f*x)**6*cos(e + f
*x)/f + 511*B*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 11*B*a**2
*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 6*B*a**2*c**5*sin(e + f*x)**4*c
os(e + f*x)**3/f + 5*B*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*B*a**
2*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**2*c**5*sin(e + f*x)
```

```
**3*cos(e + f*x)**3/(6*f) - 25*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*
f) - 24*B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 20*B*a**2*c**5*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*c**5*sin(e + f*x)**2*cos(e +
 f*x)/f + 35*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 5*B*a**2*c*
*5*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 15*B*a**2*c**5*sin(e + f*x)*cos(e
+ f*x)**3/(8*f) + 3*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) - 48*B*a**2
*c**5*cos(e + f*x)**7/(35*f) + 8*B*a**2*c**5*cos(e + f*x)**5/(3*f) - 2*B*a*
*2*c**5*cos(e + f*x)**3/(3*f) - B*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**5, True))
```

Giac [A]
time = 0.49, size = 278, normalized size = 1.21

$$-\frac{Ba^2c^5\sin(8\,fx+8\,e)}{1024\,f}+\frac{9}{128}\left(8\,Aa^2c^5-3\,Ba^2c^5\right)x-\frac{(Aa^2c^5-3\,Ba^2c^5)\cos(7\,fx+7\,e)}{448\,f}+\frac{(11\,Aa^2c^5-Ba^2c^5)\cos(5\,fx+5\,e)}{320\,f}+\frac{(13\,Aa^2c^5-7\,Ba^2c^5)\cos(3\,fx+3\,e)}{64\,f}+\frac{(27\,Aa^2c^5-17\,Ba^2c^5)\cos(fx+e)}{64\,f}-\frac{(Aa^2c^5-Ba^2c^5)\sin(6\,fx+6\,e)}{64\,f}-\frac{(2\,Aa^2c^5-7\,Ba^2c^5)\sin(4\,fx+4\,e)}{128\,f}+\frac{(19\,Aa^2c^5-3\,Ba^2c^5)\sin(2\,fx+2\,e)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x
- 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^5
- B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(3*f
*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*(A*a
^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*c^5)*
sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.13, size = 661, normalized size = 2.89

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}\left(6\,A\,a^2\,c^5-2\,B\,a^2\,c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(30\,A\,a^2\,c^5-10\,B\,a^2\,c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(22\,A\,a^2\,c^5-18\,B\,a^2\,c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}\left(46\,A\,a^2\,c^5-26\,B\,a^2\,c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\left(\frac{74\,A\,a^2\,c^5}{5}-\frac{14\,B\,a^2\,c^5}{5}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{15}\left(\frac{7\,A\,a^2\,c^5}{8}+\frac{27\,B\,a^2\,c^5}{64}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{158\,A\,a^2\,c^5}{35}-\frac{138\,B\,a^2\,c^5}{35}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}\left(\frac{218\,A\,a^2\,c^5}{5}-\frac{158\,B\,a^2\,c^5}{5}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\left(\frac{75\,A\,a^2\,c^5}{8}-\frac{305\,B\,a^2\,c^5}{64}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{13}\left(\frac{75\,A\,a^2\,c^5}{8}-\frac{305\,B\,a^2\,c^5}{64}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\left(\frac{55\,A\,a^2\,c^5}{8}-\frac{437\,B\,a^2\,c^5}{64}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}\left(\frac{55\,A\,a^2\,c^5}{8}-\frac{437\,B\,a^2\,c^5}{64}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}\left(\frac{13\,A\,a^2\,c^5}{8}-\frac{919\,B\,a^2\,c^5}{64}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}\left(\frac{13\,A\,a^2\,c^5}{8}-\frac{919\,B\,a^2\,c^5}{64}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{7\,A\,a^2\,c^5}{8}+\frac{27\,B\,a^2\,c^5}{64}\right)+\frac{46\,A\,a^2\,c^5}{35}-\frac{26\,B\,a^2\,c^5}{35}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}+8\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+28\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+56\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+70\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+56\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+28\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+8\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)}+\frac{9\,a^2\,c^5\,\mathrm{atan}\left(\frac{9\,a^2\,c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8\,A-3\,B)}{64\left(\frac{9\,A\,a^2\,c^5}{8}-\frac{27\,B\,a^2\,c^5}{64}\right)}\right)(8\,A-3\,B)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^14*(6*A*a^2*c^5 - 2*B*a^2*c^5) + tan(e/2 + (f*x)/2)^10*
(30*A*a^2*c^5 - 10*B*a^2*c^5) + tan(e/2 + (f*x)/2)^12*(22*A*a^2*c^5 - 18*B*
a^2*c^5) + tan(e/2 + (f*x)/2)^8*(46*A*a^2*c^5 - 26*B*a^2*c^5) + tan(e/2 + (
f*x)/2)^4*((74*A*a^2*c^5)/5 - (14*B*a^2*c^5)/5) - tan(e/2 + (f*x)/2)^15*((7
*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^2*((158*A*a^2*c^5)/
35 - (138*B*a^2*c^5)/35) + tan(e/2 + (f*x)/2)^6*((218*A*a^2*c^5)/5 - (158*B
*a^2*c^5)/5) + tan(e/2 + (f*x)/2)^3*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64)
 - tan(e/2 + (f*x)/2)^13*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) + tan(e/2
+ (f*x)/2)^5*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^1
1*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^7*((13*A*a^2
*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^9*((13*A*a^2*c^5)/8 - (9
19*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64
```

```
) + (46*A*a^2*c^5)/35 - (26*B*a^2*c^5)/35)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*
tan(e/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 +
56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^
14 + tan(e/2 + (f*x)/2)^16 + 1)) + (9*a^2*c^5*atan((9*a^2*c^5*tan(e/2 + (f*
x)/2)*(8*A - 3*B))/(64*((9*A*a^2*c^5)/8 - (27*B*a^2*c^5)/64)))*(8*A - 3*B))
/(64*f)
```

3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=189

$$\frac{1}{16}a^2(7A-2B)c^4x+\frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^2(7A-2B)c^4\text{co}}{}$$

```
[Out] 1/16*a^2*(7*A-2*B)*c^4*x+1/30*a^2*(7*A-2*B)*c^4*cos(f*x+e)^5/f+1/16*a^2*(7*
A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(7*A-2*B)*c^4*cos(f*x+e)^3*sin(
f*x+e)/f-1/7*a^2*B*cos(f*x+e)^5*(c^2-c^2*sin(f*x+e))^2/f+1/42*a^2*(7*A-2*B)
*cos(f*x+e)^5*(c^4-c^4*sin(f*x+e))/f
```

Rubi [A]
time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\frac{a^2c^4(7A-2B)\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2c^4x(7A-2B)-\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(7*A - 2*B)*c^4*x)/16 + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^5)/(30*f) +
(a^2*(7*A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(7*A - 2*B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^2 - c^2*S
in[e + f*x])^2)/(7*f) + (a^2*(7*A - 2*B)*Cos[e + f*x]^5*(c^4 - c^4*Sin[e +
f*x]))/(42*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^4\, dx = \left(a^2c^2\right) \int \cos^4(e + fx)(A + B\sin(e + fx$$

$$= -\frac{a^2B\cos^5(e + fx)\left(c^2 - c^2\sin(e + fx)\right)}{7f}$$

$$= -\frac{a^2B\cos^5(e + fx)\left(c^2 - c^2\sin(e + fx)\right)}{7f}$$

$$= \frac{a^2(7A - 2B)c^4\cos^5(e + fx)}{30f} - \frac{a^2B\cos^5}{}$$

$$= \frac{a^2(7A - 2B)c^4\cos^5(e + fx)}{30f} + \frac{a^2(7A -}{}$$

$$= \frac{a^2(7A - 2B)c^4\cos^5(e + fx)}{30f} + \frac{a^2(7A -}{}$$

$$= \frac{1}{16}a^2(7A - 2B)c^4x + \frac{a^2(7A - 2B)c^4\cos}{30f}$$

Mathematica [A]
time = 0.98, size = 163, normalized size = 0.86

$$\frac{a^2c^4(2940Ae - 840Be + 2940Afx - 840Bfx + 105(16A - 11B)\cos(e + fx) + 105(8A - 5B)\cos(3(e + fx)) + 168A\cos(5(e + fx)) - 63B\cos(5(e + fx)) + 15B\cos(7(e + fx)) + 1785A\sin(2(e + fx)) - 210B\sin(2(e + fx)) + 105A\sin(4(e + fx)) + 210B\sin(4(e + fx)) - 35A\sin(6(e + fx)) + 70B\sin(6(e + fx)))}{6720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*C
os[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] - 6
3*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] - 21
0*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)] - 35
*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)]))/(6720*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(177) = 354$.
time = 0.27, size = 463, normalized size = 2.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^2*A*c^4*(f*x+e)+a^2*A*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*
sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/7*B*a^2*c^4*(16/5+sin(f*x+e)^6+6/
5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^4*cos(f*x+e)+2*a^2*A*c^
```

```
4*cos(f*x+e)-2*B*a^2*c^4*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^2*A*c
^4*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/3*B*a^2*c^4*(2+sin(f*x+e)^2
)*cos(f*x+e)-a^2*A*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)-4/3*a^2*A*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+1/5*B*a^2*c^4*(8/3+sin(
f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+4*B*a^2*c^4*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/5*a^2*A*c^4*(8/3+sin(f*x+e)^4+4/3*si
n(f*x+e)^2)*cos(f*x+e)-2*B*a^2*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/
8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(188) = 376$.
time = 0.29, size = 495, normalized size = 2.62

$$\frac{896 \left(3 \cos \left(fx+e\right)^{5}-10 \cos \left(fx+e\right)^{3}+15 \cos \left(fx+e\right)\right) A a^{2} c^{4}+8960 \left(\cos \left(fx+e\right)^{3}-3 \cos \left(fx+e\right)\right) A a^{2} c^{4}+35 \left(4 \sin \left(2 fx+2 e\right)^{3}+60 fx+60 e+9 \sin \left(4 fx+4 e\right)-48 \sin \left(2 fx+2 e\right)\right) A a^{2} c^{4}-210 \left(12 fx+12 e+\sin \left(4 fx+4 e\right)-8 \sin \left(2 fx+2 e\right)\right) A a^{2} c^{4}-1680 \left(2 fx+2 e-\sin \left(2 fx+2 e\right)\right) A a^{2} c^{4}+6720 \left(fx+e\right) A a^{2} c^{4}+192 \left(5 \cos \left(fx+e\right)^{7}-21 \cos \left(fx+e\right)^{5}+35 \cos \left(fx+e\right)^{3}-35 \cos \left(fx+e\right)\right) B a^{2} c^{4}+448 \left(3 \cos \left(fx+e\right)^{5}-10 \cos \left(fx+e\right)^{3}+15 \cos \left(fx+e\right)\right) B a^{2} c^{4}-2240 \left(\cos \left(fx+e\right)^{3}-3 \cos \left(fx+e\right)\right) B a^{2} c^{4}-70 \left(4 \sin \left(2 fx+2 e\right)^{3}+60 fx+60 e+9 \sin \left(4 fx+4 e\right)-48 \sin \left(2 fx+2 e\right)\right) B a^{2} c^{4}+840 \left(12 fx+12 e+\sin \left(4 fx+4 e\right)-8 \sin \left(2 fx+2 e\right)\right) B a^{2} c^{4}-3360 \left(2 fx+2 e-\sin \left(2 fx+2 e\right)\right) B a^{2} c^{4}+13440 A a^{2} c^{4} \cos \left(fx+e\right)-6720 B a^{2} c^{4} \cos \left(fx+e\right)}{6720 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*
c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x +
2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2*c^
4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^4 -
 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^2*c^4
 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f
*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x
 + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4 - 70*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 + 13440*A*a^2
*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

Fricas [A]
time = 0.39, size = 141, normalized size = 0.75

$$\frac{240 \, B a^{2} c^{4} \cos \left(fx+e\right)^{7}+672 \, {\left(A-B\right)} a^{2} c^{4} \cos \left(fx+e\right)^{5}+105 \, {\left(7 \, A-2 \, B\right)} a^{2} c^{4} fx-35 \, {\left(8 \, {\left(A-2 \, B\right)} a^{2} c^{4} \cos \left(fx+e\right)^{5}-2 \, {\left(7 \, A-2 \, B\right)} a^{2} c^{4} \cos \left(fx+e\right)^{3}-3 \, {\left(7 \, A-2 \, B\right)} a^{2} c^{4} \cos \left(fx+e\right)\right)} \sin \left(fx+e\right)}{1680 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5 +
 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B)*a^2*c^4*cos(f*x + e)^5 - 2*(
7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. $2(172) = 344$.
time = 0.81, size = 1210, normalized size = 6.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*c*
*4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*cos(e
+ f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e + f*
x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f
) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8
*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a**2*c**4*s
in(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/
(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**4*sin
(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(15*f) - 8*A*
a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f - 5*B*a**2*c
**4*x*sin(e + f*x)**6/8 - 15*B*a**2*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/
8 + 3*B*a**2*c**4*x*sin(e + f*x)**4/2 - 15*B*a**2*c**4*x*sin(e + f*x)**2*co
s(e + f*x)**4/8 + 3*B*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2 - B*a**2*
c**4*x*sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8 + 3*B*a**2*c**4*
x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a**2*c**4*sin(e + f
*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(8*f) -
 2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B*a**2*c**4*sin(e + f*x)
**4*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) -
5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(2*f) - 8*B*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**5/(5*f) + 4*B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3
/(3*f) + B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e +
 f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(2
*f) + B*a**2*c**4*sin(e + f*x)*cos(e + f*x)/f - 16*B*a**2*c**4*cos(e + f*x)
**7/(35*f) + 8*B*a**2*c**4*cos(e + f*x)**5/(15*f) + 2*B*a**2*c**4*cos(e + f
*x)**3/(3*f) - B*a**2*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**4, True))
```

Giac [A]
time = 0.47, size = 244, normalized size = 1.29

$$\frac{Ba^2c^4\cos(7\,fx+7\,e)}{448\,f}+\frac{1}{16}\,(7\,Aa^2c^4-2\,Ba^2c^4)x+\frac{(8\,Aa^2c^4-3\,Ba^2c^4)\cos(5\,fx+5\,e)}{320\,f}+\frac{(8\,Aa^2c^4-5\,Ba^2c^4)\cos(3\,fx+3\,e)}{64\,f}+\frac{(16\,Aa^2c^4-11\,Ba^2c^4)\cos(fx+e)}{64\,f}-\frac{(Aa^2c^4-2\,Ba^2c^4)\sin(6\,fx+6\,e)}{192\,f}+\frac{(Aa^2c^4+2\,Ba^2c^4)\sin(4\,fx+4\,e)}{64\,f}+\frac{(17\,Aa^2c^4-2\,Ba^2c^4)\sin(2\,fx+2\,e)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x + 1
/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 - 5
*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(f*x
 + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*a^2*
c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c^4)*s
in(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.89, size = 553, normalized size = 2.93

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(4\,A\,a^2\,c^4-2\,B\,a^2\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}\left(12\,A\,a^2\,c^4-2\,B\,a^2\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(8\,A\,a^2\,c^4-8\,B\,a^2\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{8\,A\,a^2\,c^4}{5}-\frac{8\,B\,a^2\,c^4}{5}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{13}\left(\frac{9\,A\,a^2\,c^4}{8}+\frac{B\,a^2\,c^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}\left(16\,A\,a^2\,c^4-16\,B\,a^2\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\left(\frac{29\,A\,a^2\,c^4}{6}-\frac{11\,B\,a^2\,c^4}{3}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}\left(\frac{29\,A\,a^2\,c^4}{6}-\frac{11\,B\,a^2\,c^4}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\left(\frac{44\,A\,a^2\,c^4}{5}-\frac{14\,B\,a^2\,c^4}{5}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\left(\frac{23\,A\,a^2\,c^4}{24}+\frac{31\,B\,a^2\,c^4}{12}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}\left(\frac{23\,A\,a^2\,c^4}{24}+\frac{31\,B\,a^2\,c^4}{12}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{9\,A\,a^2\,c^4}{8}+\frac{B\,a^2\,c^4}{4}\right)+\frac{4\,A\,a^2\,c^4}{5}-\frac{18\,B\,a^2\,c^4}{35}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+35\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)}+\frac{a^2\,c^4\,\mathrm{atan}\left(\frac{a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(7\,A-2\,B\right)}{8\,\left(\frac{7\,A\,a^2\,c^4}{8}-\frac{B\,a^2\,c^4}{4}\right)}\right)\,\left(7\,A-2\,B\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^12*(4*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^8*(
12*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^10*(8*A*a^2*c^4 - 8*B*a^2*
c^4) + tan(e/2 + (f*x)/2)^2*((8*A*a^2*c^4)/5 - (8*B*a^2*c^4)/5) - tan(e/2 +
 (f*x)/2)^13*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + tan(e/2 + (f*x)/2)^6*(16*A
*a^2*c^4 - 16*B*a^2*c^4) + tan(e/2 + (f*x)/2)^3*((29*A*a^2*c^4)/6 - (11*B*a
^2*c^4)/3) - tan(e/2 + (f*x)/2)^11*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)/3) +
tan(e/2 + (f*x)/2)^4*((44*A*a^2*c^4)/5 - (14*B*a^2*c^4)/5) + tan(e/2 + (f*x
)/2)^5*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) - tan(e/2 + (f*x)/2)^9*((23*
A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) + tan(e/2 + (f*x)/2)*((9*A*a^2*c^4)/8 +
(B*a^2*c^4)/4) + (4*A*a^2*c^4)/5 - (18*B*a^2*c^4)/35)/(f*(7*tan(e/2 + (f*x)
/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f
*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (
f*x)/2)^14 + 1)) + (a^2*c^4*atan((a^2*c^4*tan(e/2 + (f*x)/2)*(7*A - 2*B))/(
8*((7*A*a^2*c^4)/8 - (B*a^2*c^4)/4)))*(7*A - 2*B))/(8*f)
```

3.28 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=147

$$\frac{1}{16}a^2(6A-B)c^3x+\frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^2(6A-B)c^3\cos^3(e}{24}$$

```
[Out] 1/16*a^2*(6*A-B)*c^3*x+1/30*a^2*(6*A-B)*c^3*cos(f*x+e)^5/f+1/16*a^2*(6*A-B)
*c^3*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(6*A-B)*c^3*cos(f*x+e)^3*sin(f*x+e)/f
-1/6*a^2*B*cos(f*x+e)^5*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A]
time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2748, 2715, 8}

$$\frac{a^2c^3(6A-B)\cos^5(e+fx)}{30f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2c^3x(6A-B)-\frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A - B)*c^3*x)/16 + (a^2*(6*A - B)*c^3*Cos[e + f*x]^5)/(30*f) + (a^2
*(6*A - B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(6*A - B)*c^3*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^3 - c^3*Sin[e + f
*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+
$$
$$= -\frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx\right.}{6f}$$
$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f} - \frac{a^2B\cos^5}{}$$
$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f} + \frac{a^2(6A-}{}$$
$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f} + \frac{a^2(6A-}{}$$
$$= \frac{1}{16}a^2(6A-B)c^3x + \frac{a^2(6A-B)c^3\cos^5}{30f}$$

Mathematica [A]
time = 0.68, size = 137, normalized size = 0.93

$$\frac{a^2c^3(360Ae-60Be+360Afx-60Bfx+120(A-B)\cos(e+fx)+60(A-B)\cos(3(e+fx))+12A\cos(5(e+fx))-12B\cos(5(e+fx))+240A\sin(2(e+fx))-15B\sin(2(e+fx))+30A\sin(4(e+fx))+15B\sin(4(e+fx))+5B\sin(6(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*x
] + 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
```

```
*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(137) = 274$.
time = 0.21, size = 365, normalized size = 2.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(-2/3*a^2*A*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*c^3*(-1/2*cos(f*x+e
)*sin(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c^3*cos(f*x+e)+a^2*A*c^3*(f*x+e)+2*B*a^2*
c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*B*a^2
*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*
f*x+1/2*e)-B*a^2*c^3*cos(f*x+e)+a^2*A*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^2*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)
^2)*cos(f*x+e)+1/5*a^2*A*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)
-B*a^2*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)
+5/16*f*x+5/16*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(145) = 290$.
time = 0.30, size = 387, normalized size = 2.63

$$\frac{64\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)Aa^{2}c^{3}+640\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aa^{2}c^{3}+30\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)Aa^{2}c^{3}-480\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^{2}c^{3}+960\left(fx+e\right)Aa^{2}c^{3}-64\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)Ba^{2}c^{3}-640\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Ba^{2}c^{3}-5\left(4\sin\left(2fx+2e\right)^{3}+60fx+60e+9\sin\left(4fx+4e\right)-48\sin\left(2fx+2e\right)\right)Ba^{2}c^{3}+60\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)Ba^{2}c^{3}-240\left(2fx+2e-\sin\left(2fx+2e\right)\right)Ba^{2}c^{3}+960Aa^{2}c^{3}\cos\left(fx+e\right)-960Ba^{2}c^{3}\cos\left(fx+e\right)}{960f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c^
3 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^3 - 640*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
 + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^2*c^3 + 960*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e))/f
```

Fricas [A]
time = 0.37, size = 119, normalized size = 0.81

$$\frac{48\,(A-B)a^{2}c^{3}\cos\left(fx+e\right)^{5}+15\,(6\,A-B)a^{2}c^{3}fx+5\left(8\,Ba^{2}c^{3}\cos\left(fx+e\right)^{5}+2\,(6\,A-B)a^{2}c^{3}\cos\left(fx+e\right)^{3}+3\,(6\,A-B)a^{2}c^{3}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8*
B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A - B)
*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(128) = 256$.
time = 0.57, size = 910, normalized size = 6.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*cos
(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*c**
3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e + f*
x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a**2*c
**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3*cos(e
+ f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**3*cos(e
+ f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15*B*a**2*c*
*3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c**3*x*cos(e
+ f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3*x*cos(e + f*
x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**2*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c**
3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*sin(e + f*x)**2*cos
(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*
c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**3*cos(e + f
*x)**3/(3*f) - B*a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**3, True))
```

Giac [A]
time = 0.47, size = 208, normalized size = 1.41

$$\frac{Ba^2c^3\sin(6fx+6e)}{192f}+\frac{1}{16}(6Aa^2c^3-Ba^2c^3)x+\frac{(Aa^2c^3-Ba^2c^3)\cos(5fx+5e)}{80f}+\frac{(Aa^2c^3-Ba^2c^3)\cos(3fx+3e)}{16f}+\frac{(Aa^2c^3-Ba^2c^3)\cos(fx+e)}{8f}+\frac{(2Aa^2c^3+Ba^2c^3)\sin(4fx+4e)}{64f}+\frac{(16Aa^2c^3-Ba^2c^3)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/192*B*a^2*c^3*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^2*c^3 - B*a^2*c^3)*x + 1/8
0*(A*a^2*c^3 - B*a^2*c^3)*cos(5*f*x + 5*e)/f + 1/16*(A*a^2*c^3 - B*a^2*c^3)
*cos(3*f*x + 3*e)/f + 1/8*(A*a^2*c^3 - B*a^2*c^3)*cos(f*x + e)/f + 1/64*(2*
A*a^2*c^3 + B*a^2*c^3)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^2*c^3 - B*a^2*c^3)
*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.15, size = 542, normalized size = 3.69

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(4Aa^2c^3-4Ba^2c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(2Aa^2c^3-2Ba^2c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(4Aa^2c^3-4Ba^2c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(2Aa^2c^3-2Ba^2c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{2Aa^2c^3}{5}-\frac{2Ba^2c^3}{5}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(\frac{Aa^2c^3}{2}+\frac{13Ba^2c^3}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(\frac{Aa^2c^3}{2}+\frac{13Ba^2c^3}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}\left(\frac{5Aa^2c^3}{4}+\frac{Ba^2c^3}{8}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{7Aa^2c^3}{4}-\frac{47Ba^2c^3}{24}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(\frac{7Aa^2c^3}{4}-\frac{47Ba^2c^3}{24}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{5Aa^2c^3}{4}+\frac{Ba^2c^3}{8}\right)+\frac{2Aa^2c^3}{5}-\frac{2Ba^2c^3}{5}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+20\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}+\frac{a^2c^3\,\mathrm{atan}\left(\frac{a^2c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(6A-B)}{8\left(\frac{3Aa^2c^3}{4}-\frac{Ba^2c^3}{8}\right)}\right)(6A-B)}{8f}-\frac{a^2c^3(6A-B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^4*(4*A*a^2*c^3 - 4*B*a^2*c^3) + tan(e/2 + (f*x)/2)^8*(2
*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c^3 - 4*B*a^2*c^3
) + tan(e/2 + (f*x)/2)^10*(2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^
2*((2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5) + tan(e/2 + (f*x)/2)^5*((A*a^2*c^3)/2
 + (13*B*a^2*c^3)/4) - tan(e/2 + (f*x)/2)^7*((A*a^2*c^3)/2 + (13*B*a^2*c^3)
/4) - tan(e/2 + (f*x)/2)^11*((5*A*a^2*c^3)/4 + (B*a^2*c^3)/8) + tan(e/2 + (
f*x)/2)^3*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) - tan(e/2 + (f*x)/2)^9*((7*
A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) + tan(e/2 + (f*x)/2)*((5*A*a^2*c^3)/4 + (
B*a^2*c^3)/8) + (2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5)/(f*(6*tan(e/2 + (f*x)/2)
^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)
/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*c^3*at
an((a^2*c^3*tan(e/2 + (f*x)/2)*(6*A - B))/(8*((3*A*a^2*c^3)/4 - (B*a^2*c^3)
/8)))*(6*A - B))/(8*f) - (a^2*c^3*(6*A - B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=89

$$\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 3/8*a^2*A*c^2*x-1/5*a^2*B*c^2*cos(f*x+e)^5/f+3/8*a^2*A*c^2*cos(f*x+e)*sin(f
*x+e)/f+1/4*a^2*A*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2748, 2715, 8}

$$\frac{a^2Ac^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2Ac^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*A*c^2*x)/8 - (a^2*B*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^2*A*c^2*Cos[e +
 f*x]*Sin[e + f*x])/(8*f) + (a^2*A*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+f$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)}{5f} + \left(a^2Ac^2\right)\int \cos^4($$
$$= -\frac{a^2Bc^2\cos^5(e+fx)}{5f} + \frac{a^2Ac^2\cos^3(e+}{4}$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)}{5f} + \frac{3a^2Ac^2\cos(e+}{8}$$
$$= \frac{3}{8}a^2Ac^2x - \frac{a^2Bc^2\cos^5(e+fx)}{5f} + \frac{3a^2Ac}{}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.61

$$\frac{a^2c^2(-32B\cos^5(e+fx)+5A(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx))))}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + S
in[4*(e + f*x)])))/(160*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(81) = 162$.
time = 0.13, size = 166, normalized size = 1.87

method	result
risch	$\frac{3a^2Ac^2x}{8} - \frac{Ba^2c^2\cos(fx+e)}{8f} - \frac{Ba^2c^2\cos(5fx+5e)}{80f} + \frac{a^2Ac^2\sin(4fx+4e)}{32f} - \frac{Ba^2c^2\cos(3fx+3e)}{16f} + \frac{a^2Ac^2\sin}{4}$
derivativedivides	$\frac{-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+a^2Ac^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2Ba^2c^2}{}}{f}$

default	$\frac{-\frac{B a^2 c^2 \left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+a^2 A c^2 \left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2B a^2}{f}$
norman	$-\frac{2B a^2 c^2}{5f}-\frac{4B a^2 c^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2B a^2 c^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2 A c^2 x}{8}+\frac{5a^2 A c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{a^2 A c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(-1/5*B*a^2*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^
2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*B*a^2*c
^2*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*c^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*
f*x+1/2*e)-B*a^2*c^2*cos(f*x+e)+a^2*A*c^2*(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(86) = 172$.
time = 0.28, size = 176, normalized size = 1.98

$$\frac{15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)Aa^2c^2-240\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c^2+480\left(fx+e\right)Aa^2c^2-32\left(3\,\cos\left(fx+e\right)^5-10\,\cos\left(fx+e\right)^3+15\,\cos\left(fx+e\right)\right)Ba^2c^2-320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba^2c^2-480\,Ba^2c^2\cos\left(fx+e\right)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^2
 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c^2
 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^2 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*cos(f*x + e
))/f
```

Fricas [A]
time = 0.37, size = 79, normalized size = 0.89

$$-\frac{8\,Ba^2c^2\cos\left(fx+e\right)^5-15\,Aa^2c^2fx-5\left(2\,Aa^2c^2\cos\left(fx+e\right)^3+3\,Aa^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(f
*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(87) = 174$.
time = 0.33, size = 372, normalized size = 4.18

$$\begin{cases} \frac{3Aa^2c^2x\sin^4(e+fx)}{8} + \frac{3Aa^2c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^2x\sin^2(e+fx) + \frac{3Aa^2c^2x\cos^4(e+fx)}{8} - Aa^2c^2x\cos^2(e+fx) + Aa^2c^2x - \frac{5Aa^2c^2\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3Aa^2c^2\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{Aa^2c^2\sin(e+fx)\cos(e+fx)}{f} - \frac{Ba^2c^2\sin^4(e+fx)\cos(e+fx)}{f} - \frac{4Ba^2c^2\sin^2(e+fx)\cos^3(e+fx)}{3f} + \frac{2Ba^2c^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{8Ba^2c^2\cos^5(e+fx)}{15f} + \frac{4Ba^2c^2\cos^3(e+fx)}{3f} - \frac{Ba^2c^2\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A+B\sin(e))(a\sin(e)+a)^2(-c\sin(e)+c)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*cos
(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**2*c
**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos(e +
 f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2*sin(
e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)**3/
(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2*cos(e
+ f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e
+ f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**2
, True))
```

Giac [A]
time = 0.46, size = 118, normalized size = 1.33

$$\frac{3}{8}Aa^2c^2x - \frac{Ba^2c^2\cos(5fx+5e)}{80f} - \frac{Ba^2c^2\cos(3fx+3e)}{16f} - \frac{Ba^2c^2\cos(fx+e)}{8f} + \frac{Aa^2c^2\sin(4fx+4e)}{32f} + \frac{Aa^2c^2\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3*
f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x + 4*
e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.21, size = 238, normalized size = 2.67

$$\frac{3\,A\,a^2\,c^2\,x}{8} - \frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{a^2\,c^2\,\left(80\,B-75\,A\,\left(e+f\,x\right)\right)}{40}+\frac{15\,A\,a^2\,c^2\,\left(e+f\,x\right)}{8}\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{a^2\,c^2\,\left(160\,B-150\,A\,\left(e+f\,x\right)\right)}{40}+\frac{15\,A\,a^2\,c^2\,\left(e+f\,x\right)}{4}\right)+\frac{a^2\,c^2\,\left(16\,B-15\,A\,\left(e+f\,x\right)\right)}{40}+\frac{3\,A\,a^2\,c^2\,\left(e+f\,x\right)}{8}-\frac{A\,a^2\,c^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{2}+\frac{A\,a^2\,c^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{2}+\frac{5\,A\,a^2\,c^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{4}-\frac{5\,A\,a^2\,c^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (3*A*a^2*c^2*x)/8 - (tan(e/2 + (f*x)/2)^8*((a^2*c^2*(80*B - 75*A*(e + f*x))
)/40 + (15*A*a^2*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^2*(160*B
- 150*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/4) + (a^2*c^2*(16*B - 15*
A*(e + f*x)))/40 + (3*A*a^2*c^2*(e + f*x))/8 - (A*a^2*c^2*tan(e/2 + (f*x)/2
)^3)/2 + (A*a^2*c^2*tan(e/2 + (f*x)/2)^7)/2 + (5*A*a^2*c^2*tan(e/2 + (f*x)/
2)^9)/4 - (5*A*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f*(tan(e/2 + (f*x)/2)^2 + 1)
^5)
```

3.30 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=98

$$\frac{1}{8}a^2(4A+B)cx-\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}+\frac{a^2(4A+B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{Bc\cos^3(e+fx)\,(a^2+a$$

```
[Out] 1/8*a^2*(4*A+B)*c*x-1/12*a^2*(4*A+B)*c*cos(f*x+e)^3/f+1/8*a^2*(4*A+B)*c*cos
(f*x+e)*sin(f*x+e)/f-1/4*B*c*cos(f*x+e)^3*(a^2+a^2*sin(f*x+e))/f
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3046, 2939, 2748, 2715, 8}

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f}+\frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx(4A+B)-\frac{Bc\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(4*A + B)*c*x)/8 - (a^2*(4*A + B)*c*Cos[e + f*x]^3)/(12*f) + (a^2*(4*A
 + B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (B*c*Cos[e + f*x]^3*(a^2 + a^2*S
in[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx=(ac)\int \cos^2(e+fx)(a+a\sin(e+fx))(A$$

$$=-\frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f}$$

$$=-\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}-\frac{Bc\cos^3(e$$

$$=-\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}+\frac{a^2(4A+$$

$$=\frac{1}{8}a^2(4A+B)cx-\frac{a^2(4A+B)c\cos^3(e+$$

Mathematica [A]
time = 0.58, size = 67, normalized size = 0.68

$$-\frac{a^2c(-12(4A+B)fx+24(A+B)\cos(e+fx)+8(A+B)\cos(3(e+fx))-24A\sin(2(e+fx))+3B\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] -1/96*(a^2*c*(-12*(4*A + B)*f*x + 24*(A + B)*Cos[e + f*x] + 8*(A + B)*Cos[3
*(e + f*x)] - 24*A*Sin[2*(e + f*x)] + 3*B*Sin[4*(e + f*x)]))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(90) = 180.
time = 0.13, size = 186, normalized size = 1.90

method	result
risch	$\frac{a^2cxA}{2}+\frac{a^2cxB}{8}-\frac{a^2c\cos(fx+e)A}{4f}-\frac{a^2c\cos(fx+e)B}{4f}-\frac{B\,a^2c\sin(4fx+4e)}{32f}-\frac{a^2c\cos(3fx+3e)A}{12f}-\frac{a^2c\cos($
derivativedivides	$\frac{-a^2Ac\cos(fx+e)+a^2Ac(fx+e)+B\,a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-B\,a^2c\cos(fx+e)-a^2Ac\left(-\frac{\cos(fx+e)\sin(fx+e}{2}\right.}{}$
default	$\frac{-a^2Ac\cos(fx+e)+a^2Ac(fx+e)+B\,a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-B\,a^2c\cos(fx+e)-a^2Ac\left(-\frac{\cos(fx+e)\sin(fx+e}{2}\right.}{}$
norman	$\frac{(\frac{1}{2}a^2Ac+\frac{1}{8}B\,a^2c)x+(2a^2Ac+\frac{1}{2}B\,a^2c)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2a^2Ac+\frac{1}{2}B\,a^2c)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^2Ac+\frac{3}{4}B\,a^2c)x\left(\text{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(-a^2*A*c*cos(f*x+e)+a^2*A*c*(f*x+e)+B*a^2*c*(-1/2*cos(f*x+e)*sin(f*x+e
)+1/2*f*x+1/2*e)-B*a^2*c*cos(f*x+e)-a^2*A*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)+1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)+1/3*a^2*A*c*(2+sin(f*x+
e)^2)*cos(f*x+e)-B*a^2*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(95) = 190$.
time = 0.27, size = 193, normalized size = 1.97

$$-\frac{32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^2c+24\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^2c-96\left(fx+e\right)Aa^2c+32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Ba^2c+3\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)Ba^2c-24\left(2fx+2e-\sin\left(2fx+2e\right)\right)Ba^2c+96Aa^2c\cos\left(fx+e\right)+96Ba^2c\cos\left(fx+e\right)}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*cos(f
*x + e) + 96*B*a^2*c*cos(f*x + e))/f
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.83

$$-\frac{8\left(A+B\right)a^2c\cos\left(fx+e\right)^3-3\left(4A+B\right)a^2cfx+3\left(2Ba^2c\cos\left(fx+e\right)^3-\left(4A+B\right)a^2c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2*
c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(90) = 180$.
time = 0.24, size = 396, normalized size = 4.04

$$\begin{cases} -\frac{Aa^2cx\sin^2(e+fx)}{2} - \frac{Aa^2cx\cos^2(e+fx)}{2} + Aa^2cx + \frac{Aa^2c\sin^2(e+fx)\cos(e+fx)}{f} + \frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{2Aa^2c\cos^3(e+fx)}{3f} - \frac{Aa^2c\cos(e+fx)}{f} - \frac{3Ba^2cx\sin^4(e+fx)}{8} - \frac{3Ba^2cx\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{Ba^2cx\sin^2(e+fx)}{2} - \frac{3Ba^2cx\cos^4(e+fx)}{8} + \frac{Ba^2cx\cos^2(e+fx)}{2} + \frac{5Ba^2c\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{Ba^2c\sin^2(e+fx)\cos(e+fx)}{f} + \frac{3Ba^2c\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{Ba^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{2Ba^2c\cos^3(e+fx)}{3f} - \frac{Ba^2c\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A + B\sin(e))(a\sin(e) + a)^2(-c\sin(e) + c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*x)
*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e + f
*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x)**4/
8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a**2*
c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

Giac [A]
time = 0.39, size = 111, normalized size = 1.13

$$-\frac{Ba^2c\sin(4fx+4e)}{32f} + \frac{Aa^2c\sin(2fx+2e)}{4f} + \frac{1}{8}(4Aa^2c + Ba^2c)x - \frac{(Aa^2c + Ba^2c)\cos(3fx+3e)}{12f} - \frac{(Aa^2c + Ba^2c)\cos(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4*
A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*(A
*a^2*c + B*a^2*c)*cos(f*x + e)/f
```

Mupad [B]
time = 13.70, size = 339, normalized size = 3.46

$$\frac{a^2\,c\,\mathrm{atan}\left(\frac{a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,A+B\right)}{4\,\left(A\,a^2\,c+\frac{B\,a^2\,c}{4}\right)}\right)\,\left(4\,A+B\right)}{4\,f}-\frac{a^2\,c\,\left(4\,A+B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(2\,A\,a^2\,c+2\,B\,a^2\,c\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a^2\,c-\frac{B\,a^2\,c}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(2\,A\,a^2\,c+2\,B\,a^2\,c\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{2\,A\,a^2\,c}{3}+\frac{2\,B\,a^2\,c}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(A\,a^2\,c-\frac{B\,a^2\,c}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(A\,a^2\,c+\frac{7\,B\,a^2\,c}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(A\,a^2\,c+\frac{7\,B\,a^2\,c}{4}\right)+\frac{2\,A\,a^2\,c}{3}+\frac{2\,B\,a^2\,c}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*atan((a^2*c*tan(e/2 + (f*x)/2)*(4*A + B))/(4*(A*a^2*c + (B*a^2*c)/4)
))*(4*A + B))/(4*f) - (a^2*c*(4*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
)/(4*f) - (tan(e/2 + (f*x)/2)^4*(2*A*a^2*c + 2*B*a^2*c) - tan(e/2 + (f*x)/2
)*(A*a^2*c - (B*a^2*c)/4) + tan(e/2 + (f*x)/2)^6*(2*A*a^2*c + 2*B*a^2*c) +
tan(e/2 + (f*x)/2)^2*((2*A*a^2*c)/3 + (2*B*a^2*c)/3) + tan(e/2 + (f*x)/2)^7
*(A*a^2*c - (B*a^2*c)/4) - tan(e/2 + (f*x)/2)^3*(A*a^2*c + (7*B*a^2*c)/4) +
 tan(e/2 + (f*x)/2)^5*(A*a^2*c + (7*B*a^2*c)/4) + (2*A*a^2*c)/3 + (2*B*a^2*
c)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*
x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{3a^2(2A+3B)x}{2c}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}$$

```
[Out] -3/2*a^2*(2*A+3*B)*x/c+3/2*a^2*(2*A+3*B)*cos(f*x+e)/c/f+a^2*(A+B)*c^2*cos(f
*x+e)^5/f/(c-c*sin(f*x+e))^3+1/2*a^2*(2*A+3*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+
e))
```

Rubi [A]
time = 0.20, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2758, 2761, 8}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{3a^2x(2A+3B)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*(2*A + 3*B)*x)/(2*c) + (3*a^2*(2*A + 3*B)*Cos[e + f*x])/(2*c*f) + (
a^2*(A + B)*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) + (a^2*(2*A + 3*
B)*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} - \left(a^2(2A+3B)c\right)\int \frac{\cos^4(}{(c-c\sin}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} + \frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))} -$$

$$= \frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} +$$

$$= -\frac{3a^2(2A+3B)x}{2c} + \frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+}{f(c-}$$

Mathematica [A]
time = 0.84, size = 191, normalized size = 1.63

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A+3B)(e+fx)-4(A+3B)\cos(e+fx)-B\sin(2(e+fx)))-\sin\left(\frac{1}{2}(e+fx)\right)(4A(8+3e+3fx)+2B(16+9e+9fx)-4(A+3B)\cos(e+fx)-B\sin(2(e+fx)))\right)}{4cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e + f
*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f
*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x) -
4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

Maple [A]
time = 0.26, size = 123, normalized size = 1.05

method	result
derivativedivides	$\frac{2a^2\left(-\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{fc}$
default	$\frac{2a^2\left(-\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{fc}$
risch	$-\frac{3a^2xA}{c}-\frac{9a^2xB}{2c}+\frac{a^2e^{i(fx+e)}A}{2cf}+\frac{3a^2e^{i(fx+e)}B}{2cf}+\frac{a^2e^{-i(fx+e)}A}{2cf}+\frac{3a^2e^{-i(fx+e)}B}{2cf}+\frac{8a^2A}{fc\left(e^{i(fx+e)}-i\right)}+$
norman	$\frac{-\frac{2a^2A+5Ba^2}{cf}+\frac{3a^2(2A+3B)x}{2c}-\frac{\left(2a^2A+3Ba^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(4a^2A+8Ba^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(6a^2A+4Ba^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a^2/c*(-(1/2*B*tan(1/2*f*x+1/2*e)^3+(-A-3*B)*tan(1/2*f*x+1/2*e)^2-1/2*B
*tan(1/2*f*x+1/2*e)-A-3*B)/(1+tan(1/2*f*x+1/2*e)^2)^2-3/2*(2*A+3*B)*arctan(
tan(1/2*f*x+1/2*e))-(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(119) = 238$.
time = 0.51, size = 678, normalized size = 5.79

$$-\frac{2Aa^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+4Ba^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+Ba^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-4}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+4Aa^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)+2Ba^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{2Aa^2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -(2*A*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c) + 4*B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) +
```

```
1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + B*a^2*((sin(f*x + e
)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c -
 c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/c) + 4*A*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
- 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*a^2*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*a
^2/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 0.37, size = 185, normalized size = 1.58

$$\frac{Ba^2\cos\left(fx+e\right)^3-3\left(2A+3B\right)a^2fx+2\left(A+3B\right)a^2\cos\left(fx+e\right)^2+8\left(A+B\right)a^2-\left(3\left(2A+3B\right)a^2fx-\left(10A+13B\right)a^2\right)\cos\left(fx+e\right)+\left(3\left(2A+3B\right)a^2fx+Ba^2\cos\left(fx+e\right)^2-\left(2A+5B\right)a^2\cos\left(fx+e\right)+8\left(A+B\right)a^2\right)\sin\left(fx+e\right)}{2\left(cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*x
 + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos(f
*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^2*c
os(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x
+ e) + c*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(104) = 208$.
time = 2.38, size = 2365, normalized size = 20.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*a**2*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2
*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/
2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x*tan(e/2 + f*x/2)**4/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 12*
A*a**2*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/
2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c
*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*A*a**2*f*x*tan
```

```
(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
 - 2*c*f) + 6*A*a**2*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)
**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
 + f*x/2) - 2*c*f) - 16*A*a**2*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)*
*5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
+ f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*tan(e/2 + f*x/2)**
3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 +
f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 3
6*A*a**2*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f
*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*ta
n(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)
**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 20*A*a**2/(2*c*f*tan(e/2 +
 f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*
tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/
2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
 - 2*c*f) + 9*B*a**2*f*x*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*f*x*tan(e/2 + f*x/2)**3/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*
B*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 +
f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*t
an(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 9*B*a**2*f*x/(2*c*f*
tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*
tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f
*x/2) - 2*c*f) + 14*B*a**2*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 -
 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 42*B*a**2*tan(e/2 + f*x/2)**2/(
2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x
/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 10*B
*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)*
*4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
+ f*x/2) - 2*c*f) - 28*B*a**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*t
an(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-
c*sin(e) + c), True))
```

Giac [A]

time = 0.51, size = 163, normalized size = 1.39

$$-\frac{\frac{3\,(2\,Aa^2+3\,Ba^2)(fx+e)}{c}+\frac{16\,(Aa^2+Ba^2)}{c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-1)}+\frac{2\left(Ba^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3-2\,Aa^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-6\,Ba^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-Ba^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-2\,Aa^2-6\,Ba^2\right)}{\left(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+1\right)^2c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(tan(1/2*f*
x + 1/2*e) - 1)) + 2*(B*a^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*tan(1/2*f*x +
1/2*e)^2 - 6*B*a^2*tan(1/2*f*x + 1/2*e)^2 - B*a^2*tan(1/2*f*x + 1/2*e) - 2*
A*a^2 - 6*B*a^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

Mupad [B]
time = 14.70, size = 244, normalized size = 2.09

$$\frac{10\,A\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,a^2+5\,B\,a^2\right)+14\,B\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(2\,A\,a^2+7\,B\,a^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(8\,A\,a^2+9\,B\,a^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(18\,A\,a^2+21\,B\,a^2\right)}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}-\frac{3\,a^2\,\mathrm{atan}\left(\frac{3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A+3\,B\right)}{6\,A\,a^2+9\,B\,a^2}\right)\,\left(2\,A+3\,B\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x)),x)
```

```
[Out] (10*A*a^2 - tan(e/2 + (f*x)/2)*(2*A*a^2 + 5*B*a^2) + 14*B*a^2 - tan(e/2 + (
f*x)/2)^3*(2*A*a^2 + 7*B*a^2) + tan(e/2 + (f*x)/2)^4*(8*A*a^2 + 9*B*a^2) +
tan(e/2 + (f*x)/2)^2*(18*A*a^2 + 21*B*a^2))/(f*(c - c*tan(e/2 + (f*x)/2) +
2*c*tan(e/2 + (f*x)/2)^2 - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^
4 - c*tan(e/2 + (f*x)/2)^5)) - (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(2*A +
 3*B))/(6*A*a^2 + 9*B*a^2))*(2*A + 3*B))/(c*f)
```

3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{a^2(A+4B)x}{c^2}-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] a^2*(A+4*B)*x/c^2-a^2*(A+4*B)*cos(f*x+e)/c^2/f+1/3*a^2*(A+B)*c^2*cos(f*x+e)
^5/f/(c-c*sin(f*x+e))^4-2/3*a^2*(A+4*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

Rubi [A]
time = 0.19, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2759, 2761, 8}

$$-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{a^2x(A+4B)}{c^2}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(A + 4*B)*x)/c^2 - (a^2*(A + 4*B)*Cos[e + f*x])/(c^2*f) + (a^2*(A + B)
*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - (2*a^2*(A + 4*B)*Cos[e
+ f*x]^3)/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{1}{3}\left(a^2(A+4B)c\right)\int \frac{\cos^4(}{(c-c\sin} \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2} + \\
&= -\frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \\
&= \frac{a^2(A+4B)x}{c^2} - \frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2}{3f(c-c\sin}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(109) = 218.
time = 0.42, size = 238, normalized size = 2.18

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3(A+4B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-3B\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-8(2A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A + B
)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*
Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4*(c - c*Sin[e + f*x])^2)
```

Maple [A]
time = 0.32, size = 107, normalized size = 0.98

method	result
derivativedivides	$\frac{2a^2\left(-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
default	$\frac{2a^2\left(-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
risch	$\frac{a^2xA}{c^2}+\frac{4a^2xB}{c^2}-\frac{B\,a^2e^{i(fx+e)}}{2c^2f}-\frac{B\,a^2e^{-i(fx+e)}}{2c^2f}-\frac{8\left(-3iA\,a^2e^{i(fx+e)}+3A\,a^2e^{2i(fx+e)}-9iB\,a^2e^{i(fx+e)}+6B\,a^2e^{2}\right.}{3\left(e^{i(fx+e)}-i\right)^3f\,c^2}$
norman	$\frac{\frac{8B\,a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{a^2(A+4B)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{8a^2A+38B\,a^2}{3cf}-\frac{a^2(A+4B)x}{c}-\frac{2\left(4a^2A+13B\,a^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(4a^2A+\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^2*(-B/(1+tan(1/2*f*x+1/2*e)^2)+(A+4*B)*arctan(tan(1/2*f*x+1/2*e))
-1/3*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1
)^2+4*B/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(112) = 224.
time = 0.53, size = 909, normalized size = 8.34

$$\frac{2\left(2Ba^2\left(\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{11\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-5}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)+Aa^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)+2Ba^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)-\frac{Aa^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{2Aa^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{Ba^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(
```

```
cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + A*a
^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e)/(cos(f*x
 + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
 + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(112) = 224$.
time = 0.37, size = 247, normalized size = 2.27

$$-\frac{3\,Ba^2\cos\left(fx+e\right)^3+6\,(A+4\,B)a^2fx+4\,(A+B)a^2-(3\,(A+4\,B)a^2fx+(8\,A+23\,B)a^2)\cos\left(fx+e\right)^2+(3\,(A+4\,B)a^2fx-2\,(2\,A+11\,B)a^2)\cos\left(fx+e\right)-\left(6\,(A+4\,B)a^2fx-3\,Ba^2\cos\left(fx+e\right)^2-4\,(A+B)a^2+(3\,(A+4\,B)a^2fx-2\,(4\,A+13\,B)a^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+(c^2f\cos\left(fx+e\right)+2\,c^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A
+ 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x -
2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*x +
 e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(f*x
+ e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. $2(100) = 200$.
time = 4.87, size = 2474, normalized size = 22.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*A*a**2*f*x*t
```

```
an(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2
)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c*
*2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c*
*2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2
 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
3*c**2*f) - 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5
 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*A*a**2*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)
**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*A*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)*
*5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2
*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*A*a**2*
tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/
2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c
**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 24*A*a**2*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2/(3*c**2*f*tan(
e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)
**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f)
 + 12*B*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2
*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 36*B*a**2*f*x*tan(e/2
 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) + 48*B*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*t
an(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x
/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 48*B*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 36*B*a**2*f*x*tan(
e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) - 12*B*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*B*a**2*tan(
e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**
4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*
f*tan(e/2 + f*x/2) - 3*c**2*f) - 78*B*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*ta
n(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*
f) + 74*B*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
```

```
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 90*B*a**2*tan(e/2 + f*x
/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*
f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f) + 38*B*a**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan
(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))
```

Giac [A]
time = 0.43, size = 135, normalized size = 1.24

$$\frac{\frac{3\,(Aa^2+4\,Ba^2)(fx+e)}{c^2}-\frac{6\,Ba^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2}+\frac{8\left(3\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2+4\,Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*e)
 - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1/2*
e) - 1)^3))/f
```

Mupad [B]
time = 14.15, size = 246, normalized size = 2.26

$$\frac{2\,a^2\,\mathrm{atan}\left(\frac{2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A+4\,B)}{2\,A\,a^2+8\,B\,a^2}\right)\,(A+4\,B)}{c^2\,f}-\frac{\frac{8\,A\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(8\,A\,a^2+30\,B\,a^2)+\frac{38\,B\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(8\,A\,a^2+26\,B\,a^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{8\,A\,a^2}{3}+\frac{74\,B\,a^2}{3}\right)+8\,B\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f\,\left(-c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (2*a^2*atan((2*a^2*tan(e/2 + (f*x)/2)*(A + 4*B))/(2*A*a^2 + 8*B*a^2))*(A +
4*B))/(c^2*f) - ((8*A*a^2)/3 - tan(e/2 + (f*x)/2)*(8*A*a^2 + 30*B*a^2) + (3
8*B*a^2)/3 - tan(e/2 + (f*x)/2)^3*(8*A*a^2 + 26*B*a^2) + tan(e/2 + (f*x)/2)
^2*((8*A*a^2)/3 + (74*B*a^2)/3) + 8*B*a^2*tan(e/2 + (f*x)/2)^4)/(f*(4*c^2*t
an(e/2 + (f*x)/2)^2 - 4*c^2*tan(e/2 + (f*x)/2)^3 + 3*c^2*tan(e/2 + (f*x)/2)
^4 - c^2*tan(e/2 + (f*x)/2)^5 + c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=112

$$-\frac{a^2Bx}{c^3}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))}$$

```
[Out] -a^2*B*x/c^3+1/5*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^2*B*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3+2*a^2*B*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2759, 8}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^2B\cos(e+fx)}{f\,(c^3-c^3\sin(e+fx))}-\frac{a^2Bx}{c^3}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^2*B*x)/c^3) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x
])^5) - (2*a^2*B*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^2*B*Co
s[e + f*x])/(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \left(a^2Bc\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx)}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{(a^2}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{}{f(}$$

$$= -\frac{a^2Bx}{c^3} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^2B\cos^3(e+f}{3f(c-c\sin(e+f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 278 vs. 2(112) = 224.
time = 0.48, size = 278, normalized size = 2.48

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(3A+8B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-15B(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+24(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-8(3A+8B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+2(3A+43B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 -
 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin[(e
 + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e +
 f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e +
f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4*(c - c*Sin[e + f*x])^3)
```

Maple [A]
time = 0.40, size = 126, normalized size = 1.12

method	result
derivativedivides	$\frac{2a^2\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\cdots}{\left(\tan\left(\frac{f}{2}\cdots\right.\right.}\right.}{f\,c^3}$
default	$\frac{2a^2\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\cdots}{\left(\tan\left(\frac{f}{2}\cdots\right.\right.}\right.}{f\,c^3}$
risch	$-\frac{a^2Bx}{c^3}+\frac{-4A\,a^2\mathrm{e}^{2i(fx+e)}+2A\,a^2\mathrm{e}^{4i(fx+e)}-\frac{100B\,a^2\mathrm{e}^{2i(fx+e)}}{3}-24iB\,a^2\mathrm{e}^{3i(fx+e)}+\frac{56iB\,a^2\mathrm{e}^{i(fx+e)}}{3}+10B\,a^2\mathrm{e}^{4i(fx+e)}\cdots}{\left(\mathrm{e}^{i(fx+e)}-i\right)^5f\,c^3}$
norman	$\frac{\frac{a^2xB}{c}+\frac{8B\,a^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{48B\,a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{6a^2A+46B\,a^2}{15cf}+\frac{40B\,a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}+\frac{64B\,a^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{112B\,a^2\cdots}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^3*(-B*arctan(tan(1/2*f*x+1/2*e))-(A+B)/(tan(1/2*f*x+1/2*e)-1)-1/5
*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(24*A+16*B)/(tan(1/2*f*x+1/2*e)-1
)^3-1/4*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^4-4*A/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. 2(117) = 234.
time = 0.53, size = 1237, normalized size = 11.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(B*a^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3
) + A*a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
```

```
cos(f*x + e) + 1)^5) - 6*A*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) -
3*B*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*A*a^2*(5*sin(f*x + e)
/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*
c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*a^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
)/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(117) = 234$.
time = 0.37, size = 289, normalized size = 2.58

$$\frac{60\,Ba^2fx - \left(15\,Ba^2fx - (3\,A + 43\,B)a^2\right)\cos\left(fx+e\right)^3 - 12\,(A+B)a^2 - \left(45\,Ba^2fx - (9\,A - 11\,B)a^2\right)\cos\left(fx+e\right)^2 + 6\left(5\,Ba^2fx - (A + 11\,B)a^2\right)\cos\left(fx+e\right) - \left(60\,Ba^2fx + 12\,(A+B)a^2 - \left(15\,Ba^2fx + (3\,A + 43\,B)a^2\right)\cos\left(fx+e\right)^2 + 6\left(5\,Ba^2fx + (A - 9\,B)a^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\left(c^3f\cos\left(fx+e\right)^3 + 3\,c^3f\cos\left(fx+e\right)^2 - 2\,c^3f\cos\left(fx+e\right) - 4\,c^3f - \left(c^3f\cos\left(fx+e\right)^2 - 2\,c^3f\cos\left(fx+e\right) - 4\,c^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12*
(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2
*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (15*
B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*B)*a
^2)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e
)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.
time = 9.58, size = 1647, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 -
75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f
*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 60*A*a**2*
tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*
x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 +
 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 6*A*a**2/(15*c**3*f*tan(e/2 + f*
x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 -
150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
15*B*a**2*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*
f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2
 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*B*a**2*f*x*tan(
e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*B*a**2*f*x*tan(e/2 + f*x/2)**3/(
15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*
tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 +
f*x/2) - 15*c**3*f) + 150*B*a**2*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2
 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)*
*3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*
f) - 75*B*a**2*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*B*a**2*f*x/(1
5*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*t
an(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f
*x/2) - 15*c**3*f) - 30*B*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x
/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 1
50*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1
20*B*a**2*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*ta
n(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f
*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 340*B*a**2*tan(e/2 + f
*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 1
50*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) + 200*B*a**2*tan(e/2 + f*x/2)/(15*c**3*f*tan(
e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/
2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c*
*3*f) - 46*B*a**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/
2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 7
5*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin
(e) + a)**2/(-c*sin(e) + c)**3, True))
```

Giac [A]

time = 0.48, size = 159, normalized size = 1.42

$$-\frac{\frac{15\,(fx+e)Ba^2}{c^3}+\frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+170\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-100\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Aa^2+23\,Ba^2\right)}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 15*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*tan(
1/2*f*x + 1/2*e)^2 + 170*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*tan(1/2*f
*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B]

time = 14.68, size = 233, normalized size = 2.08

$$-\frac{B\,a^2\,x}{c^3}-\frac{\frac{a^2\,\left(6\,A+46\,B-15\,B\,\left(e+f\,x\right)\right)}{15}-{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{a^2\,\left(120\,B-150\,B\,\left(e+f\,x\right)\right)}{15}+10\,B\,a^2\,\left(e+f\,x\right)\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{a^2\,\left(30\,A+30\,B-75\,B\,\left(e+f\,x\right)\right)}{15}+5\,B\,a^2\,\left(e+f\,x\right)\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{a^2\,\left(60\,A+340\,B-150\,B\,\left(e+f\,x\right)\right)}{15}+10\,B\,a^2\,\left(e+f\,x\right)\right)-\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{a^2\,\left(200\,B-75\,B\,\left(e+f\,x\right)\right)}{15}+5\,B\,a^2\,\left(e+f\,x\right)\right)+B\,a^2\,\left(e+f\,x\right)}{c^3\,f\,{\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] - (B*a^2*x)/c^3 - ((a^2*(6*A + 46*B - 15*B*(e + f*x)))/15 - tan(e/2 + (f*x)
/2)^3*((a^2*(120*B - 150*B*(e + f*x)))/15 + 10*B*a^2*(e + f*x)) + tan(e/2 +
 (f*x)/2)^4*((a^2*(30*A + 30*B - 75*B*(e + f*x)))/15 + 5*B*a^2*(e + f*x)) +
 tan(e/2 + (f*x)/2)^2*((a^2*(60*A + 340*B - 150*B*(e + f*x)))/15 + 10*B*a^2
*(e + f*x)) - tan(e/2 + (f*x)/2)*((a^2*(200*B - 75*B*(e + f*x)))/15 + 5*B*a
^2*(e + f*x)) + B*a^2*(e + f*x))/(c^3*f*(tan(e/2 + (f*x)/2) - 1)^5)
```

3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] 1/7*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*(A-6*B)*c*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

Rubi [A]
time = 0.15, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 2750}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c(A-6B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*(A - 6
*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6} + \frac{1}{7}\left(a^2(A-6B)c\right)\int \frac{\cos^4(}{(c-c\sin}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6} + \frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 191 vs. $2(75) = 150$.
time = 0.65, size = 191, normalized size = 2.55

$$-\frac{a^2\left(-35(A+4B)\cos\left(\frac{1}{2}(e+fx)\right)+7(2A+13B)\cos\left(\frac{3}{2}(e+fx)\right)+35B\cos\left(\frac{5}{2}(e+fx)\right)+A\cos\left(\frac{7}{2}(e+fx)\right)-6B\cos\left(\frac{7}{2}(e+fx)\right)-70A\sin\left(\frac{1}{2}(e+fx)\right)+70B\sin\left(\frac{1}{2}(e+fx)\right)-35A\sin\left(\frac{3}{2}(e+fx)\right)+35B\sin\left(\frac{3}{2}(e+fx)\right)+7A\sin\left(\frac{5}{2}(e+fx)\right)-7B\sin\left(\frac{5}{2}(e+fx)\right)\right)}{140c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] -1/140*(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f*
x))/2] + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*(e
 + f*x))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(3*(
e + f*x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7*B*S
in[(5*(e + f*x))/2]))/(c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(71) = 142$.
time = 0.46, size = 161, normalized size = 2.15

method	result
derivativedivides	$\frac{2a^2\left(-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{96A+96B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{128A+112B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{42A+}{3\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,c^4}$
default	$\frac{2a^2\left(-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{96A+96B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{128A+112B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{42A+}{3\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,c^4}$
risch	$-\frac{2\left(-35A\,a^2e^{4i(fx+e)}-140B\,a^2e^{4i(fx+e)}+7iA\,a^2e^{i(fx+e)}+a^2A-70iA\,a^2e^{3i(fx+e)}+70iB\,a^2e^{3i(fx+e)}+91B\,a^2e^{2i(fx+}\right.}{35\left(e^{i(fx+e)}-i\right)^7}$

norman	$\frac{\frac{\left(10a^2A-10B\,a^2\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{12a^2A-2B\,a^2}{35cf}-\frac{2a^2A\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(a^2A-B\,a^2\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(2a^2A-2B\,a^2\right)\tan}{5cf}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^4*(-1/7*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(10*A+2*B)/(tan(
1/2*f*x+1/2*e)-1)^2-1/6*(96*A+96*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/5*(128*A+112
*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+1/2*e)-1)-1/3*(42*A+18*B)/(tan(
1/2*f*x+1/2*e)-1)^3-1/4*(96*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1709 vs. $2(77) = 154$.
time = 0.37, size = 1709, normalized size = 22.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(2*A*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/
(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7) + B*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7) - 3*A*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2
10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x +
 e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^
4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*A*a^2*
```

```
(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7) - 8*B*a^2*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 4
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 6*B*a^2*(7*
sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(77) = 154$.
time = 0.35, size = 279, normalized size = 3.72

$$-\frac{(A-6B)a^2\cos(fx+e)^4+(4A+11B)a^2\cos(fx+e)^3+(13A+27B)a^2\cos(fx+e)^2-10(A+B)a^2\cos(fx+e)-20(A+B)a^2-((A-6B)a^2\cos(fx+e)^3-(3A+17B)a^2\cos(fx+e)^2+10(A+B)a^2\cos(fx+e)+20(A+B)a^2)\sin(fx+e)}{35\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+4c^4f\cos(fx+e)+8c^4f+(c^4f\cos(fx+e)^3+4c^4f\cos(fx+e)^2-4c^4f\cos(fx+e)-8c^4f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (13
*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*a^
2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10*(A
 + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^
4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e)
+ 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*
x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(66) = 132$.
time = 18.24, size = 2008, normalized size = 26.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*A*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 -
245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4
*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e
/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*A*a**2*tan(e
/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)
**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 12
25*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f
*tan(e/2 + f*x/2) - 35*c**4*f) - 280*A*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*
tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 +
 f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)
**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**
4*f) + 140*A*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*
c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*t
an(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 +
 f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*A*a**2*tan(e/2
+ f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6
 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*
c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*ta
n(e/2 + f*x/2) - 35*c**4*f) + 14*A*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2
 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)
**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 7
35*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) -
12*A*a**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 +
 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c*
*4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(
e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**5/(35*c**4*f*tan(e/
2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2
)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 -
735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) -
 70*B*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*
tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2
+ f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)
**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 140*B*a**2*tan(e/2 + f*x/2
)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*
c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*
tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 +
 f*x/2) - 35*c**4*f) - 28*B*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f
*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5
- 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c
**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 14*B
*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2
+ f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)
**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 24
5*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 2*B*a**2/(35*c**4*f*tan(e/2 + f*x/
2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1
```

```
225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4
*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f), Ne(f, 0)
), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(77) = 154.
time = 0.45, size = 229, normalized size = 3.05

$$-\frac{2\left(35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+35\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+140\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+35\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-70\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+70\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+91\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,Aa^2-Ba^2\right)}{35\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 +
35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2*f*x
 + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2*e) +
 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

Mupad [B]
time = 13.35, size = 269, normalized size = 3.59

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{109\,A\,a^2}{4}+\frac{11\,B\,a^2}{4}-\frac{27\,A\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{5\,A\,a^2\,\cos(3\,e+3\,f\,x)}{8}-\frac{13\,B\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{5\,B\,a^2\,\cos(3\,e+3\,f\,x)}{8}+\frac{7\,A\,a^2\,\sin(2\,e+2\,f\,x)}{2}+\frac{7\,A\,a^2\,\sin(3\,e+3\,f\,x)}{8}-\frac{7\,B\,a^2\,\sin(2\,e+2\,f\,x)}{2}-\frac{7\,B\,a^2\,\sin(3\,e+3\,f\,x)}{8}-\frac{121\,A\,a^2\,\cos(e+f\,x)}{8}-\frac{9\,B\,a^2\,\cos(e+f\,x)}{8}-\frac{105\,A\,a^2\,\sin(e+f\,x)}{8}+\frac{105\,B\,a^2\,\sin(e+f\,x)}{8}\right)}{35\,c^4\,f\,\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\frac{21\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{8}-\frac{7\,\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{8}+\frac{\sqrt{2}\,\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((109*A*a^2)/4 + (11*B*a^2)/4 - (27*A*a^2*cos(2*e + 2
*f*x))/4 + (5*A*a^2*cos(3*e + 3*f*x))/8 - (13*B*a^2*cos(2*e + 2*f*x))/4 + (
5*B*a^2*cos(3*e + 3*f*x))/8 + (7*A*a^2*sin(2*e + 2*f*x))/2 + (7*A*a^2*sin(3
*e + 3*f*x))/8 - (7*B*a^2*sin(2*e + 2*f*x))/2 - (7*B*a^2*sin(3*e + 3*f*x))/
8 - (121*A*a^2*cos(e + f*x))/8 - (9*B*a^2*cos(e + f*x))/8 - (105*A*a^2*sin(
e + f*x))/8 + (105*B*a^2*sin(e + f*x))/8))/(35*c^4*f*((35*2^(1/2)*cos(e/2 +
 pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8 - (7*2
^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 - pi/4 + (
7*f*x)/2))/8))
```

3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5} dx$

Optimal. Leaf size=115

$$\frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

```
[Out] 1/9*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+1/63*a^2*(2*A-7*B)*c*co
s(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/315*a^2*(2*A-7*B)*cos(f*x+e)^5/f/(c-c*sin
(f*x+e))^5
```

Rubi [A]
time = 0.19, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{a^2c(2A-7B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (a^2*(2*A -
 7*B)*c*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (a^2*(2*A - 7*B)*Co
s[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{1}{9}\left(a^2(2A-7B)c\right)\int \frac{\cos^4}{(c-c\text{s}} \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(115) = 230.
time = 0.78, size = 261, normalized size = 2.27

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(315(2A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-63(4A+11B)\cos\left(\frac{3}{2}(e+fx)\right)-315B\cos\left(\frac{5}{2}(e+fx)\right)-18A\cos\left(\frac{7}{2}(e+fx)\right)+63B\cos\left(\frac{7}{2}(e+fx)\right)+882A\sin\left(\frac{1}{2}(e+fx)\right)+63B\sin\left(\frac{1}{2}(e+fx)\right)+420A\sin\left(\frac{3}{2}(e+fx)\right)+105B\sin\left(\frac{3}{2}(e+fx)\right)-72A\sin\left(\frac{5}{2}(e+fx)\right)-63B\sin\left(\frac{5}{2}(e+fx)\right)+2A\sin\left(\frac{9}{2}(e+fx)\right)-7B\sin\left(\frac{9}{2}(e+fx)\right)\right)}{2520c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(-1+\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -1/2520*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(31
5*(2*A + 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 315
*B*Cos[(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f*x)
)/2] + 882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e + f
*x))/2] + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*B*Sin
```

```
[(5*(e + f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/2]))/(
c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)
```

Maple [A]
time = 0.56, size = 205, normalized size = 1.78

method	result
risch	$\frac{2ia^2(420iA\,e^{6i(fx+e)}+105iB\,e^{6i(fx+e)}+315B\,e^{7i(fx+e)}-882iA\,e^{4i(fx+e)}-630A\,e^{5i(fx+e)}-63iB\,e^{4i(fx+e)}-945B\,e^{5i(fx}}{315f\,c^5(e^{i(fx+}}$
derivativedivides	$\frac{2a^2\left(-\frac{544A+448B}{6(\tan(\frac{fx}{2}+\frac{e}{2})-1)^6}-\frac{480A+448B}{7(\tan(\frac{fx}{2}+\frac{e}{2})-1)^7}-\frac{64A+64B}{9(\tan(\frac{fx}{2}+\frac{e}{2})-1)^9}-\frac{404A+276B}{5(\tan(\frac{fx}{2}+\frac{e}{2})-1)^5}-\frac{A}{\tan(\frac{fx}{2}+\frac{e}{2})-1}-\frac{64A+22}{3(\tan(\frac{fx}{2}+\frac{e}{}}\right.}{f\,c^5}$
default	$\frac{2a^2\left(-\frac{544A+448B}{6(\tan(\frac{fx}{2}+\frac{e}{2})-1)^6}-\frac{480A+448B}{7(\tan(\frac{fx}{2}+\frac{e}{2})-1)^7}-\frac{64A+64B}{9(\tan(\frac{fx}{2}+\frac{e}{2})-1)^9}-\frac{404A+276B}{5(\tan(\frac{fx}{2}+\frac{e}{2})-1)^5}-\frac{A}{\tan(\frac{fx}{2}+\frac{e}{2})-1}-\frac{64A+22}{3(\tan(\frac{fx}{2}+\frac{e}{}}\right.}{f\,c^5}$
norman	$\frac{(4a^2A-2B\,a^2)(\tan^{13}(\frac{fx}{2}+\frac{e}{2}))}{cf}+\frac{(28a^2A-12B\,a^2)(\tan^{11}(\frac{fx}{2}+\frac{e}{2}))}{cf}-\frac{94a^2A-14B\,a^2}{315cf}-\frac{2a^2A(\tan^{14}(\frac{fx}{2}+\frac{e}{2}))}{cf}+\frac{(24a^2A-14B\,a^2}{35}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^5*(-1/6*(544*A+448*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/7*(480*A+448*B)/
(tan(1/2*f*x+1/2*e)-1)^7-1/9*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/5*(404*
A+276*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+1/2*e)-1)-1/3*(64*A+22*B)/
(tan(1/2*f*x+1/2*e)-1)^3-1/8*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/4*(20
0*A+104*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(12*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2273 vs. 2(118) = 236.
time = 0.36, size = 2273, normalized size = 19.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
```

```
4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) - 10*A*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
 10*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) + 14*A*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/
(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^
9) + 28*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) +
 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(c
```

```
os(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(
cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(118) = 236$.
time = 0.37, size = 355, normalized size = 3.09

$$\frac{(2A-7B)a^2\cos(fx+e)^5 - 4(2A-7B)a^2\cos(fx+e)^4 - 5(5A+14B)a^2\cos(fx+e)^3 - 5(17A+35B)a^2\cos(fx+e)^2 + 70(A+B)a^2\cos(fx+e) + 140(A+B)a^2 + ((2A-7B)a^2\cos(fx+e)^4 + 5(2A-7B)a^2\cos(fx+e)^3 - 15(A+7B)a^2\cos(fx+e)^2 + 70(A+B)a^2\cos(fx+e) + 140(A+B)a^2)\sin(fx+e)}{315\left(c^5f\cos(fx+e)^5 + 5c^5f\cos(fx+e)^4 - 8c^5f\cos(fx+e)^3 - 20c^5f\cos(fx+e)^2 + 8c^5f\cos(fx+e) + 16c^5f - (c^5f\cos(fx+e)^4 - 4c^5f\cos(fx+e)^3 - 12c^5f\cos(fx+e)^2 + 8c^5f\cos(fx+e) + 16c^5f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 -
5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 + 70
*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x + e)
^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2 + 7
0*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e))/(c^5*f*cos(f*x
+ e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
 + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(102) = 204$.
time = 33.15, size = 3262, normalized size = 28.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9
- 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 2646
0*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**
5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*t
an(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260*A*a*
*2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f
*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)
**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 +
 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*A*a**2*tan(e/2 + f*x/2)*
*6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 1134
0*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**
```

5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*t
an(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2
 + f*x/2) - 315*c**5*f) + 5040*A*a**2*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e
/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f
*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)
5 - 39690*c5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 -
 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5
*f) - 6804*A*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 3276*A*a**2*ta
n(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f
*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)
6 + 39690*c5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 +
 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835
*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 2124*A*a**2*tan(e/2 + f*x/2)**2/(3
15*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**
5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*t
an(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/
2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*
x/2) - 315*c**5*f) + 216*A*a**2*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/
2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7
 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 396
90*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c*
*5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 94*
A*a**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 +
11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690
*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5
*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan
(e/2 + f*x/2) - 315*c**5*f) - 630*B*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*ta
n(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2
+ f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x
/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**
3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c
5*f) - 210*B*a2*tan(e/2 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
5*f*tan(e/2 + f*x/2)6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a**2*
tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 +
 f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/
2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4
 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 28
35*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2 + f*x/2)**4/(

```
315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c*
*5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*
tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e
/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f
*x/2) - 315*c**5*f) - 1386*B*a**2*tan(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 +
 f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2
)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5
- 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 113
40*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f)
- 126*B*a**2*tan(e/2 + f*x/2)**2/(315*c**5*f*ta...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(118) = 236.
time = 0.50, size = 301, normalized size = 2.62

$$-\frac{2\left(315\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-630\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+315\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2310\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+105\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-2520\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+945\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3402\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+63\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1638\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+693\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+1062\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+63\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-108\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+63\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+47\,Aa^{2}-7\,Ba^{2}\right)}{315\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/315*(315*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*tan(1/2*f*x + 1/2*e)^7
 + 315*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*tan(1/2*f*x + 1/2*e)^6 + 1
05*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 945*B
*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 63*B*a^2*
tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 1062*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*tan(1/2*f
*x + 1/2*e)^2 - 108*A*a^2*tan(1/2*f*x + 1/2*e) + 63*B*a^2*tan(1/2*f*x + 1/2
*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B]
time = 13.36, size = 331, normalized size = 2.88

$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{265\,A\,a^2\,\cos(2\,e+2\,f\,x)}{2}-\frac{49\,B\,a^2}{8}-\frac{4967\,A\,a^2}{16}-\frac{89\,A\,a^2\,\cos(3\,e+3\,f\,x)}{4}-\frac{49\,A\,a^2\,\cos(4\,e+4\,f\,x)}{16}+\frac{35\,B\,a^2\,\cos(2\,e+2\,f\,x)}{4}-\frac{7\,B\,a^2\,\cos(3\,e+3\,f\,x)}{8}+\frac{7\,B\,a^2\,\cos(4\,e+4\,f\,x)}{8}-\frac{567\,A\,a^2\,\sin(2\,e+2\,f\,x)}{8}-\frac{243\,A\,a^2\,\sin(3\,e+3\,f\,x)}{8}+\frac{45\,A\,a^2\,\sin(4\,e+4\,f\,x)}{16}+\frac{63\,B\,a^2\,\sin(2\,e+2\,f\,x)}{2}+\frac{63\,B\,a^2\,\sin(3\,e+3\,f\,x)}{8}+\frac{625\,A\,a^2\,\cos(e+f\,x)}{4}+\frac{35\,B\,a^2\,\cos(e+f\,x)}{8}+\frac{2205\,A\,a^2\,\sin(e+f\,x)}{8}-\frac{945\,B\,a^2\,\sin(e+f\,x)}{8}\right)}{315\,c^5\,f\,\left(\frac{63\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\frac{21\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{9\,\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}+\frac{9\,\sqrt{2}\,\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{16}+\frac{\sqrt{2}\,\cos\left(\frac{9\,e}{2}+\frac{\pi}{4}+\frac{9\,f\,x}{2}\right)}{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^5,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((265*A*a^2*cos(2*e + 2*f*x))/2 - (49*B*a^2)/8 - (49
67*A*a^2)/16 - (89*A*a^2*cos(3*e + 3*f*x))/4 - (49*A*a^2*cos(4*e + 4*f*x))/
16 + (35*B*a^2*cos(2*e + 2*f*x))/4 - (7*B*a^2*cos(3*e + 3*f*x))/8 + (7*B*a^
2*cos(4*e + 4*f*x))/8 - (567*A*a^2*sin(2*e + 2*f*x))/8 - (243*A*a^2*sin(3*e
 + 3*f*x))/8 + (45*A*a^2*sin(4*e + 4*f*x))/16 + (63*B*a^2*sin(2*e + 2*f*x))
/2 + (63*B*a^2*sin(3*e + 3*f*x))/8 + (625*A*a^2*cos(e + f*x))/4 + (35*B*a^2
*cos(e + f*x))/8 + (2205*A*a^2*sin(e + f*x))/8 - (945*B*a^2*sin(e + f*x))/8
```

```
))/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((
3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))
/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2
+ pi/4 + (9*f*x)/2))/16))
```

3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=156

$$\frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}$$

```
[Out] 1/11*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/99*a^2*(3*A-8*B)*c*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/693*a^2*(3*A-8*B)*cos(f*x+e)^5/f/(c-c*si
n(f*x+e))^6+2/3465*a^2*(3*A-8*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

Rubi [A]
time = 0.23, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{a^2c(3A-8B)\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*(3*A
- 8*B)*c*Cos[e + f*x]^5)/(99*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(3*A - 8*B)
*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(3*A - 8*B)*Cos[e
+ f*x]^5)/(3465*c*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{1}{11}\left(a^2(3A-8B)c\right)\int \frac{\cos}{(c-c}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

Mathematica [A]
time = 1.00, size = 285, normalized size = 1.83

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(231(27A+28B)\cos\left(\frac{1}{2}(e+fx)\right)-2475(A+2B)\cos\left(\frac{3}{2}(e+fx)\right)-2310B\cos\left(\frac{5}{2}(e+fx)\right)-165A\cos\left(\frac{7}{2}(e+fx)\right)+440B\cos\left(\frac{7}{2}(e+fx)\right)+3A\cos\left(\frac{11}{2}(e+fx)\right)-8B\cos\left(\frac{11}{2}(e+fx)\right)+7623A\sin\left(\frac{1}{2}(e+fx)\right)+2772B\sin\left(\frac{1}{2}(e+fx)\right)+3465A\sin\left(\frac{3}{2}(e+fx)\right)+2310B\sin\left(\frac{3}{2}(e+fx)\right)-495A\sin\left(\frac{5}{2}(e+fx)\right)-990B\sin\left(\frac{5}{2}(e+fx)\right)+33A\sin\left(\frac{9}{2}(e+fx)\right)-88B\sin\left(\frac{9}{2}(e+fx)\right)\right)}{27720c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(-1+\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A
+ 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*Cos
[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))/2]
 + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(e +
```

```
f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310*B*Si
n[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f*x))/2
] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c^6*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)
```

Maple [A]
time = 0.65, size = 249, normalized size = 1.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^6*(-1/7*(2376*A+1896*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/9*(1536*A+1472
*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/5*(932*A+528*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(
tan(1/2*f*x+1/2*e)-1)-1/4*(352*A+152*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/3*(90*A+
26*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(2304*A+2048*B)/(tan(1/2*f*x+1/2*e)-1)^8
-1/6*(1752*A+1208*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/10*(640*A+640*B)/(tan(1/2*f
*x+1/2*e)-1)^10-1/11*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/2*(14*A+2*B)
/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2838 vs. $2(160) = 320$.
time = 0.40, size = 2838, normalized size = 18.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*A*a^2*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
```

^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^2*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*B*a^
2*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*A*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11) + 8*B*a^2*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -

```
5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(
cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 -
 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(160) = 320$.
time = 0.39, size = 431, normalized size = 2.76

$$-\frac{2\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^6+12\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^5-25\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^4-35\,(6\,A+17\,B)a^2\cos\left(fx+e\right)^3-35\,(21\,A+43\,B)a^2\cos\left(fx+e\right)^2+630\,(A+B)a^2\cos\left(fx+e\right)+1260\,(A+B)a^2-\left(2\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^5-10\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^4-35\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^3+35\,(3\,A+25\,B)a^2\cos\left(fx+e\right)^2-630\,(A+B)a^2\cos\left(fx+e\right)-1260\,(A+B)a^2\right)\sin\left(fx+e\right)}{3465\,\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-18\,c^6f\cos\left(fx+e\right)^4+20\,c^6f\cos\left(fx+e\right)^3+48\,c^6f\cos\left(fx+e\right)^2-16\,c^6f\cos\left(fx+e\right)-32\,c^6f+\left(c^6f\cos\left(fx+e\right)^5+6\,c^6f\cos\left(fx+e\right)^4-12\,c^6f\cos\left(fx+e\right)^3-32\,c^6f\cos\left(fx+e\right)^2+16\,c^6f\cos\left(fx+e\right)+32\,c^6f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e)
^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)^3
 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) + 126
0*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*cos(
f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*cos(f*
x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x + e))/(
c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 2
0*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) -
32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*
x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(
f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(141) = 282$.
time = 56.21, size = 4816, normalized size = 30.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-6930*A*a**2*tan(e/2 + f*x/2)**10/(3465*c**6*f*tan(e/2 + f*x/2)*
*11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9
 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 -
 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 -
1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 19
0575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6
*f) + 20790*A*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 -
38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571
```

725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 16008
30*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 114345
0*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c
6*f*tan(e/2 + f*x/2)2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) -
83160*A*a**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*
c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c*
*6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**
6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6
*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*
tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 138600
*A*a**2*tan(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*
f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*
tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*t
an(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*ta
n(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e
/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 224532*A*a*
*2*tan(e/2 + f*x/2)**6/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan
(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e
/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/
2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2
 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 +
f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 196812*A*a**2*ta
n(e/2 + f*x/2)**5/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2
+ f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 +
f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f
*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*
x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2
)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 162360*A*a**2*tan(e/2
 + f*x/2)**4/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x
/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2
)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)
6 + 1600830*c6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)*
*4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2
+ 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 67320*A*a**2*tan(e/2 + f*x
/2)**3/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**1
0 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 +
 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 +
1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 5
71725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 3811
5*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 29370*A*a**2*tan(e/2 + f*x/2)**2
/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 19
0575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 11434
50*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 160083
0*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*
c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6

```
*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 3102*A*a**2*tan(e/2 + f*x/2)/(3465*c**
6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*
f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*
tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*t
an(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan
(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2
 + f*x/2) - 3465*c**6*f) - 912*A*a**2/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 3
8115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 5717
25*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 160083
0*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450
*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c*
*6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 6
930*B*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c*
*6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(160) = 320$.
time = 0.48, size = 373, normalized size = 2.39

$$-\frac{2\left(3465\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-10395\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+3465\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+41580\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-1155\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-69300\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+16170\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+112266\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-6006\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-98406\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+22176\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+81180\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-3960\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-33660\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+8910\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+14685\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+110\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1551\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+671\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+456\,Aa^2-61\,Ba^2\right)}{3465\,c^6f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/3465*(3465*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 10395*A*a^2*tan(1/2*f*x + 1/2
*e)^9 + 3465*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*tan(1/2*f*x + 1/2*e
)^8 - 1155*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*tan(1/2*f*x + 1/2*e)^
7 + 16170*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*tan(1/2*f*x + 1/2*e)^
6 - 6006*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*tan(1/2*f*x + 1/2*e)^5
+ 22176*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*tan(1/2*f*x + 1/2*e)^4 -
 3960*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 8
910*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 110
*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*tan(1/2*f*x + 1/2*e) + 671*B*a^2
*tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(tan(1/2*f*x + 1/2*e)
- 1)^11)
```

Mupad [B]
time = 13.54, size = 423, normalized size = 2.71

$$\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{38163\,Aa^2}{8}-\frac{1283\,Ba^2}{8}-\frac{11931\,Aa^2\cos(2e+2fx)}{4}+\frac{9609\,Aa^2\cos(3e+3fx)}{16}+\frac{1383\,Aa^2\cos(4e+4fx)}{8}-\frac{225\,Aa^2\cos(5e+5fx)}{16}+\frac{631\,Ba^2\cos(2e+2fx)}{4}-\frac{1583\,Ba^2\cos(3e+3fx)}{32}-\frac{223\,Ba^2\cos(4e+4fx)}{8}+\frac{45\,Ba^2\cos(5e+5fx)}{32}+1386\,Aa^2\sin(2e+2fx)+\frac{14949\,Aa^2\sin(3e+3fx)}{16}-\frac{561\,Aa^2\sin(4e+4fx)}{4}-\frac{231\,Aa^2\sin(5e+5fx)}{16}-\frac{3003\,Ba^2\sin(2e+2fx)}{8}-\frac{4653\,Ba^2\sin(3e+3fx)}{32}+\frac{209\,Ba^2\sin(4e+4fx)}{16}+\frac{77\,Ba^2\sin(5e+5fx)}{32}-2091\,Aa^2\cos(e+fx)+\frac{281\,Ba^2\cos(e+fx)}{16}-\frac{22869\,Aa^2\sin(e+fx)}{4}+\frac{23331\,Ba^2\sin(e+fx)}{16}\right)}{3465\,c^6f\left(\frac{231\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{16}-\frac{165\sqrt{2}\cos\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{16}-\frac{165\sqrt{2}\cos\left(\frac{5e}{2}+\frac{\pi}{4}+\frac{5fx}{2}\right)}{32}+\frac{55\sqrt{2}\cos\left(\frac{7e}{2}-\frac{\pi}{4}+\frac{7fx}{2}\right)}{32}+\frac{11\sqrt{2}\cos\left(\frac{9e}{2}+\frac{\pi}{4}+\frac{9fx}{2}\right)}{32}-\frac{\sqrt{2}\cos\left(\frac{11e}{2}-\frac{\pi}{4}+\frac{11fx}{2}\right)}{32}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^6,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((38163*A*a^2)/8 - (1283*B*a^2)/8 - (11931*A*a^2*cos(
2*e + 2*f*x))/4 + (9609*A*a^2*cos(3*e + 3*f*x))/16 + (1383*A*a^2*cos(4*e +
4*f*x))/8 - (225*A*a^2*cos(5*e + 5*f*x))/16 + (631*B*a^2*cos(2*e + 2*f*x))/
4 - (1583*B*a^2*cos(3*e + 3*f*x))/32 - (223*B*a^2*cos(4*e + 4*f*x))/8 + (45
*B*a^2*cos(5*e + 5*f*x))/32 + 1386*A*a^2*sin(2*e + 2*f*x) + (14949*A*a^2*si
n(3*e + 3*f*x))/16 - (561*A*a^2*sin(4*e + 4*f*x))/4 - (231*A*a^2*sin(5*e +
5*f*x))/16 - (3003*B*a^2*sin(2*e + 2*f*x))/8 - (4653*B*a^2*sin(3*e + 3*f*x)
)/32 + (209*B*a^2*sin(4*e + 4*f*x))/16 + (77*B*a^2*sin(5*e + 5*f*x))/32 - 2
091*A*a^2*cos(e + f*x) + (281*B*a^2*cos(e + f*x))/16 - (22869*A*a^2*sin(e +
 f*x))/4 + (23331*B*a^2*sin(e + f*x))/16))/(3465*c^6*f*((231*2^(1/2)*cos(e/
2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/16
- (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2)*cos((7*e)/
2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32
 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=197

$$\frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}$$

```
[Out] 1/13*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^9+1/143*a^2*(4*A-9*B)*c*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/429*a^2*(4*A-9*B)*cos(f*x+e)^5/f/(c-c*s
in(f*x+e))^7+2/3003*a^2*(4*A-9*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^6+2/150
15*a^2*(4*A-9*B)*cos(f*x+e)^5/c^2/f/(c-c*sin(f*x+e))^5
```

Rubi [A]
time = 0.28, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}+\frac{a^2c(4A-9B)\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + (a^2*(4*A
 - 9*B)*c*Cos[e + f*x]^5)/(143*f*(c - c*Sin[e + f*x])^8) + (a^2*(4*A - 9*B)*
Cos[e + f*x]^5)/(429*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(4*A - 9*B)*Cos[e +
 f*x]^5)/(3003*c*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(4*A - 9*B)*Cos[e + f*x
]^5)/(15015*c^2*f*(c - c*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{1}{13}\left(a^2(4A-9B)c\right)\int \frac{\cos}{(c-c}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}$$

Mathematica [A]
time = 2.32, size = 313, normalized size = 1.59

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(6006(8A+7B)\cos\left(\frac{1}{2}(e+fx)\right)-1716(11A+19B)\cos\left(\frac{3}{2}(e+fx)\right)-15015B\cos\left(\frac{5}{2}(e+fx)\right)-1144A\cos\left(\frac{7}{2}(e+fx)\right)+2574B\cos\left(\frac{7}{2}(e+fx)\right)+52A\cos\left(\frac{11}{2}(e+fx)\right)-117B\cos\left(\frac{11}{2}(e+fx)\right)+54912A\sin\left(\frac{1}{2}(e+fx)\right)+26598B\sin\left(\frac{1}{2}(e+fx)\right)+24024A\sin\left(\frac{3}{2}(e+fx)\right)+21021B\sin\left(\frac{3}{2}(e+fx)\right)-2860A\sin\left(\frac{5}{2}(e+fx)\right)-8580B\sin\left(\frac{5}{2}(e+fx)\right)+312A\sin\left(\frac{9}{2}(e+fx)\right)-702B\sin\left(\frac{9}{2}(e+fx)\right)-4A\sin\left(\frac{13}{2}(e+fx)\right)+9B\sin\left(\frac{13}{2}(e+fx)\right)\right)}{240240c^7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(-1+\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -1/240240*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(
6006*(8*A + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/2]
 - 15015*B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*Cos[
(7*(e + f*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x))/2]
 + 54912*A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(3*(e
+ f*x))/2] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/2] - 8
580*B*Sin[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[(9*(e +
 f*x))/2] - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))/(c^7*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)
```

Maple [A]
time = 0.54, size = 293, normalized size = 1.49

method	result
risch	$-\frac{4ia^2\left(2860iA\,e^{4i(fx+e)}-312iA\,e^{2i(fx+e)}+15015B\,e^{9i(fx+e)}-26598iB\,e^{6i(fx+e)}-48048A\,e^{7i(fx+e)}-9iB-42042B\,e^{7i(}\right.}{}$
derivativedivides	$\frac{2a^2\left(-\frac{7744A+5368B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{10896A+9360B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4320A+2568B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{1816A+884B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{1536A+15}{12\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{}$
default	$\frac{2a^2\left(-\frac{7744A+5368B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{10896A+9360B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4320A+2568B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{1816A+884B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{1536A+15}{12\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2/c^7*(-1/7*(7744*A+5368*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/9*(10896*A+936
0*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/6*(4320*A+2568*B)/(tan(1/2*f*x+1/2*e)-1)^6-
1/5*(1816*A+884*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+1/2*e)-1)-1/12*(
1536*A+1536*B)/(tan(1/2*f*x+1/2*e)-1)^12-1/3*(120*A+30*B)/(tan(1/2*f*x+1/2*
e)-1)^3-1/10*(8320*A+7680*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/4*(560*A+208*B)/(t
an(1/2*f*x+1/2*e)-1)^4-1/8*(10560*A+8256*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/11*(
4480*A+4352*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/13*(256*A+256*B)/(tan(1/2*f*x+1/
2*e)-1)^13-1/2*(16*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3402 vs. $2(202) = 404$.
time = 0.43, size = 3402, normalized size = 17.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

[Out] -2/45045*(2*A*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 4*B*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) +
 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
 + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^2*(3796*sin(f*x + e)/(cos(f*x + e)
 + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin
(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(
f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 70*A*a^2*(6
11*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)

```
^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 2
7027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1
) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos
(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(
f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^2*(611*sin(f*x + e)/(cos(f*x +
e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(c...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(202) = 404$.
time = 0.37, size = 503, normalized size = 2.55

$$\frac{2 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{7} - 12 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{6} - 49 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{5} + 70 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{4} + 105 \, {\left(7 \, A + 20 \, B\right)} a^{2} \cos \left(f x + e\right)^{3} + 105 \, {\left(25 \, A + 51 \, B\right)} a^{2} \cos \left(f x + e\right)^{2} - 2310 \, {\left(A + B\right)} a^{2} \cos \left(f x + e\right) - 4620 \, {\left(A + B\right)} a^{2} + {\left(2 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{6} + 14 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{5} - 35 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{4} - 105 \, {\left(4 \, A - 9 \, B\right)} a^{2} \cos \left(f x + e\right)^{3} + 105 \, {\left(3 \, A + 29 \, B\right)} a^{2} \cos \left(f x + e\right)^{2} - 2310 \, {\left(A + B\right)} a^{2} \cos \left(f x + e\right) - 4620 \, {\left(A + B\right)} a^{2}\right)} \sin \left(f x + e\right)}{15015 \, {\left(c^{7} f \cos \left(f x + e\right)^{7} + 7 \, c^{7} f \cos \left(f x + e\right)^{6} - 18 \, c^{7} f \cos \left(f x + e\right)^{5} - 56 \, c^{7} f \cos \left(f x + e\right)^{4} + 48 \, c^{7} f \cos \left(f x + e\right)^{3} + 112 \, c^{7} f \cos \left(f x + e\right)^{2} - 32 \, c^{7} f \cos \left(f x + e\right) - 64 \, c^{7} f - {\left(c^{7} f \cos \left(f x + e\right)^{6} - 6 \, c^{7} f \cos \left(f x + e\right)^{5} - 24 \, c^{7} f \cos \left(f x + e\right)^{4} + 32 \, c^{7} f \cos \left(f x + e\right)^{3} + 80 \, c^{7} f \cos \left(f x + e\right)^{2} - 32 \, c^{7} f \cos \left(f x + e\right) - 64 \, c^{7} f\right)} \sin \left(f x + e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] 1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e)
^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^4
+ 105*(7*A + 20*B)*a^2*cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*cos(f*x + e)^
2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a^2*c
os(f*x + e)^6 + 14*(4*A - 9*B)*a^2*cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2*cos(
f*x + e)^4 - 105*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 105*(3*A + 29*B)*a^2*cos(
f*x + e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2)*sin(f*x + e)
)/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5
- 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e
)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*co
s(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f
*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

<u>**Sympy**</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(178) = 356$.

time = 93.87, size = 6669, normalized size = 33.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Piecewise((-30030*A*a**2*tan(e/2 + f*x/2)**12/(15015*c**7*f*tan(e/2 + f*x/2
)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2
)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x
/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*
x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f
*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f
*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x
/2) - 15015*c**7*f) + 120120*A*a**2*tan(e/2 + f*x/2)**11/(15015*c**7*f*tan(
e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(
e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*ta
n(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*t
an(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*
tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*
tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*ta
n(e/2 + f*x/2) - 15015*c**7*f) - 540540*A*a**2*tan(e/2 + f*x/2)**10/(15015*
c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*
c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 1073572
5*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 257657
40*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324
305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294
290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 19519
5*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 1201200*A*a**2*tan(e/2 + f*x/2)
**9/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12
 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**1
0 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)*
*8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)
**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2
)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)
**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 2348346*A*a**2*tan(e
/2 + f*x/2)**8/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 +
 f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2
+ f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/
2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e
/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(
e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e
/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) + 2930928*A
```

```
*a**2*tan(e/2 + f*x/2)**7/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*
f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7
*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c*
*7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c
**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*
c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c
**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f)
- 3119688*A*a**2*tan(e/2 + f*x/2)**6/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 1
95195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4
294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 -
19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 -
 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5
- 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 + f*x/2)**3
- 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f*x/2) - 150
15*c**7*f) + 2189616*A*a**2*tan(e/2 + f*x/2)**5/(15015*c**7*f*tan(e/2 + f*x
/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x
/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 + f
*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2 +
f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/2 +
 f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e/2 +
 f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/2 + f
*x/2) - 15015*c**7*f) - 1319890*A*a**2*tan(e/2 + f*x/2)**4/(15015*c**7*f*ta
n(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*ta
n(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*
tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f
*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*
f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*
f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*
tan(e/2 + f*x/2) - 15015*c**7*f) + 467896*A*a**2*tan(e/2 + f*x/2)**3/(15015
*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170
*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*t...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(202) = 404.
time = 0.51, size = 445, normalized size = 2.26

$$-\frac{2\left(15015\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{12}-60060\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+15015\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+270270\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-15015\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-600600\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+105105\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+1174173\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-93093\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-1465464\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+234234\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+1559844\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-131274\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-1094808\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+181038\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+659945\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-47190\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-233948\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+45903\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+77454\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1599\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-7904\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2769\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1763\,Aa^2-213\,Ba^2\right)}{15015\,c^7f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x + 1
/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x +
 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*f*x
```

```
 + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1/2*f
*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan(1/2*
f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*tan(1/
2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a^2*tan(
1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A*a^2*tan
(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*A*a^2*tan
(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*A*a^2*tan(
1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*a^2*tan(1/2
*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 - 213*B*a^2)/(
c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

Mupad [B]
time = 14.05, size = 500, normalized size = 2.54

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{994249Aa^2}{32}-\frac{63639Ba^2}{32}-\frac{1609013Aa^2\cos(2e+2fx)}{64}+\frac{85687Aa^2\cos(3e+3fx)}{16}+\frac{79591Aa^2\cos(4e+4fx)}{32}-\frac{5261Aa^2\cos(5e+5fx)}{16}-\frac{1771Aa^2\cos(6e+6fx)}{64}+\frac{140553Ba^2\cos(2e+2fx)}{64}-\frac{4431Ba^2\cos(3e+3fx)}{8}-\frac{10161Ba^2\cos(4e+4fx)}{32}+36Ba^2\cos(5e+5fx)+\frac{231Ba^2\cos(6e+6fx)}{64}+\frac{636207Aa^2\sin(2e+2fx)}{64}+\frac{309309Aa^2\sin(3e+3fx)}{32}-\frac{7007Aa^2\sin(4e+4fx)}{4}-\frac{12389Aa^2\sin(5e+5fx)}{32}+\frac{1755Aa^2\sin(6e+6fx)}{64}-\frac{121407Ba^2\sin(2e+2fx)}{64}-\frac{39039Ba^2\sin(3e+3fx)}{32}+\frac{3003Ba^2\sin(4e+4fx)}{16}+\frac{1599Ba^2\sin(5e+5fx)}{32}-\frac{195Ba^2\sin(6e+6fx)}{64}-\frac{93221Aa^2\cos(e+fx)}{8}+\frac{3291Ba^2\cos(e+fx)}{8}-\frac{704847Aa^2\sin(e+fx)}{16}+\frac{125697Ba^2\sin(e+fx)}{16}\right)}{15015c^7f\left(\frac{1287\sqrt{2}\cos\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{64}-\frac{429\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{16}+\frac{715\sqrt{2}\cos\left(\frac{5e}{2}+\frac{\pi}{4}+\frac{5fx}{2}\right)}{64}-\frac{143\sqrt{2}\cos\left(\frac{7e}{2}-\frac{\pi}{4}+\frac{7fx}{2}\right)}{32}-\frac{39\sqrt{2}\cos\left(\frac{9e}{2}+\frac{\pi}{4}+\frac{9fx}{2}\right)}{32}+\frac{13\sqrt{2}\cos\left(\frac{11e}{2}-\frac{\pi}{4}+\frac{11fx}{2}\right)}{64}+\frac{\sqrt{2}\cos\left(\frac{13e}{2}+\frac{\pi}{4}+\frac{13fx}{2}\right)}{64}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^7,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((994249*A*a^2)/32 - (63639*B*a^2)/32 - (1609013*A*a
^2*cos(2*e + 2*f*x))/64 + (85687*A*a^2*cos(3*e + 3*f*x))/16 + (79591*A*a^2*
cos(4*e + 4*f*x))/32 - (5261*A*a^2*cos(5*e + 5*f*x))/16 - (1771*A*a^2*cos(6
*e + 6*f*x))/64 + (140553*B*a^2*cos(2*e + 2*f*x))/64 - (4431*B*a^2*cos(3*e
+ 3*f*x))/8 - (10161*B*a^2*cos(4*e + 4*f*x))/32 + 36*B*a^2*cos(5*e + 5*f*x)
 + (231*B*a^2*cos(6*e + 6*f*x))/64 + (636207*A*a^2*sin(2*e + 2*f*x))/64 + (
309309*A*a^2*sin(3*e + 3*f*x))/32 - (7007*A*a^2*sin(4*e + 4*f*x))/4 - (1238
9*A*a^2*sin(5*e + 5*f*x))/32 + (1755*A*a^2*sin(6*e + 6*f*x))/64 - (121407*B
*a^2*sin(2*e + 2*f*x))/64 - (39039*B*a^2*sin(3*e + 3*f*x))/32 + (3003*B*a^2
*sin(4*e + 4*f*x))/16 + (1599*B*a^2*sin(5*e + 5*f*x))/32 - (195*B*a^2*sin(6
*e + 6*f*x))/64 - (93221*A*a^2*cos(e + f*x))/8 + (3291*B*a^2*cos(e + f*x))/
8 - (704847*A*a^2*sin(e + f*x))/16 + (125697*B*a^2*sin(e + f*x))/16))/(1501
5*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*c
os(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2)
)/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((
9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)
/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

3.38 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$

Optimal. Leaf size=265

$$\frac{11}{256}a^3(10A-3B)c^6x+\frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f}+\frac{11a^3(10A-3B)c^6\cos(e+fx)\sin(e+fx)}{256f}+\frac{11a^3(10$$

```
[Out] 11/256*a^3*(10*A-3*B)*c^6*x+11/560*a^3*(10*A-3*B)*c^6*cos(f*x+e)^7/f+11/256
*a^3*(10*A-3*B)*c^6*cos(f*x+e)*sin(f*x+e)/f+11/384*a^3*(10*A-3*B)*c^6*cos(f
*x+e)^3*sin(f*x+e)/f+11/480*a^3*(10*A-3*B)*c^6*cos(f*x+e)^5*sin(f*x+e)/f-1/
10*a^3*B*cos(f*x+e)^7*(c^2-c^2*sin(f*x+e))^3/f+1/90*a^3*(10*A-3*B)*cos(f*x+
e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/720*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^6-c^6*s
in(f*x+e))/f
```

Rubi [A]
time = 0.27, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\frac{11a^3c^6(10A-3B)\cos^7(e+fx)}{560f}+\frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f}+\frac{11a^3c^6(10A-3B)\sin(e+fx)\cos^5(e+fx)}{480f}+\frac{11a^3c^6(10A-3B)\sin(e+fx)\cos^3(e+fx)}{384f}+\frac{11a^3c^6(10A-3B)\sin(e+fx)\cos(e+fx)}{256f}+\frac{11}{256}a^3c^6x(10A-3B)+\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f}-\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (11*a^3*(10*A - 3*B)*c^6*x)/256 + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^7)/
(560*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]*Sin[e + f*x])/(256*f) + (11
*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(384*f) + (11*a^3*(10*A
- 3*B)*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(480*f) - (a^3*B*Cos[e + f*x]^7*(c^
2 - c^2*Sin[e + f*x])^3)/(10*f) + (a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^3 - c
^3*Sin[e + f*x])^2)/(90*f) + (11*a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^6 - c^6
*Sin[e + f*x]))/(720*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+f$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{10f}$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{10f}$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{10f}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} - \frac{a^3B}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3}{}$$

$$= \frac{11}{256}a^3(10A-3B)c^6x + \frac{11a^3(10A-3B}{56}$$

Mathematica [A]
time = 2.62, size = 255, normalized size = 0.96

$$\frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6(27720(10A-3B)(e+fx)+5040(33A-19B)\cos(e+fx)+3360(29A-15B)\cos(3(e+fx))+10080(3A-B)\cos(5(e+fx))+360(9A+5B)\cos(7(e+fx))-280(A-3B)\cos(9(e+fx))+1260(144A-25B)\sin(2(e+fx))+2520(6A+7B)\sin(4(e+fx))-210(32A-51B)\sin(6(e+fx))-315(6A-5B)\sin(8(e+fx))-126B\sin(10(e+fx)))}{645120f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{12}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
6,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*x
) + 5040*(33*A - 19*B)*Cos[e + f*x] + 3360*(29*A - 15*B)*Cos[3*(e + f*x)] +
 10080*(3*A - B)*Cos[5*(e + f*x)] + 360*(9*A + 5*B)*Cos[7*(e + f*x)] - 280*
(A - 3*B)*Cos[9*(e + f*x)] + 1260*(144*A - 25*B)*Sin[2*(e + f*x)] + 2520*(6
*A + 7*B)*Sin[4*(e + f*x)] - 210*(32*A - 51*B)*Sin[6*(e + f*x)] - 315*(6*A
- 5*B)*Sin[8*(e + f*x)] - 126*B*Sin[10*(e + f*x)]))/(645120*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^12*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(249) = 498.
time = 0.49, size = 651, normalized size = 2.46 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^3*A*c^6*(f*x+e)-1/9*a^3*A*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+
48/35*sin(f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^6*(-1/10*(sin(f*x
+e)^9+9/8*sin(f*x+e)^7+21/16*sin(f*x+e)^5+105/64*sin(f*x+e)^3+315/128*sin(f
*x+e))*cos(f*x+e)+63/256*f*x+63/256*e)-8/7*B*a^3*c^6*(16/5+sin(f*x+e)^6+6/5
*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-6*B*a^3*c^6*(-1/6*(sin(f*x+e)^5+
5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*B*a^3*c^6
*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^6*(-1/8*(sin(f*x+
e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/12
8*f*x+35/128*e)+1/3*B*a^3*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*s
in(f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c^6*cos(f*x+e)+3*a^3*A*c^6
*cos(f*x+e)-3*B*a^3*c^6*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-8/3*a^3*
A*c^6*(2+sin(f*x+e)^2)*cos(f*x+e)+8*B*a^3*c^6*(-1/4*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+8*a^3*A*c^6*(-1/6*(sin(f*x+e)^5+5/4*sin(f*
x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*a^3*A*c^6*(8/3+sin(
f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-6*a^3*A*c^6*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(265) = 530$.
time = 0.30, size = 713, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -1/645120*(2048*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*A*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c^6 - 1720320*(cos(f*x +
e)^3 - 3*cos(f*x + e))*A*a^3*c^6 + 630*(128*sin(2*f*x + 2*e)^3 + 840*f*x +
840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A
*a^3*c^6 - 26880*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e)
 - 48*sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + sin(4*f*x + 4*e
) - 8*sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 6144*(35*c
os(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^5 - 420*cos(f*x + e)^
3 + 315*cos(f*x + e))*B*a^3*c^6 - 147456*(5*cos(f*x + e)^7 - 21*cos(f*x + e
)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^6 + 63*(32*sin(2*f*x +
2*e)^5 - 640*sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*sin(8*f*x + 8*e) -
 600*sin(4*f*x + 4*e) + 2560*sin(2*f*x + 2*e))*B*a^3*c^6 + 20160*(4*sin(2*f
```

```
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a
^3*c^6 - 161280*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a
^3*c^6 + 483840*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^6 - 1935360*A*a^3*
c^6*cos(f*x + e) + 645120*B*a^3*c^6*cos(f*x + e))/f
```

Fricas [**A**]
time = 0.40, size = 189, normalized size = 0.71

$$-\frac{8960\,(A-3\,B)a^3c^6\cos\left(fx+e\right)^9-46080\,(A-B)a^3c^6\cos\left(fx+e\right)^7-3465\,(10\,A-3\,B)a^3c^6fx+21\,\left(384\,Ba^3c^6\cos\left(fx+e\right)^9+48\,(30\,A-41\,B)a^3c^6\cos\left(fx+e\right)^7-88\,(10\,A-3\,B)a^3c^6\cos\left(fx+e\right)^5-110\,(10\,A-3\,B)a^3c^6\cos\left(fx+e\right)^3-165\,(10\,A-3\,B)a^3c^6\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{80640\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/80640*(8960*(A - 3*B)*a^3*c^6*cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*cos
(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*cos(f*x + e
)^9 + 48*(30*A - 41*B)*a^3*c^6*cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*cos
(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*cos(f*x + e)^3 - 165*(10*A - 3*B)*a^
3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. 2(252) = 504.
time = 2.13, size = 1948, normalized size = 7.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(e
 + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A*a
**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e + f*
x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**3*c*
*6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x)**2*
cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105
*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6/2 - 9*
A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e + f*x)*
*8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) -
8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*c**6*sin(e
 + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)**5*cos(e +
 f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) + 6*A*a*
*3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e + f*x)**3*co
s(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**3/(3*f
) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A*a**3*c**6*sin(
e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e + f*x)**2*cos(e +
f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f + 105*A*a**3*c**6*
```

```
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(e + f*x)*cos(e + f
*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 128*A*a**
3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(e + f*x)**5/(5*f) - 16*
A*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*A*a**3*c**6*cos(e + f*x)/f + 63*B*a**
3*c**6*x*sin(e + f*x)**10/256 + 315*B*a**3*c**6*x*sin(e + f*x)**8*cos(e + f
*x)**2/256 + 315*B*a**3*c**6*x*sin(e + f*x)**6*cos(e + f*x)**4/128 - 15*B*a
**3*c**6*x*sin(e + f*x)**6/8 + 315*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*
x)**6/128 - 45*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*B*a**3*c
**6*x*sin(e + f*x)**4 + 315*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**8/2
56 - 45*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/8 + 6*B*a**3*c**6*x*s
in(e + f*x)**2*cos(e + f*x)**2 - 3*B*a**3*c**6*x*sin(e + f*x)**2/2 + 63*B*a
**3*c**6*x*cos(e + f*x)**10/256 - 15*B*a**3*c**6*x*cos(e + f*x)**6/8 + 3*B*
a**3*c**6*x*cos(e + f*x)**4 - 3*B*a**3*c**6*x*cos(e + f*x)**2/2 - 193*B*a**
3*c**6*sin(e + f*x)**9*cos(e + f*x)/(256*f) + 3*B*a**3*c**6*sin(e + f*x)**8
*cos(e + f*x)/f - 237*B*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)**3/(128*f) +
 8*B*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/f - 8*B*a**3*c**6*sin(e + f*
x)**6*cos(e + f*x)/f - 21*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**5/(10*f
) + 33*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 48*B*a**3*c**6*sin(
e + f*x)**4*cos(e + f*x)**5/(5*f) - 16*B*a**3*c**6*sin(e + f*x)**4*cos(e +
f*x)**3/f + 6*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f - 147*B*a**3*c**6*
sin(e + f*x)**3*cos(e + f*x)**7/(128*f) + 5*B*a**3*c**6*sin(e + f*x)**3*cos
(e + f*x)**3/f - 5*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/f + 192*B*a**3*
c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 64*B*a**3*c**6*sin(e + f*x)**
2*cos(e + f*x)**5/(5*f) + 8*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**3/f -
 63*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**9/(256*f) + 15*B*a**3*c**6*sin(e
 + f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
f + 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)/(2*f) + 128*B*a**3*c**6*cos(e +
 f*x)**9/(105*f) - 128*B*a**3*c**6*cos(e + f*x)**7/(35*f) + 16*B*a**3*c**6*
cos(e + f*x)**5/(5*f) - B*a**3*c**6*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*si
n(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

<u>Giac</u> [A]
time = 0.49, size = 347, normalized size = 1.31

$$-\frac{Ba^3c^6\sin(10\,fx+10\,e)}{5120\,f}+\frac{11}{256}\,(10\,Aa^3c^6-3\,Ba^3c^6)x-\frac{(Aa^3c^6-3\,Ba^3c^6)\cos(9\,fx+9\,e)}{2304\,f}+\frac{(9\,Aa^3c^6+5\,Ba^3c^6)\cos(7\,fx+7\,e)}{1792\,f}+\frac{(3\,Aa^3c^6-Ba^3c^6)\cos(5\,fx+5\,e)}{64\,f}+\frac{(29\,Aa^3c^6-15\,Ba^3c^6)\cos(3\,fx+3\,e)}{192\,f}+\frac{(33\,Aa^3c^6-19\,Ba^3c^6)\cos(fx+e)}{128\,f}-\frac{(6\,Aa^3c^6-5\,Ba^3c^6)\sin(8\,fx+8\,e)}{2048\,f}-\frac{(32\,Aa^3c^6-51\,Ba^3c^6)\sin(6\,fx+6\,e)}{3072\,f}+\frac{(6\,Aa^3c^6+7\,Ba^3c^6)\sin(4\,fx+4\,e)}{256\,f}+\frac{(144\,Aa^3c^6-25\,Ba^3c^6)\sin(2\,fx+2\,e)}{512\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/5120*B*a^3*c^6*sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^6
)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*cos(9*f*x + 9*e)/f + 1/1792*(9*A*a^3
*c^6 + 5*B*a^3*c^6)*cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*cos
(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*cos(3*f*x + 3*e)/f +
1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*cos(f*x + e)/f - 1/2048*(6*A*a^3*c^6 -
```

```
5*B*a^3*c^6)*sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6)*sin(
6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*sin(4*f*x + 4*e)/f + 1/5
12*(144*A*a^3*c^6 - 25*B*a^3*c^6)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.87, size = 812, normalized size = 3.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^18*(6*A*a^3*c^6 - 2*B*a^3*c^6) + tan(e/2 + (f*x)/2)^16*
(22*A*a^3*c^6 - 18*B*a^3*c^6) + tan(e/2 + (f*x)/2)^8*(84*A*a^3*c^6 - 28*B*a
^3*c^6) + tan(e/2 + (f*x)/2)^14*((136*A*a^3*c^6)/3 - 8*B*a^3*c^6) + tan(e/2
 + (f*x)/2)^4*((136*A*a^3*c^6)/7 - (24*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^1
0*(116*A*a^3*c^6 - 60*B*a^3*c^6) - tan(e/2 + (f*x)/2)^19*((73*A*a^3*c^6)/64
 + (33*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^2*((202*A*a^3*c^6)/63 - (58*B*a
^3*c^6)/21) + tan(e/2 + (f*x)/2)^12*((328*A*a^3*c^6)/3 - 72*B*a^3*c^6) + ta
n(e/2 + (f*x)/2)^7*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) - tan(e/2 + (f
*x)/2)^13*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) + tan(e/2 + (f*x)/2)^6*
((456*A*a^3*c^6)/7 - (344*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^5*((449*A*a^3*
c^6)/48 - (577*B*a^3*c^6)/160) - tan(e/2 + (f*x)/2)^15*((449*A*a^3*c^6)/48
- (577*B*a^3*c^6)/160) + tan(e/2 + (f*x)/2)^3*((2117*A*a^3*c^6)/192 - (705*
B*a^3*c^6)/128) - tan(e/2 + (f*x)/2)^17*((2117*A*a^3*c^6)/192 - (705*B*a^3*
c^6)/128) + tan(e/2 + (f*x)/2)^9*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64)
 - tan(e/2 + (f*x)/2)^11*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64) + tan(e
/2 + (f*x)/2)*((73*A*a^3*c^6)/64 + (33*B*a^3*c^6)/128) + (58*A*a^3*c^6)/63
- (10*B*a^3*c^6)/21)/(f*(10*tan(e/2 + (f*x)/2)^2 + 45*tan(e/2 + (f*x)/2)^4
+ 120*tan(e/2 + (f*x)/2)^6 + 210*tan(e/2 + (f*x)/2)^8 + 252*tan(e/2 + (f*x)
/2)^10 + 210*tan(e/2 + (f*x)/2)^12 + 120*tan(e/2 + (f*x)/2)^14 + 45*tan(e/2
 + (f*x)/2)^16 + 10*tan(e/2 + (f*x)/2)^18 + tan(e/2 + (f*x)/2)^20 + 1)) + (
11*a^3*c^6*atan((11*a^3*c^6*tan(e/2 + (f*x)/2)*(10*A - 3*B))/(128*((55*A*a^
3*c^6)/64 - (33*B*a^3*c^6)/128)))*(10*A - 3*B))/(128*f)
```

3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=222

$$\frac{5}{128}a^3(9A-2B)c^5x+\frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f}+\frac{5a^3(9A-2B)c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{5a^3(9A-2B$$

```
[Out] 5/128*a^3*(9*A-2*B)*c^5*x+1/56*a^3*(9*A-2*B)*c^5*cos(f*x+e)^7/f+5/128*a^3*(
9*A-2*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(9*A-2*B)*c^5*cos(f*x+e)^3*s
in(f*x+e)/f+1/48*a^3*(9*A-2*B)*c^5*cos(f*x+e)^5*sin(f*x+e)/f-1/9*a^3*B*c^3*
cos(f*x+e)^7*(c-c*sin(f*x+e))^2/f+1/72*a^3*(9*A-2*B)*cos(f*x+e)^7*(c^5-c^5*
sin(f*x+e))/f
```

Rubi [A]
time = 0.22, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$\frac{a^3c^5(9A-2B)\cos^7(e+fx)}{56f}+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}+\frac{a^3c^5(9A-2B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^5(9A-2B)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5a^3c^5(9A-2B)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}a^3c^5x(9A-2B)-\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (5*a^3*(9*A - 2*B)*c^5*x)/128 + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^7)/(56*f)
 + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(9*A
- 2*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(9*A - 2*B)*c^5*Cos[
e + f*x]^5*Sin[e + f*x])/(48*f) - (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e +
f*x])^2)/(9*f) + (a^3*(9*A - 2*B)*Cos[e + f*x]^7*(c^5 - c^5*Sin[e + f*x]))/
(72*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+
$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx)}{9f}$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx)}{9f}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} - \frac{a^3Bc^3\,c}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{a^3(9A}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A}{}$$
$$= \frac{5}{128}a^3(9A-2B)c^5x + \frac{a^3(9A-2B)c^5}{56f}$$

Mathematica [A]
time = 1.61, size = 232, normalized size = 1.05

$$\frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5(2520(9A-2B)(e+fx)+504(20A-13B)\cos(e+fx)+336(18A-11B)\cos(3(e+fx))+1008(2A-B)\cos(5(e+fx))+36(8A-B)\cos(7(e+fx))+28B\cos(9(e+fx))+2016(8A-B)\sin(2(e+fx))+504(5A+2B)\sin(4(e+fx))+672B\sin(6(e+fx))-63(A-2B)\sin(8(e+fx)))}{64512f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x)
+ 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 100
8*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[9*(
e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e + f*
x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(208) = 416$.
time = 0.43, size = 611, normalized size = 2.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^3*A*c^5*(f*x+e)-2/7*B*a^3*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/
5*sin(f*x+e)^2)*cos(f*x+e)-2/7*a^3*A*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^
4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*B*a^3*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+
e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-a
^3*A*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(
f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+1/9*B*a^3*c^5*(128/35+sin(f*x+e)^8+
8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c^
5*cos(f*x+e)+2*a^3*A*c^5*cos(f*x+e)-2*B*a^3*c^5*(-1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-2*a^3*A*c^5*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2/3*
B*a^3*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^3*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+
e)+6*B*a^3*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e
)+6/5*a^3*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-6*B*a^3*c^5*
(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5
/16*e)+2*a^3*A*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*co
s(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(221) = 442$.
time = 0.30, size = 665, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] 1/322560*(18432*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^3*c^5 + 129024*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^3*c^5 + 645120*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^3*c^5 - 105*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e)
 + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*sin(2*f
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a
^3*c^5 - 161280*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*(f*x +
e)*A*a^3*c^5 + 1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x +
 e)^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^5 + 18432*(5*cos(f*x
 + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^
5 - 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^5 + 210*(128*sin(2*f*x
 + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 7
68*sin(2*f*x + 2*e))*B*a^3*c^5 - 10080*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^5 + 60480*(12*f*x + 1
2*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^5 - 161280*(2*f*x + 2*
e - sin(2*f*x + 2*e))*B*a^3*c^5 + 645120*A*a^3*c^5*cos(f*x + e) - 322560*B*
a^3*c^5*cos(f*x + e))/f
```

Fricas [A]

time = 0.41, size = 165, normalized size = 0.74

$$\frac{896\,Ba^3c^5\cos\left(fx+e\right)^9+2304\,(A-B)a^3c^5\cos\left(fx+e\right)^7+315\,(9\,A-2\,B)a^3c^5fx-21\,\left(48\,(A-2\,B)a^3c^5\cos\left(fx+e\right)^7-8\,(9\,A-2\,B)a^3c^5\cos\left(fx+e\right)^5-10\,(9\,A-2\,B)a^3c^5\cos\left(fx+e\right)^3-15\,(9\,A-2\,B)a^3c^5\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8064\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7
+ 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 - 8
*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^3
 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(209) = 418$.
time = 1.58, size = 1753, normalized size = 7.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e +
 f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a**
3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f*x)
**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32
 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*sin(e
 + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*cos(e +
 f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a**3*c**5
*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)**6*cos(e
 + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*A*
a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(e + f*x)**
4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*
a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c**5*sin(e + f
*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f - 6*A*a**3*c**5*
sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**
7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) + A*a**3*c**5*
sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)**7/(35*f) + 16*A*
a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f*x)**3/f + 2*A*a**
3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8/64 + 35*B*a**3*c**
5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c**5*x*sin(e + f*x)**6/8
 + 105*B*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/32 - 45*B*a**3*c**5*x*
sin(e + f*x)**4*cos(e + f*x)**2/8 + 9*B*a**3*c**5*x*sin(e + f*x)**4/4 + 35*
B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/16 - 45*B*a**3*c**5*x*sin(e +
```

```
f*x)**2*cos(e + f*x)**4/8 + 9*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**
2/2 - B*a**3*c**5*x*sin(e + f*x)**2 + 35*B*a**3*c**5*x*cos(e + f*x)**8/64 -
 15*B*a**3*c**5*x*cos(e + f*x)**6/8 + 9*B*a**3*c**5*x*cos(e + f*x)**4/4 - B
*a**3*c**5*x*cos(e + f*x)**2 + B*a**3*c**5*sin(e + f*x)**8*cos(e + f*x)/f -
 93*B*a**3*c**5*sin(e + f*x)**7*cos(e + f*x)/(64*f) + 8*B*a**3*c**5*sin(e +
 f*x)**6*cos(e + f*x)**3/(3*f) - 2*B*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)
/f - 511*B*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(192*f) + 33*B*a**3*c*
*5*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 16*B*a**3*c**5*sin(e + f*x)**4*cos(
e + f*x)**5/(5*f) - 4*B*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f - 385*B
*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(192*f) + 5*B*a**3*c**5*sin(e +
f*x)**3*cos(e + f*x)**3/f - 15*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)/(4*
f) + 64*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 16*B*a**3*c**5
*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**5*sin(e + f*x)**2*cos(
e + f*x)/f - 35*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**7/(64*f) + 15*B*a**3
*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) - 9*B*a**3*c**5*sin(e + f*x)*cos(e
 + f*x)**3/(4*f) + B*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f + 128*B*a**3*c**
5*cos(e + f*x)**9/(315*f) - 32*B*a**3*c**5*cos(e + f*x)**7/(35*f) + 4*B*a**
3*c**5*cos(e + f*x)**3/(3*f) - B*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(A
 + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**5, True))
```

Giac [A]
time = 0.48, size = 301, normalized size = 1.36

$$\frac{Ba^3c^5\cos(9fx+9e)}{2304f}+\frac{Ba^3c^5\sin(6fx+6e)}{96f}+\frac{5}{128}(9Aa^3c^5-2Ba^3c^5)x+\frac{(8Aa^3c^5-Ba^3c^5)\cos(7fx+7e)}{1792f}+\frac{(2Aa^3c^5-Ba^3c^5)\cos(5fx+5e)}{64f}+\frac{(18Aa^3c^5-11Ba^3c^5)\cos(3fx+3e)}{192f}+\frac{(20Aa^3c^5-13Ba^3c^5)\cos(fx+e)}{128f}-\frac{(Aa^3c^5-2Ba^3c^5)\sin(8fx+8e)}{1024f}+\frac{(5Aa^3c^5+2Ba^3c^5)\sin(4fx+4e)}{128f}+\frac{(8Aa^3c^5-Ba^3c^5)\sin(2fx+2e)}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] 1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f + 5
/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos(7
*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/192*(
18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 - 13*
B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x + 8*
e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8*A*a^3
*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.93, size = 705, normalized size = 3.18

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}(4Aa^3c^5-2Ba^3c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}(8Aa^3c^5-8Ba^3c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{8Aa^3c^5}{7}-\frac{8Ba^3c^5}{7}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}(32Aa^3c^5-4Ba^3c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}(24Aa^3c^5-24Ba^3c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(24Aa^3c^5-\frac{16Ba^3c^5}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(40Aa^3c^5-40Ba^3c^5)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\left(\frac{88Aa^3c^5}{7}-\frac{32Ba^3c^5}{7}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{17}\left(\frac{83Aa^3c^5}{64}+\frac{5Ba^3c^5}{32}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\left(\frac{149Aa^3c^5}{32}+\frac{83Ba^3c^5}{16}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{13}\left(\frac{149Aa^3c^5}{32}+\frac{83Ba^3c^5}{16}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\left(\frac{189Aa^3c^5}{32}-\frac{191Ba^3c^5}{48}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{15}\left(\frac{189Aa^3c^5}{32}-\frac{191Ba^3c^5}{48}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}\left(\frac{409Aa^3c^5}{32}-\frac{145Ba^3c^5}{16}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}\left(\frac{409Aa^3c^5}{32}-\frac{145Ba^3c^5}{16}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{83Aa^3c^5}{64}+\frac{5Ba^3c^5}{32}\right)+\frac{4Aa^3c^5}{7}-\frac{22Ba^3c^5}{63}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{18}+9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}+36\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+84\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+126\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+126\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+84\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+36\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)}+\frac{5a^3c^5\,\mathrm{atan}\left(\frac{5a^3c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(9A-2B)}{64\left(\frac{45Aa^3c^5}{64}-\frac{5Ba^3c^5}{32}\right)}\right)(9A-2B)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^16*(4*A*a^3*c^5 - 2*B*a^3*c^5) + tan(e/2 + (f*x)/2)^14*
(8*A*a^3*c^5 - 8*B*a^3*c^5) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c^5)/7 - (8*B*
a^3*c^5)/7) + tan(e/2 + (f*x)/2)^8*(32*A*a^3*c^5 - 4*B*a^3*c^5) + tan(e/2 +
 (f*x)/2)^6*(24*A*a^3*c^5 - 24*B*a^3*c^5) + tan(e/2 + (f*x)/2)^12*(24*A*a^3
*c^5 - (16*B*a^3*c^5)/3) + tan(e/2 + (f*x)/2)^10*(40*A*a^3*c^5 - 40*B*a^3*c
^5) + tan(e/2 + (f*x)/2)^4*((88*A*a^3*c^5)/7 - (32*B*a^3*c^5)/7) - tan(e/2
+ (f*x)/2)^17*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + tan(e/2 + (f*x)/2)^5
*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^13*((149*A*a
^3*c^5)/32 + (83*B*a^3*c^5)/16) + tan(e/2 + (f*x)/2)^3*((189*A*a^3*c^5)/32
- (191*B*a^3*c^5)/48) - tan(e/2 + (f*x)/2)^15*((189*A*a^3*c^5)/32 - (191*B*
a^3*c^5)/48) + tan(e/2 + (f*x)/2)^7*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/1
6) - tan(e/2 + (f*x)/2)^11*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/16) + tan(
e/2 + (f*x)/2)*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + (4*A*a^3*c^5)/7 - (
22*B*a^3*c^5)/63)/(f*(9*tan(e/2 + (f*x)/2)^2 + 36*tan(e/2 + (f*x)/2)^4 + 84
*tan(e/2 + (f*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^8 + 126*tan(e/2 + (f*x)/2)^1
0 + 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^14 + 9*tan(e/2 + (f*x)
/2)^16 + tan(e/2 + (f*x)/2)^18 + 1)) + (5*a^3*c^5*atan((5*a^3*c^5*tan(e/2 +
 (f*x)/2)*(9*A - 2*B))/(64*((45*A*a^3*c^5)/64 - (5*B*a^3*c^5)/32)))*(9*A -
2*B))/(64*f)
```

3.40 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=181

$$\frac{5}{128}a^3(8A-B)c^4x+\frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{5a^3(8A-B)c^4\cos(e+fx)\sin(e+fx)}{128f}+\frac{5a^3(8A-B)c^4\cos$$

```
[Out] 5/128*a^3*(8*A-B)*c^4*x+1/56*a^3*(8*A-B)*c^4*cos(f*x+e)^7/f+5/128*a^3*(8*A-
B)*c^4*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(8*A-B)*c^4*cos(f*x+e)^3*sin(f*x+e
)/f+1/48*a^3*(8*A-B)*c^4*cos(f*x+e)^5*sin(f*x+e)/f-1/8*a^3*B*cos(f*x+e)^7*(
c^4-c^4*sin(f*x+e))/f
```

Rubi [A]
time = 0.17, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2748, 2715, 8}

$$\frac{a^3c^4(8A-B)\cos^7(e+fx)}{56f}+\frac{a^3c^4(8A-B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^4(8A-B)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5a^3c^4(8A-B)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}a^3c^4x(8A-B)-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*(8*A - B)*c^4*x)/128 + (a^3*(8*A - B)*c^4*Cos[e + f*x]^7)/(56*f) + (
5*a^3*(8*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(8*A - B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(8*A - B)*c^4*Cos[e + f*x]^5
*Sin[e + f*x])/(48*f) - (a^3*B*Cos[e + f*x]^7*(c^4 - c^4*Sin[e + f*x]))/(8*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx\right.}{8f}$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} - \frac{a^3B\cos^7}{}$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} + \frac{a^3(8A-}{}$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} + \frac{5a^3(8A}{}$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} + \frac{5a^3(8A}{}$$

$$= \frac{5}{128}a^3(8A-B)c^4x + \frac{a^3(8A-B)c^4\cos}{56f}$$

Mathematica [A]
time = 1.20, size = 209, normalized size = 1.15

$$\frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4(840(8A-B)(e+fx)+1680(A-B)\cos(e+fx)+1008(A-B)\cos(3(e+fx))+336(A-B)\cos(5(e+fx))+48(A-B)\cos(7(e+fx))+336(15A-B)\sin(2(e+fx))+168(6A+B)\sin(4(e+fx))+112(A+B)\sin(6(e+fx))+21B\sin(8(e+fx)))}{21504f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) + 1
680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Cos[
5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f*x)
] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*B*Si
n[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(169) = 338$.
time = 0.35, size = 568, normalized size = 3.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^3*A*c^4*(f*x+e)+3/5*a^3*A*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*co
s(f*x+e)-3*B*a^3*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*
cos(f*x+e)+5/16*f*x+5/16*e)-a^3*A*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+
15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/7*B*a^3*c^4*(16/5+sin(f*x+e)
^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-1/7*a^3*A*c^4*(16/5+sin(f*
x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^4*(-1/8*(sin(f
*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35
/128*f*x+35/128*e)-B*a^3*c^4*cos(f*x+e)+a^3*A*c^4*cos(f*x+e)-B*a^3*c^4*(-1/
2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3*a^3*A*c^4*(-1/2*cos(f*x+e)*sin(f*x
+e)+1/2*f*x+1/2*e)+B*a^3*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^4*(2+sin(f
*x+e)^2)*cos(f*x+e)+3*B*a^3*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x
+e)+3/8*f*x+3/8*e)+3*a^3*A*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)-3/5*B*a^3*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(179) = 358$.
time = 0.30, size = 615, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
```

```
15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3
*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c
^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^
5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f*x + e)^
5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*
c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*x + e))/f
```

Fricas [A]
time = 0.39, size = 143, normalized size = 0.79

$$\frac{384\,(A-B)a^3c^4\cos(fx+e)^7+105\,(8\,A-B)a^3c^4fx+7\,\left(48\,Ba^3c^4\cos(fx+e)^7+8\,(8\,A-B)a^3c^4\cos(fx+e)^5+10\,(8\,A-B)a^3c^4\cos(fx+e)^3+15\,(8\,A-B)a^3c^4\cos(fx+e)\right)\sin(fx+e)}{2688\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7*
(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8*A
 - B)*a^3*c^4*cos(f*x + e)^3 + 15*(8*A - B)*a^3*c^4*cos(f*x + e))*sin(f*x +
 e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(163) = 326$.
time = 1.17, size = 1579, normalized size = 8.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*cos(e
 + f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f +
 11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*sin(e +
 f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c**4*sin(e
```

```
 + f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A*a**3*c**4*
sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**5
/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**3*c**4*
sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)**7/(35*f) + 8
*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f*x)**3/f + A*a*
*3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/128 + 35*B*a**3*c
**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*x*sin(e + f*x)**6
/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 45*B*a**3*c**4
*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*sin(e + f*x)**4/8 +
 35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 45*B*a**3*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 - B*a**3*c**4*x*sin(e + f*x)**2/2 + 35*B*a**3*c**4*x*cos(e + f*x)*
*8/128 - 15*B*a**3*c**4*x*cos(e + f*x)**6/16 + 9*B*a**3*c**4*x*cos(e + f*x)
**4/8 - B*a**3*c**4*x*cos(e + f*x)**2/2 - 93*B*a**3*c**4*sin(e + f*x)**7*co
s(e + f*x)/(128*f) + B*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f - 511*B*a**
3*c**4*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 33*B*a**3*c**4*sin(e + f*x
)**5*cos(e + f*x)/(16*f) + 2*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
- 3*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 385*B*a**3*c**4*sin(e + f*
x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)*
*3/(2*f) - 15*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*B*a**3*c**
4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**3/f + 3*B*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 35*B*a**3*c
**4*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 15*B*a**3*c**4*sin(e + f*x)*cos(
e + f*x)**5/(16*f) - 9*B*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + B*a
**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*B*a**3*c**4*cos(e + f*x)**7/(
35*f) - 8*B*a**3*c**4*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**4*cos(e + f*x)**3
/f - B*a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3*(-c*sin(e) + c)**4, True))
```

<u>Giac</u> [A]
time = 0.48, size = 273, normalized size = 1.51

$$\frac{Ba^3c^4\sin\left(8\,fx+8\,e\right)}{1024\,f}+\frac{5}{128}\left(8\,Aa^3c^4-Ba^3c^4\right)x+\frac{\left(Aa^3c^4-Ba^3c^4\right)\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{\left(Aa^3c^4-Ba^3c^4\right)\cos\left(5\,fx+5\,e\right)}{64\,f}+\frac{3\left(Aa^3c^4-Ba^3c^4\right)\cos\left(3\,fx+3\,e\right)}{64\,f}+\frac{5\left(Aa^3c^4-Ba^3c^4\right)\cos\left(fx+e\right)}{64\,f}+\frac{\left(Aa^3c^4+Ba^3c^4\right)\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{\left(6\,Aa^3c^4+Ba^3c^4\right)\sin\left(4\,fx+4\,e\right)}{128\,f}+\frac{\left(15\,Aa^3c^4-Ba^3c^4\right)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x + 1
/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3*c
^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f +
5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*c^4)
*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)/f +
1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.78, size = 661, normalized size = 3.65

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(6\,A\,a^3\,c^4-6\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(2\,A\,a^3\,c^4-2\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(6\,A\,a^3\,c^4-6\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}\,\left(2\,A\,a^3\,c^4-2\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{2\,A\,a^3\,c^4}{7}-\frac{2\,B\,a^3\,c^4}{7}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(10\,A\,a^3\,c^4-10\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}\,\left(10\,A\,a^3\,c^4-10\,B\,a^3\,c^4\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}\,\left(\frac{11\,A\,a^3\,c^4}{8}+\frac{5\,B\,a^3\,c^4}{64}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{61\,A\,a^3\,c^4}{24}-\frac{397\,B\,a^3\,c^4}{192}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}\,\left(\frac{61\,A\,a^3\,c^4}{24}-\frac{397\,B\,a^3\,c^4}{192}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(\frac{113\,A\,a^3\,c^4}{24}+\frac{895\,B\,a^3\,c^4}{192}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}\,\left(\frac{113\,A\,a^3\,c^4}{24}+\frac{895\,B\,a^3\,c^4}{192}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(\frac{85\,A\,a^3\,c^4}{24}-\frac{1765\,B\,a^3\,c^4}{192}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\,\left(\frac{85\,A\,a^3\,c^4}{24}-\frac{1765\,B\,a^3\,c^4}{192}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{11\,A\,a^3\,c^4}{8}+\frac{5\,B\,a^3\,c^4}{64}\right)+\frac{2\,A\,a^3\,c^4}{7}-\frac{2\,B\,a^3\,c^4}{7}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{16}+8\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}+28\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+56\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+70\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+56\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+28\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+8\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{5\,a^3\,c^4\,\mathrm{atan}\left(\frac{5\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,A-B\right)}{64\,\left(\frac{5\,A\,a^3\,c^4}{8}-\frac{5\,B\,a^3\,c^4}{64}\right)}\right)\,\left(8\,A-B\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^4*(6*A*a^3*c^4 - 6*B*a^3*c^4) + tan(e/2 + (f*x)/2)^12*(
2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)^6*(6*A*a^3*c^4 - 6*B*a^3*c^
4) + tan(e/2 + (f*x)/2)^14*(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)
^2*((2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7) + tan(e/2 + (f*x)/2)^8*(10*A*a^3*c^4
 - 10*B*a^3*c^4) + tan(e/2 + (f*x)/2)^10*(10*A*a^3*c^4 - 10*B*a^3*c^4) - ta
n(e/2 + (f*x)/2)^15*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + tan(e/2 + (f*x)
/2)^3*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^13*((6
1*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^5*((113*A*a^3*c
^4)/24 + (895*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^11*((113*A*a^3*c^4)/24 +
 (895*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^7*((85*A*a^3*c^4)/24 - (1765*B*a
^3*c^4)/192) - tan(e/2 + (f*x)/2)^9*((85*A*a^3*c^4)/24 - (1765*B*a^3*c^4)/1
92) + tan(e/2 + (f*x)/2)*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + (2*A*a^3*c
^4)/7 - (2*B*a^3*c^4)/7)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e/2 + (f*x)/2)
^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*tan(e/2 + (f*x)
/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*
x)/2)^16 + 1)) + (5*a^3*c^4*atan((5*a^3*c^4*tan(e/2 + (f*x)/2)*(8*A - B))/(
64*((5*A*a^3*c^4)/8 - (5*B*a^3*c^4)/64)))*(8*A - B))/(64*f)
```

3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=117

$$\frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos(e+fx)\sin(e+fx)}{16f}+\frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3Ac^3\,\mathrm{c}}{}$$

```
[Out] 5/16*a^3*A*c^3*x-1/7*a^3*B*c^3*cos(f*x+e)^7/f+5/16*a^3*A*c^3*cos(f*x+e)*sin
(f*x+e)/f+5/24*a^3*A*c^3*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*A*c^3*cos(f*x+e)
^5*sin(f*x+e)/f
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2748, 2715, 8}

$$\frac{a^3Ac^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3Ac^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3Ac^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*A*c^3*x)/16 - (a^3*B*c^3*Cos[e + f*x]^7)/(7*f) + (5*a^3*A*c^3*Cos[e
+ f*x]*Sin[e + f*x])/(16*f) + (5*a^3*A*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(24
*f) + (a^3*A*c^3*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = (a^3c^3)\int \cos^6(e+fx)(A+B\sin(e+$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + (a^3Ac^3)\int \cos$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{a^3Ac^3\cos^5(e+$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos^3(e$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos(e-$$
$$= \frac{5}{16}a^3Ac^3x - \frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3$$

Mathematica [A]
time = 0.15, size = 64, normalized size = 0.55

$$\frac{a^3c^3(-192B\cos^7(e+fx)+7A(60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))))}{1344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)]
+ 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(107) = 214$.
time = 0.20, size = 263, normalized size = 2.25

method	result

risch	$\frac{5a^3Ac^3x}{16} - \frac{5B\,a^3c^3\cos(fx+e)}{64f} - \frac{B\,a^3c^3\cos(7fx+7e)}{448f} + \frac{a^3Ac^3\sin(6fx+6e)}{192f} - \frac{B\,a^3c^3\cos(5fx+5e)}{64f} + \frac{3a^3Ac^3\,\mathrm{s}}{\,}$
derivativedivides	$\frac{\frac{B\,a^3c^3\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}-a^3Ac^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)}{6}\right.}{\,}$
default	$\frac{\frac{B\,a^3c^3\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}-a^3Ac^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)}{6}\right.}{\,}$
norman	$\frac{-\frac{2B\,a^3c^3}{7f}-\frac{6B\,a^3c^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{10B\,a^3c^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2B\,a^3c^3\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{5a^3Ac^3x}{16}+\frac{11a^3Ac^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+}{\,}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(1/7*B*a^3*c^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*co
s(f*x+e)-a^3*A*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*co
s(f*x+e)+5/16*f*x+5/16*e)-3/5*B*a^3*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)
*cos(f*x+e)+3*a^3*A*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*
f*x+3/8*e)+B*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^3*(-1/2*cos(f*x+
e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^3*cos(f*x+e)+a^3*A*c^3*(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(114) = 228$.
time = 0.29, size = 284, normalized size = 2.43

$$-\frac{35\left(4\sin(2fx+2e)^3+60fx+60e+9\sin(4fx+4e)-48\sin(2fx+2e)\right)Aa^3c^3-630\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)Aa^3c^3+5040\left(2fx+2e-\sin(2fx+2e)\right)Aa^3c^3-6720(fx+e)Aa^3c^3+192\left(5\cos(fx+e)^7-21\cos(fx+e)^5+35\cos(fx+e)^3-35\cos(fx+e)\right)Ba^3c^3+1344\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)Ba^3c^3+6720\left(\cos(fx+e)^3-3\cos(fx+e)\right)Ba^3c^3+6720Ba^3c^3\cos(fx+e)}{6720f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^3
 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 3
5*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f
```

Fricas [A]
time = 0.38, size = 97, normalized size = 0.83

$$-\frac{48 \, B a^{3} c^{3} \cos \left(f x + e\right)^{7} - 105 \, A a^{3} c^{3} f x - 7 \, {\left(8 \, A a^{3} c^{3} \cos \left(f x + e\right)^{5} + 10 \, A a^{3} c^{3} \cos \left(f x + e\right)^{3} + 15 \, A a^{3} c^{3} \cos \left(f x + e\right)\right)} \sin \left(f x + e\right)}{336 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*co
s(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(116) = 232$.
time = 0.72, size = 682, normalized size = 5.83

$$\begin{cases} - \frac{5 A a^{3} c^{3} x \sin^{6}{\left(e + f x \right)}}{16} - \frac{15 A a^{3} c^{3} x \sin^{4}{\left(e + f x \right)} \cos^{2}{\left(e + f x \right)}}{16} + \frac{9 A a^{3} c^{3} x \sin^{4}{\left(e + f x \right)}}{8} - \frac{15 A a^{3} c^{3} x \sin^{2}{\left(e + f x \right)} \cos^{4}{\left(e + f x \right)}}{16} + \frac{9 A a^{3} c^{3} x \sin^{2}{\left(e + f x \right)} \cos^{2}{\left(e + f x \right)}}{4} - \frac{3 A a^{3} c^{3} x \sin^{2}{\left(e + f x \right)}}{2} - \frac{5 A a^{3} c^{3} x \cos^{6}{\left(e + f x \right)}}{16} + \frac{9 A a^{3} c^{3} x \cos^{4}{\left(e + f x \right)}}{8} - \frac{3 A a^{3} c^{3} x \cos^{2}{\left(e + f x \right)}}{2} + A a^{3} c^{3} x + \frac{11 A a^{3} c^{3} \sin^{5}{\left(e + f x \right)} \cos{\left(e + f x \right)}}{16 f} + \frac{5 A a^{3} c^{3} \sin^{3}{\left(e + f x \right)} \cos^{3}{\left(e + f x \right)}}{6 f} - \frac{15 A a^{3} c^{3} \sin^{3}{\left(e + f x \right)} \cos{\left(e + f x \right)}}{8 f} + \frac{5 A a^{3} c^{3} \sin{\left(e + f x \right)} \cos^{5}{\left(e + f x \right)}}{16 f} - \frac{9 A a^{3} c^{3} \sin{\left(e + f x \right)} \cos^{3}{\left(e + f x \right)}}{8 f} + \frac{3 A a^{3} c^{3} \sin{\left(e + f x \right)} \cos{\left(e + f x \right)}}{2 f} + \frac{B a^{3} c^{3} \sin^{6}{\left(e + f x \right)} \cos{\left(e + f x \right)}}{f} + \frac{2 B a^{3} c^{3} \sin^{4}{\left(e + f x \right)} \cos^{3}{\left(e + f x \right)}}{f} - \frac{3 B a^{3} c^{3} \sin^{4}{\left(e + f x \right)} \cos{\left(e + f x \right)}}{f} + \frac{8 B a^{3} c^{3} \sin^{2}{\left(e + f x \right)} \cos^{5}{\left(e + f x \right)}}{5 f} - \frac{4 B a^{3} c^{3} \sin^{2}{\left(e + f x \right)} \cos^{3}{\left(e + f x \right)}}{f} + \frac{3 B a^{3} c^{3} \sin^{2}{\left(e + f x \right)} \cos{\left(e + f x \right)}}{f} + \frac{16 B a^{3} c^{3} \cos^{7}{\left(e + f x \right)}}{35 f} - \frac{8 B a^{3} c^{3} \cos^{5}{\left(e + f x \right)}}{5 f} + \frac{2 B a^{3} c^{3} \cos^{3}{\left(e + f x \right)}}{f} - \frac{B a^{3} c^{3} \cos{\left(e + f x \right)}}{f} & \text{for}\: f \neq 0 \\x \left(A + B \sin{\left(e \right)}\right) \left(a \sin{\left(e \right)} + a\right)^{3} \left(- c \sin{\left(e \right)} + c\right)^{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*cos(e
 + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/
(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c*
*3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*cos(e +
f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**
3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)**6*cos(e
+ f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)
**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f + 3*B*a**3*c**3
*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)**7/(35*f) - 8
*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f*x)**3/f - B*a*
*3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*
sin(e) + c)**3, True))
```

Giac [A]
time = 0.49, size = 162, normalized size = 1.38

$$\frac{5}{16} \, A a^{3} c^{3} x - \frac{B a^{3} c^{3} \cos \left(7 \, f x + 7 \, e\right)}{448 \, f} - \frac{B a^{3} c^{3} \cos \left(5 \, f x + 5 \, e\right)}{64 \, f} - \frac{3 \, B a^{3} c^{3} \cos \left(3 \, f x + 3 \, e\right)}{64 \, f} - \frac{5 \, B a^{3} c^{3} \cos \left(f x + e\right)}{64 \, f} + \frac{A a^{3} c^{3} \sin \left(6 \, f x + 6 \, e\right)}{192 \, f} + \frac{3 \, A a^{3} c^{3} \sin \left(4 \, f x + 4 \, e\right)}{64 \, f} + \frac{15 \, A a^{3} c^{3} \sin \left(2 \, f x + 2 \, e\right)}{64 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos(
5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f*x
 + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x + 4
*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f
```

Mupad [B]

time = 14.29, size = 325, normalized size = 2.78

$$\frac{5\,A\,a^3\,c^3\,x}{16}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(\frac{a^3\,c^3\,\left(672\,B-735\,A\,\left(e+f\,x\right)\right)}{336}+\frac{35\,A\,a^3\,c^3\,\left(e+f\,x\right)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{a^3\,c^3\,\left(2016\,B-2205\,A\,\left(e+f\,x\right)\right)}{336}+\frac{105\,A\,a^3\,c^3\,\left(e+f\,x\right)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{a^3\,c^3\,\left(3360\,B-3675\,A\,\left(e+f\,x\right)\right)}{336}+\frac{175\,A\,a^3\,c^3\,\left(e+f\,x\right)}{16}\right)+\frac{a^3\,c^3\,\left(96\,B-105\,A\,\left(e+f\,x\right)\right)}{336}+\frac{5\,A\,a^3\,c^3\,\left(e+f\,x\right)}{16}-\frac{7\,A\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6}-\frac{85\,A\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{24}+\frac{85\,A\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{24}+\frac{7\,A\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{6}+\frac{11\,A\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}-\frac{11\,A\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*A*a^3*c^3*x)/16 - (tan(e/2 + (f*x)/2)^12*((a^3*c^3*(672*B - 735*A*(e + f
*x)))/336 + (35*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^3*(
2016*B - 2205*A*(e + f*x)))/336 + (105*A*a^3*c^3*(e + f*x))/16) + tan(e/2 +
 (f*x)/2)^8*((a^3*c^3*(3360*B - 3675*A*(e + f*x)))/336 + (175*A*a^3*c^3*(e
+ f*x))/16) + (a^3*c^3*(96*B - 105*A*(e + f*x)))/336 + (5*A*a^3*c^3*(e + f*
x))/16 - (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^3)/6 - (85*A*a^3*c^3*tan(e/2 + (f*
x)/2)^5)/24 + (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (7*A*a^3*c^3*tan(e/2
 + (f*x)/2)^11)/6 + (11*A*a^3*c^3*tan(e/2 + (f*x)/2)^13)/8 - (11*A*a^3*c^3*
tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=138

$$\frac{1}{16}a^3(6A+B)c^2x-\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(6A+B)c^2\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^3(6A+B)c^2\cos^3(\ldots}{24\ldots}$$

```
[Out] 1/16*a^3*(6*A+B)*c^2*x-1/30*a^3*(6*A+B)*c^2*cos(f*x+e)^5/f+1/16*a^3*(6*A+B)
*c^2*cos(f*x+e)*sin(f*x+e)/f+1/24*a^3*(6*A+B)*c^2*cos(f*x+e)^3*sin(f*x+e)/f
-1/6*B*c^2*cos(f*x+e)^5*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2748, 2715, 8}

$$-\frac{a^3c^2(6A+B)\cos^5(e+fx)}{30f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^3c^2x(6A+B)-\frac{Bc^2\cos^5(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(6*A + B)*c^2*x)/16 - (a^3*(6*A + B)*c^2*Cos[e + f*x]^5)/(30*f) + (a^3
*(6*A + B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^3*(6*A + B)*c^2*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (B*c^2*Cos[e + f*x]^5*(a^3 + a^3*Sin[e + f
*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx)$$

$$= -\frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} - \frac{Bc^2\cos^5}{}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} + \frac{a^3(6A+}{}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} + \frac{a^3(6A+}{}$$

$$= \frac{1}{16}a^3(6A+B)c^2x - \frac{a^3(6A+B)c^2\cos^5(}{30f}$$

Mathematica [A]

time = 0.67, size = 133, normalized size = 0.96

$$\frac{a^3c^2(360Ae+60Be+360Afx+60Bfx-120(A+B)\cos(e+fx)-60(A+B)\cos(3(e+fx))-12A\cos(5(e+fx))-12B\cos(5(e+fx))+240A\sin(2(e+fx))+15B\sin(2(e+fx))+30A\sin(4(e+fx))-15B\sin(4(e+fx))-5B\sin(6(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*x
] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
```

```
*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(128) = 256$.
time = 0.21, size = 364, normalized size = 2.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(-2*a^3*A*c^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^3*A*c^2*cos(
f*x+e)+a^3*A*c^2*(f*x+e)+2/3*B*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^
2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^2*cos(f*x+e)+2/3*a^3*A
*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-2*B*a^3*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*
x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^3*A*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^3*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)
^2)*cos(f*x+e)-1/5*a^3*A*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)
+B*a^3*c^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)
+5/16*f*x+5/16*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(135) = 270$.
time = 0.29, size = 387, normalized size = 2.80

$$-\frac{64\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))Aa^3c^2+640\,(\cos(fx+e)^3-3\,\cos(fx+e))Aa^3c^2-30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Aa^3c^2+480\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^3c^2-960\,(fx+e)Aa^3c^2+64\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))Ba^3c^2+640\,(\cos(fx+e)^3-3\,\cos(fx+e))Ba^3c^2-5\,(4\,\sin(2\,fx+2\,e)^3+60\,fx+60\,e+9\,\sin(4\,fx+4\,e)-48\,\sin(2\,fx+2\,e))Ba^3c^2+60\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Ba^3c^2-240\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Ba^3c^2+960\,Aa^3c^2\cos(fx+e)+960\,Ba^3c^2\cos(fx+e)}{960\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c
^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 - 30*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2 + 480*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a^3*c^2 - 960*(f*x + e)*A*a^3*c^2 + 64*(3*cos(f*x + e)^5 - 1
0*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2 + 640*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^2 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*
x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^2 + 60*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e)
)*B*a^3*c^2 + 960*A*a^3*c^2*cos(f*x + e) + 960*B*a^3*c^2*cos(f*x + e))/f
```

Fricas [A]
time = 0.38, size = 111, normalized size = 0.80

$$-\frac{48\,(A+B)a^3c^2\cos(fx+e)^5-15\,(6\,A+B)a^3c^2fx+5\,\left(8\,Ba^3c^2\cos(fx+e)^5-2\,(6\,A+B)a^3c^2\cos(fx+e)^3-3\,(6\,A+B)a^3c^2\cos(fx+e)\right)\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(8
*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A + B
)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(128) = 256$.
time = 0.57, size = 910, normalized size = 6.59

$$\begin{cases} \frac{3Aa^3c^2x\sin^4(e+fx)}{8} + \frac{3Aa^3c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^3c^2x\sin^2(e+fx) + \frac{3Aa^3c^2x\cos^4(e+fx)}{8} - Aa^3c^2x\cos^2(e+fx) + Aa^3c^2x - \frac{Aa^3c^2\sin^4(e+fx)\cos(e+fx)}{f} - \frac{5Aa^3c^2\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{4Aa^3c^2\sin^2(e+fx)\cos^3(e+fx)}{3f} + \frac{2Aa^3c^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Aa^3c^2\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{Aa^3c^2\sin(e+fx)\cos(e+fx)}{f} - \frac{8Aa^3c^2\cos^5(e+fx)}{15f} + \frac{4Aa^3c^2\cos^3(e+fx)}{3f} - \frac{Aa^3c^2\cos(e+fx)}{f} + \frac{5Ba^3c^2x\sin^6(e+fx)}{16} + \cdots & \text{for}\: f \neq 0 \\ x(A+B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*cos
(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*c**
2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e + f*
x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a**3*c
**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2*cos(e
+ f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**2*cos(e
+ f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15*B*a**3*c*
*2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c**2*x*cos(e
+ f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2*x*cos(e + f*
x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**3*c**2*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**3*c**
2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*sin(e + f*x)**2*cos
(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*B*a**3*
c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*B*a**3*c**2*cos(e + f
*x)**3/(3*f) - B*a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**3*(-c*sin(e) + c)**2, True))
```

Giac [A]
time = 0.53, size = 204, normalized size = 1.48

$$-\frac{Ba^3c^2\sin(6\,fx+6\,e)}{192\,f} + \frac{1}{16}\,(6\,Aa^3c^2 + Ba^3c^2)x - \frac{(Aa^3c^2 + Ba^3c^2)\cos(5\,fx+5\,e)}{80\,f} - \frac{(Aa^3c^2 + Ba^3c^2)\cos(3\,fx+3\,e)}{16\,f} - \frac{(Aa^3c^2 + Ba^3c^2)\cos(fx+e)}{8\,f} + \frac{(2\,Aa^3c^2 - Ba^3c^2)\sin(4\,fx+4\,e)}{64\,f} + \frac{(16\,Aa^3c^2 + Ba^3c^2)\sin(2\,fx+2\,e)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*c^2*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^3*c^2 + B*a^3*c^2)*x - 1/
80*(A*a^3*c^2 + B*a^3*c^2)*cos(5*f*x + 5*e)/f - 1/16*(A*a^3*c^2 + B*a^3*c^2
)*cos(3*f*x + 3*e)/f - 1/8*(A*a^3*c^2 + B*a^3*c^2)*cos(f*x + e)/f + 1/64*(2
*A*a^3*c^2 - B*a^3*c^2)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^3*c^2 + B*a^3*c^2
)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.17, size = 536, normalized size = 3.88

$$\frac{a^3\,c^2\,\mathrm{atan}\left(\frac{a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,A+B\right)}{8\,\left(\frac{3\,A\,a^3\,c^2}{4}+\frac{B\,a^3\,c^2}{8}\right)}\right)\,\left(6\,A+B\right)}{8\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(4\,A\,a^3\,c^2+4\,B\,a^3\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(2\,A\,a^3\,c^2+2\,B\,a^3\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(4\,A\,a^3\,c^2+4\,B\,a^3\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(2\,A\,a^3\,c^2+2\,B\,a^3\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{2\,A\,a^3\,c^2}{5}+\frac{2\,B\,a^3\,c^2}{5}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(\frac{A\,a^3\,c^2}{2}-\frac{13\,B\,a^3\,c^2}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(\frac{A\,a^3\,c^2}{2}-\frac{13\,B\,a^3\,c^2}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}\,\left(\frac{5\,A\,a^3\,c^2}{4}-\frac{B\,a^3\,c^2}{8}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{7\,A\,a^3\,c^2}{4}+\frac{47\,B\,a^3\,c^2}{24}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9\,\left(\frac{7\,A\,a^3\,c^2}{4}+\frac{47\,B\,a^3\,c^2}{24}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,A\,a^3\,c^2}{4}-\frac{B\,a^3\,c^2}{8}\right)+\frac{2\,A\,a^3\,c^2}{5}+\frac{2\,B\,a^3\,c^2}{5}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+20\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}-\frac{a^3\,c^2\,\left(6\,A+B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a^3*c^2*atan((a^3*c^2*tan(e/2 + (f*x)/2)*(6*A + B))/(8*((3*A*a^3*c^2)/4 +
(B*a^3*c^2)/8)))*(6*A + B))/(8*f) - (tan(e/2 + (f*x)/2)^4*(4*A*a^3*c^2 + 4*
B*a^3*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2 + (
f*x)/2)^6*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^10*(2*A*a^3*c^2
 + 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^2*((2*A*a^3*c^2)/5 + (2*B*a^3*c^2)/5) -
 tan(e/2 + (f*x)/2)^5*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/
2)^7*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^11*((5*A*a^3*c
^2)/4 - (B*a^3*c^2)/8) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c^2)/4 + (47*B*a^3*
c^2)/24) + tan(e/2 + (f*x)/2)^9*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/24) - tan
(e/2 + (f*x)/2)*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) + (2*A*a^3*c^2)/5 + (2*B*
a^3*c^2)/5)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e
/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e
/2 + (f*x)/2)^12 + 1)) - (a^3*c^2*(6*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

3.43 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=140

$$\frac{1}{8}a^3(5A+2B)cx-\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f}+\frac{a^3(5A+2B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{aBc\cos^3(e+fx)(a+$$

```
[Out] 1/8*a^3*(5*A+2*B)*c*x-1/12*a^3*(5*A+2*B)*c*cos(f*x+e)^3/f+1/8*a^3*(5*A+2*B)
*c*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^2/f-1/20
*(5*A+2*B)*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3046, 2939, 2757, 2748, 2715, 8}

$$-\frac{a^3c(5A+2B)\cos^3(e+fx)}{12f}-\frac{c(5A+2B)\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{20f}+\frac{a^3c(5A+2B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^3cx(5A+2B)-\frac{aBc\cos^3(e+fx)(a\sin(e+fx)+a)^2}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(5*A + 2*B)*c*x)/8 - (a^3*(5*A + 2*B)*c*Cos[e + f*x]^3)/(12*f) + (a^3*
(5*A + 2*B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(a +
 a*Sin[e + f*x])^2)/(5*f) - ((5*A + 2*B)*c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e
+ f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f}$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f}$$

$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} - \frac{aBc\cos}{}$$

$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} + \frac{a^3(5A}{}$$

$$= \frac{1}{8}a^3(5A+2B)cx - \frac{a^3(5A+2B)c\cos^3(}{12f}$$

Mathematica [A]

time = 0.56, size = 95, normalized size = 0.68

$$\frac{a^3c(-60(4A+3B)\cos(e+fx)-10(8A+5B)\cos(3(e+fx))+6B\cos(5(e+fx))+15(4(5A+2B)fx+8A\sin(2(e+fx))-(A+2B)\sin(4(e+fx))))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a^3*c*(-60*(4*A + 3*B)*Cos[e + f*x] - 10*(8*A + 5*B)*Cos[3*(e + f*x)] + 6*
B*Cos[5*(e + f*x)] + 15*(4*(5*A + 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A + 2*
B)*Sin[4*(e + f*x)])))/(480*f)
```

Maple [A]
time = 0.17, size = 208, normalized size = 1.49

method	result
risch	$\frac{5a^3cxA}{8}+\frac{a^3cxB}{4}-\frac{a^3c\cos(fx+e)A}{2f}-\frac{3a^3c\cos(fx+e)B}{8f}+\frac{Ba^3c\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)a^3Ac}{32f}-\frac{\sin(4fx}{}$
derivativedivides	$\frac{-2a^3Ac\cos(fx+e)+a^3Ac(fx+e)+2Ba^3c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Ba^3c\cos(fx+e)+\frac{2a^3Ac\left(2+\sin^2(fx+e)\right)\cos(f}{3}}{}$
default	$\frac{-2a^3Ac\cos(fx+e)+a^3Ac(fx+e)+2Ba^3c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Ba^3c\cos(fx+e)+\frac{2a^3Ac\left(2+\sin^2(fx+e)\right)\cos(f}{3}}{}$
norman	$\frac{(\frac{5}{8}a^3Ac+\frac{1}{4}Ba^3c)x+(\frac{5}{8}a^3Ac+\frac{1}{4}Ba^3c)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}a^3Ac+\frac{5}{2}Ba^3c)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}a^3Ac+\frac{5}{2}Ba^3c)x(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(-2*a^3*A*c*cos(f*x+e)+a^3*A*c*(f*x+e)+2*B*a^3*c*(-1/2*cos(f*x+e)*sin(f
*x+e)+1/2*f*x+1/2*e)-B*a^3*c*cos(f*x+e)+2/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*
x+e)-2*B*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e
)-a^3*A*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/5
*B*a^3*c*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A]
time = 0.28, size = 216, normalized size = 1.54

$$-\frac{320\left(\cos(fx+e)^3-3\cos(fx+e)\right)Aa^3c+15\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)Aa^3c-480(fx+e)Aa^3c-32\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)Ba^3c+30\left(12fx+12e+\sin(4fx+4e)-8\sin(2fx+2e)\right)Ba^3c-240\left(2fx+2e-\sin(2fx+2e)\right)Ba^3c+960Aa^3c\cos(fx+e)+480Ba^3c\cos(fx+e)}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c - 3
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*(12
*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f*x +
 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^3*c*c
os(f*x + e))/f
```

Fricas [A]
time = 0.36, size = 105, normalized size = 0.75

$$\frac{24\,Ba^3c\cos\left(fx+e\right)^5-80\,(A+B)a^3c\cos\left(fx+e\right)^3+15\,(5\,A+2\,B)a^3cfx-15\,\left(2\,(A+2\,B)a^3c\cos\left(fx+e\right)^3-(5\,A+2\,B)a^3c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5*
A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a^3
*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(128) = 256$.
time = 0.36, size = 486, normalized size = 3.47

$$\begin{cases} -\frac{3Aa^3cx\sin^4(e+fx)}{8}-\frac{3Aa^3cx\sin^2(e+fx)\cos^2(e+fx)}{4}-\frac{3Aa^3cx\cos^4(e+fx)}{8}+Aa^3cx+\frac{5Aa^3c\sin^3(e+fx)\cos(e+fx)}{8f}+\frac{2Aa^3c\sin^2(e+fx)\cos(e+fx)}{f}+\frac{3Aa^3c\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{4Aa^3c\cos^3(e+fx)}{3f}-\frac{2Aa^3c\cos(e+fx)}{f}-\frac{3Ba^3cx\sin^4(e+fx)}{4}-\frac{3Ba^3cx\sin^2(e+fx)\cos^2(e+fx)}{2}+Ba^3cx\sin^2(e+fx)-\frac{3Ba^3cx\cos^4(e+fx)}{4}+Ba^3cx\cos^2(e+fx)+\frac{Ba^3c\sin^4(e+fx)\cos(e+fx)}{f}+\frac{5Ba^3c\sin^3(e+fx)\cos(e+fx)}{4f}+\frac{4Ba^3c\sin^2(e+fx)\cos^3(e+fx)}{3f}+\frac{3Ba^3c\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{Ba^3c\sin(e+fx)\cos(e+fx)}{f}+\frac{8Ba^3c\cos^5(e+fx)}{15f}-\frac{Ba^3c\cos(e+fx)}{f} & \text{for } f\neq 0 \\ x(A+B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3*c
*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e + f
*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**4/4 -
 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e + f*x)**
2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*a**3*c*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*c*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
*3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))
```

Giac [A]
time = 0.57, size = 145, normalized size = 1.04

$$\frac{Ba^3c\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{Aa^3c\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(5\,Aa^3c+2\,Ba^3c\right)x-\frac{\left(8\,Aa^3c+5\,Ba^3c\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(4\,Aa^3c+3\,Ba^3c\right)\cos\left(fx+e\right)}{8\,f}-\frac{\left(Aa^3c+2\,Ba^3c\right)\sin\left(4\,fx+4\,e\right)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f - 1
/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*sin(
4*f*x + 4*e)/f
```

Mupad [B]
time = 13.66, size = 390, normalized size = 2.79

$$\frac{a^3\,c\,\mathrm{atan}\left(\frac{a^3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,A+2\,B\right)}{4\,\left(\frac{5\,A\,a^3\,c}{4}+\frac{B\,a^3\,c}{2}\right)}\right)\,\left(5\,A+2\,B\right)}{4\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(4\,A\,a^3\,c+2\,B\,a^3\,c\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,A\,a^3\,c}{4}-\frac{B\,a^3\,c}{2}\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{7\,A\,a^3\,c}{2}+3\,B\,a^3\,c\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(\frac{7\,A\,a^3\,c}{2}+3\,B\,a^3\,c\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9\,\left(\frac{3\,A\,a^3\,c}{4}-\frac{B\,a^3\,c}{2}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(8\,A\,a^3\,c+8\,B\,a^3\,c\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{8\,A\,a^3\,c}{3}+\frac{8\,B\,a^3\,c}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{16\,A\,a^3\,c}{3}+\frac{4\,B\,a^3\,c}{3}\right)+\frac{4\,A\,a^3\,c}{3}+\frac{14\,B\,a^3\,c}{15}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}-\frac{a^3\,c\,\left(5\,A+2\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^3*c*atan((a^3*c*tan(e/2 + (f*x)/2)*(5*A + 2*B))/(4*((5*A*a^3*c)/4 + (B*a
^3*c)/2)))*(5*A + 2*B))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^3*c + 2*B*a^3*
c) - tan(e/2 + (f*x)/2)*((3*A*a^3*c)/4 - (B*a^3*c)/2) - tan(e/2 + (f*x)/2)^
3*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^7*((7*A*a^3*c)/2 + 3*B*a
^3*c) + tan(e/2 + (f*x)/2)^9*((3*A*a^3*c)/4 - (B*a^3*c)/2) + tan(e/2 + (f*x
)/2)^6*(8*A*a^3*c + 8*B*a^3*c) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c)/3 + (8*B
*a^3*c)/3) + tan(e/2 + (f*x)/2)^4*((16*A*a^3*c)/3 + (4*B*a^3*c)/3) + (4*A*a
^3*c)/3 + (14*B*a^3*c)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/
2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2
)^10 + 1)) - (a^3*c*(5*A + 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$-\frac{5a^3(3A+4B)x}{2c}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf}+\frac{a^3(A+B)c^3\cos^7}{f(c-c\sin(e+}$$

```
[Out] -5/2*a^3*(3*A+4*B)*x/c+5/3*a^3*(3*A+4*B)*cos(f*x+e)^3/c/f-5/2*a^3*(3*A+4*B)
*cos(f*x+e)*sin(f*x+e)/c/f+a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^4+
2*a^3*(3*A+4*B)*c^3*cos(f*x+e)^5/f/(c^2-c^2*sin(f*x+e))^2
```

Rubi [A]
time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f\,(c^2-c^2\sin(e+fx))^2}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\sin(e+fx)\cos(e+fx)}{2cf}-\frac{5a^3x(3A+4B)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-5*a^3*(3*A + 4*B)*x)/(2*c) + (5*a^3*(3*A + 4*B)*Cos[e + f*x]^3)/(3*c*f) -
 (5*a^3*(3*A + 4*B)*Cos[e + f*x]*Sin[e + f*x])/(2*c*f) + (a^3*(A + B)*c^3*C
os[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) + (2*a^3*(3*A + 4*B)*c^3*Cos[e +
f*x]^5)/(f*(c^2 - c^2*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} - \left(a^3(3A+4B)c^2\right)\int \frac{\cos^6(e}{(c-c\sin} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2} \\
&= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \\
&= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+fx)\,\mathrm{si}}{2cf} \\
&= -\frac{5a^3(3A+4B)x}{2c} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A}{}
\end{aligned}$$

Mathematica [A]

time = 0.92, size = 223, normalized size = 1.43

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A+4B)(e+fx)-3(16A+31B)\cos(e+fx)+B\cos(3(e+fx))-3(A+4B)\sin(2(e+fx)))-\sin\left(\frac{1}{2}(e+fx)\right)(24B(8+5e+5fx)+6A(32+15e+15fx)-3(16A+31B)\cos(e+fx)+B\cos(3(e+fx))-3(A+4B)\sin(2(e+fx)))\right)}{12cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3*(
e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 + 5*e
 + 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

Maple [A]
time = 0.29, size = 152, normalized size = 0.97

method	result
derivativedivides	$\frac{2a^3\left(-\frac{\left(\frac{A}{2}+2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}-2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{fc}$
default	$\frac{2a^3\left(-\frac{\left(\frac{A}{2}+2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}-2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{fc}$
risch	$-\frac{15a^3xA}{2c}-\frac{10a^3xB}{c}+\frac{2a^3e^{i(fx+e)}A}{cf}+\frac{31a^3e^{i(fx+e)}B}{8cf}+\frac{2a^3e^{-i(fx+e)}A}{cf}+\frac{31a^3e^{-i(fx+e)}B}{8cf}+\frac{16a^3A}{fc(e^{i(fx+e)}}$
norman	$\frac{-\frac{17a^3A+20Ba^3}{cf}+\frac{5a^3(3A+4B)x}{2c}-\frac{\left(5a^3A+2Ba^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(17a^3A+18Ba^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(21a^3A+34Ba^3\right)\left(\tan^9\right.}{3cf}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a^3/c*(-((1/2*A+2*B)*tan(1/2*f*x+1/2*e)^5+(-4*A-7*B)*tan(1/2*f*x+1/2*e)
^4+(-8*A-16*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A-2*B)*tan(1/2*f*x+1/2*e)-4*A-23/
3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3-5/2*(3*A+4*B)*arctan(tan(1/2*f*x+1/2*e))-(8
*A+8*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(159) = 318$.
time = 0.52, size = 1241, normalized size = 7.96

$$-\frac{Ba^3\left(\frac{\frac{7\sin(fx+e)}{\cos(fx+e)+1}-\frac{39\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{24\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{24\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{9\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-16}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{3c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{c\sin(fx+e)^7}{(\cos(fx+e)+1)^7}}+\frac{9\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+18Aa^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+18Ba^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+3Aa^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-4}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+9Ba^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-4}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+18Aa^3\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)+6Ba^3\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{6Aa^3}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*a^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a
^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((sin(f*x + e)/(c
os(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*s
in(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/c) + 9*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
 + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 6*B*a^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*a^3/(c - c*sin(f*x + e)/(cos(f*x +
 e) + 1)))/f
```

<u>Fricas</u> [A]
time = 0.37, size = 228, normalized size = 1.46

$$-\frac{2\,Ba^3\cos\left(fx+e\right)^4-(3\,A+10\,B)a^3\cos\left(fx+e\right)^3+15\,(3\,A+4\,B)a^3fx-24\,(A+2\,B)a^3\cos\left(fx+e\right)^2-48\,(A+B)a^3+3\,(5\,(3\,A+4\,B)a^3fx-(23\,A+28\,B)a^3)\cos\left(fx+e\right)-\left(2\,Ba^3\cos\left(fx+e\right)^3+15\,(3\,A+4\,B)a^3fx+3\,(A+4\,B)a^3\cos\left(fx+e\right)^2-3\,(7\,A+12\,B)a^3\cos\left(fx+e\right)+48\,(A+B)a^3\right)\sin\left(fx+e\right)}{6\,(cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*a^3*cos(f*x + e)^4 - (3*A + 10*B)*a^3*cos(f*x + e)^3 + 15*(3*A +
4*B)*a^3*f*x - 24*(A + 2*B)*a^3*cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3*A
 + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*cos(f*x + e) - (2*B*a^3*cos(f*x + e)^3
```

```
 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*cos(f*x + e)^2 - 3*(7*A + 12*B)
*a^3*cos(f*x + e) + 48*(A + B)*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*s
in(f*x + e) + c*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. $2(144) = 288$.
time = 4.62, size = 4255, normalized size = 27.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f
*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*
tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 + f*x/2)
**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x
*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6
 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2
 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f)
 - 135*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*ta
n(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4
 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
+ f*x/2) - 6*c*f) + 135*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x
/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*ta
n(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*tan(e/2 + f*x/2)/(6*c*f*
tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**
5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/
2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x/(6*c*f*tan(
e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 102*A*a**3*tan(e/2 + f*x/2)**
6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 +
 f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c
*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 54*A*a**3*tan(e/
2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c
*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/
2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 336*
```

```
A*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x
/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*t
an(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
6*c*f) + 96*A*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 378*A*a**3*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 42*A*a**3*tan(e/2 + f*x/2)/(6*c*f*tan(e/2
 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*
c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x
/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 144*A*a**3/(6*c*f*tan(e/2 + f*x/
2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan
(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
+ 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*
f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)
**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(
e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 60*B*a**3*f*x*tan(e/2 +
 f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*
tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)*
*3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 180*B*a
**3*f*x*tan(e/2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*
x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*
tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6
*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*
x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*t
an(e/2 + f*x/2) - 6*c*f) - 180*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)
**2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/...
```

Giac [A]
time = 0.52, size = 234, normalized size = 1.50

$$-\frac{\frac{15\,(3\,Aa^3+4\,Ba^3)(fx+e)}{c}+\frac{96\,(Aa^3+Ba^3)}{c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-1)}+\frac{2\left(3\,Aa^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+12\,Ba^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-24\,Aa^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4-42\,Ba^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4-48\,Aa^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-96\,Ba^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-3\,Aa^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-12\,Ba^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-24\,Aa^3-46\,Ba^3\right)}{\left(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+1\right)^3c}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(tan(1/2*f
*x + 1/2*e) - 1)) + 2*(3*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*tan(1/2*f*
x + 1/2*e)^5 - 24*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*tan(1/2*f*x + 1/2
*e)^4 - 48*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*a^3*tan(1/2*f*x + 1/2*e) - 12*B*a^3*tan(1/2*f*x + 1/2*e) - 24*A*a^3 -
46*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f
```

Mupad [B]
time = 14.12, size = 323, normalized size = 2.07

$$\frac{24\,A\,a^3-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\,A\,a^3+\frac{34\,B\,a^3}{3}\right)+\frac{94\,B\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(9\,A\,a^3+18\,B\,a^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(17\,A\,a^3+20\,B\,a^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(16\,A\,a^3+32\,B\,a^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(56\,A\,a^3+62\,B\,a^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(63\,A\,a^3+76\,B\,a^3\right)}{f\,\left(-c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}-\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,A+4\,B\right)}{15\,A\,a^3+20\,B\,a^3}\right)\,\left(3\,A+4\,B\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x)),x)
```

```
[Out] (24*A*a^3 - tan(e/2 + (f*x)/2)*(7*A*a^3 + (34*B*a^3)/3) + (94*B*a^3)/3 - ta
n(e/2 + (f*x)/2)^5*(9*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^6*(17*A*a^3 +
20*B*a^3) - tan(e/2 + (f*x)/2)^3*(16*A*a^3 + 32*B*a^3) + tan(e/2 + (f*x)/2)
^4*(56*A*a^3 + 62*B*a^3) + tan(e/2 + (f*x)/2)^2*(63*A*a^3 + 76*B*a^3))/(f*(
c - c*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 - 3*c*tan(e/2 + (f*x)/2
)^3 + 3*c*tan(e/2 + (f*x)/2)^4 - 3*c*tan(e/2 + (f*x)/2)^5 + c*tan(e/2 + (f*
x)/2)^6 - c*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*
(3*A + 4*B))/(15*A*a^3 + 20*B*a^3))*(3*A + 4*B))/(c*f)
```

3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=163

$$\frac{5a^3(2A+5B)x}{2c^2}-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}-\frac{5a^3}{6}$$

```
[Out] 5/2*a^3*(2*A+5*B)*x/c^2-5/2*a^3*(2*A+5*B)*cos(f*x+e)/c^2/f+1/3*a^3*(A+B)*c^
3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^5-2/3*a^3*(2*A+5*B)*c*cos(f*x+e)^5/f/(c-c
*sin(f*x+e))^3-5/6*a^3*(2*A+5*B)*cos(f*x+e)^3/f/(c^2-c^2*sin(f*x+e))
```

Rubi [A]

time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2758, 2761, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{5a^3x(2A+5B)}{2c^2}-\frac{2a^3c(2A+5B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (5*a^3*(2*A + 5*B)*x)/(2*c^2) - (5*a^3*(2*A + 5*B)*Cos[e + f*x])/(2*c^2*f)
+ (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - (2*a^3*(2
*A + 5*B)*c*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^3) - (5*a^3*(2*A + 5*
B)*Cos[e + f*x]^3)/(6*f*(c^2 - c^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{1}{3}\left(a^3(2A+5B)c^2\right)\int \frac{\cos^6}{(c-c\,\mathrm{si}}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3}$$
$$= -\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}$$
$$= \frac{5a^3(2A+5B)x}{2c^2} - \frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B}{3f(c-c}$$

Mathematica [A]
time = 0.61, size = 280, normalized size = 1.72

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(32(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+30(2A+5B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-12(A+5B)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-32(7A+13B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)-3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sin(2(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

Maple [A]
time = 0.33, size = 166, normalized size = 1.02

method	result
derivativedivides	$\frac{2a^3\left(\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A+5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right.}{f\,c^2}$

default	$\frac{2a^3\left(\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A+5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{f\,c^2}$
risch	$\frac{5a^3xA}{c^2}+\frac{25a^3xB}{2c^2}+\frac{iB\,a^3\mathrm{e}^{2i(fx+e)}}{8c^2f}-\frac{a^3\mathrm{e}^{i(fx+e)}A}{2c^2f}-\frac{5a^3\mathrm{e}^{i(fx+e)}B}{2c^2f}-\frac{a^3\mathrm{e}^{-i(fx+e)}A}{2c^2f}-\frac{5a^3\mathrm{e}^{-i(fx+e)}B}{2c^2f}-$
norman	$\frac{\frac{\left(8a^3A+25B\,a^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{46a^3A+118B\,a^3}{3cf}-\frac{5a^3(2A+5B)x}{2c}-\frac{\left(34a^3A+77B\,a^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(38a^3A+93B\,a^3\right)\tan}{cf}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^2*((1/2*B*tan(1/2*f*x+1/2*e)^3+(-A-5*B)*tan(1/2*f*x+1/2*e)^2-1/2*
B*tan(1/2*f*x+1/2*e)-A-5*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A+5*B)*arctan
(tan(1/2*f*x+1/2*e))-(-4*A-12*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(16*A+16*B)/(ta
n(1/2*f*x+1/2*e)-1)^3-1/2*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1504 vs. $2(163) = 326$.
time = 0.55, size = 1504, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*a^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) - 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 7*c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*c^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/c^2) + 4*A*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/c^2) + 12*B*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x
```

```
 + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^
2) + 6*A*a^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 6*B*a^3*((9*sin(f*x + e
)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*
c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c^2) - 2*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3) + 6*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 -
 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*a^3*(3*sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3))/f
```

Fricas [**A**]
time = 0.37, size = 298, normalized size = 1.83

$$\frac{3\,Ba^3\cos\left(fx+e\right)^4-6\,(A+4\,B)a^3\cos\left(fx+e\right)^3-30\,(2\,A+5\,B)a^3fx-16\,(A+B)a^3+(15\,(2\,A+5\,B)a^3fx+(62\,A+131\,B)a^3)\cos\left(fx+e\right)^2-(15\,(2\,A+5\,B)a^3fx-2\,(26\,A+71\,B)a^3)\cos\left(fx+e\right)-(3\,Ba^3\cos\left(fx+e\right)^3-30\,(2\,A+5\,B)a^3fx+3\,(2\,A+9\,B)a^3\cos\left(fx+e\right)^2+16\,(A+B)a^3-(15\,(2\,A+5\,B)a^3fx-2\,(34\,A+79\,B)a^3)\cos\left(fx+e\right))\sin\left(fx+e\right)}{6\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+(c^2f\cos\left(fx+e\right)+2\,c^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5*
B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^3)
*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x +
e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a^3*c
os(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 79*B)*
a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(151) = 302$.
time = 8.98, size = 4665, normalized size = 28.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((30*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7
- 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f
*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 +
f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 90*A*a**3*f*x*tan(e/2
+ f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 +
 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*
tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f
*x/2) - 6*c**2*f) + 150*A*a**3*f*x*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 +
f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 -
 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*
tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 210*A*a**3*f
*x*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 +
f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 +
 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*
tan(e/2 + f*x/2) - 6*c**2*f) + 210*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f
*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 +
f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 1
50*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f
*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 +
f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 +
 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 90*A*a**3*f*x*tan(e/2 + f*x/2)/(6
*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan
(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/
2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2
*f) - 30*A*a**3*f*x/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x
/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42
*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan
(e/2 + f*x/2) - 6*c**2*f) + 48*A*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2
 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**
5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2
*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 204*A*a**
3*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f
*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 +
42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*t
an(e/2 + f*x/2) - 6*c**2*f) + 212*A*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(
e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2
)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c
**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 432*A*
a**3*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
+ f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4
 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*
f*tan(e/2 + f*x/2) - 6*c**2*f) + 256*A*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*t
an(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
```

```
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 228
*A*a**3*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
+ f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4
 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*
f*tan(e/2 + f*x/2) - 6*c**2*f) + 92*A*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 -
18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*t
an(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*
x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 75*B*a**3*f*x*tan(e/2 +
f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 3
0*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*ta
n(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x
/2) - 6*c**2*f) - 225*B*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*
x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 4
2*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*ta
n(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 375*B*a**3*f*x
*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*
x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 4
2*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*ta
n(e/2 + f*x/2) - 6*c**2*f) - 525*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*t
an(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/...
```

Giac [A]
time = 0.56, size = 233, normalized size = 1.43

$$\frac{\frac{15\,(2\,Aa^3+5\,Ba^3)(fx+e)}{c^2}+\frac{6\left(Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-10\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Aa^3-10\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c^2}+\frac{16\left(3\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-24\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,Aa^3+11\,Ba^3\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a^3
*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*e)^2
 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A*a^3
+ 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B]
time = 14.00, size = 341, normalized size = 2.09

$$\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A+5\,B)}{10\,A\,a^3+25\,B\,a^3}\right)\,(2\,A+5\,B)}{c^2\,f}-\frac{\frac{46\,A\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(38\,A\,a^3+93\,B\,a^3)+\frac{118\,B\,a^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6(8\,A\,a^3+25\,B\,a^3)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5(34\,A\,a^3+77\,B\,a^3)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3(72\,A\,a^3+166\,B\,a^3)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{106\,A\,a^3}{3}+\frac{328\,B\,a^3}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{128\,A\,a^3}{3}+\frac{359\,B\,a^3}{3}\right)}{f\,\left(-c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-5\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+7\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-7\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+5\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(2*A + 5*B))/(10*A*a^3 + 25*B*a^3))*(
2*A + 5*B))/(c^2*f) - ((46*A*a^3)/3 - tan(e/2 + (f*x)/2)*(38*A*a^3 + 93*B*a
^3) + (118*B*a^3)/3 + tan(e/2 + (f*x)/2)^6*(8*A*a^3 + 25*B*a^3) - tan(e/2 +
 (f*x)/2)^5*(34*A*a^3 + 77*B*a^3) - tan(e/2 + (f*x)/2)^3*(72*A*a^3 + 166*B*
a^3) + tan(e/2 + (f*x)/2)^4*((106*A*a^3)/3 + (328*B*a^3)/3) + tan(e/2 + (f*
x)/2)^2*((128*A*a^3)/3 + (359*B*a^3)/3))/(f*(5*c^2*tan(e/2 + (f*x)/2)^2 - 7
*c^2*tan(e/2 + (f*x)/2)^3 + 7*c^2*tan(e/2 + (f*x)/2)^4 - 5*c^2*tan(e/2 + (f
*x)/2)^5 + 3*c^2*tan(e/2 + (f*x)/2)^6 - c^2*tan(e/2 + (f*x)/2)^7 + c^2 - 3*
c^2*tan(e/2 + (f*x)/2)))
```

3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{a^3(A+6B)x}{c^3}+\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}+\frac{2a^3(A+$$

```
[Out] -a^3*(A+6*B)*x/c^3+a^3*(A+6*B)*cos(f*x+e)/c^3/f+1/5*a^3*(A+B)*c^3*cos(f*x+e
)^7/f/(c-c*sin(f*x+e))^6-2/15*a^3*(A+6*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))
^4+2/3*a^3*(A+6*B)*c^3*cos(f*x+e)^3/f/(c^3-c^3*sin(f*x+e))^2
```

Rubi [A]

time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2759, 2761, 8}

$$\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}+\frac{2a^3c^3(A+6B)\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{a^3x(A+6B)}{c^3}-\frac{2a^3c(A+6B)\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(A + 6*B)*x)/c^3) + (a^3*(A + 6*B)*Cos[e + f*x])/(c^3*f) + (a^3*(A +
 B)*c^3*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - (2*a^3*(A + 6*B)*c*C
os[e + f*x]^5)/(15*f*(c - c*Sin[e + f*x])^4) + (2*a^3*(A + 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(c^3 - c^3*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{1}{5}\left(a^3(A+6B)c^2\right)\int \frac{\cos^6}{(c-c\mathrm{s}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}$$

$$= \frac{a^3(A+6B)\cos(e+fx)}{c^3f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2}{}$$

$$= -\frac{a^3(A+6B)x}{c^3} + \frac{a^3(A+6B)\cos(e+fx)}{c^3f} + \frac{a^3(A+B)c}{5f(c-c\mathrm{s}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(153) = 306.
time = 0.73, size = 316, normalized size = 2.07

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(11A+21B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-15(A+6B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+15B\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+48(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-8(11A+21B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+4(23A+93B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B*Co
s[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e + f*
x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*
x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*
(c - c*Sin[e + f*x])^3)
```

Maple [A]
time = 0.44, size = 157, normalized size = 1.03

method	result
derivativedivides	$\frac{2a^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4}{3(\tan(}\right.}{f\,c^3}$
default	$\frac{2a^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4}{3(\tan(}\right.}{f\,c^3}$
risch	$-\frac{a^3xA}{c^3}-\frac{6a^3xB}{c^3}+\frac{B\,a^3\mathrm{e}^{i(fx+e)}}{2c^3f}+\frac{B\,a^3\mathrm{e}^{-i(fx+e)}}{2c^3f}+\frac{-\frac{112A\,a^3\mathrm{e}^{2i(fx+e)}}{3}-24iA\,a^3\mathrm{e}^{3i(fx+e)}+\frac{56iA\,a^3\mathrm{e}^{i(fx+e)}}{3}+12}{}$
norman	$\frac{-\frac{52a^3A+282B\,a^3}{15cf}-\frac{71a^3(A+6B)x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{84a^3(A+6B)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{84a^3(A+6B)x\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{71a^3(A+6B}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^3*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A+6*B)*arctan(tan(1/2*f*x+1/2*e))-
(2*A+6*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(8*A-8*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5
*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(40*A+24*B)/(tan(1/2*f*x+1/2*e)-1
)^3-1/4*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1831 vs. 2(157) = 314.
time = 0.61, size = 1831, normalized size = 11.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*a^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*c^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1
1*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c^3) + A*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/c^3) + A*a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 9*A*a^3*(5*sin(f*x + e)/(cos(
f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 3*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) +
6*A*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5) + 6*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(157) = 314$.
time = 0.37, size = 351, normalized size = 2.29

$$\frac{15\,Ba^{3}\cos\left(fx+e\right)^{4}+60\,(A+6\,B)a^{3}fx-24\,(A+B)a^{3}-\left(15\,(A+6\,B)a^{3}fx-(46\,A+231\,B)a^{3}\right)\cos\left(fx+e\right)^{3}-\left(45\,(A+6\,B)a^{3}fx+2\,(A+66\,B)a^{3}\right)\cos\left(fx+e\right)^{2}+6\left(5\,(A+6\,B)a^{3}fx-2\,(6\,A+31\,B)a^{3}\right)\cos\left(fx+e\right)-\left(15\,Ba^{3}\cos\left(fx+e\right)^{3}+60\,(A+6\,B)a^{3}fx+24\,(A+B)a^{3}-\left(15\,(A+6\,B)a^{3}fx+2\,(23\,A+108\,B)a^{3}\right)\cos\left(fx+e\right)^{2}+6\left(5\,(A+6\,B)a^{3}fx-2\,(4\,A+29\,B)a^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\left(c^{3}f\cos\left(fx+e\right)^{3}+3\,c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)-4\,c^{3}f-\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)-4\,c^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*a^3*cos(f*x + e)^4 + 60*(A + 6*B)*a^3*f*x - 24*(A + B)*a^3 - (15
*(A + 6*B)*a^3*f*x - (46*A + 231*B)*a^3)*cos(f*x + e)^3 - (45*(A + 6*B)*a^3
*f*x + 2*(A + 66*B)*a^3)*cos(f*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(6*A +
 31*B)*a^3)*cos(f*x + e) - (15*B*a^3*cos(f*x + e)^3 + 60*(A + 6*B)*a^3*f*x
+ 24*(A + B)*a^3 - (15*(A + 6*B)*a^3*f*x + 2*(23*A + 108*B)*a^3)*cos(f*x +
e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(4*A + 29*B)*a^3)*cos(f*x + e))*sin(f*x +
 e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e)
- 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x
 + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.
time = 17.18, size = 4665, normalized size = 30.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*A*a**3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**
7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c*
*3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(
e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*A*a**3*f*x*t
an(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x
/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 +
225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) - 165*A*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**
3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/
2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2
)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**
3*f) + 225*A*a**3*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 -
75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f
*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2
+ f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 225*A*a**3*f*x*tan(
e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)
```

```
**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225
*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*ta
n(e/2 + f*x/2) - 15*c**3*f) + 165*A*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f
*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 +
 f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**
3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f
) - 75*A*a**3*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3
*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/
2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2
)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*A*a**3*f*x/(15*c**3*f*t
an(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f
*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3
- 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f)
- 60*A*a**3*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*
tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 +
 f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**
2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 120*A*a**3*tan(e/2 + f*x/2)**
5/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3
*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/
2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2)
 - 15*c**3*f) - 460*A*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c
**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 320*A*a**3*tan
(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2
)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 22
5*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f) - 452*A*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*ta
n(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*
x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 -
 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) +
 200*A*a**3*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan
(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*
x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 +
 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 52*A*a**3/(15*c**3*f*tan(e/2 + f
*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 -
 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 90*B*a**
3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/
2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2
)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75
*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 450*B*a**3*f*x*tan(e/2 + f*x/2)**6/
(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f
*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2
+ f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
```

```
 15*c**3*f) - 990*B*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2
)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225
*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1350*B*a**3*
f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2
+ f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - ...
```

Giac [A]
time = 0.58, size = 226, normalized size = 1.48

$$\frac{\frac{30 \, B a^{3}}{{\left(\tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} + 1\right)} c^{3}} - \frac{15 \, {\left(A a^{3} + 6 \, B a^{3}\right)} {\left(f x + e\right)}}{c^{3}} - \frac{4 \, {\left(15 \, A a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{4} + 45 \, B a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{4} - 30 \, A a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} - 210 \, B a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} + 100 \, A a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} + 420 \, B a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} - 50 \, A a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) - 270 \, B a^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 13 \, A a^{3} + 63 \, B a^{3}\right)}}{c^{3} {\left(\tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) - 1\right)}^{5}}}{15 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^3) - 15*(A*a^3 + 6*B*a^3)*(f
*x + e)/c^3 - 4*(15*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*a^3*tan(1/2*f*x + 1
/2*e)^4 - 30*A*a^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*a^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 420*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 5
0*A*a^3*tan(1/2*f*x + 1/2*e) - 270*B*a^3*tan(1/2*f*x + 1/2*e) + 13*A*a^3 +
63*B*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B]
time = 14.00, size = 336, normalized size = 2.20

$$\frac{\frac{52\,A\,a^3}{15}-\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{40\,A\,a^3}{3}+82\,B\,a^3\right)+\frac{94\,B\,a^3}{5}+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(4\,A\,a^3+12\,B\,a^3\right)-{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(8\,A\,a^3+58\,B\,a^3\right)-{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{64\,A\,a^3}{3}+148\,B\,a^3\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{92\,A\,a^3}{3}+134\,B\,a^3\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{452\,A\,a^3}{15}+\frac{744\,B\,a^3}{5}\right)}{f\,\left(-c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+5\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-11\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+15\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-15\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+11\,c^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-5\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c^3\right)}-\frac{2\,a^3\,\mathrm{atan}\left(\frac{2\,a^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A+6\,B\right)}{2\,A\,a^3+12\,B\,a^3}\right)\,\left(A+6\,B\right)}{c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] ((52*A*a^3)/15 - tan(e/2 + (f*x)/2)*((40*A*a^3)/3 + 82*B*a^3) + (94*B*a^3)/
5 + tan(e/2 + (f*x)/2)^6*(4*A*a^3 + 12*B*a^3) - tan(e/2 + (f*x)/2)^5*(8*A*a
^3 + 58*B*a^3) - tan(e/2 + (f*x)/2)^3*((64*A*a^3)/3 + 148*B*a^3) + tan(e/2
+ (f*x)/2)^4*((92*A*a^3)/3 + 134*B*a^3) + tan(e/2 + (f*x)/2)^2*((452*A*a^3)
/15 + (744*B*a^3)/5))/(f*(11*c^3*tan(e/2 + (f*x)/2)^2 - 15*c^3*tan(e/2 + (f
*x)/2)^3 + 15*c^3*tan(e/2 + (f*x)/2)^4 - 11*c^3*tan(e/2 + (f*x)/2)^5 + 5*c^
3*tan(e/2 + (f*x)/2)^6 - c^3*tan(e/2 + (f*x)/2)^7 + c^3 - 5*c^3*tan(e/2 + (
f*x)/2))) - (2*a^3*atan((2*a^3*tan(e/2 + (f*x)/2)*(A + 6*B))/(2*A*a^3 + 12*
B*a^3))*(A + 6*B))/(c^3*f)
```

3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=151

$$\frac{a^3Bx}{c^4}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] a^3*B*x/c^4+1/7*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7-2/5*a^3*B*c
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5+2/3*a^3*B*c^2*cos(f*x+e)^3/f/(c^2-c^2*si
n(f*x+e))^3-2*a^3*B*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Rubi [A]
time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2759, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}+\frac{a^3Bx}{c^4}+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*B*x)/c^4 + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^
7) - (2*a^3*B*c*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) + (2*a^3*B*c^2
*Cos[e + f*x]^3)/(3*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a^3*B*Cos[e + f*x])/
(f*(c^4 - c^4*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \left(a^3Bc^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + (a^3 \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2}{3cf} \\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2}{3cf} \\
&= \frac{a^3Bx}{c^4} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 356 vs. 2(151) = 302.
time = 0.81, size = 356, normalized size = 2.36

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(120(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12(15A+29B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+2(45A+199B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+105B(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7+240(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-24(15A+29B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+4(45A+199B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)-2(15A+337B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
```

```
^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e +
f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x)/2]
 - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2
] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/
2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(
c - c*Sin[e + f*x])^4)
```

Maple [A]
time = 0.52, size = 177, normalized size = 1.17

method	result
derivativedivides	$\frac{2a^3\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160}{4(\tan(}\right.}{f\,c^4}$
default	$\frac{2a^3\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160}{4(\tan(}\right.}{f\,c^4}$
risch	$\frac{a^3Bx}{c^4}-\frac{2\left(-15a^3A-337B\,a^3+6160iB\,a^3\mathrm{e}^{3i(fx+e)}-1624iB\,a^3\mathrm{e}^{i(fx+e)}+315A\,a^3\mathrm{e}^{2i(fx+e)}+4557B\,a^3\mathrm{e}^{2i(fx+e)}+105\right.}{105\left(\mathrm{e}^{i(fx+e)}-i\right)^7}$
norman	$-\frac{25a^3xB\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{63a^3xB\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{125a^3xB\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{203a^3xB\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{304B\,a^3\tan\left(\frac{fx}{2}+\right.}{15cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^4*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)-1)-1/2*
(12*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(60*A+20*B)/(tan(1/2*f*x+1/2*e)-1)^
3-1/7*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/4*(160*A+96*B)/(tan(1/2*f*x+1/
2*e)-1)^4-1/6*(192*A+192*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/5*(240*A+208*B)/(tan
(1/2*f*x+1/2*e)-1)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2304 vs. 2(157) = 314.
time = 0.57, size = 2304, normalized size = 15.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(5*B*a^3*((203*sin(f*x + e)/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f*x
```

+ e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f
*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^4) + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1)
- 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*
c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x + e)/(cos(f*x + e)
+ 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^3*(49*sin(f*x + e)/(cos(f
*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7) - 12*A*a^3*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x
+ e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*
c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 12*B*a^3*(14*sin(f*x
+ e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2
)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7) + 6*A*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4
- 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +

```
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + 18*B*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*
c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(157) = 314$.
time = 0.34, size = 379, normalized size = 2.51

$$\frac{840\,Ba^3fx + \left(105\,Ba^3fx + (15\,A + 337\,B)a^3\right)\cos\left(fx + e\right)^4 + 120\,(A + B)a^3 - \left(315\,Ba^3fx + (45\,A - 613\,B)a^3\right)\cos\left(fx + e\right)^3 - 24\left(35\,Ba^3fx + (5\,A + 26\,B)a^3\right)\cos\left(fx + e\right)^2 + 60\left(7\,Ba^3fx + (A - 13\,B)a^3\right)\cos\left(fx + e\right) - \left(840\,Ba^3fx - 120\,(A + B)a^3 - \left(105\,Ba^3fx - (15\,A + 337\,B)a^3\right)\cos\left(fx + e\right)^3 - 12\left(35\,Ba^3fx - (5\,A - 23\,B)a^3\right)\cos\left(fx + e\right)^2 + 60\left(7\,Ba^3fx - (A + 15\,B)a^3\right)\cos\left(fx + e\right)\right)\sin\left(fx + e\right)}{105\left(c^4f\cos\left(fx + e\right)^4 - 3\,c^4f\cos\left(fx + e\right)^3 - 8\,c^4f\cos\left(fx + e\right)^2 + 4\,c^4f\cos\left(fx + e\right) + 8\,c^4f + \left(c^4f\cos\left(fx + e\right)^3 + 4\,c^4f\cos\left(fx + e\right)^2 - 4\,c^4f\cos\left(fx + e\right) - 8\,c^4f\right)\sin\left(fx + e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4
+ 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 - 2
4*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (A -
 13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^3*f*
x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*B)*a^
3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*sin(f*x
 + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e
)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(
f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. $2(141) = 282$.
time = 30.30, size = 2951, normalized size = 19.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*a**3*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7
- 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c
**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*t
an(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 1050*A*a**
3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2
+ f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2
)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 +
```

735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 630*A*a**3*tan(e/2 + f*x/2)**2/
(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**
4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan
(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f
*x/2) - 105*c**4*f) - 30*A*a**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*
f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 105*B*a**3*f*x*tan(e/
2 + f*x/2)**7/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)
6 + 2205*c4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3
675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4
*f*tan(e/2 + f*x/2) - 105*c**4*f) - 735*B*a**3*f*x*tan(e/2 + f*x/2)**6/(105
*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*
tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2
 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2
) - 105*c**4*f) + 2205*B*a**3*f*x*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 +
 f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)*
*5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 22
05*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) -
 3675*B*a**3*f*x*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*
c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*
tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2
 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 3675*B*a**3*f*x*
tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 +
f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)*
*4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 73
5*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 2205*B*a**3*f*x*tan(e/2 + f*x/2)*
*2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*
c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*
tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2
+ f*x/2) - 105*c**4*f) + 735*B*a**3*f*x*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/
2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/
2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 -
 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f
) - 105*B*a**3*f*x/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f
*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**
4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735
*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 210*B*a**3*tan(e/2 + f*x/2)**6/(10
5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f) - 1680*B*a**3*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*
x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5
 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*
c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 38

```
50*B*a**3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*
tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2
 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/
2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 7840*B*a**3*tan(e/2 + f
*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 +
 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c
**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*ta
n(e/2 + f*x/2) - 105*c**4*f) + 5334*B*a**3*tan(e/2 + f*x/2)**2/(105*c**4*f*
tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2
+ f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2
)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*
c**4*f) - 2128*B*a**3*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x/2)**7 - 73
5*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*
f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e
/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - ...
```

Giac [A]
time = 0.66, size = 213, normalized size = 1.41

$$\frac{\frac{105\,(fx+e)Ba^3}{c^4}-\frac{2\left(105\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-105\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+840\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+525\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1925\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+3920\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+315\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2667\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1064\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,Aa^3-167\,Ba^3\right)}{c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105*
B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a^3
*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3*ta
n(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*tan(1/
2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*a^3)/
(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [B]
time = 15.98, size = 316, normalized size = 2.09

$$\frac{B\,a^3\,x}{c^4}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(\frac{a^3\,\left(1680\,B-2205\,B\,\left(e+f\,x\right)\right)}{105}+21\,B\,a^3\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{a^3\,\left(7840\,B-3675\,B\,\left(e+f\,x\right)\right)}{105}+35\,B\,a^3\,\left(e+f\,x\right)\right)+\frac{a^3\,\left(30\,A-334\,B+105\,B\,\left(e+f\,x\right)\right)}{105}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{a^3\,\left(210\,A-210\,B+735\,B\,\left(e+f\,x\right)\right)}{105}-7\,B\,a^3\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a^3\,\left(630\,A-5334\,B+2205\,B\,\left(e+f\,x\right)\right)}{105}-21\,B\,a^3\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{a^3\,\left(1050\,A-3850\,B+3675\,B\,\left(e+f\,x\right)\right)}{105}-35\,B\,a^3\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{a^3\,\left(2128\,B-735\,B\,\left(e+f\,x\right)\right)}{105}+7\,B\,a^3\,\left(e+f\,x\right)\right)-B\,a^3\,\left(e+f\,x\right)}{c^4\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (B*a^3*x)/c^4 - (tan(e/2 + (f*x)/2)^5*((a^3*(1680*B - 2205*B*(e + f*x)))/10
5 + 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^3*((a^3*(7840*B - 3675*B*(e +
f*x)))/105 + 35*B*a^3*(e + f*x)) + (a^3*(30*A - 334*B + 105*B*(e + f*x)))/1
05 + tan(e/2 + (f*x)/2)^6*((a^3*(210*A - 210*B + 735*B*(e + f*x)))/105 - 7*
B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^2*((a^3*(630*A - 5334*B + 2205*B*(e +
```

```
 f*x)))/105 - 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^4*((a^3*(1050*A - 38
50*B + 3675*B*(e + f*x)))/105 - 35*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)*((
a^3*(2128*B - 735*B*(e + f*x)))/105 + 7*B*a^3*(e + f*x)) - B*a^3*(e + f*x))
/(c^4*f*(tan(e/2 + (f*x)/2) - 1)^7)
```

3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=77

$$\frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

```
[Out] 1/9*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*(A-8*B)*c^2*co
s(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

Rubi [A]
time = 0.16, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3046, 2938, 2750}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2(A-8B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*(A - 8
*B)*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx\\ &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{1}{9}\left(a^3(A-8B)c^2\right)\int \frac{\cos^6(}{(c-c\sin}\\ &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(77) = 154.
time = 1.64, size = 283, normalized size = 3.68

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(315(A-B)\cos\left(\frac{1}{2}(e+fx)\right)-189(A-B)\cos\left(\frac{3}{2}(e+fx)\right)-63A\cos\left(\frac{5}{2}(e+fx)\right)+63B\cos\left(\frac{5}{2}(e+fx)\right)+9A\cos\left(\frac{7}{2}(e+fx)\right)-9B\cos\left(\frac{7}{2}(e+fx)\right)+189A\sin\left(\frac{1}{2}(e+fx)\right)+693B\sin\left(\frac{1}{2}(e+fx)\right)+105A\sin\left(\frac{3}{2}(e+fx)\right)+483B\sin\left(\frac{3}{2}(e+fx)\right)-27A\sin\left(\frac{5}{2}(e+fx)\right)-225B\sin\left(\frac{5}{2}(e+fx)\right)-63B\sin\left(\frac{7}{2}(e+fx)\right)-A\sin\left(\frac{9}{2}(e+fx)\right)+8B\sin\left(\frac{9}{2}(e+fx)\right)\right)}{504c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(-1+\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -1/504*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(315
*(A - B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(5*
(e + f*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9*B*
Cos[(7*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2] + 10
5*A*Sin[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e + f*
x))/2] - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*Sin[(9*
(e + f*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(-1 + Sin[e + f*x])^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(73) = 146.
time = 0.60, size = 205, normalized size = 2.66

method	result
derivativedivides	$\frac{2a^3\left(-\frac{14A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{680A+440B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{128A+128B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{A}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^5}$

default	$\frac{2a^3\left(-\frac{14A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{680A+440B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{128A+128B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{}{\tan\left(\frac{f}{2}\right.}\right.}{f\,c^5}$
risch	$\frac{-\frac{2a^3A}{63}+\frac{16B\,a^3}{63}-\frac{2iA\,a^3e^{i(fx+e)}}{7}+6iA\,a^3e^{3i(fx+e)}-2iB\,a^3e^{7i(fx+e)}+10iB\,a^3e^{5i(fx+e)}-\frac{6A\,a^3e^{2i(fx+e)}}{7}-\frac{50B\,a^3e^{2i(fx}}{7}}{}$
norman	$\frac{-\frac{16a^3A-2B\,a^3}{63cf}-\frac{2a^3A\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(a^3A-B\,a^3\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(2a^3A-2B\,a^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7cf}+\frac{6\left(3a^3A-3B\,a^3\right)\left(\tan\right.}{cf}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^5*(-1/2*(14*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/7*(928*A+864*B)/(ta
n(1/2*f*x+1/2*e)-1)^7-1/4*(304*A+144*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(680*A
+440*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/9*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)^9
-A/(tan(1/2*f*x+1/2*e)-1)-1/6*(992*A+800*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/3*(8
6*A+26*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(512*A+512*B)/(tan(1/2*f*x+1/2*e)-1)
^8)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2941 vs. $2(79) = 158$.
time = 0.38, size = 2941, normalized size = 38.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
```

+ e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 30*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 8*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 126*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 42*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(co

```
s(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)
+ 42*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(79) = 158$.
time = 0.53, size = 351, normalized size = 4.56

$$-\frac{(A-8B)a^3\cos(fx+e)^5-(4A+31B)a^3\cos(fx+e)^4+(19A+37B)a^3\cos(fx+e)^3+4(13A+22B)a^3\cos(fx+e)^2-28(A+B)a^3\cos(fx+e)-56(A+B)a^3+\left((A-8B)a^3\cos(fx+e)^4+(5A+23B)a^3\cos(fx+e)^3+12(2A+5B)a^3\cos(fx+e)^2-28(A+B)a^3\cos(fx+e)-56(A+B)a^3\right)\sin(fx+e)}{63\left(c^5f\cos(fx+e)^5+5c^5f\cos(fx+e)^4-8c^5f\cos(fx+e)^3-20c^5f\cos(fx+e)^2+8c^5f\cos(fx+e)+16c^5f-\left(c^5f\cos(fx+e)^4-4c^5f\cos(fx+e)^3-12c^5f\cos(fx+e)^2+8c^5f\cos(fx+e)+16c^5f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (19
*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(A
+ B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 + (5
*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*(A +
 B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5
+ 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2
 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*
x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f
*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(68) = 136$.
time = 51.28, size = 3262, normalized size = 42.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*A*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9 -
 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c*
*5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*ta
n(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 +
 f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*A*a**3*tan(e/2
+ f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8
 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938
*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f
*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2
```

+ f*x/2) - 63*c**5*f) - 966*A*a**3*tan(e/2 + f*x/2)**6/(63*c**5*f*tan(e/2
+ f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)
7 - 5292*c5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7
938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**
5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 630*A*
a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/
2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x
/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4
+ 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c
5*f*tan(e/2 + f*x/2) - 63*c5*f) - 1386*A*a**3*tan(e/2 + f*x/2)**4/(63*c
5*f*tan(e/2 + f*x/2)9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*ta
n(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 +
 f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)
3 - 2268*c5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c*
*5*f) + 378*A*a**3*tan(e/2 + f*x/2)**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f
*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/
2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x
/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 450*A*a**3*tan(e/2 + f*
x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**
5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan
(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f
*x/2) - 63*c**5*f) + 18*A*a**3*tan(e/2 + f*x/2)/(63*c**5*f*tan(e/2 + f*x/2)
9 - 567*c5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 52
92*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5
*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(
e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 16*A*a**3/(63*
c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*t
an(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2
+ f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2
)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c
5*f) - 126*B*a3*tan(e/2 + f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 56
7*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*
f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e
/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*
x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 210*B*a**3*tan(e/2 + f
*x/2)**6/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 +
2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c*
*5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*ta
n(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 +
f*x/2) - 63*c**5*f) - 630*B*a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f
*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7
 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938
*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f

```
*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**
3*tan(e/2 + f*x/2)**4/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +
 f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)
**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5
292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5
*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**3*tan(e/2 + f*x/2)**3/(63*c**5*
f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/
2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x
/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3
- 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f
) - 54*B*a**3*tan(e/2 + f*x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(
e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(79) = 158.
time = 0.65, size = 301, normalized size = 3.91

$$-\frac{2\left(63\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-63\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+63\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+483\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+105\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-315\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+315\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+693\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+189\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-189\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+189\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+225\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+27\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-9\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,Aa^{3}-Ba^{3}\right)}{63\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*B*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^3*t
an(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*tan(1/
2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/2*f*x
 + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x + 1/2
*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e) + 8*A*a
^3 - B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B]
time = 13.39, size = 346, normalized size = 4.49

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{1013\,A\,a^3}{16}+\frac{149\,B\,a^3}{16}-\frac{113\,A\,a^3\,\cos(2\,e+2\,f\,x)}{4}+\frac{37\,A\,a^3\,\cos(3\,e+3\,f\,x)}{8}+\frac{7\,A\,a^3\,\cos(4\,e+4\,f\,x)}{16}-\frac{41\,B\,a^3\,\cos(2\,e+2\,f\,x)}{4}+\frac{19\,B\,a^3\,\cos(3\,e+3\,f\,x)}{8}+\frac{7\,B\,a^3\,\cos(4\,e+4\,f\,x)}{16}+\frac{63\,A\,a^3\,\sin(2\,e+2\,f\,x)}{8}+\frac{9\,A\,a^3\,\sin(3\,e+3\,f\,x)}{2}-\frac{9\,A\,a^3\,\sin(4\,e+4\,f\,x)}{16}-\frac{63\,B\,a^3\,\sin(2\,e+2\,f\,x)}{8}-\frac{9\,B\,a^3\,\sin(3\,e+3\,f\,x)}{2}+\frac{9\,B\,a^3\,\sin(4\,e+4\,f\,x)}{16}-\frac{257\,A\,a^3\,\cos(e+f\,x)}{8}-\frac{23\,B\,a^3\,\cos(e+f\,x)}{8}-\frac{63\,A\,a^3\,\sin(e+f\,x)}{2}+\frac{63\,B\,a^3\,\sin(e+f\,x)}{2}\right)}{63\,c^5\,f\,\left(\frac{63\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\frac{21\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{9\,\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}+\frac{9\,\sqrt{2}\,\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{16}+\frac{\sqrt{2}\,\cos\left(\frac{9\,e}{2}+\frac{\pi}{4}+\frac{9\,f\,x}{2}\right)}{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((1013*A*a^3)/16 + (149*B*a^3)/16 - (113*A*a^3*cos(2*
e + 2*f*x))/4 + (37*A*a^3*cos(3*e + 3*f*x))/8 + (7*A*a^3*cos(4*e + 4*f*x))/
16 - (41*B*a^3*cos(2*e + 2*f*x))/4 + (19*B*a^3*cos(3*e + 3*f*x))/8 + (7*B*a
^3*cos(4*e + 4*f*x))/16 + (63*A*a^3*sin(2*e + 2*f*x))/8 + (9*A*a^3*sin(3*e
```

```
+ 3*f*x))/2 - (9*A*a^3*sin(4*e + 4*f*x))/16 - (63*B*a^3*sin(2*e + 2*f*x))/8
 - (9*B*a^3*sin(3*e + 3*f*x))/2 + (9*B*a^3*sin(4*e + 4*f*x))/16 - (257*A*a^
3*cos(e + f*x))/8 - (23*B*a^3*cos(e + f*x))/8 - (63*A*a^3*sin(e + f*x))/2 +
 (63*B*a^3*sin(e + f*x))/2))/(63*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/
2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*
e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/1
6 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=118

$$\frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3(2A-9B)c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+1/99*a^3*(2*A-9*B)*c^2
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/693*a^3*(2*A-9*B)*c*cos(f*x+e)^7/f/(c-
c*sin(f*x+e))^7
```

Rubi [A]
time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3c^2(2A-9B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c(2A-9B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (a^3*(2*A
- 9*B)*c^2*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (a^3*(2*A - 9*B)
*c*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{1}{11}\left(a^3(2A-9B)c^2\right)\int \frac{\cos}{(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(118) = 236.
time = 1.84, size = 313, normalized size = 2.65

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(462(11A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-594(5A+2B)\cos\left(\frac{3}{2}(e+fx)\right)-924A\cos\left(\frac{5}{2}(e+fx)\right)-693B\cos\left(\frac{5}{2}(e+fx)\right)+110A\cos\left(\frac{7}{2}(e+fx)\right)+198B\cos\left(\frac{7}{2}(e+fx)\right)-2A\cos\left(\frac{11}{2}(e+fx)\right)+9B\cos\left(\frac{11}{2}(e+fx)\right)+4158A\sin\left(\frac{1}{2}(e+fx)\right)+5544B\sin\left(\frac{1}{2}(e+fx)\right)+2310A\sin\left(\frac{3}{2}(e+fx)\right)+4158B\sin\left(\frac{3}{2}(e+fx)\right)-594A\sin\left(\frac{5}{2}(e+fx)\right)-2178B\sin\left(\frac{5}{2}(e+fx)\right)-693B\sin\left(\frac{7}{2}(e+fx)\right)-22A\sin\left(\frac{9}{2}(e+fx)\right)+99B\sin\left(\frac{9}{2}(e+fx)\right)\right)}{11088c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(-1+\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A
+ 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Cos[
(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/2]
+ 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*(e +
 f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A*Sin[
```

```
(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*x))/2]
 - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*Sin[(9*(
e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(112) = 224$.
time = 0.44, size = 249, normalized size = 2.11

method	result
risch	$-\frac{2ia^3\left(924iA\,e^{8i(fx+e)}+1188iB\,e^{4i(fx+e)}+693B\,e^{9i(fx+e)}+2970iA\,e^{4i(fx+e)}-2310A\,e^{7i(fx+e)}+693iB\,e^{8i(fx+e)}-41\right.}{}$
derivativedivides	$\frac{2a^3\left(-\frac{1280A+1280B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{504A+200B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{4272A+3344B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{3008A+2880B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{2}{6(\mathrm{ta}}\right.}{f}$
default	$\frac{2a^3\left(-\frac{1280A+1280B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{504A+200B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{4272A+3344B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{3008A+2880B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{2}{6(\mathrm{ta}}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^6*(-1/10*(1280*A+1280*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/4*(504*A+200
*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/7*(4272*A+3344*B)/(tan(1/2*f*x+1/2*e)-1)^7-1
/8*(4352*A+3840*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/9*(3008*A+2880*B)/(tan(1/2*f*
x+1/2*e)-1)^9-1/6*(2960*A+1968*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/5*(1460*A+780*
B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(16*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-A/(tan(
1/2*f*x+1/2*e)-1)-1/11*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/3*(116*A+3
0*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3694 vs. $2(121) = 242$.
time = 0.49, size = 3694, normalized size = 31.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
```

+ e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*A*a^3*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^3*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*A*a^3*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*B*a^3*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e)

```
 + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 -
11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4
62*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*A*a^3*(253*sin(f*x + e)
/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(121) = 242$.
time = 0.59, size = 429, normalized size = 3.64

$$\frac{(2A-9B)a^3\cos(fx+e)^6+6(2A-9B)a^3\cos(fx+e)^5-(25A+234B)a^3\cos(fx+e)^4+7(23A+45B)a^3\cos(fx+e)^3+28(16A+27B)a^3\cos(fx+e)^2-252(A+B)a^3\cos(fx+e)-504(A+B)a^3-((2A-9B)a^3\cos(fx+e)^5-5(2A-9B)a^3\cos(fx+e)^4-7(5A+27B)a^3\cos(fx+e)^3-28(7A+18B)a^3\cos(fx+e)^2+252(A+B)a^3\cos(fx+e)+504(A+B)a^3)\sin(fx+e)}{693\left(c^6f\cos(fx+e)^6-5c^6f\cos(fx+e)^5-18c^6f\cos(fx+e)^4+20c^6f\cos(fx+e)^3+48c^6f\cos(fx+e)^2-16c^6f\cos(fx+e)-32c^6f+(c^6f\cos(fx+e)^5+6c^6f\cos(fx+e)^4-12c^6f\cos(fx+e)^3-32c^6f\cos(fx+e)^2+16c^6f\cos(fx+e)+32c^6f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 -
(25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 + 28
*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(A +
 B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x + e)^
4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x + e)^2
+ 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e))/(c^6*f*cos(
f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*co
s(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f +
 (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 -
 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(105) = 210$.
time = 85.31, size = 4816, normalized size = 40.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
[Out] Piecewise((-1386*A*a**3*tan(e/2 + f*x/2)**10/(693*c**6*f*tan(e/2 + f*x/2)**
11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320
166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690
*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**
6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 2772
*A*a**3*tan(e/2 + f*x/2)**9/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*
tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan
(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/
2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 +
 f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x
/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 16170*A*a**3*tan(e/2
+ f*x/2)**8/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)
**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8
 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 +
320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114
345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c*
*6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 21252*A*a**3*tan(e/2 + f*x/2)**7/(693
*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**
6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f
*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*ta
n(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e
/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f
*x/2) - 693*c**6*f) - 42504*A*a**3*tan(e/2 + f*x/2)**6/(693*c**6*f*tan(e/2
+ f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*
x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2
)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**
5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 -
 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6
*f) + 30492*A*a**3*tan(e/2 + f*x/2)**5/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7
623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345
*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c*
*6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*
f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*ta
n(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 30888*A*a*
*3*tan(e/2 + f*x/2)**4/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e
/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2
+ f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f
*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/
2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**
2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 9900*A*a**3*tan(e/2 + f*x/
2)**3/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 +
 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228
```

```
690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166
*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c*
*6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*t
an(e/2 + f*x/2) - 693*c**6*f) - 5918*A*a**3*tan(e/2 + f*x/2)**2/(693*c**6*f
*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan
(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/
2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 +
 f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*
x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) -
 693*c**6*f) + 352*A*a**3*tan(e/2 + f*x/2)/(693*c**6*f*tan(e/2 + f*x/2)**11
 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 11
4345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32016
6*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c
**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*
f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 158*A*
a**3/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 +
38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 2286
90*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*
c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**
6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*ta
n(e/2 + f*x/2) - 693*c**6*f) - 1386*B*a**3*tan(e/2 + f*x/2)**9/(693*c**6*f*
tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(
e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2
 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(121) = 242.
time = 0.68, size = 373, normalized size = 3.16

$$-\frac{2\left(693\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-1386\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+693\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+8085\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+693\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-10626\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+4158\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+21252\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+1386\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-15246\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+5544\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+15444\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+1188\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-4950\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2178\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2959\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+198\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-176\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+99\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+79\,Aa^{3}-9\,Ba^{3}\right)}{693\,c^{6}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/693*(693*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*A*a^3*tan(1/2*f*x + 1/2*e)
^9 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*tan(1/2*f*x + 1/2*e)^8 +
 693*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 41
58*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 1386
*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 5544*B
*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 1188*B*a
^3*tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 2178*B*a^3*
tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 198*B*a^3*tan(
1/2*f*x + 1/2*e)^2 - 176*A*a^3*tan(1/2*f*x + 1/2*e) + 99*B*a^3*tan(1/2*f*x
+ 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(tan(1/2*f*x + 1/2*e) - 1)^11)
```

Mupad [B]
time = 13.54, size = 408, normalized size = 3.46

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(565\,A\,a^3\,\cos\left(2\,e+2\,f\,x\right)-\frac{837\,B\,a^3}{16}-922\,A\,a^3-\frac{3527\,A\,a^3\,\cos(3\,e+3\,f\,x)}{32}-29\,A\,a^3\,\cos\left(4\,e+4\,f\,x\right)+\frac{81\,A\,a^3\,\cos(5\,e+5\,f\,x)}{32}+\frac{225\,B\,a^3\,\cos(2\,e+2\,f\,x)}{4}-\frac{207\,B\,a^3\,\cos(3\,e+3\,f\,x)}{16}+\frac{9\,B\,a^3\,\cos(4\,e+4\,f\,x)}{16}-\frac{9\,B\,a^3\,\cos(5\,e+5\,f\,x)}{16}-\frac{1617\,A\,a^3\,\sin(2\,e+2\,f\,x)}{8}-\frac{5049\,A\,a^3\,\sin(3\,e+3\,f\,x)}{32}+\frac{407\,A\,a^3\,\sin(4\,e+4\,f\,x)}{16}+\frac{77\,A\,a^3\,\sin(5\,e+5\,f\,x)}{32}+\frac{693\,B\,a^3\,\sin(2\,e+2\,f\,x)}{8}+\frac{99\,B\,a^3\,\sin(3\,e+3\,f\,x)}{2}-\frac{99\,B\,a^3\,\sin(4\,e+4\,f\,x)}{16}+\frac{6635\,A\,a^3\,\cos(e+f\,x)}{16}+18\,B\,a^3\,\cos\left(e+f\,x\right)+\frac{13629\,A\,a^3\,\sin(e+f\,x)}{16}-\frac{693\,B\,a^3\,\sin(e+f\,x)}{2}\right)}{693\,c^6\,f\,\left(\frac{231\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{16}-\frac{165\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{16}-\frac{165\,\sqrt{2}\,\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{32}+\frac{55\,\sqrt{2}\,\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{32}+\frac{11\,\sqrt{2}\,\cos\left(\frac{9\,e}{2}+\frac{\pi}{4}+\frac{9\,f\,x}{2}\right)}{32}-\frac{\sqrt{2}\,\cos\left(\frac{11\,e}{2}-\frac{\pi}{4}+\frac{11\,f\,x}{2}\right)}{32}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*(565*A*a^3*cos(2*e + 2*f*x) - (837*B*a^3)/16 - 922*A
*a^3 - (3527*A*a^3*cos(3*e + 3*f*x))/32 - 29*A*a^3*cos(4*e + 4*f*x) + (81*A
*a^3*cos(5*e + 5*f*x))/32 + (225*B*a^3*cos(2*e + 2*f*x))/4 - (207*B*a^3*cos
(3*e + 3*f*x))/16 + (9*B*a^3*cos(4*e + 4*f*x))/16 - (9*B*a^3*cos(5*e + 5*f*
x))/16 - (1617*A*a^3*sin(2*e + 2*f*x))/8 - (5049*A*a^3*sin(3*e + 3*f*x))/32
 + (407*A*a^3*sin(4*e + 4*f*x))/16 + (77*A*a^3*sin(5*e + 5*f*x))/32 + (693*
B*a^3*sin(2*e + 2*f*x))/8 + (99*B*a^3*sin(3*e + 3*f*x))/2 - (99*B*a^3*sin(4
*e + 4*f*x))/16 + (6635*A*a^3*cos(e + f*x))/16 + 18*B*a^3*cos(e + f*x) + (1
3629*A*a^3*sin(e + f*x))/16 - (693*B*a^3*sin(e + f*x))/2))/(693*c^6*f*((231
*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 +
(3*f*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(
1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 +
 (9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=156

$$\frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3(3A-10B)c\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}+\frac{2a^3(3A-10B)\cdots}{9009f(c-c\sin\cdots}$$

```
[Out] 1/13*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/143*a^3*(3*A-10*B)*
c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/1287*a^3*(3*A-10*B)*c*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^8+2/9009*a^3*(3*A-10*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
7
```

Rubi [A]
time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3c^2(3A-10B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}+\frac{2a^3c(3A-10B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (a^3*(3*A
 - 10*B)*c^2*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(3*A -
 10*B)*c*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(3*A - 10
*B)*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}\,dx\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{1}{13}\left(a^3(3A-10B)c^2\right)\int \frac{\mathrm{co}}{(c-c}\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}\\
&= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(156) = 312.
time = 3.34, size = 339, normalized size = 2.17

$$
-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6006(9A+5B)\cos\left(\frac{1}{2}(e+fx)\right)-7722(4A+3B)\cos\left(\frac{3}{2}(e+fx)\right)-9009A\cos\left(\frac{5}{2}(e+fx)\right)-12012B\cos\left(\frac{5}{2}(e+fx)\right)+858A\cos\left(\frac{7}{2}(e+fx)\right)+3146B\cos\left(\frac{7}{2}(e+fx)\right)-39A\cos\left(\frac{11}{2}(e+fx)\right)+130B\cos\left(\frac{11}{2}(e+fx)\right)+48906A\sin\left(\frac{1}{2}(e+fx)\right)+47190B\sin\left(\frac{1}{2}(e+fx)\right)+27027A\sin\left(\frac{3}{2}(e+fx)\right)+36036B\sin\left(\frac{3}{2}(e+fx)\right)-6864A\sin\left(\frac{5}{2}(e+fx)\right)-19162B\sin\left(\frac{5}{2}(e+fx)\right)-6006B\sin\left(\frac{7}{2}(e+fx)\right)-234A\sin\left(\frac{9}{2}(e+fx)\right)+780B\sin\left(\frac{9}{2}(e+fx)\right)+3A\sin\left(\frac{13}{2}(e+fx)\right)-10B\sin\left(\frac{13}{2}(e+fx)\right)\right)}{144144c^7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(-1+\sin(e+fx))^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -1/144144*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(
6006*(9*A + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2] -
 9009*A*Cos[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[(7*
(e + f*x))/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2] +
130*B*Cos[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(e + f
*x)/2] + 27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2] - 6864
*A*Sin[(5*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[(7*(e +
 f*x))/2] - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2] + 3*A*S
in[(13*(e + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)
```

Maple [A]
time = 0.54, size = 293, normalized size = 1.88

method	result
derivativedivides	$2a^3\left(-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{13112A+8840B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{768A+264B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{20256A+17248B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2700A+\ldots}{5\left(\tan\left(\frac{fx}{2}+\ldots\right.\right.}\right.\ldots$
default	$2a^3\left(-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{13112A+8840B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{768A+264B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{20256A+17248B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2700A+\ldots}{5\left(\tan\left(\frac{fx}{2}+\ldots\right.\right.}\right.\ldots$
risch	$-\frac{4(3a^3A-10Ba^3-130iBa^3e^{i(fx+e)}+9009iAa^3e^{9i(fx+e)}+30888iAa^3e^{5i(fx+e)}-30030iBa^3e^{7i(fx+e)}+23166iBa^3e\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^7*(-1/2*(18*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/7*(13112*A+8840*B)/
(tan(1/2*f*x+1/2*e)-1)^7-1/4*(768*A+264*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/9*(20
256*A+17248*B)/(tan(1/2*f*x+1/2*e)-1)^9-A/(tan(1/2*f*x+1/2*e)-1)-1/5*(2700*
A+1240*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/11*(8832*A+8576*B)/(tan(1/2*f*x+1/2*e)
-1)^11-1/13*(512*A+512*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/10*(16000*A+14720*B)/
(tan(1/2*f*x+1/2*e)-1)^10-1/6*(6888*A+3928*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/3*
(150*A+34*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(18816*A+14464*B)/(tan(1/2*f*x+1/
2*e)-1)^8-1/12*(3072*A+3072*B)/(tan(1/2*f*x+1/2*e)-1)^12)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4446 vs. $2(160) = 320$.
time = 0.47, size = 4446, normalized size = 28.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

```
[Out] -2/45045*(6*A*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 6*B*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) +
 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
 + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^3*(3796*sin(f*x + e)/(cos(f*x + e)
 + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin
(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(
f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 105*A*a^3*(
611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1
```

```
)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x +
 e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)
^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) +
1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(co
s(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin
(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^3*(611*sin(f*x + e)/(cos(f*x +
 e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(160) = 320$.

time = 0.59, size = 503, normalized size = 3.22

$$-\frac{2\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^7-12\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^6-49\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^5+7\,(30\,A+329\,B)a^3\cos\left(fx+e\right)^4-63\,(27\,A+53\,B)a^3\cos\left(fx+e\right)^3-252\,(19\,A+32\,B)a^3\cos\left(fx+e\right)^2+2772\,(A+B)a^3\cos\left(fx+e\right)+5544\,(A+B)a^3+\left(2\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^6+14\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^5-35\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^4-63\,(5\,A+31\,B)a^3\cos\left(fx+e\right)^3-252\,(8\,A+21\,B)a^3\cos\left(fx+e\right)^2+2772\,(A+B)a^3\cos\left(fx+e\right)+5544\,(A+B)a^3\right)\sin\left(fx+e\right)}{9009\,\left(c^7f\cos\left(fx+e\right)^7+7\,c^7f\cos\left(fx+e\right)^6-18\,c^7f\cos\left(fx+e\right)^5-56\,c^7f\cos\left(fx+e\right)^4+48\,c^7f\cos\left(fx+e\right)^3+112\,c^7f\cos\left(fx+e\right)^2-32\,c^7f\cos\left(fx+e\right)-64\,c^7f-\left(c^7f\cos\left(fx+e\right)^6-6\,c^7f\cos\left(fx+e\right)^5-24\,c^7f\cos\left(fx+e\right)^4+32\,c^7f\cos\left(fx+e\right)^3+80\,c^7f\cos\left(fx+e\right)^2-32\,c^7f\cos\left(fx+e\right)-64\,c^7f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] -1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x +
e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x +
e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f*x
+ e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 10*B)
*a^3*cos(f*x + e)^6 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 35*(3*A - 10*B)*
a^3*cos(f*x + e)^4 - 63*(5*A + 31*B)*a^3*cos(f*x + e)^3 - 252*(8*A + 21*B)*
a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3)*sin(
f*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x
 + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos
(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*
c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 +
80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(143) = 286$.
time = 135.35, size = 6669, normalized size = 42.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Piecewise((-18018*A*a**3*tan(e/2 + f*x/2)**12/(9009*c**7*f*tan(e/2 + f*x/2)
**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)*
*11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)
**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f*x/2
)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 + f*x/
2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f*x/2
)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/2) -
 9009*c**7*f) + 54054*A*a**3*tan(e/2 + f*x/2)**11/(9009*c**7*f*tan(e/2 + f*
x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x
/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*
x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f
*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 +
f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 + f
*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f*x/
2) - 9009*c**7*f) - 306306*A*a**3*tan(e/2 + f*x/2)**10/(9009*c**7*f*tan(e/2
 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2
+ f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2
 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/
2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e
/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/
2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 +
 f*x/2) - 9009*c**7*f) + 594594*A*a**3*tan(e/2 + f*x/2)**9/(9009*c**7*f*tan
(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(
e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan
(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*ta
n(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*t
an(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*ta
n(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e
/2 + f*x/2) - 9009*c**7*f) - 1297296*A*a**3*tan(e/2 + f*x/2)**8/(9009*c**7*
f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f
*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*
f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7
*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**
7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7
*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*
tan(e/2 + f*x/2) - 9009*c**7*f) + 1477476*A*a**3*tan(e/2 + f*x/2)**7/(9009*
```

```
c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c
**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*
c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444
*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 1159458
3*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574
*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c*
*7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 1714284*A*a**3*tan(e/2 + f*x/2)**6/(
9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702
702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 644
1435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 154
59444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11
594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 25
76574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 1171
17*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 1096524*A*a**3*tan(e/2 + f*x/2)
**5/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12
+ 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10
+ 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8
+ 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6
 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4
 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 +
 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) - 735306*A*a**3*tan(e/2 + f*
x/2)**4/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)*
*12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)*
*10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)
**8 + 15459444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2
)**6 + 11594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2
)**4 + 2576574*c**7*f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)*
*2 + 117117*c**7*f*tan(e/2 + f*x/2) - 9009*c**7*f) + 225654*A*a**3*tan(e/2
+ f*x/2)**3/(9009*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x
/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x
/2)**10 + 6441435*c**7*f*tan(e/2 + f*x/2)**9 - ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(160) = 320$.
time = 0.68, size = 445, normalized size = 2.85

$$-\frac{2\left(9009\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-27027\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+9009\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+153153\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}+3003\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-297297\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+69069\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+648648\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-9009\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-738738\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+150150\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+857142\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-16302\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-548262\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+115830\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+367653\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-286\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-112827\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+30745\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+45513\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1443\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-3081\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1261\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+930\,Aa^{3}-97\,Ba^{3}\right)}{9009\,c^{7}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/9009*(9009*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 27027*A*a^3*tan(1/2*f*x + 1/2
*e)^11 + 9009*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 153153*A*a^3*tan(1/2*f*x + 1/
2*e)^10 + 3003*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 297297*A*a^3*tan(1/2*f*x + 1
```

```
/2*e)^9 + 69069*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*tan(1/2*f*x + 1
/2*e)^8 - 9009*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*tan(1/2*f*x + 1/
2*e)^7 + 150150*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*tan(1/2*f*x + 1
/2*e)^6 - 16302*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*tan(1/2*f*x + 1
/2*e)^5 + 115830*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*tan(1/2*f*x +
1/2*e)^4 - 286*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*tan(1/2*f*x + 1/
2*e)^3 + 30745*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*tan(1/2*f*x + 1/2
*e)^2 + 1443*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*tan(1/2*f*x + 1/2*e)
 + 1261*B*a^3*tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7*f*(tan(1/2*
f*x + 1/2*e) - 1)^13)
```

Mupad [B]
time = 13.79, size = 500, normalized size = 3.21

$$\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{2363\,B\,a^3}{32}-\frac{279183\,A\,a^3}{16}+\frac{220269\,A\,a^3\cos(2e+2fx)}{16}-\frac{46095\,A\,a^3\cos(3e+3fx)}{16}-\frac{20829\,A\,a^3\cos(4e+4fx)}{16}+\frac{2811\,A\,a^3\cos(5e+5fx)}{16}+\frac{231\,A\,a^3\cos(6e+6fx)}{16}-\frac{8995\,B\,a^3\cos(2e+2fx)}{64}+\frac{497\,B\,a^3\cos(3e+3fx)}{16}+\frac{3725\,B\,a^3\cos(4e+4fx)}{32}-\frac{361\,B\,a^3\cos(5e+5fx)}{16}-\frac{77\,B\,a^3\cos(6e+6fx)}{64}-\frac{19305\,A\,a^3\sin(2e+2fx)}{4}-\frac{81081\,A\,a^3\sin(3e+3fx)}{16}+\frac{15015\,A\,a^3\sin(4e+4fx)}{16}+\frac{3237\,A\,a^3\sin(5e+5fx)}{16}-\frac{117\,A\,a^3\sin(6e+6fx)}{8}+\frac{77649\,B\,a^3\sin(2e+2fx)}{64}+\frac{27027\,B\,a^3\sin(3e+3fx)}{32}-\frac{1001\,B\,a^3\sin(4e+4fx)}{8}-\frac{559\,B\,a^3\sin(5e+5fx)}{32}+\frac{117\,B\,a^3\sin(6e+6fx)}{64}+\frac{26979\,A\,a^3\cos(e+fx)}{4}+40\,B\,a^3\cos(e+fx)+\frac{173745\,A\,a^3\sin(e+fx)}{8}-\frac{80223\,B\,a^3\sin(e+fx)}{16}\right)}{9009\,c^7\,f\left(\frac{1287\sqrt{2}\cos\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{64}-\frac{429\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{16}+\frac{715\sqrt{2}\cos\left(\frac{5e}{2}+\frac{\pi}{4}+\frac{5fx}{2}\right)}{64}-\frac{143\sqrt{2}\cos\left(\frac{7e}{2}-\frac{\pi}{4}+\frac{7fx}{2}\right)}{32}-\frac{39\sqrt{2}\cos\left(\frac{9e}{2}+\frac{\pi}{4}+\frac{9fx}{2}\right)}{32}+\frac{13\sqrt{2}\cos\left(\frac{11e}{2}-\frac{\pi}{4}+\frac{11fx}{2}\right)}{64}+\frac{\sqrt{2}\cos\left(\frac{13e}{2}+\frac{\pi}{4}+\frac{13fx}{2}\right)}{64}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^7,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((2363*B*a^3)/32 - (279183*A*a^3)/16 + (220269*A*a^3*
cos(2*e + 2*f*x))/16 - (46095*A*a^3*cos(3*e + 3*f*x))/16 - (20829*A*a^3*cos
(4*e + 4*f*x))/16 + (2811*A*a^3*cos(5*e + 5*f*x))/16 + (231*A*a^3*cos(6*e +
 6*f*x))/16 - (8995*B*a^3*cos(2*e + 2*f*x))/64 + (497*B*a^3*cos(3*e + 3*f*x
))/16 + (3725*B*a^3*cos(4*e + 4*f*x))/32 - (361*B*a^3*cos(5*e + 5*f*x))/16
- (77*B*a^3*cos(6*e + 6*f*x))/64 - (19305*A*a^3*sin(2*e + 2*f*x))/4 - (8108
1*A*a^3*sin(3*e + 3*f*x))/16 + (15015*A*a^3*sin(4*e + 4*f*x))/16 + (3237*A*
a^3*sin(5*e + 5*f*x))/16 - (117*A*a^3*sin(6*e + 6*f*x))/8 + (77649*B*a^3*si
n(2*e + 2*f*x))/64 + (27027*B*a^3*sin(3*e + 3*f*x))/32 - (1001*B*a^3*sin(4*
e + 4*f*x))/8 - (559*B*a^3*sin(5*e + 5*f*x))/32 + (117*B*a^3*sin(6*e + 6*f*
x))/64 + (26979*A*a^3*cos(e + f*x))/4 + 40*B*a^3*cos(e + f*x) + (173745*A*a
^3*sin(e + f*x))/8 - (80223*B*a^3*sin(e + f*x))/16))/(9009*c^7*f*((1287*2^(
1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/2 + pi/4 + (f
*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64 - (143*2^(1/2
)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e)/2 + pi/4 + (9
*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/64 + (2^(1/2)
*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$

Optimal. Leaf size=197

$$\frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}+\frac{2a^3(4A-11B)\,c}{6435f(c-c\sin}$$

```
[Out] 1/15*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+1/195*a^3*(4*A-11*B)*
c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/715*a^3*(4*A-11*B)*c*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^9+2/6435*a^3*(4*A-11*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
8+2/45045*a^3*(4*A-11*B)*cos(f*x+e)^7/c/f/(c-c*sin(f*x+e))^7
```

Rubi [A]

time = 0.28, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 2750}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{a^3c^2(4A-11B)\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+\frac{a^3c(4A-11B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (a^3*(4*A
 - 11*B)*c^2*Cos[e + f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (a^3*(4*A -
11*B)*c*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(4*A - 11*B
)*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(4*A - 11*B)*Cos
[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{1}{15}\left(a^3(4A-11B)c^2\right)\int \frac{\mathrm{co}}{(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}$$

Mathematica [**A**]
time = 4.58, size = 365, normalized size = 1.85

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6435(72A+47B)\cos\left(\frac{1}{2}(e+fx)\right)-10010(26A+23B)\cos\left(\frac{3}{2}(e+fx)\right)-72072A\cos\left(\frac{5}{2}(e+fx)\right)-117117B\cos\left(\frac{5}{2}(e+fx)\right)+5460A\cos\left(\frac{7}{2}(e+fx)\right)+30030B\cos\left(\frac{7}{2}(e+fx)\right)-420A\cos\left(\frac{11}{2}(e+fx)\right)+1155B\cos\left(\frac{11}{2}(e+fx)\right)+4A\cos\left(\frac{15}{2}(e+fx)\right)-11B\cos\left(\frac{15}{2}(e+fx)\right)+437580A\sin\left(\frac{1}{2}(e+fx)\right)+373230B\sin\left(\frac{1}{2}(e+fx)\right)+240240A\sin\left(\frac{3}{2}(e+fx)\right)+285285B\sin\left(\frac{3}{2}(e+fx)\right)-60060A\sin\left(\frac{5}{2}(e+fx)\right)-150150B\sin\left(\frac{5}{2}(e+fx)\right)-45045B\sin\left(\frac{7}{2}(e+fx)\right)-1820A\sin\left(\frac{9}{2}(e+fx)\right)+5005B\sin\left(\frac{9}{2}(e+fx)\right)+60A\sin\left(\frac{13}{2}(e+fx)\right)-165B\sin\left(\frac{13}{2}(e+fx)\right)\right)}{1441440c^8f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(-1+\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^8,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(6435*(72*A
 + 47*B)*Cos[(e + f*x)/2] - 10010*(26*A + 23*B)*Cos[(3*(e + f*x))/2] - 7207
2*A*Cos[(5*(e + f*x))/2] - 117117*B*Cos[(5*(e + f*x))/2] + 5460*A*Cos[(7*(e
 + f*x))/2] + 30030*B*Cos[(7*(e + f*x))/2] - 420*A*Cos[(11*(e + f*x))/2] +
1155*B*Cos[(11*(e + f*x))/2] + 4*A*Cos[(15*(e + f*x))/2] - 11*B*Cos[(15*(e
+ f*x))/2] + 437580*A*Sin[(e + f*x)/2] + 373230*B*Sin[(e + f*x)/2] + 240240
*A*Sin[(3*(e + f*x))/2] + 285285*B*Sin[(3*(e + f*x))/2] - 60060*A*Sin[(5*(e
 + f*x))/2] - 150150*B*Sin[(5*(e + f*x))/2] - 45045*B*Sin[(7*(e + f*x))/2]
- 1820*A*Sin[(9*(e + f*x))/2] + 5005*B*Sin[(9*(e + f*x))/2] + 60*A*Sin[(13*
(e + f*x))/2] - 165*B*Sin[(13*(e + f*x))/2]))/(1441440*c^8*f*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^8)
```

Maple [A]
time = 0.62, size = 337, normalized size = 1.71

method	result
risch	$\frac{4ia^3(-1155iB\,e^{2i(fx+e)}+420iA\,e^{2i(fx+e)}+45045B\,e^{11i(fx+e)}-302445iB\,e^{8i(fx+e)}-240240A\,e^{9i(fx+e)}+11iB-28528}{}$
derivativedivides	$\frac{2a^3\left(-\frac{1104A+336B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32288A+19176B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{13824A+6936B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{84112A+63856B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{}{\tan\left(\frac{f}{2}\right.}\right.}{}$
default	$\frac{2a^3\left(-\frac{1104A+336B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32288A+19176B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{13824A+6936B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{84112A+63856B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{}{\tan\left(\frac{f}{2}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3/c^8*(-1/4*(1104*A+336*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/7*(32288*A+1917
6*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(13824*A+6936*B)/(tan(1/2*f*x+1/2*e)-1)^6
-1/9*(84112*A+63856*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/5*(4536*A+1836*B)/(tan(1/
2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+1/2*e)-1)-1/15*(1024*A+1024*B)/(tan(1/2*f*
x+1/2*e)-1)^15-1/2*(20*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/14*(7168*A+7168*B)
/(tan(1/2*f*x+1/2*e)-1)^14-1/10*(94144*A+78144*B)/(tan(1/2*f*x+1/2*e)-1)^10
-1/11*(81344*A+72512*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/13*(24320*A+23808*B)/(t
an(1/2*f*x+1/2*e)-1)^13-1/8*(58816*A+40000*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/3*
(188*A+38*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/12*(52736*A+49664*B)/(tan(1/2*f*x+1
/2*e)-1)^12)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5197 vs. 2(202) = 404.
time = 0.54, size = 5197, normalized size = 26.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="maxima")
```

```
[Out] 2/45045*(3*A*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891
345*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 2867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*s
in(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 643
5*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x
+ e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^1
4 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + B*a^3*(17715*sin(f*x + e)/
(cos(f*x + e) + 1) - 78960*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 342160*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891345*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3043040*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 3912480*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3687255
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2867865*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 - 1585584*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 720720*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 195195*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
+ 45045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x
 + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 45
5*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(c
os(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^
8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e)
 + 1)^15) - 7*A*a^3*(7845*sin(f*x + e)/(cos(f*x + e) + 1) - 54915*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6
68850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1444443*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3063060*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 + 2357355*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1414413*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10 + 630630*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 210210
```

```
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e
) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 952)/(c^8 - 15*c^8
*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005
*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(
f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x +
e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13
 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(
f*x + e) + 1)^15) - 12*A*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(c
os(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6
435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(202) = 404$.
time = 0.39, size = 573, normalized size = 2.91

$$\frac{2 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{8} + 16 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{7} - 49 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{6} - 168 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{5} + 105 \, {\left(7 \, A + 88 \, B\right)} a^{3} \cos\left(f x + e\right)^{4} - 231 \, {\left(31 \, A + 61 \, B\right)} a^{3} \cos\left(f x + e\right)^{3} - 924 \, {\left(22 \, A + 37 \, B\right)} a^{3} \cos\left(f x + e\right)^{2} + 12012 \, {\left(A + B\right)} a^{3} \cos\left(f x + e\right) + 24024 \, {\left(A + B\right)} a^{3} - {\left(2 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{7} - 14 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{6} - 63 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{5} + 105 \, {\left(4 \, A - 11 \, B\right)} a^{3} \cos\left(f x + e\right)^{4} + 1155 \, {\left(A + 7 \, B\right)} a^{3} \cos\left(f x + e\right)^{3} + 2772 \, {\left(3 \, A + 8 \, B\right)} a^{3} \cos\left(f x + e\right)^{2} - 12012 \, {\left(A + B\right)} a^{3} \cos\left(f x + e\right) - 24024 \, {\left(A + B\right)} a^{3}\right)} \sin\left(f x + e\right)}{45045 \, {\left(c^{8} f \cos\left(f x + e\right)^{8} - 7 \, c^{8} f \cos\left(f x + e\right)^{7} - 32 \, c^{8} f \cos\left(f x + e\right)^{6} + 56 \, c^{8} f \cos\left(f x + e\right)^{5} + 160 \, c^{8} f \cos\left(f x + e\right)^{4} - 112 \, c^{8} f \cos\left(f x + e\right)^{3} - 256 \, c^{8} f \cos\left(f x + e\right)^{2} + 64 \, c^{8} f \cos\left(f x + e\right) + 128 \, c^{8} f + {\left(c^{8} f \cos\left(f x + e\right)^{7} + 8 \, c^{8} f \cos\left(f x + e\right)^{6} - 24 \, c^{8} f \cos\left(f x + e\right)^{5} - 80 \, c^{8} f \cos\left(f x + e\right)^{4} + 80 \, c^{8} f \cos\left(f x + e\right)^{3} + 192 \, c^{8} f \cos\left(f x + e\right)^{2} - 64 \, c^{8} f \cos\left(f x + e\right) - 128 \, c^{8} f\right)} \sin\left(f x + e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="fricas")
```

```
[Out] 1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x +
e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x +
e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f*x
+ e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f*x +
 e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A - 11
*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*A - 11
*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3*A + 8*
B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A + B)*a^3)
*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32*c^8*f*co
```

```
s(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4 - 112*c^8
*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x + e) + 128*
c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8*f*cos(f*x +
 e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 192*c^8*f*cos(f
*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8821 vs. $2(178) = 356$.
time = 211.77, size = 8821, normalized size = 44.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)
```

```
[Out] Piecewise((-90090*A*a**3*tan(e/2 + f*x/2)**14/(45045*c**8*f*tan(e/2 + f*x/2
)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2
)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*
x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2
 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(
e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*t
an(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*
tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*ta
n(e/2 + f*x/2) - 45045*c**8*f) + 360360*A*a**3*tan(e/2 + f*x/2)**13/(45045*
c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*
c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 614864
25*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 22
5450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 +
 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**
6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)
**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)
**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 2132130*A*a**3*tan(e
/2 + f*x/2)**12/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2
+ f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/
2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*ta
n(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*
f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c*
*8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*
c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*
c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f)
 + 5405400*A*a**3*tan(e/2 + f*x/2)**11/(45045*c**8*f*tan(e/2 + f*x/2)**15 -
 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 -
 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**1
1 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/
2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f
```

```
*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2
+ f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2
 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 +
 f*x/2) - 45045*c**8*f) - 13351338*A*a**3*tan(e/2 + f*x/2)**10/(45045*c**8*
f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*
f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c*
*8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 2254502
25*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 2898
64575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 1
35270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 +
 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 +
 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 20420400*A*a**3*tan(e/2 +
 f*x/2)**9/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x
/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f
*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2
 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan
(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*
tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*
f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*
f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 28
249650*A*a**3*tan(e/2 + f*x/2)**8/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 6756
75*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 2049
5475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 1
35270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9
 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)
**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x
/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*
x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/
2) - 45045*c**8*f) + 26357760*A*a**3*tan(e/2 + f*x/2)**7/(45045*c**8*f*tan(
e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(
e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*t
an(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**
8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*
c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 1352701
35*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495
475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 67567
5*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 220...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(202) = 404.

time = 0.67, size = 517, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="giac")
```

```
[Out] -2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x +
1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f*x
 + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1/2*
f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*tan(
1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*A*a^3
*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 14124825*A
*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 1317888
0*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 110
26015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/2*e)^6 -
6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x + 1/2*e)^5
+ 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*x + 1/2*e)^
4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*f*x + 1/2*e)
^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*f*x + 1/2*e)^
2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x + 1/2*e) + 42
43*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

Mupad [B]
time = 13.96, size = 577, normalized size = 2.93

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{544369\,A\,a^3}{4}-\frac{21791\,B\,a^3}{4}-\frac{257861\,A\,a^3\cos(2\,e+2\,f\,x)}{2}+\frac{3497111\,A\,a^3\cos(3\,e+3\,f\,x)}{128}+\frac{72047\,A\,a^3\cos(4\,e+4\,f\,x)}{4}-\frac{378579\,A\,a^3\cos(5\,e+5\,f\,x)}{128}-\frac{1059\,A\,a^3\cos(6\,e+6\,f\,x)}{2}+\frac{4251\,A\,a^3\cos(7\,e+7\,f\,x)}{128}+\frac{219769\,B\,a^3\cos(2\,e+2\,f\,x)}{32}-\frac{191389\,B\,a^3\cos(3\,e+3\,f\,x)}{128}-1672\,B\,a^3\cos\left(4\,e+4\,f\,x\right)+\frac{38841\,B\,a^3\cos(5\,e+5\,f\,x)}{128}+\frac{1551\,B\,a^3\cos(6\,e+6\,f\,x)}{32}-\frac{429\,B\,a^3\cos(7\,e+7\,f\,x)}{128}+\frac{2633345\,A\,a^3\sin(2\,e+2\,f\,x)}{64}+\frac{7210775\,A\,a^3\sin(3\,e+3\,f\,x)}{128}-\frac{89375\,A\,a^3\sin(4\,e+4\,f\,x)}{8}-\frac{504205\,A\,a^3\sin(5\,e+5\,f\,x)}{128}+\frac{29765\,A\,a^3\sin(6\,e+6\,f\,x)}{64}+\frac{4235\,A\,a^3\sin(7\,e+7\,f\,x)}{128}-\frac{451165\,B\,a^3\sin(2\,e+2\,f\,x)}{64}-\frac{854425\,B\,a^3\sin(3\,e+3\,f\,x)}{128}+\frac{9295\,B\,a^3\sin(4\,e+4\,f\,x)}{8}+\frac{46475\,B\,a^3\sin(5\,e+5\,f\,x)}{128}-\frac{3025\,B\,a^3\sin(6\,e+6\,f\,x)}{64}-\frac{385\,B\,a^3\sin(7\,e+7\,f\,x)}{128}-\frac{5734111\,A\,a^3\cos(e+f\,x)}{128}+\frac{126929\,B\,a^3\cos(e+f\,x)}{128}-\frac{25501905\,A\,a^3\sin(e+f\,x)}{128}+\frac{3970395\,B\,a^3\sin(e+f\,x)}{128}\right)}{45045\,c^8\,f\left(\frac{6435\,\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{128}-\frac{5005\,\sqrt{2}\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{128}-\frac{3003\,\sqrt{2}\cos\left(\frac{5\,e}{2}+\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{128}+\frac{1365\,\sqrt{2}\cos\left(\frac{7\,e}{2}-\frac{\pi}{4}+\frac{7\,f\,x}{2}\right)}{128}+\frac{455\,\sqrt{2}\cos\left(\frac{9\,e}{2}+\frac{\pi}{4}+\frac{9\,f\,x}{2}\right)}{128}-\frac{105\,\sqrt{2}\cos\left(\frac{11\,e}{2}-\frac{\pi}{4}+\frac{11\,f\,x}{2}\right)}{128}-\frac{15\,\sqrt{2}\cos\left(\frac{13\,e}{2}+\frac{\pi}{4}+\frac{13\,f\,x}{2}\right)}{128}+\frac{\sqrt{2}\cos\left(\frac{15\,e}{2}-\frac{\pi}{4}+\frac{15\,f\,x}{2}\right)}{128}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^8,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((544369*A*a^3)/4 - (21791*B*a^3)/4 - (257861*A*a^3*c
os(2*e + 2*f*x))/2 + (3497111*A*a^3*cos(3*e + 3*f*x))/128 + (72047*A*a^3*co
s(4*e + 4*f*x))/4 - (378579*A*a^3*cos(5*e + 5*f*x))/128 - (1059*A*a^3*cos(6
*e + 6*f*x))/2 + (4251*A*a^3*cos(7*e + 7*f*x))/128 + (219769*B*a^3*cos(2*e
+ 2*f*x))/32 - (191389*B*a^3*cos(3*e + 3*f*x))/128 - 1672*B*a^3*cos(4*e + 4
*f*x) + (38841*B*a^3*cos(5*e + 5*f*x))/128 + (1551*B*a^3*cos(6*e + 6*f*x))/
32 - (429*B*a^3*cos(7*e + 7*f*x))/128 + (2633345*A*a^3*sin(2*e + 2*f*x))/64
 + (7210775*A*a^3*sin(3*e + 3*f*x))/128 - (89375*A*a^3*sin(4*e + 4*f*x))/8
- (504205*A*a^3*sin(5*e + 5*f*x))/128 + (29765*A*a^3*sin(6*e + 6*f*x))/64 +
 (4235*A*a^3*sin(7*e + 7*f*x))/128 - (451165*B*a^3*sin(2*e + 2*f*x))/64 - (
854425*B*a^3*sin(3*e + 3*f*x))/128 + (9295*B*a^3*sin(4*e + 4*f*x))/8 + (464
75*B*a^3*sin(5*e + 5*f*x))/128 - (3025*B*a^3*sin(6*e + 6*f*x))/64 - (385*B*
a^3*sin(7*e + 7*f*x))/128 - (5734111*A*a^3*cos(e + f*x))/128 + (126929*B*a^
3*cos(e + f*x))/128 - (25501905*A*a^3*sin(e + f*x))/128 + (3970395*B*a^3*si
n(e + f*x))/128))/(45045*c^8*f*((6435*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/12
8 - (5005*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/128 - (3003*2^(1/2)*cos(
(5*e)/2 + pi/4 + (5*f*x)/2))/128 + (1365*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*
x)/2))/128 + (455*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/128 - (105*2^(1/
2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/128 - (15*2^(1/2)*cos((13*e)/2 + pi/4
 + (13*f*x)/2))/128 + (2^(1/2)*cos((15*e)/2 - pi/4 + (15*f*x)/2))/128))
```

3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=190

$$-\frac{35(4A-5B)c^4x}{8a}-\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af}-\frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af}-\frac{a^4(A-B)c^4\,\mathrm{c}\cdots}{f(a+a\sin(\cdots}$$

```
[Out] -35/8*(4*A-5*B)*c^4*x/a-35/12*(4*A-5*B)*c^4*cos(f*x+e)^3/a/f-35/8*(4*A-5*B)
*c^4*cos(f*x+e)*sin(f*x+e)/a/f-a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e)
)^5-2*a^2*(4*A-5*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^3-7/4*(4*A-5*B)*c^4
*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
```

Rubi [A]
time = 0.25, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3046, 2938, 2759, 2758, 2761, 2715, 8}

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}-\frac{2a^2c^4(4A-5B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{35c^4(4A-5B)\cos^3(e+fx)}{12af}-\frac{7c^4(4A-5B)\cos^5(e+fx)}{4f(a\sin(e+fx)+a)}-\frac{35c^4(4A-5B)\sin(e+fx)\cos(e+fx)}{8af}-\frac{35c^4x(4A-5B)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-35*(4*A - 5*B)*c^4*x)/(8*a) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]^3)/(12*a*f
) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(8*a*f) - (a^4*(A - B)*c
^4*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5) - (2*a^2*(4*A - 5*B)*c^4*Cos[
e + f*x]^7)/(f*(a + a*Sin[e + f*x])^3) - (7*(4*A - 5*B)*c^4*Cos[e + f*x]^5)
/(4*f*(a + a*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
```

```
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = (a^4c^4)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - (a^3(4A-5B)c^4)\int \frac{\cos}{(a+a}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+f}{f(a+a\sin(e+fx))^3}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+f}{f(a+a\sin(e+fx))^3}$$
$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{a^4(A-B)c^4\cos^9(e+fx}{f(a+a\sin(e+fx))^5}$$
$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(e+}{8af}$$
$$= -\frac{35(4A-5B)c^4x}{8a} - \frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4}{}$$

Mathematica [A]
time = 1.54, size = 274, normalized size = 1.44

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(3072(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-420(4A-5B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-24(47A-75B)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+8(A-5B)\cos(3(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+24(5A-12B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(2(e+fx))+3B\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(4(e+fx))\right)}{96af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(3072*(A - B)
*Sin[(e + f*x)/2] - 420*(4*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 24*(47*A - 75*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]) + 8*(A - 5*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) +
24*(5*A - 12*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)] + 3*
B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[4*(e + f*x)]))/(96*a*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.32, size = 209, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*c^4/a*(-((5/2*A-47/8*B)*tan(1/2*f*x+1/2*e)^7+(11*A-15*B)*tan(1/2*f*x+1/
2*e)^6+(5/2*A-55/8*B)*tan(1/2*f*x+1/2*e)^5+(35*A-55*B)*tan(1/2*f*x+1/2*e)^4
```

```
+(-5/2*A+55/8*B)*tan(1/2*f*x+1/2*e)^3+(107/3*A-175/3*B)*tan(1/2*f*x+1/2*e)^
2+(-5/2*A+47/8*B)*tan(1/2*f*x+1/2*e)+35/3*A-55/3*B)/(1+tan(1/2*f*x+1/2*e)^2
)^4-35/8*(4*A-5*B)*arctan(tan(1/2*f*x+1/2*e))-(16*A-16*B)/(tan(1/2*f*x+1/2*
e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1962 vs. $2(191) = 382$.
time = 0.53, size = 1962, normalized size = 10.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/12*(B*c^4*((19*sin(f*x + e)/(cos(f*x + e) + 1) + 211*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 91*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 219*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 165*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 165*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 64)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 6*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 4*a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a*sin(f*x + e)^8/(cos(f*x + e) + 1)^
8 + a*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 45*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a) - 4*A*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 9*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) + 16*B*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)
 + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4
8*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*
```

```
x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 72*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f
*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a) - 144*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 96*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)
/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 96
*A*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1))) + 24*B*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 24*A*c^4/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1)))/f
```

Fricas [**A**]
time = 0.37, size = 273, normalized size = 1.44

$$-\frac{6\,Bc^4\cos\left(fx+e\right)^5-8\,(A-5\,B)c^4\cos\left(fx+e\right)^4+(52\,A-113\,B)c^4\cos\left(fx+e\right)^3+105\,(4\,A-5\,B)c^4fx+96\,(3\,A-5\,B)c^4\cos\left(fx+e\right)^2+384\,(A-B)c^4+3\,(35\,(4\,A-5\,B)c^4fx+(204\,A-239\,B)c^4)\cos\left(fx+e\right)-(6\,Bc^4\cos\left(fx+e\right)^4+2\,(4\,A-17\,B)c^4\cos\left(fx+e\right)^3-105\,(4\,A-5\,B)c^4fx+3\,(20\,A-49\,B)c^4\cos\left(fx+e\right)^2-3\,(76\,A-111\,B)c^4\cos\left(fx+e\right)+384\,(A-B)c^4)\sin\left(fx+e\right)}{24\,(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(6*B*c^4*cos(f*x + e)^5 - 8*(A - 5*B)*c^4*cos(f*x + e)^4 + (52*A - 11
3*B)*c^4*cos(f*x + e)^3 + 105*(4*A - 5*B)*c^4*f*x + 96*(3*A - 5*B)*c^4*cos(
f*x + e)^2 + 384*(A - B)*c^4 + 3*(35*(4*A - 5*B)*c^4*f*x + (204*A - 239*B)*
c^4)*cos(f*x + e) - (6*B*c^4*cos(f*x + e)^4 + 2*(4*A - 17*B)*c^4*cos(f*x +
e)^3 - 105*(4*A - 5*B)*c^4*f*x + 3*(20*A - 49*B)*c^4*cos(f*x + e)^2 - 3*(76
*A - 111*B)*c^4*cos(f*x + e) + 384*(A - B)*c^4)*sin(f*x + e))/(a*f*cos(f*x
+ e) + a*f*sin(f*x + e) + a*f)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6690 vs. $2(173) = 346$.
time = 8.62, size = 6690, normalized size = 35.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-420*A*c**4*f*x*tan(e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9
+ 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2
+ f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 9
6*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f
*x/2) + 24*a*f) - 420*A*c**4*f*x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/
2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*ta
n(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)*
*4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e
/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2
 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96
*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 +
f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*
f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**6/(24*a*f*
tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)*
*7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan
(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
+ 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**5/(
24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 +
f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*
a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x
/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/
2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan
(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5
 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/
2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2
 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*
a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*
x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f
*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*
tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**
8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e
/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 +
 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*c**
4*f*x*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2
)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*ta
n(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**
3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*
c**4*f*x/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*
tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)
**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan
(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 888*A*c**4*tan(e/2 +
 f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*
f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/
2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*t
an(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 648*A*c**4*tan(e/2
```

```
 + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*
a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*
x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f
*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 3720*A*c**4*tan(
e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 +
96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 +
 f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*
a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1800*A*c**4*t
an(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8
 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/
2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 +
96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 6168*A*c**
4*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)
**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan
(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3
 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1592*A*
c**4*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x
/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*
tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)
**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 4664
*A*c**4*tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 +
f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f...
```

Giac [A]
time = 0.57, size = 343, normalized size = 1.81

$$-\frac{\frac{105\,(4\,Ac^4-5\,Bc^4)(fx+e)}{a}+\frac{768\,(Ac^4-Bc^4)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-141\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+264\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-360\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-165\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+840\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1320\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+165\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+856\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1400\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+141\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+280\,Ac^4-440\,Bc^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^4a}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/24*(105*(4*A*c^4 - 5*B*c^4)*(f*x + e)/a + 768*(A*c^4 - B*c^4)/(a*(tan(1/
2*f*x + 1/2*e) + 1)) + 2*(60*A*c^4*tan(1/2*f*x + 1/2*e)^7 - 141*B*c^4*tan(1
/2*f*x + 1/2*e)^7 + 264*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 360*B*c^4*tan(1/2*f*
x + 1/2*e)^6 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 165*B*c^4*tan(1/2*f*x + 1/
2*e)^5 + 840*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1320*B*c^4*tan(1/2*f*x + 1/2*e)
^4 - 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*B*c^4*tan(1/2*f*x + 1/2*e)^3 + 8
56*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1400*B*c^4*tan(1/2*f*x + 1/2*e)^2 - 60*A*
c^4*tan(1/2*f*x + 1/2*e) + 141*B*c^4*tan(1/2*f*x + 1/2*e) + 280*A*c^4 - 440
*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a))/f
```

Mupad [B]
time = 14.78, size = 397, normalized size = 2.09

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{55\,A\,c^4}{3}-\frac{299\,B\,c^4}{12}\right)+\frac{166\,A\,c^4}{3}-\frac{206\,B\,c^4}{3}+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7\left(27\,A\,c^4-\frac{167\,B\,c^4}{4}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8\left(37\,A\,c^4-\frac{175\,B\,c^4}{4}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5\left(75\,A\,c^4-\frac{495\,B\,c^4}{4}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6\left(155\,A\,c^4-\frac{687\,B\,c^4}{4}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4\left(257\,A\,c^4-\frac{1153\,B\,c^4}{4}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3\left(\frac{199\,A\,c^4}{3}-\frac{1235\,B\,c^4}{12}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2\left(\frac{583\,A\,c^4}{3}-\frac{2795\,B\,c^4}{12}\right)}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^9+a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8+4\,a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7+4\,a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6+6\,a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5+6\,a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4+4\,a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3+4\,a\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+a\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+a\right)}-\frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(4\,A-5\,B\right)}{140\,A\,c^4-175\,B\,c^4}\right)\,\left(4\,A-5\,B\right)}{4\,a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((55*A*c^4)/3 - (299*B*c^4)/12) + (166*A*c^4)/3 - (20
6*B*c^4)/3 + tan(e/2 + (f*x)/2)^7*(27*A*c^4 - (167*B*c^4)/4) + tan(e/2 + (f
*x)/2)^8*(37*A*c^4 - (175*B*c^4)/4) + tan(e/2 + (f*x)/2)^5*(75*A*c^4 - (495
*B*c^4)/4) + tan(e/2 + (f*x)/2)^6*(155*A*c^4 - (687*B*c^4)/4) + tan(e/2 + (
f*x)/2)^4*(257*A*c^4 - (1153*B*c^4)/4) + tan(e/2 + (f*x)/2)^3*((199*A*c^4)/
3 - (1235*B*c^4)/12) + tan(e/2 + (f*x)/2)^2*((583*A*c^4)/3 - (2795*B*c^4)/1
2))/(f*(a + a*tan(e/2 + (f*x)/2) + 4*a*tan(e/2 + (f*x)/2)^2 + 4*a*tan(e/2 +
 (f*x)/2)^3 + 6*a*tan(e/2 + (f*x)/2)^4 + 6*a*tan(e/2 + (f*x)/2)^5 + 4*a*tan
(e/2 + (f*x)/2)^6 + 4*a*tan(e/2 + (f*x)/2)^7 + a*tan(e/2 + (f*x)/2)^8 + a*t
an(e/2 + (f*x)/2)^9)) - (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(4*A - 5*B)
)/(140*A*c^4 - 175*B*c^4))*(4*A - 5*B))/(4*a*f)
```

3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=157

$$-\frac{5(3A-4B)c^3x}{2a}-\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af}-\frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af}-\frac{a^3(A-B)c^3\cos^7(}{f(a+a\sin(e+}$$

```
[Out] -5/2*(3*A-4*B)*c^3*x/a-5/3*(3*A-4*B)*c^3*cos(f*x+e)^3/a/f-5/2*(3*A-4*B)*c^3
*cos(f*x+e)*sin(f*x+e)/a/f-a^3*(A-B)*c^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4-
2*a^3*(3*A-4*B)*c^3*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^2
```

Rubi [A]
time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{2a^3c^3(3A-4B)\cos^5(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{5c^3(3A-4B)\cos^3(e+fx)}{3af}-\frac{5c^3(3A-4B)\sin(e+fx)\cos(e+fx)}{2af}-\frac{5c^3x(3A-4B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-5*(3*A - 4*B)*c^3*x)/(2*a) - (5*(3*A - 4*B)*c^3*Cos[e + f*x]^3)/(3*a*f) -
 (5*(3*A - 4*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - (a^3*(A - B)*c^3*C
os[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) - (2*a^3*(3*A - 4*B)*c^3*Cos[e +
f*x]^5)/(f*(a^2 + a^2*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\left(a^2(3A-4B)c^3\right)\int \frac{\cos^6}{(a+a\,\mathrm{s}}\\ &= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}\\ &= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}\\ &= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af}-\frac{5(3A-4B)c^3\cos(e+fx)\,\mathrm{s}}{2af}\\ &= -\frac{5(3A-4B)c^3x}{2a}-\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af}-\frac{5(3A-}{}\end{aligned}$$

Mathematica [A]

time = 0.91, size = 220, normalized size = 1.40

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A-4B)(e+fx)+(48A-93B)\cos(e+fx)+B\cos(3(e+fx))-3(A-4B)\sin(2(e+fx)))+\sin\left(\frac{1}{2}(e+fx)\right)(-24B(-8+5e+5fx)+6A(-32+15e+15fx)+(48A-93B)\cos(e+fx)+B\cos(3(e+fx))-3(A-4B)\sin(2(e+fx)))\right)}{12af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*(e
 + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 + 5*
e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.29, size = 152, normalized size = 0.97

method	result
derivativedivides	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-5\ldots}{fa}$
default	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-5\ldots}{fa}$
risch	$-\frac{15c^3xA}{2a}+\frac{10c^3xB}{a}-\frac{2c^3e^{i(fx+e)}A}{af}+\frac{31c^3e^{i(fx+e)}B}{8af}-\frac{2c^3e^{-i(fx+e)}A}{af}+\frac{31c^3e^{-i(fx+e)}B}{8af}-\frac{16c^3A}{fa\left(e^{i(fx+e)}\ldots\right.}$
norman	$\frac{-\frac{72Ac^3-94Bc^3}{3af}-\frac{5(3A-4B)c^3x}{2a}-\frac{\left(9Ac^3-18Bc^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(17Ac^3-20Bc^3\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(21Ac^3-34Bc^3\right)\tan\ldots}{3af}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*c^3/a*(-((1/2*A-2*B)*tan(1/2*f*x+1/2*e)^5+(4*A-7*B)*tan(1/2*f*x+1/2*e)^
4+(8*A-16*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+2*B)*tan(1/2*f*x+1/2*e)+4*A-23/3*
B)/(1+tan(1/2*f*x+1/2*e)^2)^3-5/2*(3*A-4*B)*arctan(tan(1/2*f*x+1/2*e))-(8*A
-8*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(158) = 316$.
time = 0.58, size = 1222, normalized size = 7.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/3*(B*c^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*c^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 9*B*c^3*((sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) - 18*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 18*B*c^3*((sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x
 + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 18*A*c^3*(a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1))) + 6*B*c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*c^3/(a + a*sin(f*x + e)/(cos(f*x +
e) + 1)))/f
```

Fricas [A]
time = 0.36, size = 228, normalized size = 1.45

$$-\frac{2\,Bc^3\cos\left(fx+e\right)^4+\left(3\,A-10\,B\right)c^3\cos\left(fx+e\right)^3+15\left(3\,A-4\,B\right)c^3fx+24\left(A-2\,B\right)c^3\cos\left(fx+e\right)^2+48\left(A-B\right)c^3+3\left(5\left(3\,A-4\,B\right)c^3fx+\left(23\,A-28\,B\right)c^3\right)\cos\left(fx+e\right)+\left(2\,Bc^3\cos\left(fx+e\right)^3+15\left(3\,A-4\,B\right)c^3fx-3\left(A-4\,B\right)c^3\cos\left(fx+e\right)^2+3\left(7\,A-12\,B\right)c^3\cos\left(fx+e\right)-48\left(A-B\right)c^3\right)\sin\left(fx+e\right)}{6\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*c^3*cos(f*x + e)^4 + (3*A - 10*B)*c^3*cos(f*x + e)^3 + 15*(3*A -
4*B)*c^3*f*x + 24*(A - 2*B)*c^3*cos(f*x + e)^2 + 48*(A - B)*c^3 + 3*(5*(3*A
 - 4*B)*c^3*f*x + (23*A - 28*B)*c^3)*cos(f*x + e) + (2*B*c^3*cos(f*x + e)^3
```

```
 + 15*(3*A - 4*B)*c^3*f*x - 3*(A - 4*B)*c^3*cos(f*x + e)^2 + 3*(7*A - 12*B)
*c^3*cos(f*x + e) - 48*(A - B)*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*s
in(f*x + e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. $2(139) = 278$.

time = 4.66, size = 4255, normalized size = 27.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)
**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x
*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 135*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 102*A*c**3*tan(e/2 + f*x/2)**
6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c**3*tan(e/
2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a
*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/
2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 336*
```

```
A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) - 96*A*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) - 378*A*c**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 42*A*c**3*tan(e/2 + f*x/2)/(6*a*f*tan(e/2
 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*
a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x
/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 144*A*c**3/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 +
 f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*
tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)*
*3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c
**3*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*
x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*
tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)
**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/...
```

Giac [A]

time = 0.52, size = 235, normalized size = 1.50

$$-\frac{\frac{15\,(3\,Ac^3-4\,Bc^3)(fx+e)}{a}+\frac{96\,(Ac^3-Bc^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-12\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+24\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+48\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-96\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,Ac^3-46\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*c^3 - 4*B*c^3)*(f*x + e)/a + 96*(A*c^3 - B*c^3)/(a*(tan(1/2*f
*x + 1/2*e) + 1)) + 2*(3*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 12*B*c^3*tan(1/2*f*
x + 1/2*e)^5 + 24*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*c^3*tan(1/2*f*x + 1/2
*e)^4 + 48*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*c^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*c^3*tan(1/2*f*x + 1/2*e) + 12*B*c^3*tan(1/2*f*x + 1/2*e) + 24*A*c^3 -
46*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

Mupad [B]
time = 14.01, size = 319, normalized size = 2.03

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\,A\,c^3-\frac{34\,B\,c^3}{3}\right)+24\,A\,c^3-\frac{94\,B\,c^3}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(9\,A\,c^3-18\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(17\,A\,c^3-20\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(16\,A\,c^3-32\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(56\,A\,c^3-62\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(63\,A\,c^3-76\,B\,c^3\right)}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+3\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}-\frac{5\,c^3\,\mathrm{atan}\left(\frac{5\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,A-4\,B\right)}{15\,A\,c^3-20\,B\,c^3}\right)\,\left(3\,A-4\,B\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(7*A*c^3 - (34*B*c^3)/3) + 24*A*c^3 - (94*B*c^3)/3 +
tan(e/2 + (f*x)/2)^5*(9*A*c^3 - 18*B*c^3) + tan(e/2 + (f*x)/2)^6*(17*A*c^3
- 20*B*c^3) + tan(e/2 + (f*x)/2)^3*(16*A*c^3 - 32*B*c^3) + tan(e/2 + (f*x)/
2)^4*(56*A*c^3 - 62*B*c^3) + tan(e/2 + (f*x)/2)^2*(63*A*c^3 - 76*B*c^3))/(f
*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)
/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e/2 + (
f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7)) - (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2
)*(3*A - 4*B))/(15*A*c^3 - 20*B*c^3))*(3*A - 4*B))/(a*f)
```

3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{3(2A-3B)c^2x}{2a}-\frac{3(2A-3B)c^2\cos(e+fx)}{2af}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}$$

```
[Out] -3/2*(2*A-3*B)*c^2*x/a-3/2*(2*A-3*B)*c^2*cos(f*x+e)/a/f-a^2*(A-B)*c^2*cos(f
*x+e)^5/f/(a+a*sin(f*x+e))^3-1/2*(2*A-3*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+
e))
```

Rubi [A]
time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2758, 2761, 8}

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{3c^2(2A-3B)\cos(e+fx)}{2af}-\frac{c^2(2A-3B)\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{3c^2x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*(2*A - 3*B)*c^2*x)/(2*a) - (3*(2*A - 3*B)*c^2*Cos[e + f*x])/(2*a*f) - (
a^2*(A - B)*c^2*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3) - ((2*A - 3*B)*c
^2*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \left(a(2A-3B)c^2\right)\int \frac{\cos^4}{(a+a\,\mathrm{s}}$$
$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}$$
$$= -\frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}$$
$$= -\frac{3(2A-3B)c^2x}{2a} - \frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-}{f(a+}$$

Mathematica [A]
time = 0.88, size = 188, normalized size = 1.59

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A-3B)(e+fx)+4(A-3B)\cos(e+fx)+B\sin(2(e+fx)))+\sin\left(\frac{1}{2}(e+fx)\right)(4A(-8+3e+3fx)-2B(-16+9e+9fx)+4(A-3B)\cos(e+fx)+B\sin(2(e+fx)))\right)}{4af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] -1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[
(e + f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*
(e + f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e + 9
*f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(a*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.26, size = 119, normalized size = 1.01

method	result
derivativedivides	$\frac{2c^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots}\cdots\right.}{fa}$
default	$\frac{2c^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots}\cdots\right.}{fa}$
risch	$-\frac{3c^2xA}{a}+\frac{9c^2xB}{2a}-\frac{c^2\mathrm{e}^{i(fx+e)}A}{2af}+\frac{3c^2\mathrm{e}^{i(fx+e)}B}{2af}-\frac{c^2\mathrm{e}^{-i(fx+e)}A}{2af}+\frac{3c^2\mathrm{e}^{-i(fx+e)}B}{2af}-\frac{8c^2A}{fa\left(\mathrm{e}^{i(fx+e)}+i\right)}+\cdots$
norman	$\frac{\frac{\left(6Ac^2-4Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(8Ac^2-9Bc^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(20Ac^2-15Bc^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(22Ac^2-20Bc^2\right)\left(\tan^5\left(\frac{fx}{2}+\cdots\right)\right)}{af}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*c^2/a*(-(-1/2*B*tan(1/2*f*x+1/2*e)^3+(A-3*B)*tan(1/2*f*x+1/2*e)^2+1/2*B
*tan(1/2*f*x+1/2*e)+A-3*B)/(1+tan(1/2*f*x+1/2*e)^2)^2-3/2*(2*A-3*B)*arctan(
tan(1/2*f*x+1/2*e))-(4*A-4*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(117) = 234$.
time = 0.53, size = 662, normalized size = 5.61

$$\frac{Bc^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-2Ac^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+4Bc^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-4Ac^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+2Bc^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{2Ac^2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
```

```
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 4*B*
c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) - 4*A*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*c^2*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 0.37, size = 187, normalized size = 1.58

$$\frac{Bc^2\cos(fx+e)^3 - 3(2A-3B)c^2fx - 2(A-3B)c^2\cos(fx+e)^2 - 8(A-B)c^2 - (3(2A-3B)c^2fx + (10A-13B)c^2)\cos(fx+e) - (3(2A-3B)c^2fx + Bc^2\cos(fx+e)^2 + (2A-5B)c^2\cos(fx+e) - 8(A-B)c^2)\sin(fx+e)}{2(af\cos(fx+e) + af\sin(fx+e) + af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*x
 + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos(f
*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^2*c
os(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(99) = 198$.
time = 2.45, size = 2365, normalized size = 20.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2
*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/
2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan(e/2 + f*x/2)**4/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*
A*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) - 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan
```

```
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 6*A*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) - 16*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*tan(e/2 + f*x/2)**
3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 3
6*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f
*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*ta
n(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 20*A*c**2/(2*a*f*tan(e/2 +
 f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*
tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/
2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*f*x*tan(e/2 + f*x/2)**3/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*
B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 +
f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*t
an(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*
tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 14*B*c**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 42*B*c**2*tan(e/2 + f*x/2)**2/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 10*B
*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) + 28*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*t
an(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**2/(
a*sin(e) + a), True))
```

Giac [A]

time = 0.51, size = 164, normalized size = 1.39

$$-\frac{\frac{3\,(2\,Ac^2-3\,Bc^2)(fx+e)}{a}+\frac{16\,(Ac^2-Bc^2)}{a(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}-\frac{2\left(Bc^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3-2\,Ac^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+6\,Bc^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-Bc^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-2\,Ac^2+6\,Bc^2\right)}{\left(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+1\right)^2a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f*
x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x +
1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e)^2 - B*c^2*tan(1/2*f*x + 1/2*e) - 2*
A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

Mupad [B]
time = 14.58, size = 241, normalized size = 2.04

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,c^2-5\,B\,c^2\right)+10\,A\,c^2-14\,B\,c^2+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,A\,c^2-7\,B\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(8\,A\,c^2-9\,B\,c^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(18\,A\,c^2-21\,B\,c^2\right)}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+2\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+2\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}-\frac{3\,c^2\,\mathrm{atan}\left(\frac{3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A-3\,B\right)}{6\,A\,c^2-9\,B\,c^2}\right)\,\left(2\,A-3\,B\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(2*A*c^2 - 5*B*c^2) + 10*A*c^2 - 14*B*c^2 + tan(e/2 +
 (f*x)/2)^3*(2*A*c^2 - 7*B*c^2) + tan(e/2 + (f*x)/2)^4*(8*A*c^2 - 9*B*c^2)
+ tan(e/2 + (f*x)/2)^2*(18*A*c^2 - 21*B*c^2))/(f*(a + a*tan(e/2 + (f*x)/2)
+ 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2
)^4 + a*tan(e/2 + (f*x)/2)^5)) - (3*c^2*atan((3*c^2*tan(e/2 + (f*x)/2)*(2*A
 - 3*B))/(6*A*c^2 - 9*B*c^2))*(2*A - 3*B))/(a*f)
```

3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=57

$$-\frac{(A-2B)cx}{a}+\frac{Bc\cos(e+fx)}{af}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] -(A-2*B)*c*x/a+B*c*cos(f*x+e)/a/f-2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3046, 2936, 2718}

$$-\frac{2c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx(A-2B)}{a}+\frac{Bc\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 2*B)*c*x)/a) + (B*c*Cos[e + f*x])/(a*f) - (2*(A - B)*c*Cos[e + f*x]
)/(f*(a + a*Sin[e + f*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))} - \frac{c\int (aA-2aB+aB\sin(e+fx$$
$$= -\frac{(A-2B)cx}{a} - \frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))} - \frac{(Bc)\int \sin(e+}{a}$$
$$= -\frac{(A-2B)cx}{a} + \frac{Bc\cos(e+fx)}{af} - \frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.
time = 0.40, size = 127, normalized size = 2.23

$$\frac{\left(-((A-2B)x)+\frac{B\cos(e)\cos(fx)}{f}-\frac{B\sin(e)\sin(fx)}{f}+\frac{4(A-B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(c-c\sin(e+fx))}{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] ((-((A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A -
B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)
```

Maple [A]
time = 0.22, size = 67, normalized size = 1.18

method	result
derivativedivides	$\frac{2c\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A-2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{fa}$
default	$\frac{2c\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A-2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{fa}$
risch	$-\frac{cxA}{a}+\frac{2cxB}{a}+\frac{Bc\,e^{i(fx+e)}}{2af}+\frac{Bc\,e^{-i(fx+e)}}{2af}-\frac{4cA}{fa\left(e^{i(fx+e)}+i\right)}+\frac{4cB}{fa\left(e^{i(fx+e)}+i\right)}$
norman	$\frac{\frac{2Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{4Ac-6Bc}{af}-\frac{(A-2B)cx}{a}+\frac{2Bc\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2(4Ac-5Bc)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(4Ac-4Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-}{\left(1+\tan^2\left(\frac{fx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*c/a*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A-2*B)*arctan(tan(1/2*f*x+1/2*e))-(2*A
-2*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(60) = 120$.
time = 0.49, size = 278, normalized size = 4.88

$$\frac{2\left(Bc\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Ac\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+Bc\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{Ac}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(60) = 120$.
time = 0.38, size = 123, normalized size = 2.16

$$-\frac{(A-2B)cfx-Bc\cos(fx+e)^2+2(A-B)c+((A-2B)cfx+(2A-3B)c)\cos(fx+e)+((A-2B)cfx-Bc\cos(fx+e)-2(A-B)c)\sin(fx+e)}{af\cos(fx+e)+af\sin(fx+e)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x + (
2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A - B
)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(49) = 98$.
time = 1.25, size = 828, normalized size = 14.53

$$\begin{cases} -\frac{Acfx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{Acfx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{Acfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{Acfx}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{4Ac\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{4Ac}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2Bcfx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2Bcfx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2Bcfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2Bcfx}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{4Bc\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2Bc\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{6Bc}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\ f \neq 0 \\ \frac{x(A+B\sin(e))(-c\sin(e)+c)}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(
e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2)**3
+ a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e/2 + f
*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*
f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2
+ a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f
*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*
x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
 + a*f) + 4*B*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2
+ f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)/(a*f*tan
(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
6*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a), T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(60) = 120.
time = 0.50, size = 122, normalized size = 2.14

$$-\frac{\frac{(Ac-2\,Bc)(fx+e)}{a}+\frac{2\left(2\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac-3\,Bc\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(1
/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f*x
 + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

Mupad [B]
time = 12.94, size = 110, normalized size = 1.93

$$-\frac{(4\,A\,c-4\,B\,c)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,c-6\,B\,c}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}-\frac{A\,c\,f\,x-2\,B\,c\,f\,x}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] - (4*A*c - 6*B*c + tan(e/2 + (f*x)/2)^2*(4*A*c - 4*B*c) - 2*B*c*tan(e/2 + (
f*x)/2))/(f*(a + a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2
+ (f*x)/2)^3)) - (A*c*f*x - 2*B*c*f*x)/(a*f)
```

3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=35

$$\frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

```
[Out] B*sec(f*x+e)/a/c/f+A*tan(f*x+e)/a/c/f
```

Rubi [A]
time = 0.10, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2748, 3852, 8}

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\,dx}{ac}\\ &= \frac{B\sec(e+fx)}{acf}+\frac{A\int \sec^2(e+fx)\,dx}{ac}\\ &= \frac{B\sec(e+fx)}{acf}-\frac{A\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{acf}\\ &= \frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$\frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),
x]
```

```
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Maple [A]
time = 0.17, size = 57, normalized size = 1.63

method	result	size
risch	$\frac{2iA+2B\,e^{i(fx+e)}}{\left(e^{i(fx+e)}-i\right)\left(e^{i(fx+e)}+i\right)acf}$	56
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{acf}$	57
default	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{acf}$	57
norman	$\frac{-\frac{2B}{acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}-\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f/a/c*(-(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)-(1/2*A-1/2*B)/(tan(1/2*f*x+1
/2*e)+1))
```

Maxima [A]
time = 0.29, size = 37, normalized size = 1.06

$$\frac{\frac{A\tan(fx+e)}{ac} + \frac{B}{ac\cos(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] (A*tan(f*x + e)/(a*c) + B/(a*c*cos(f*x + e)))/f
```

Fricas [A]
time = 0.33, size = 30, normalized size = 0.86

$$\frac{A\sin(fx+e)+B}{acf\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] (A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.
time = 0.80, size = 83, normalized size = 2.37

$$\begin{cases} -\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} - \frac{2B}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B/
(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e
) + a)*(-c*sin(e) + c)), True))
```

Giac [A]
time = 0.51, size = 41, normalized size = 1.17

$$-\frac{2\left(A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+B\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)
```

Mupad [B]
time = 12.55, size = 39, normalized size = 1.11

$$-\frac{2\left(B+A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,c\,f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*(B + A*tan(e/2 + (f*x)/2)))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=63

$$\frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{(2A-B)\tan(e+fx)}{3ac^2f}$$

```
[Out] 1/3*(A+B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+1/3*(2*A-B)*tan(f*x+e)/a/c^2/
f
```

Rubi [A]
time = 0.14, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 3852, 8}

$$\frac{(2A-B)\tan(e+fx)}{3ac^2f}+\frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + ((2*A - B)*Tan[e
+ f*x])/(3*a*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{(A+B)\sec(e+fx)}{3af\,(c^2-c^2\sin(e+fx))}+\frac{(2A-B)\int \sec^2(e+fx)\,dx}{3ac^2}\\ &= \frac{(A+B)\sec(e+fx)}{3af\,(c^2-c^2\sin(e+fx))}-\frac{(2A-B)\text{Subst}(\int 1\,dx,x,-\tan(}{3ac^2f}\\ &= \frac{(A+B)\sec(e+fx)}{3af\,(c^2-c^2\sin(e+fx))}+\frac{(2A-B)\tan(e+fx)}{3ac^2f}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 108, normalized size = 1.71

$$\frac{\cos(e+fx)(6B-2(A+B)\cos(e+fx)+(4A-2B)\cos(2(e+fx))+8A\sin(e+fx)-4B\sin(e+fx)+A\sin(2(e+fx))+B\sin(2(e+fx)))}{12ac^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2
),x]
```

```
[Out] (Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)]
+ 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e + f
*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.23, size = 93, normalized size = 1.48

method	result
risch	$-\frac{2i\left(4iA\,\mathrm{e}^{i(fx+e)}-2iB\,\mathrm{e}^{i(fx+e)}+3B\,\mathrm{e}^{2i(fx+e)}+2A-B\right)}{3\left(\mathrm{e}^{i(fx+e)}-i\right)^3\left(\mathrm{e}^{i(fx+e)}+i\right)fa\,c^2}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a\,c^2f}$
default	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a\,c^2f}$

norman	$\frac{\frac{2A-4B}{6acf}-\frac{4(2A-B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{(2A-4B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{(8A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}+\frac{(14A-16B)\left(\tan^2\right.}{6acf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a/c^2*(-(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(A+B)/(tan(1/2*f*x+1/2
*e)-1)^3-1/2*(A+B)/(tan(1/2*f*x+1/2*e)-1)^2-(3/4*A+1/4*B)/(tan(1/2*f*x+1/2*
e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(63) = 126$.
time = 0.30, size = 288, normalized size = 4.57

$$-\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin(f*x
+ e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A]
time = 0.34, size = 78, normalized size = 1.24

$$-\frac{(2A-B)\cos(fx+e)^2+(2A-B)\sin(fx+e)-A+2B}{3\left(ac^2f\cos(fx+e)\sin(fx+e)-ac^2f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*f
*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(51) = 102$.
time = 2.70, size = 578, normalized size = 9.17

$$\begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}+\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{2A}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{6B\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}+\frac{4B\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{2B}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f} & \text{for}\ f\neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c
**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*A
*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 +
 f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 + f*x/
2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c
**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/2)**4
- 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*ta
n(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*B*tan(e/2
 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3
+ 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*tan(e/2 + f*x
/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**2),
 True))
```

Giac [A]
time = 0.49, size = 102, normalized size = 1.62

$$-\frac{\frac{3\,(A-B)}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A+B}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)/(
a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B]
time = 12.32, size = 118, normalized size = 1.87

$$\frac{2\left(\frac{3\,B}{2}+A\,\cos\left(e+f\,x\right)+B\,\cos\left(e+f\,x\right)+2\,A\,\sin\left(e+f\,x\right)-B\,\sin\left(e+f\,x\right)+A\,\cos\left(2\,e+2\,f\,x\right)-\frac{B\,\cos\left(2\,e+2\,f\,x\right)}{2}-\frac{A\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{B\,\sin\left(2\,e+2\,f\,x\right)}{2}\right)}{3\,a\,c^2\,f\,\left(2\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

```
[Out] (2*((3*B)/2 + A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) - B*sin(e
+ f*x) + A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))
/2 - (B*sin(2*e + 2*f*x))/2))/(3*a*c^2*f*(2*cos(e + f*x) - sin(2*e + 2*f*x)
))
```

3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))}+\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}$$

```
[Out] 1/5*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/15*(3*A-2*B)*sec(f*x+e)/a/f
/(c^3-c^3*sin(f*x+e))+2/15*(3*A-2*B)*tan(f*x+e)/a/c^3/f
```

Rubi [A]
time = 0.18, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2751, 3852, 8}

$$\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}+\frac{(3A-2B)\sec(e+fx)}{15af\,(c^3-c^3\sin(e+fx))}+\frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(5*a*c*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*Sec[
e + f*x])/(15*a*f*(c^3 - c^3*Sin[e + f*x])) + (2*(3*A - 2*B)*Tan[e + f*x])/
(15*a*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{ac} \\
&= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2} \\
&= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + (2 \\
&= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} - (2 \\
&= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + 2(
\end{aligned}
$$

Mathematica [A]
time = 0.59, size = 157, normalized size = 1.54

$$
-\frac{\cos(e+fx)(80B+5(-9A+B)\cos(e+fx)+32(3A-2B)\cos(2(e+fx))+9A\cos(3(e+fx))-B\cos(3(e+fx))+120A\sin(e+fx)-80B\sin(e+fx)+36A\sin(2(e+fx))-4B\sin(2(e+fx))-24A\sin(3(e+fx))+16B\sin(3(e+fx)))}{240ac^3f(-1+\sin(e+fx))^3(1+\sin(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3
),x]
```

```
[Out] -1/240*(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Cos
[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e + f
```

```
*x] - 80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] - 24
*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(a*c^3*f*(-1 + Sin[e + f*x])^
3*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.32, size = 145, normalized size = 1.42

method	result
risch	$-\frac{4\left(15 i A\,{\mathrm e}^{2 i (f x+e)}+12 A\,{\mathrm e}^{i (f x+e)}+10 B\,{\mathrm e}^{3 i (f x+e)}-3 i A-8 B\,{\mathrm e}^{i (f x+e)}-10 i B\,{\mathrm e}^{2 i (f x+e)}+2 i B\right)}{15\left({\mathrm e}^{i (f x+e)}-i\right)^{5}\left({\mathrm e}^{i (f x+e)}+i\right) f a\,c^{3}}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{5\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{4A+4B}{2\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3}}}{a f\,c^{3}}$
default	$\frac{-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{5\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{4A+4B}{2\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3}}}{a f\,c^{3}}$
norman	$\frac{-\frac{12A+2B}{15acf}+\frac{2(6A-7B)\left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^{7}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2(-8B+7A)\left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5acf}+\frac{2(2A-B)\left(\tan^{6}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2(9A-4\cdots}{\cdots}}{\left(1+\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)c^{2}\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a/c^3*(-(1/8*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(2*A+2*B)/(tan(1/2*f*x
+1/2*e)-1)^5-1/4*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(5/2*A+3/2*B)/(tan(
1/2*f*x+1/2*e)-1)^2-(7/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(9/2*A+7/2*B)/
(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(103) = 206$.
time = 0.30, size = 459, normalized size = 4.50

$$-\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-2\right)}{ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6) + 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*
```

```
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*sin(f*x + e
)/(cos(f*x + e) + 1) + 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [A]
time = 0.35, size = 114, normalized size = 1.12

$$-\frac{4\left(3A-2B\right)\cos\left(fx+e\right)^{2}-\left(2\left(3A-2B\right)\cos\left(fx+e\right)^{2}-9A+6B\right)\sin\left(fx+e\right)-6A+9B}{15\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A +
 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(85) = 170$.
time = 5.57, size = 1236, normalized size = 12.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*
a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*
f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 60*A*
tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2
+ f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2
)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f*x/2)*
*3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*
a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*
f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 18*A*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(
e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f
*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) -
 15*a*c**3*f) - 12*A/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2
 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/
2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75
*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3
*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)**3/(15*a*c**3*f*
```

```
tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/
2) - 15*a*c**3*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a*c**3*f*tan(e/2 + f*x/2)*
*6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75
*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f)
 + 8*B*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(
e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f
*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 2*B/(15*a*c**3*f*t
an(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
+ f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2
) - 15*a*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) +
 c)**3), True))
```

Giac [A]
time = 0.55, size = 177, normalized size = 1.74

$$-\frac{\frac{15\,(A-B)}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-40\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-210\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+50\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+63\,A-7\,B}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A - B)/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 + 15*B*tan(1/2*f*x + 1/2*e)^4 - 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 - 40*B*tan(1/2*f*x
 + 1/2*e)^2 - 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 - 7*B)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B]
time = 12.47, size = 178, normalized size = 1.75

$$\frac{2\left(\frac{5\,B\sin(e+f\,x)}{2}-\frac{15\,A\cos(e+f\,x)}{4}-\frac{5\,B\cos(e+f\,x)}{8}-\frac{15\,A\sin(e+f\,x)}{4}-\frac{5\,B}{2}-3\,A\cos\left(2\,e+2\,f\,x\right)+\frac{3\,A\cos(3\,e+3\,f\,x)}{4}+2\,B\cos\left(2\,e+2\,f\,x\right)+\frac{B\cos(3\,e+3\,f\,x)}{8}+3\,A\sin\left(2\,e+2\,f\,x\right)+\frac{3\,A\sin(3\,e+3\,f\,x)}{4}+\frac{B\sin(2\,e+2\,f\,x)}{2}-\frac{B\sin(3\,e+3\,f\,x)}{2}\right)}{15\,a\,c^3\,f\left(\frac{\cos(3\,e+3\,f\,x)}{4}-\frac{5\cos(e+f\,x)}{4}+\sin\left(2\,e+2\,f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

```
[Out] (2*((5*B*sin(e + f*x))/2 - (15*A*cos(e + f*x))/4 - (5*B*cos(e + f*x))/8 - (
15*A*sin(e + f*x))/4 - (5*B)/2 - 3*A*cos(2*e + 2*f*x) + (3*A*cos(3*e + 3*f*
x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*x
) + (3*A*sin(3*e + 3*f*x))/4 + (B*sin(2*e + 2*f*x))/2 - (B*sin(3*e + 3*f*x)
)/2))/(15*a*c^3*f*(cos(3*e + 3*f*x)/4 - (5*cos(e + f*x))/4 + sin(2*e + 2*f*
x)))
```

3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=142

$$\frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+1/35*(4*A-3*B)*sec(f*x+e)/a/f
/(c^2-c^2*sin(f*x+e))^2+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e))+
2/35*(4*A-3*B)*tan(f*x+e)/a/c^4/f
```

Rubi [A]
time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2751, 3852, 8}

$$\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(7*a*c*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*Sec[
e + f*x])/(35*a*f*(c^2 - c^2*Sin[e + f*x])^2) + ((4*A - 3*B)*Sec[e + f*x])/
(35*a*f*(c^4 - c^4*Sin[e + f*x])) + (2*(4*A - 3*B)*Tan[e + f*x])/(35*a*c^4*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(3}{}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(}{35}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(}{35}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(}{35}$$

Mathematica [A]
time = 0.76, size = 240, normalized size = 1.69

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(560B+(-406A+182B)\cos(e+fx)+224(4A-3B)\cos(2(e+fx))+174A\cos(3(e+fx))-78B\cos(3(e+fx))-64A\cos(4(e+fx))+48B\cos(4(e+fx))+896A\sin(e+fx)-672B\sin(e+fx)+406A\sin(2(e+fx))-182B\sin(2(e+fx))-384A\sin(3(e+fx))+288B\sin(3(e+fx))-29A\sin(4(e+fx))+13B\sin(4(e+fx))\right)}{2240ac^4f(-1+\sin(e+fx))^4(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4
),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x)]
 + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] +
 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*A*Si
n[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288*B*Si
n[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(2240*a*c^
4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.36, size = 189, normalized size = 1.33

method	result
risch	$\frac{4i\left(56iA\,\mathrm{e}^{3i(fx+e)}-42iB\,\mathrm{e}^{3i(fx+e)}+35B\,\mathrm{e}^{4i(fx+e)}-24iA\,\mathrm{e}^{i(fx+e)}+56A\,\mathrm{e}^{2i(fx+e)}+18iB\,\mathrm{e}^{i(fx+e)}-42B\,\mathrm{e}^{2i(fx+e)}-4A+\right.}{35\left(\mathrm{e}^{i(fx+e)}-i\right)^7\left(\mathrm{e}^{i(fx+e)}+i\right)af\,c^4}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-}{af\,c^4}$
default	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-}{af\,c^4}$
norman	$\frac{\frac{(6A-2B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{26A-2B}{35acf}-\frac{12(4A-3B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf}-\frac{2A\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{2(4A-18B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5acf}-\frac{4(3A-}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a/c^4*(-(1/16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)-1/7*(4*A+4*B)/(tan(1/2*f
*x+1/2*e)-1)^7-1/6*(12*A+12*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(18*A+14*B)/(ta
n(1/2*f*x+1/2*e)-1)^4-1/5*(19*A+17*B)/(tan(1/2*f*x+1/2*e)-1)^5-(15/16*A+1/1
6*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(17/4*A+7/4*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3
*(45/4*A+27/4*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(144) = 288$.
time = 0.30, size = 673, normalized size = 4.74

$$-\frac{2\left(\frac{A\left(\frac{43\sin(fx+e)}{\cos(fx+e)+1}-\frac{77\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{175\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{35\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-13\right)}{ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14ac^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{14ac^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{14ac^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{6ac^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{ac^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}-\frac{B\left(\frac{6\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{56\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{70\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-1\right)}{ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14ac^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{14ac^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{14ac^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{6ac^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{ac^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}\right)}{35f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -2/35*(A*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/(
a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 56*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 70*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(a*c^4 - 6*a*c^4*sin(f*
x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a*c^4*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8))/f
```

Fricas [A]
time = 0.36, size = 150, normalized size = 1.06

$$\frac{2\,(4\,A-3\,B)\cos{(fx+e)}^4-9\,(4\,A-3\,B)\cos{(fx+e)}^2+\left(6\,(4\,A-3\,B)\cos{(fx+e)}^2-20\,A+15\,B\right)\sin{(fx+e)}+15\,A-20\,B}{35\,\left(3\,ac^4f\cos{(fx+e)}^3-4\,ac^4f\cos{(fx+e)}-\left(ac^4f\cos{(fx+e)}^3-4\,ac^4f\cos{(fx+e)}\right)\sin{(fx+e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/35*(2*(4*A - 3*B)*cos(f*x + e)^4 - 9*(4*A - 3*B)*cos(f*x + e)^2 + (6*(4*A
 - 3*B)*cos(f*x + e)^2 - 20*A + 15*B)*sin(f*x + e) + 15*A - 20*B)/(3*a*c^4*
f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f*cos(f*x + e)^3 - 4*a*c
^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2468 vs. $2(122) = 244$.
time = 11.82, size = 2468, normalized size = 17.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*A*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210
*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c*
*4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*
tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*
```

```
tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2
 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*
x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)*
*2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*A*tan(e/2 + f*x/2)*
*5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 49
0*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c
**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f
*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*tan(e/2 + f*x/2)**4/(35*a*c**4*f*t
an(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/
2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f
*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2)
 - 35*a*c**4*f) + 14*A*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/2)**8
 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c
**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) -
154*A*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*t
an(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f
*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 86*A*tan(e/2 + f*
x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 +
490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a
*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4
*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 26*A/(35*a*c**4*f*tan(e/2 + f*x/2)**8
- 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490
*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c*
*4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 7
0*B*tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan
(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2
+ f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x
/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 140*B*tan(e/2 + f*x
/2)**5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7
+ 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490
*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c*
*4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 210*B*tan(e/2 + f*x/2)**4/(35*a*c**4
*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*ta
n(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2
 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*
x/2) - 35*a*c**4*f) + 112*B*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/
2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6
 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*
f) - 42*B*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4
*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*ta
n(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2
 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 12*B*tan(e/2
```

```
+ f*x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**
7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 4
90*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*
c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 2*B/(35*a*c**4*f*tan(e/2 + f*x/2)*
*8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 -
490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f),
 Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**4), True))
```

Giac [A]
time = 0.52, size = 237, normalized size = 1.67

$$-\frac{\frac{35\,(A-B)}{ac^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{525\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+35\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1960\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+4025\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-665\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-4480\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+1120\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3143\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-791\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1176\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+392\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+243\,A-51\,B}{ac^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}}{280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/280*(35*(A - B)/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*A*tan(1/2*f*x
 + 1/2*e)^6 + 35*B*tan(1/2*f*x + 1/2*e)^6 - 1960*A*tan(1/2*f*x + 1/2*e)^5 +
280*B*tan(1/2*f*x + 1/2*e)^5 + 4025*A*tan(1/2*f*x + 1/2*e)^4 - 665*B*tan(1/
2*f*x + 1/2*e)^4 - 4480*A*tan(1/2*f*x + 1/2*e)^3 + 1120*B*tan(1/2*f*x + 1/2
*e)^3 + 3143*A*tan(1/2*f*x + 1/2*e)^2 - 791*B*tan(1/2*f*x + 1/2*e)^2 - 1176
*A*tan(1/2*f*x + 1/2*e) + 392*B*tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/(a*c^4
*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [B]
time = 13.23, size = 239, normalized size = 1.68

$$\frac{2\left(\frac{35\,B}{4}+\frac{91\,A\,\cos(e+f\,x)}{4}-\frac{7\,B\,\cos(e+f\,x)}{4}+14\,A\,\sin\left(e+f\,x\right)-\frac{21\,B\,\sin(e+f\,x)}{2}+14\,A\,\cos\left(2\,e+2\,f\,x\right)-\frac{39\,A\,\cos(3\,e+3\,f\,x)}{4}-A\,\cos\left(4\,e+4\,f\,x\right)-\frac{21\,B\,\cos(2\,e+2\,f\,x)}{2}+\frac{3\,B\,\cos(3\,e+3\,f\,x)}{4}+\frac{3\,B\,\cos(4\,e+4\,f\,x)}{4}-\frac{91\,A\,\sin(2\,e+2\,f\,x)}{4}-6\,A\,\sin\left(3\,e+3\,f\,x\right)+\frac{13\,A\,\sin(4\,e+4\,f\,x)}{8}+\frac{7\,B\,\sin(2\,e+2\,f\,x)}{4}+\frac{9\,B\,\sin(3\,e+3\,f\,x)}{2}-\frac{B\,\sin(4\,e+4\,f\,x)}{8}\right)}{35\,a\,c^4\,f\,\left(\frac{7\,\cos(e+f\,x)}{2}-\frac{3\,\cos(3\,e+3\,f\,x)}{2}-\frac{7\,\sin(2\,e+2\,f\,x)}{2}+\frac{\sin(4\,e+4\,f\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

```
[Out] (2*((35*B)/4 + (91*A*cos(e + f*x))/4 - (7*B*cos(e + f*x))/4 + 14*A*sin(e +
f*x) - (21*B*sin(e + f*x))/2 + 14*A*cos(2*e + 2*f*x) - (39*A*cos(3*e + 3*f*
x))/4 - A*cos(4*e + 4*f*x) - (21*B*cos(2*e + 2*f*x))/2 + (3*B*cos(3*e + 3*f
*x))/4 + (3*B*cos(4*e + 4*f*x))/4 - (91*A*sin(2*e + 2*f*x))/4 - 6*A*sin(3*e
 + 3*f*x) + (13*A*sin(4*e + 4*f*x))/8 + (7*B*sin(2*e + 2*f*x))/4 + (9*B*sin
(3*e + 3*f*x))/2 - (B*sin(4*e + 4*f*x)/8))/(35*a*c^4*f*((7*cos(e + f*x))/2
 - (3*cos(3*e + 3*f*x))/2 - (7*sin(2*e + 2*f*x))/2 + sin(4*e + 4*f*x)/4))
```

3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=240

$$\frac{105(4A-7B)c^5x}{8a^2}+\frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f}+\frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f}-\frac{a^5(A-B)c^5\,c}{3f(a+a\sin}$$

```
[Out] 105/8*(4*A-7*B)*c^5*x/a^2+35/4*(4*A-7*B)*c^5*cos(f*x+e)^3/a^2/f+105/8*(4*A-
7*B)*c^5*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a
*sin(f*x+e))^7+2/3*a^3*(4*A-7*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^5+6*a^
4*(4*A-7*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^3+21/4*(4*A-7*B)*c^5*co
s(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A]
time = 0.30, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3046, 2938, 2759, 2758, 2761, 2715, 8}

$$-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}+\frac{2a^3c^5(4A-7B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{35c^5(4A-7B)\cos^3(e+fx)}{4a^2f}+\frac{21c^5(4A-7B)\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}+\frac{105c^5(4A-7B)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{105c^5x(4A-7B)}{8a^2}+\frac{6a^4c^5(4A-7B)\cos^7(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (105*(4*A - 7*B)*c^5*x)/(8*a^2) + (35*(4*A - 7*B)*c^5*Cos[e + f*x]^3)/(4*a^
2*f) + (105*(4*A - 7*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f) - (a^5*(A
- B)*c^5*Cos[e + f*x]^11)/(3*f*(a + a*Sin[e + f*x])^7) + (2*a^3*(4*A - 7*B)
*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^5) + (6*a^4*(4*A - 7*B)*c^5*
Cos[e + f*x]^7)/(f*(a^2 + a^2*Sin[e + f*x])^3) + (21*(4*A - 7*B)*c^5*Cos[e
+ f*x]^5)/(4*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = (a^5c^5)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} - \frac{1}{3}\left(a^4(4A-7B)c^5\right)\int \frac{\cos}{(a+}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+}{3f(a+a\sin(e+fx))^5}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+}{3f(a+a\sin(e+fx))^5}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+}{3f(a+a\sin(e+fx))^5}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)c^5\cos(e+f}{8a^2f}$$

$$= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4}{}$$

Mathematica [A]
time = 1.32, size = 354, normalized size = 1.48

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(2048(A-B)\sin\left(\tfrac{1}{2}(e+fx)\right)-1024(A-B)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-1024(13A-19B)\sin\left(\tfrac{1}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2+1260(4A-7B)(e+fx)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3+24(95A-217B)\cos(e+fx)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3-8(A-7B)\cos(3(e+fx))\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3-24(7A-24B)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3\sin(2(e+fx))-3B\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3\sin(4(e+fx))\right)}{96a^2f\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^{10}(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B)
*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 10
24*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(9
5*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*(A -
7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7*A - 2
4*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)]))/(96*a^2*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.45, size = 252, normalized size = 1.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x,method=_RETURN VERBOSE)

[Out] 2/f*c^5/a^2*(((7/2*A-95/8*B)*tan(1/2*f*x+1/2*e)^7+(23*A-49*B)*tan(1/2*f*x+1/2*e)^6+(7/2*A-103/8*B)*tan(1/2*f*x+1/2*e)^5+(71*A-161*B)*tan(1/2*f*x+1/2*e)^4+(-7/2*A+103/8*B)*tan(1/2*f*x+1/2*e)^3+(215/3*A-497/3*B)*tan(1/2*f*x+1/2*e)^2+(-7/2*A+95/8*B)*tan(1/2*f*x+1/2*e)+71/3*A-161/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^4+105/8*(4*A-7*B)*arctan(tan(1/2*f*x+1/2*e))-1/2*(-64*A+64*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-48*A+80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(64*A-64*B)/(tan(1/2*f*x+1/2*e)+1)^3)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3244 vs. $2(239) = 478$.
time = 0.58, size = 3244, normalized size = 13.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] -1/12*(B*c^5*((603*sin(f*x + e)/(cos(f*x + e) + 1) + 1297*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2228*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2628*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3014*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2618*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1980*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1100*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 495*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 165*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 256)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 13*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 18*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 18*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 13*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7*a^2*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3*a^2*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + a^2*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 165*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 20*A*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 40*B*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 +

```
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)
 + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*A*c^5*((57*sin(f*x +
 e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 + a^2*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) + 40*B*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2
*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 160*A*c^5*
((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*
a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 160*B*c^5*((12*s
in(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 80*A*c^5*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^2) + 40*B*c^5*((9*sin(f*x ...
```

Fricas [A]

time = 0.39, size = 386, normalized size = 1.61

$$-\frac{6\,Bc^5\cos\left(fx+e\right)^6+4\,(2\,A-11\,B)c^5\cos\left(fx+e\right)^5+(76\,A-241\,B)c^5\cos\left(fx+e\right)^4-2\,(212\,A-431\,B)c^5\cos\left(fx+e\right)^3+630\,(4\,A-7\,B)c^5fx-256\,(A-B)c^5-(315\,(4\,A-7\,B)c^5fx-(2156\,A-3485\,B)c^5)\cos\left(fx+e\right)^2+(315\,(4\,A-7\,B)c^5fx+2\,(1196\,A-2141\,B)c^5)\cos\left(fx+e\right)+\left(6\,Bc^5\cos\left(fx+e\right)^5-2\,(4\,A-25\,B)c^5\cos\left(fx+e\right)^4+(68\,A-191\,B)c^5\cos\left(fx+e\right)^3+630\,(4\,A-7\,B)c^5fx+3\,(164\,A-351\,B)c^5\cos\left(fx+e\right)^2+256\,(A-B)c^5+(315\,(4\,A-7\,B)c^5fx+2\,(1324\,A-2269\,B)c^5)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A -
 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(4*
A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A - 3
485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 2141*B)
*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(f*x +
 e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x + 3*(16
4*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)*c^5*f*
x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. 2(224) = 448.
time = 28.34, size = 10608, normalized size = 44.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((1260*A*c**5*f*x*tan(e/2 + f*x/2)**11/(24*a**2*f*tan(e/2 + f*x/2)
**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 31
2*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*
tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 +
 f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**
2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*tan(e/2 + f*x
/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*
f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2
 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)
**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2
*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f*x/2)**11 +
 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2
*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/
2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2
)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72
*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c**5*f*x*tan(e/2 + f*x/2)**
```

8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a*
*2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(
e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x
/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 +
168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) +
22680*A*c**5*f*x*tan(e/2 + f*x/2)**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a
2*f*tan(e/2 + f*x/2)10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*ta
n(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f
*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4
+ 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2
*f*tan(e/2 + f*x/2) + 24*a**2*f) + 27720*A*c**5*f*x*tan(e/2 + f*x/2)**6/(24
*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*
tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
 f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**
5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a
2*f*tan(e/2 + f*x/2)2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 27720
*A*c**5*f*x*tan(e/2 + f*x/2)**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f
*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)
6 + 528*a2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312
*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*ta
n(e/2 + f*x/2) + 24*a**2*f) + 22680*A*c**5*f*x*tan(e/2 + f*x/2)**4/(24*a**2
*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e
/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/
2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 4
32*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f
*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c*
*5*f*x*tan(e/2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(
e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*
x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +
 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2
*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
 + f*x/2) + 24*a**2*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**2/(24*a**2*f*tan
(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f
*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
+ 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**
2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e
/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*
tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/
2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 +
432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2
 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2)
+ 24*a**2*f) + 1260*A*c**5*f*x/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*
tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2

```
+ f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)*
*6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*
a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan
(e/2 + f*x/2) + 24*a**2*f) + 2472*A*c**5*tan(e/2 + f*x/2)**10/(24*a**2*f*ta
n(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)...
```

Giac [A]
time = 0.54, size = 412, normalized size = 1.72

$$\frac{\frac{315\,(4\,Ac^5-7\,Bc^5)(fx+e)}{a^2}+\frac{256\left(9\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+11\,Ac^5-17\,Bc^5\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}+\frac{2\left(84\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-285\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+552\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1176\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+84\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-309\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+1704\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-3864\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-84\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+309\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+1720\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3976\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-84\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+285\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+568\,Ac^5-1288\,Bc^5\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^4a^2}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1/
2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e) -
36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x + 1/
2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f*x +
 1/2*e)^7 + 552*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*tan(1/2*f*x + 1/2
*e)^6 + 84*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*tan(1/2*f*x + 1/2*e)^5
+ 1704*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*tan(1/2*f*x + 1/2*e)^4 - 8
4*A*c^5*tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 1720*A*
c^5*tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 84*A*c^5*t
an(1/2*f*x + 1/2*e) + 285*B*c^5*tan(1/2*f*x + 1/2*e) + 568*A*c^5 - 1288*B*c
^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f
```

Mupad [B]
time = 14.83, size = 500, normalized size = 2.08

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(391\,A\,c^5-\frac{2729\,B\,c^5}{4}\right)+\frac{494\,A\,c^5}{3}-\frac{866\,B\,c^5}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(103\,A\,c^5-\frac{735\,B\,c^5}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9\,\left(323\,A\,c^5-\frac{2213\,B\,c^5}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(1332\,A\,c^5-2253\,B\,c^5\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(1632\,A\,c^5-2943\,B\,c^5\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{2002\,A\,c^5}{3}-\frac{3637\,B\,c^5}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{4420\,A\,c^5}{3}-\frac{7621\,B\,c^5}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{2489\,A\,c^5}{3}-\frac{17609\,B\,c^5}{12}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{4594\,A\,c^5}{3}-\frac{16805\,B\,c^5}{6}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(\frac{6274\,A\,c^5}{3}-\frac{21299\,B\,c^5}{6}\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+7\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+13\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+18\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+22\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+22\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+18\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+13\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+7\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}+\frac{105\,c^5\,\mathrm{atan}\left(\frac{105\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,A-7\,B\right)}{420\,A\,c^5-735\,B\,c^5}\right)\,\left(4\,A-7\,B\right)}{4\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(391*A*c^5 - (2729*B*c^5)/4) + (494*A*c^5)/3 - (866*B*c
^5)/3 + tan(e/2 + (f*x)/2)^10*(103*A*c^5 - (735*B*c^5)/4) + tan(e/2 + (f*x)
/2)^9*(323*A*c^5 - (2213*B*c^5)/4) + tan(e/2 + (f*x)/2)^7*(1332*A*c^5 - 225
3*B*c^5) + tan(e/2 + (f*x)/2)^4*(1632*A*c^5 - 2943*B*c^5) + tan(e/2 + (f*x)
/2)^8*((2002*A*c^5)/3 - (3637*B*c^5)/3) + tan(e/2 + (f*x)/2)^3*((4420*A*c^5
)/3 - (7621*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((2489*A*c^5)/3 - (17609*B*c^5
)/12) + tan(e/2 + (f*x)/2)^6*((4594*A*c^5)/3 - (16805*B*c^5)/6) + tan(e/2 +
 (f*x)/2)^5*((6274*A*c^5)/3 - (21299*B*c^5)/6))/(f*(7*a^2*tan(e/2 + (f*x)/2
)^2 + 13*a^2*tan(e/2 + (f*x)/2)^3 + 18*a^2*tan(e/2 + (f*x)/2)^4 + 22*a^2*ta
n(e/2 + (f*x)/2)^5 + 22*a^2*tan(e/2 + (f*x)/2)^6 + 18*a^2*tan(e/2 + (f*x)/2
)^7 + 13*a^2*tan(e/2 + (f*x)/2)^8 + 7*a^2*tan(e/2 + (f*x)/2)^9 + 3*a^2*tan(
```

```
e/2 + (f*x)/2)^10 + a^2*tan(e/2 + (f*x)/2)^11 + a^2 + 3*a^2*tan(e/2 + (f*x)
/2))) + (105*c^5*atan((105*c^5*tan(e/2 + (f*x)/2)*(4*A - 7*B))/(420*A*c^5 -
 735*B*c^5))*(4*A - 7*B))/(4*a^2*f)
```

3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=180

$$\frac{35(A-2B)c^4x}{2a^2}+\frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f}+\frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f}-\frac{a^4(A-B)c^4\cos^9(e+}{3f(a+a\sin(e+fx}$$

```
[Out] 35/2*(A-2*B)*c^4*x/a^2+35/3*(A-2*B)*c^4*cos(f*x+e)^3/a^2/f+35/2*(A-2*B)*c^4
*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+
e))^6+2*a^2*(A-2*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4+14*(A-2*B)*c^4*co
s(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

Rubi [A]
time = 0.25, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{35c^4(A-2B)\cos^3(e+fx)}{3a^2f}+\frac{2a^2c^4(A-2B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{35c^4(A-2B)\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{35c^4x(A-2B)}{2a^2}+\frac{14c^4(A-2B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (35*(A - 2*B)*c^4*x)/(2*a^2) + (35*(A - 2*B)*c^4*Cos[e + f*x]^3)/(3*a^2*f)
+ (35*(A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(2*a^2*f) - (a^4*(A - B)*c^4
*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^6) + (2*a^2*(A - 2*B)*c^4*Cos[e
+ f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (14*(A - 2*B)*c^4*Cos[e + f*x]^5)/(f
*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} - \left(a^3(A-2B)c^4\right)\int \frac{\cos^8(}{(a+a\sin}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}$$

$$= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} +$$

$$= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2B)c^4\cos(e+fx)\sin}{2a^2f}$$

$$= \frac{35(A-2B)c^4x}{2a^2} + \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2}{}$$

Mathematica [A]
time = 0.84, size = 311, normalized size = 1.73

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(128(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-64(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-128(5A-8B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+210(A-2B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+3(24A-71B)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+B\cos(3(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-3(A-6B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sin(2(e+fx))\right)}{12a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)*
Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(
5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(
A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A - 71*B
)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e + f*x)]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.41, size = 193, normalized size = 1.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^4/a^2*(((1/2*A-3*B)*tan(1/2*f*x+1/2*e)^5+(6*A-17*B)*tan(1/2*f*x+1/2*e
)^4+(12*A-36*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+3*B)*tan(1/2*f*x+1/2*e)+6*A-53
/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+35/2*(A-2*B)*arctan(tan(1/2*f*x+1/2*e))-1/
2*(-32*A+32*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-16*A+32*B)/(tan(1/2*f*x+1/2*e)+1)
-1/3*(32*A-32*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2276 vs. $2(181) = 362$.
time = 0.55, size = 2276, normalized size = 12.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(A*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 4*B*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*B*c^4*((57*sin(f*x + e)/(cos(f*x + e
) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2
4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^
2) + 16*A*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
```

```
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 24
*B*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 12*A*c^4*
((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*B*c^4*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^2) - 2*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) +
1)^3) + 8*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*B*c^4*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [A]
time = 0.36, size = 336, normalized size = 1.87

$$\frac{2\,Bc^4\cos\left(fx+e\right)^5-(3\,A-16\,B)c^4\cos\left(fx+e\right)^4+2\,(15\,A-38\,B)c^4\cos\left(fx+e\right)^3-210\,(A-2\,B)c^4fx+32\,(A-B)c^4+(105\,(A-2\,B)c^4fx-(193\,A-346\,B)c^4)\cos\left(fx+e\right)^2-(105\,(A-2\,B)c^4fx+2\,(97\,A-202\,B)c^4)\cos\left(fx+e\right)-(2\,Bc^4\cos\left(fx+e\right)^4+(3\,A-14\,B)c^4\cos\left(fx+e\right)^3+210\,(A-2\,B)c^4fx+3\,(11\,A-30\,B)c^4\cos\left(fx+e\right)^2+32\,(A-B)c^4+(105\,(A-2\,B)c^4fx+2\,(113\,A-218\,B)c^4)\cos\left(fx+e\right))\sin\left(fx+e\right)}{6\,\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(2*B*c^4*cos(f*x + e)^5 - (3*A - 16*B)*c^4*cos(f*x + e)^4 + 2*(15*A - 3
8*B)*c^4*cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(A
- 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*cos(f*x + e)^2 - (105*(A - 2*B)*c^4*f
*x + 2*(97*A - 202*B)*c^4)*cos(f*x + e) - (2*B*c^4*cos(f*x + e)^4 + (3*A -
14*B)*c^4*cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4*cos(
f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218*B)*c^
4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) -
 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. $2(175) = 350$.

time = 16.26, size = 7337, normalized size = 40.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((105*A*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**9
 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 +
 f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2
+ 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/2)*
*8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*
f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
 f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
630*A*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*
f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 +
 f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4
+ 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan(e/2 + f*x/2)**6/(6*a**2
*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2
+ f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5
 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1260*A*c**
4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2
 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**
6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2
*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
+ f*x/2) + 6*a**2*f) + 1260*A*c**4*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/
2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)*
*7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**
2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2
 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan
(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)
**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a*
*2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/
2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) + 630*A*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)
**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a*
*2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/
2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)*
*2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/
2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*
f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
```

```
f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
105*A*c**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
+ f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) + 198*A*c**4*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 +
18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*t
an(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*
x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 666*A*c**4*tan(e/2 + f*x/2)**7/(6*a
**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e
/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)
**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a*
*2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1066*A*
c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2
+ f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6
 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*
f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
 f*x/2) + 6*a**2*f) + 2094*A*c**4*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f
*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 +
60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*t
an(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1842*A*c**4*tan(e/2 + f*
x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*
a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(
e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2
)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*
f) + 2214*A*c**4*tan(e/2 + f*x/2)**3/(6*a**2*f*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(181) = 362$.
time = 0.53, size = 369, normalized size = 2.05

$$\frac{\frac{105\,(Ac^4-2\,Bc^4)(fx+e)}{a^2}+\frac{2\left(99\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-210\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+333\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-636\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+533\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1160\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+1047\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-1980\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+921\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1980\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+1107\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2140\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+651\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1344\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+393\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-780\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+164\,Ac^4-330\,Bc^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*tan(1/2*f*x + 1/2*e)
^8 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*tan(1/2*f*x + 1/2*e)^7 -
636*B*c^4*tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 1160*
B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 1980*B*c
^4*tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1980*B*c^4*t
```

```
an(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^4*tan(
1/2*f*x + 1/2*e)^3 + 651*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*tan(1/2*
f*x + 1/2*e)^2 + 393*A*c^4*tan(1/2*f*x + 1/2*e) - 780*B*c^4*tan(1/2*f*x + 1
/2*e) + 164*A*c^4 - 330*B*c^4)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2
*e)^2 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

Mupad [B]
time = 14.95, size = 414, normalized size = 2.30

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(131\,A\,c^4-260\,B\,c^4\right)+\frac{164\,A\,c^4}{3}-110\,B\,c^4+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(33\,A\,c^4-70\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(111\,A\,c^4-212\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(217\,A\,c^4-448\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(307\,A\,c^4-660\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(349\,A\,c^4-660\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(\frac{533\,A\,c^4}{3}-\frac{1160\,B\,c^4}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(369\,A\,c^4-\frac{2140\,B\,c^4}{3}\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+6\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+10\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+12\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+12\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+6\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}+\frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A-2\,B\right)}{35\,A\,c^4-70\,B\,c^4}\right)\,\left(A-2\,B\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(131*A*c^4 - 260*B*c^4) + (164*A*c^4)/3 - 110*B*c^4 + t
an(e/2 + (f*x)/2)^8*(33*A*c^4 - 70*B*c^4) + tan(e/2 + (f*x)/2)^7*(111*A*c^4
 - 212*B*c^4) + tan(e/2 + (f*x)/2)^2*(217*A*c^4 - 448*B*c^4) + tan(e/2 + (f
*x)/2)^4*(307*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^5*(349*A*c^4 - 660*B*
c^4) + tan(e/2 + (f*x)/2)^6*((533*A*c^4)/3 - (1160*B*c^4)/3) + tan(e/2 + (f
*x)/2)^3*(369*A*c^4 - (2140*B*c^4)/3))/(f*(6*a^2*tan(e/2 + (f*x)/2)^2 + 10*
a^2*tan(e/2 + (f*x)/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(e/2 + (
f*x)/2)^5 + 10*a^2*tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^7 + 3*a^
2*tan(e/2 + (f*x)/2)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*tan(e/2 + (
f*x)/2))) + (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(A - 2*B))/(35*A*c^4 -
70*B*c^4))*(A - 2*B))/(a^2*f)
```

3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{5(2A-5B)c^3x}{2a^2}+\frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{5(2A}{6f}$$

```
[Out] 5/2*(2*A-5*B)*c^3*x/a^2+5/2*(2*A-5*B)*c^3*cos(f*x+e)/a^2/f-1/3*a^3*(A-B)*c^
3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5+2/3*a*(2*A-5*B)*c^3*cos(f*x+e)^5/f/(a+a
*sin(f*x+e))^3+5/6*(2*A-5*B)*c^3*cos(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A]
time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2758, 2761, 8}

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{5c^3(2A-5B)\cos(e+fx)}{2a^2f}+\frac{5c^3(2A-5B)\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5c^3x(2A-5B)}{2a^2}+\frac{2ac^3(2A-5B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*(2*A - 5*B)*c^3*x)/(2*a^2) + (5*(2*A - 5*B)*c^3*Cos[e + f*x])/(2*a^2*f)
- (a^3*(A - B)*c^3*Cos[e + f*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (2*a*(2*A
 - 5*B)*c^3*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^3) + (5*(2*A - 5*B)*c
^3*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} - \frac{1}{3}\left(a^2(2A-5B)c^3\right)\int \frac{\cos}{(a+a} \cdots$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} \cdots$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} \cdots$$

$$= \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \cdots$$

$$= \frac{5(2A-5B)c^3x}{2a^2} + \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)}{3f(a+a} \cdots$$

Mathematica [A]
time = 0.59, size = 274, normalized size = 1.69

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^3\left(64(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-32(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-32(7A-13B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+30(2A-5B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+12(A-5B)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+3B\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sin(2(e+fx))\right)}{12a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*S
in[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(7*
A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*(2*
A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*B)*C
os[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.38, size = 162, normalized size = 1.00

method	result
derivativedivides	$\frac{2c^3\left(\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A-5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cdots\right.}\cdots\right.}{f\,a^2}$

default	$\frac{2c^3\left(\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A-5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-16A+16}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{f\,a^2}$
risch	$\frac{5c^3xA}{a^2}-\frac{25c^3xB}{2a^2}-\frac{iB\,c^3e^{2i(fx+e)}}{8a^2f}+\frac{c^3e^{i(fx+e)}A}{2a^2f}-\frac{5c^3e^{i(fx+e)}B}{2a^2f}+\frac{c^3e^{-i(fx+e)}A}{2a^2f}-\frac{5c^3e^{-i(fx+e)}B}{2a^2f}+iE$
norman	$\frac{\left(8A\,c^3-25B\,c^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(34A\,c^3-77B\,c^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(38A\,c^3-93B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(148A\,c^3-352B\,c^3\right)(\tan}{af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^3/a^2*((-1/2*B*tan(1/2*f*x+1/2*e)^3+(A-5*B)*tan(1/2*f*x+1/2*e)^2+1/2*
B*tan(1/2*f*x+1/2*e)+A-5*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A-5*B)*arctan
(tan(1/2*f*x+1/2*e))-1/2*(-16*A+16*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-4*A+12*B)/
(tan(1/2*f*x+1/2*e)+1)-1/3*(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1496 vs. $2(159) = 318$.
time = 0.52, size = 1496, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/3*(B*c^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) - 4*A*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*
x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a^2) + 12*B*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
```

```
x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
^2) - 6*A*c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*B*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3
*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a^2) + 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3) - 6*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*c^3*(3*sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3))/f
```

Fricas [**A**]
time = 0.38, size = 303, normalized size = 1.87

$$\frac{3\,Bc^3\cos\left(fx+e\right)^4+6\,(A-4\,B)c^3\cos\left(fx+e\right)^3-30\,(2\,A-5\,B)c^3fx+16\,(A-B)c^3+\left(15\,(2\,A-5\,B)c^3fx-(62\,A-131\,B)c^3\right)\cos\left(fx+e\right)^2-\left(15\,(2\,A-5\,B)c^3fx+2\,(26\,A-71\,B)c^3\right)\cos\left(fx+e\right)+\left(3\,Bc^3\cos\left(fx+e\right)^3-30\,(2\,A-5\,B)c^3fx-3\,(2\,A-9\,B)c^3\cos\left(fx+e\right)^2-16\,(A-B)c^3-\left(15\,(2\,A-5\,B)c^3fx+2\,(34\,A-79\,B)c^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{6\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*c^3*cos(f*x + e)^4 + 6*(A - 4*B)*c^3*cos(f*x + e)^3 - 30*(2*A - 5*
B)*c^3*f*x + 16*(A - B)*c^3 + (15*(2*A - 5*B)*c^3*f*x - (62*A - 131*B)*c^3)
*cos(f*x + e)^2 - (15*(2*A - 5*B)*c^3*f*x + 2*(26*A - 71*B)*c^3)*cos(f*x +
e) + (3*B*c^3*cos(f*x + e)^3 - 30*(2*A - 5*B)*c^3*f*x - 3*(2*A - 9*B)*c^3*c
os(f*x + e)^2 - 16*(A - B)*c^3 - (15*(2*A - 5*B)*c^3*f*x + 2*(34*A - 79*B)*
c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
 - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(148) = 296$.
time = 9.14, size = 4665, normalized size = 28.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((30*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2
+ f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 +
 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*
tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 +
f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*
tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f
*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1
50*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f
*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 +
f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 +
 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2 + f*x/2)/(6
*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan
(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/
2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) + 30*A*c**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 48*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2
 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**
5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2
*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 204*A*c**
3*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f
*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*t
an(e/2 + f*x/2) + 6*a**2*f) + 212*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 432*A*
c**3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 256*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*t
an(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
```

```
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 228
*A*c**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 92*A*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x*tan(e/2 +
f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*
x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 4
2*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*ta
n(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x
*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 525*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*t
an(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/...
```

Giac [A]
time = 0.53, size = 233, normalized size = 1.44

$$\frac{\frac{15\,(2\,Ac^3-5\,Bc^3)(fx+e)}{a^2}-\frac{6\,\left(Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^3+10\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}+\frac{16\,\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-24\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,Ac^3-11\,Bc^3\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*c^3 - 5*B*c^3)*(f*x + e)/a^2 - 6*(B*c^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 10*B*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c^3
*tan(1/2*f*x + 1/2*e) - 2*A*c^3 + 10*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*a^2) + 16*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 12*A*c^3*tan(1/2*f*x + 1/2*e) - 24*B*c^3*tan(1/2*f*x + 1/2*e) + 5*A*c^3
- 11*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 14.34, size = 336, normalized size = 2.07

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(38\,A\,c^3-93\,B\,c^3\right)+\frac{46\,A\,c^3}{3}-\frac{118\,B\,c^3}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(8\,A\,c^3-25\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(34\,A\,c^3-77\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(72\,A\,c^3-166\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{106\,A\,c^3}{3}-\frac{328\,B\,c^3}{3}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{128\,A\,c^3}{3}-\frac{359\,B\,c^3}{3}\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+5\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+7\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+7\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+5\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}+\frac{5\,c^3\,\mathrm{atan}\left(\frac{5\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A-5\,B\right)}{10\,A\,c^3-25\,B\,c^3}\right)\,\left(2\,A-5\,B\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(38*A*c^3 - 93*B*c^3) + (46*A*c^3)/3 - (118*B*c^3)/3 +
tan(e/2 + (f*x)/2)^6*(8*A*c^3 - 25*B*c^3) + tan(e/2 + (f*x)/2)^5*(34*A*c^3
- 77*B*c^3) + tan(e/2 + (f*x)/2)^3*(72*A*c^3 - 166*B*c^3) + tan(e/2 + (f*x)
/2)^4*((106*A*c^3)/3 - (328*B*c^3)/3) + tan(e/2 + (f*x)/2)^2*((128*A*c^3)/3
 - (359*B*c^3)/3))/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2
)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2*tan(e
/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)
)) + (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2)*(2*A - 5*B))/(10*A*c^3 - 25*B*c^
3))*(2*A - 5*B))/(a^2*f)
```

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{(A-4B)c^2x}{a^2}+\frac{(A-4B)c^2\cos(e+fx)}{a^2f}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}$$

```
[Out] (A-4*B)*c^2*x/a^2+(A-4*B)*c^2*cos(f*x+e)/a^2/f-1/3*a^2*(A-B)*c^2*cos(f*x+e)
^5/f/(a+a*sin(f*x+e))^4+2/3*(A-4*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A]
time = 0.19, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2759, 2761, 8}

$$\frac{c^2(A-4B)\cos(e+fx)}{a^2f}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{c^2x(A-4B)}{a^2}+\frac{2c^2(A-4B)\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((A - 4*B)*c^2*x)/a^2 + ((A - 4*B)*c^2*Cos[e + f*x])/(a^2*f) - (a^2*(A - B)
*c^2*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^4) + (2*(A - 4*B)*c^2*Cos[e
+ f*x]^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}-\frac{1}{3}\left(a(A-4B)c^2\right)\int \frac{\cos}{(a+a}\\ &= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}\\ &= \frac{(A-4B)c^2\cos(e+fx)}{a^2f}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2}{}\\ &= \frac{(A-4B)c^2x}{a^2}+\frac{(A-4B)c^2\cos(e+fx)}{a^2f}-\frac{a^2(A-B)c^2\,c}{3f(a+a\sin(}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 234 vs. $2(108) = 216$.
time = 0.40, size = 234, normalized size = 2.17

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-4(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-8(2A-5B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3(A-4B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-3B\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)(c-c\sin(e+fx))^2}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A -
 B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)*(c - c*Sin[e + f*x])^2)/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.32, size = 107, normalized size = 0.99

method	result
derivativedivides	$\frac{2c^{2}\left(-\frac{B}{1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^{2}}$
default	$\frac{2c^{2}\left(-\frac{B}{1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^{2}}$
risch	$\frac{c^{2}xA}{a^{2}}-\frac{4c^{2}xB}{a^{2}}-\frac{B\,c^{2}{\mathrm e}^{i(fx+e)}}{2a^{2}f}-\frac{B\,c^{2}{\mathrm e}^{-i(fx+e)}}{2a^{2}f}+\frac{8iA\,c^{2}{\mathrm e}^{i(fx+e)}+8A\,c^{2}{\mathrm e}^{2i(fx+e)}-24iB\,c^{2}{\mathrm e}^{i(fx+e)}-16B\,c^{2}{\mathrm e}^{2i(f}}{f\,a^{2}\left({\mathrm e}^{i(fx+e)}+i\right)^{3}}$
norman	$\frac{\frac{c^{2}(A-4B)x}{a}+\frac{\left(8A\,c^{2}-30B\,c^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c^{2}(A-4B)x\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{8A\,c^{2}-38B\,c^{2}}{3af}-\frac{8B\,c^{2}\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{2\left(4A\,c^{2}-61B\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^2/a^2*(-B/(1+tan(1/2*f*x+1/2*e)^2)+(A-4*B)*arctan(tan(1/2*f*x+1/2*e))
-1/2*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A-8*B)/(tan(1/2*f*x+1/2*e)+
1)^3-4*B/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(109) = 218$.
time = 0.60, size = 903, normalized size = 8.36

$$-\frac{2\left(2\,Bc^{2}\left(\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{9\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+5}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{4a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{4a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3a^{2}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{2}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{2}}\right)-Ac^{2}\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+4}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{2}}\right)+2\,Bc^{2}\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+4}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{2}}\right)+\frac{Ac^{2}\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}-\frac{2\,Ac^{2}\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{Bc^{2}\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
```

```
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*
c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 2*B*c^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(109) = 218$.
time = 0.36, size = 252, normalized size = 2.33

$$-\frac{3\,Bc^2\cos\left(fx+e\right)^3+6\,(A-4\,B)c^2fx-4\,(A-B)c^2-(3\,(A-4\,B)c^2fx-(8\,A-23\,B)c^2)\cos\left(fx+e\right)^2+(3\,(A-4\,B)c^2fx+2\,(2\,A-11\,B)c^2)\cos\left(fx+e\right)+\left(6\,(A-4\,B)c^2fx-3\,Bc^2\cos\left(fx+e\right)^2+4\,(A-B)c^2+(3\,(A-4\,B)c^2fx+2\,(4\,A-13\,B)c^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A
- 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x +
2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*x +
 e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(f*x
+ e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f -
(a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. $2(102) = 204$.
time = 4.96, size = 2474, normalized size = 22.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x*t
```

```
an(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a*
*2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5
 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)
**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*A*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)*
*5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*A*c**2*
tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 24*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 - 12*B*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(e/2
 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*t
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x
/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(
e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 12*B*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c**2*tan(
e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*
f*tan(e/2 + f*x/2) + 3*a**2*f) - 78*B*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) - 74*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
```

```
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 90*B*c**2*tan(e/2 + f*x
/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*
f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) - 38*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))
```

Giac [**A**]
time = 0.54, size = 136, normalized size = 1.26

$$\frac{\frac{3\,(Ac^2-4\,Bc^2)(fx+e)}{a^2}-\frac{6\,Bc^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{8\,\left(3\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Ac^2+4\,Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*e)
 + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

Mupad [**B**]
time = 14.53, size = 242, normalized size = 2.24

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,A\,c^2-30\,B\,c^2\right)+\frac{8\,A\,c^2}{3}-\frac{38\,B\,c^2}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(8\,A\,c^2-26\,B\,c^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{8\,A\,c^2}{3}-\frac{74\,B\,c^2}{3}\right)-8\,B\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}+\frac{2\,c^2\,\mathrm{atan}\left(\frac{2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A-4\,B)}{2\,A\,c^2-8\,B\,c^2}\right)\,(A-4\,B)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(8*A*c^2 - 30*B*c^2) + (8*A*c^2)/3 - (38*B*c^2)/3 + tan
(e/2 + (f*x)/2)^3*(8*A*c^2 - 26*B*c^2) + tan(e/2 + (f*x)/2)^2*((8*A*c^2)/3
- (74*B*c^2)/3) - 8*B*c^2*tan(e/2 + (f*x)/2)^4)/(f*(4*a^2*tan(e/2 + (f*x)/2
)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2
 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2))) + (2*c^2*atan((2*c^2*tan(e
/2 + (f*x)/2)*(A - 4*B))/(2*A*c^2 - 8*B*c^2))*(A - 4*B))/(a^2*f)
```

3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{Bcx}{a^2}+\frac{(A-7B)c\cos(e+fx)}{3a^2f(1+\sin(e+fx))}-\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

```
[Out] -B*c*x/a^2+1/3*(A-7*B)*c*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-2/3*(A-B)*c*cos(f*
x+e)/f/(a+a*sin(f*x+e))^2
```

Rubi [A]
time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3046, 2936, 2814, 2727}

$$\frac{c(A-7B)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{Bcx}{a^2}-\frac{2c(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*c*x)/a^2) + ((A - 7*B)*c*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) -
(2*(A - B)*c*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{c\int \frac{aA-4aB+3aB\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= -\frac{Bcx}{a^2} - \frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{((A-7B)c)\int \frac{1}{a+a\sin(e+}}{3a}$$

$$= -\frac{Bcx}{a^2} - \frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(A-7B)c\cos(e+fx}{3f\left(a^2+a^2\sin(e+fx)\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 156 vs. 2(72) = 144.
time = 0.40, size = 156, normalized size = 2.17

$$\frac{c\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A-3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)-14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e+\frac{3fx}{2}\right)+24B\sin\left(\frac{fx}{2}\right)-9Bfx\sin\left(e+\frac{fx}{2}\right)-3Bfx\sin\left(e+\frac{3fx}{2}\right)\right)}{6a^2f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*f
*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin[
(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^2*f
*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A]
time = 0.27, size = 86, normalized size = 1.19

method	result
risch	$-\frac{Bcx}{a^2}+\frac{2Ac\,e^{2i(fx+e)}-8iBc\,e^{i(fx+e)}-6Bc\,e^{2i(fx+e)}-\frac{2Ac}{3}+\frac{14Bc}{3}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$

derivativedivides	$\frac{2c\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}\right)}{f\,a^{2}}$
default	$\frac{2c\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}\right)}{f\,a^{2}}$
norman	$\frac{-\frac{2Ac+10Bc}{3af}-\frac{cxB}{a}-\frac{16Bc\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8Bc\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{(14Ac+22Bc)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(10Ac+26B}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*c/a^2*(-B*arctan(tan(1/2*f*x+1/2*e))-1/2*(-4*A+4*B)/(tan(1/2*f*x+1/2*e)
+1)^2-(A+B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(4*A-4*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(72) = 144$.
time = 0.57, size = 490, normalized size = 6.81

$$-\frac{2\left(Bc\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+4}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{2}}\right)+\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}-\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{Bc\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(72) = 144$.
time = 0.40, size = 174, normalized size = 2.42

$$\frac{6\,Bcfx-(3\,Bcfx+(A-7\,B)c)\cos(fx+e)^{2}+2\,(A-B)c+(3\,Bcfx+(A+5\,B)c)\cos(fx+e)+(6\,Bcfx-2\,(A-B)c+(3\,Bcfx-(A-7\,B)c)\cos(fx+e))\sin(fx+e)}{3\left(a^{2}f\cos(fx+e)^{2}-a^{2}f\cos(fx+e)-2\,a^{2}f-(a^{2}f\cos(fx+e)+2\,a^{2}f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c + (
3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c*f
*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(70) = 140$.
time = 2.51, size = 702, normalized size = 9.75

$$\begin{cases} -\frac{6Ac\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{2Ac}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{3Bcfx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{9Bcfx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{9Bcfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{3Bcfx}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6Bc\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{24Bc\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{10Bc}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} & \text{for}\ f \neq 0 \\ \frac{x(A+B\sin(e))(-c\sin(e)+c)}{(a\sin(e)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(3*
a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*B*c*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)
**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*a**2*f*tan(e/2 +
 f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) - 6*B*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c*tan(e
/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 +
9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*B*c/(3*a**2*f*tan(e/2 + f*x/2)**
3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), N
e(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))
```

Giac [A]
time = 0.48, size = 92, normalized size = 1.28

$$-\frac{\frac{3\,(fx+e)Bc}{a^2} + \frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ac+5\,Bc\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/2
*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/2*
f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 12.83, size = 133, normalized size = 1.85

$$-\frac{B\,c\,x}{a^2}-\frac{\left(\frac{c\,\left(6\,A+6\,B+9\,B\,\left(e+f\,x\right)\right)}{3}-3\,B\,c\,\left(e+f\,x\right)\right)\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(\frac{c\,\left(24\,B+9\,B\,\left(e+f\,x\right)\right)}{3}-3\,B\,c\,\left(e+f\,x\right)\right)\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{c\,\left(2\,A+10\,B+3\,B\,\left(e+f\,x\right)\right)}{3}-B\,c\,\left(e+f\,x\right)}{a^2\,f\,{\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] - (B*c*x)/a^2 - (tan(e/2 + (f*x)/2)^2*((c*(6*A + 6*B + 9*B*(e + f*x)))/3 -
3*B*c*(e + f*x)) + tan(e/2 + (f*x)/2)*((c*(24*B + 9*B*(e + f*x)))/3 - 3*B*c
*(e + f*x)) + (c*(2*A + 10*B + 3*B*(e + f*x)))/3 - B*c*(e + f*x))/(a^2*f*(t
an(e/2 + (f*x)/2) + 1)^3)
```

3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=62

$$-\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}+\frac{(2A+B)\tan(e+fx)}{3a^2cf}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+1/3*(2*A+B)*tan(f*x+e)/a^2/c
/f
```

Rubi [A]
time = 0.14, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 3852, 8}

$$\frac{(2A+B)\tan(e+fx)}{3a^2cf}-\frac{(A-B)\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/3*((A - B)*Sec[e + f*x])/(c*f*(a^2 + a^2*Sin[e + f*x])) + ((2*A + B)*Tan
[e + f*x])/(3*a^2*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{ac}\\ &= -\frac{(A-B)\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}+\frac{(2A+B)\int \sec^2(e+fx)\,dx}{3a^2c}\\ &= -\frac{(A-B)\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}-\frac{(2A+B)\mathrm{Subst}(\int 1\,dx,x,-\mathrm{ta}}{3a^2cf}\\ &= -\frac{(A-B)\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}+\frac{(2A+B)\tan(e+fx)}{3a^2cf}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 110, normalized size = 1.77

$$\frac{\cos(e+fx)(-6B-2(A-B)\cos(e+fx)+2(2A+B)\cos(2(e+fx))-8A\sin(e+fx)-4B\sin(e+fx)-A\sin(2(e+fx))+B\sin(2(e+fx)))}{12a^2cf(-1+\sin(e+fx))(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)]
 - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e +
f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.23, size = 97, normalized size = 1.56

method	result
risch	$\frac{2i\left(4iA\,e^{i(fx+e)}+2iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2A-B\right)}{3\left(e^{i(fx+e)}+i\right)^3\left(e^{i(fx+e)}-i\right)a^2cf}$
derivativedivides	$\frac{-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a^2cf}$
default	$\frac{-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a^2cf}$

norman	$\frac{-\frac{2A+4B}{6acf}-\frac{4(2A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}+\frac{A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{(2A+4B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{(8A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}-\frac{(14A+16B)\left(\tan\right.}{6acf}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^2/c*(-1/2*(-A+B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(A-B)/(tan(1/2*f*x+1/2*
e)+1)^3-(3/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)-(1/4*B+1/4*A)/(tan(1/2*f*x+1/2
*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(61) = 122$.
time = 0.29, size = 287, normalized size = 4.63

$$\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(f*x +
 e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*
c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A]
time = 0.36, size = 74, normalized size = 1.19

$$-\frac{(2A+B)\cos(fx+e)^2-(2A+B)\sin(fx+e)-A-2B}{3\left(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*f
*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(51) = 102$.
time = 2.67, size = 578, normalized size = 9.32

$$\begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} + \frac{2A}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{6B\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{4B\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{2B}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} & \text{for}\ f \neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*A
*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 +
 f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 + f*x/
2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**
2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/2)**4
+ 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*ta
n(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*B*tan(e/2
 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3
- 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*tan(e/2 + f*x
/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)),
 True))
```

Giac [A]
time = 0.45, size = 102, normalized size = 1.65

$$-\frac{\frac{3(A+B)}{a^2c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)} + \frac{9A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+12A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7A-B}{a^2c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)/(
a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 12.29, size = 117, normalized size = 1.89

$$\frac{2\left(\frac{3B}{2}-A\cos(e+fx)+B\cos(e+fx)+2A\sin(e+fx)+B\sin(e+fx)-A\cos(2e+2fx)-\frac{B\cos(2e+2fx)}{2}-\frac{A\sin(2e+2fx)}{2}+\frac{B\sin(2e+2fx)}{2}\right)}{3a^2cf\left(2\cos(e+fx)+\sin(2e+2fx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

```
[Out] (2*((3*B)/2 - A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) + B*sin(e
+ f*x) - A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))
/2 + (B*sin(2*e + 2*f*x))/2))/(3*a^2*c*f*(2*cos(e + f*x) + sin(2*e + 2*f*x)
))
```

3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=62

$$\frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{A\tan^3(e+fx)}{3a^2c^2f}$$

```
[Out] 1/3*B*sec(f*x+e)^3/a^2/c^2/f+A*tan(f*x+e)/a^2/c^2/f+1/3*A*tan(f*x+e)^3/a^2/
c^2/f
```

Rubi [A]
time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2748, 3852}

$$\frac{A\tan^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*Tan[e + f*x])/(a^2*c^2*f) + (A*Tan[e
+ f*x]^3)/(3*a^2*c^2*f)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\,dx}{a^2c^2}\\ &= \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\int \sec^4(e+fx)\,dx}{a^2c^2}\\ &= \frac{B\sec^3(e+fx)}{3a^2c^2f}-\frac{A\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{a^2c^2f}\\ &= \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{A\tan^3(e+fx)}{3a^2c^2f}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 0.85

$$\frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(a
^2*c^2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(58) = 116.
time = 0.20, size = 145, normalized size = 2.34

method	result
risch	$\frac{4iA\,e^{2i(fx+e)}+\frac{8B\,e^{3i(fx+e)}}{3}+\frac{4iA}{3}}{\left(e^{i(fx+e)}-i\right)^3\left(e^{i(fx+e)}+i\right)^3a^2c^2f}$
derivativedivides	$\frac{-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a^2c^2f}$
default	$\frac{-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a^2c^2f}$
norman	$\frac{-\frac{2B}{3acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}-\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^2\left(\frac{fx}{2}+\right.\right.}{3acf}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2/c^2*(-1/2*(-1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/2*A-1/2*B)
/(tan(1/2*f*x+1/2*e)+1)^3-(1/2*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(1/2*A+1
/2*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^2-(
1/2*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [A]
time = 0.28, size = 50, normalized size = 0.81

$$\frac{\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)A}{a^2c^2}+\frac{B}{a^2c^2\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x + e
)^3))/f
```

Fricas [A]
time = 0.36, size = 44, normalized size = 0.71

$$\frac{\left(2\,A\cos\left(fx+e\right)^2+A\right)\sin\left(fx+e\right)+B}{3\,a^2c^2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(56) = 112$.
time = 2.42, size = 469, normalized size = 7.56

$$\begin{cases} -\frac{6A\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f}+\frac{4A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f}-\frac{6A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f}-\frac{6B\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f}-\frac{2B}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f} & \text{for}\: f \neq 0 \\ \frac{x\left(A+B\sin\left(e\right)\right)}{\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
```

```
2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) - 6*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) - 2*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f
*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne(f, 0)), (
x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.40

$$-\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+B\right)}{3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}a^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/2
*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 -
1)^3*a^2*c^2*f)
```

Mupad [B]
time = 12.38, size = 82, normalized size = 1.32

$$-\frac{2\left(3\,A\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,B\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,A\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,A\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+B\right)}{3\,a^2\,c^2\,f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(B + 3*A*tan(e/2 + (f*x)/2) - 2*A*tan(e/2 + (f*x)/2)^3 + 3*A*tan(e/2 +
(f*x)/2)^5 + 3*B*tan(e/2 + (f*x)/2)^4))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^2
- 1)^3)
```

3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{(A+B)\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}$$

```
[Out] 1/5*(A+B)*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+1/5*(4*A-B)*tan(f*x+e)/a^
2/c^3/f+1/15*(4*A-B)*tan(f*x+e)^3/a^2/c^3/f
```

Rubi [A]
time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2938, 3852}

$$\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(A+B)\sec^3(e+fx)}{5a^2f\,(c^3-c^3\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + ((4*A - B)*Ta
n[e + f*x])/(5*a^2*c^3*f) + ((4*A - B)*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx &= \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^2c^2}\\
&= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{(4A-B)\int \sec^4(e+fx)\,dx}{5a^2c^3}\\
&= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}-\frac{(4A-B)\mathrm{Subst}\left(\int \left(1+x^2\right)\,d\right.}{5a^2c^3f}\\
&= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(4A}{}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 237 vs. $2(93) = 186$.
time = 0.70, size = 237, normalized size = 2.55

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-240B+54(A+B)\cos(e+fx)-32(4A-B)\cos(2(e+fx))+18A\cos(3(e+fx))+18B\cos(3(e+fx))-64A\cos(4(e+fx))+16B\cos(4(e+fx))-384A\sin(e+fx)+96B\sin(e+fx)-18A\sin(2(e+fx))-18B\sin(2(e+fx))-128A\sin(3(e+fx))+32B\sin(3(e+fx))-9A\sin(4(e+fx))-9B\sin(4(e+fx))\right)}{960a^2c^3f(-1+\sin(e+fx))^3(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*A*
Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*Cos
[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e + f*
x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)
] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1 + Sin[
e + f*x])^3*(1 + Sin[e + f*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(87) = 174$.
time = 0.34, size = 183, normalized size = 1.97

method	result
risch	$-\frac{4i\left(24iA\,e^{3i(fx+e)}-6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}+8A\,e^{2i(fx+e)}-2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}+4A-B\right)}{15\left(e^{i(fx+e)}+i\right)^3\left(e^{i(fx+e)}-i\right)^5 f\,a^2c^3}$
derivativedivides	$\frac{-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{5A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}}{a^2fc^3}$

default	$\frac{-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{5A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\cdots}{a^2 f c^3}$
norman	$\frac{\frac{6A-4B}{10acf}-\frac{4(4A-B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}-\frac{A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{(14A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10acf}+\frac{(6A-4B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}+\frac{(2A-8B)\left(\tan\cdots\right.}{3a\cdots}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2/c^3*(-1/2*(-1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/4*A-1/4*B)
/(tan(1/2*f*x+1/2*e)+1)^3-(5/16*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(A+B)/
(tan(1/2*f*x+1/2*e)-1)^5-1/4*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(3/2*A+
B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(5/2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^3-(11/16
*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(92) = 184$.

time = 0.30, size = 705, normalized size = 7.58

$$\frac{2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+3\right)}{a^2c^3-\frac{2a^2c^3\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{6a^2c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{6a^2c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^2c^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{2a^2c^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^2c^3\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}-\frac{B\left(\frac{6\sin(fx+e)}{\cos(fx+e)+1}-\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{8\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{10\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)}{a^2c^3-\frac{2a^2c^3\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{6a^2c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{6a^2c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^2c^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{2a^2c^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^2c^3\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/(a^2*c^
3 - 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*
c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) - 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)/(a^2*c^3 - 2*a^2*c^3*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

Fricas [A]
time = 0.36, size = 123, normalized size = 1.32

$$-\frac{2\,(4\,A-B)\cos\left(fx+e\right)^{4}-(4\,A-B)\cos\left(fx+e\right)^{2}+\left(2\,(4\,A-B)\cos\left(fx+e\right)^{2}+4\,A-B\right)\sin\left(fx+e\right)-A+4\,B}{15\,\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-a^{2}c^{3}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B)
*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e)^
3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. $2(82) = 164$.
time = 11.35, size = 2674, normalized size = 28.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 30*A*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 50*A*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 42*A*tan(e/2 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
```

```
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 18*A*tan(e/2 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*
a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a
**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a*
*2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c
**3*f) - 6*A/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 30*B*tan(e/2
 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 20*B*tan(e/2
 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 10*B*tan(e/2
 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 16*B*tan(e/2
 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 18*B*tan(e/2
 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 12*B*tan(e/2
 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*
x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x
/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/
2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 6*B/(15*a**2*c*
*3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**
3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3
*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*
f*tan(e/2 + f*x/2) - 15*a**2*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**2*(-c*sin(e) + c)**3), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(92) = 184$.
time = 0.50, size = 235, normalized size = 2.53

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A-7\,B\right)}{a^{2}c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+45\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-480\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-60\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+650\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+70\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-400\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-20\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+113\,A+13\,B}{a^{2}c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 - 9*B*tan(1/2*f*x + 1/2*e)^2 + 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A - 7*B)/(a^2*c^3*(ta
n(1/2*f*x + 1/2*e) + 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 + 45*B*tan(1/2*f
*x + 1/2*e)^4 - 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 + 70*B*tan(1/2*f*x + 1/2*e)^2 - 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A + 13*B)/(a^2*c^3*(tan(1/
2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B]
time = 12.45, size = 183, normalized size = 1.97

$$\frac{\left(\frac{8\,A}{15}-\frac{2\,B}{15}-\frac{16\,A\,\sin(e+f\,x)}{15}+\frac{4\,B\,\sin(e+f\,x)}{15}\right)\,\cos\left(e+f\,x\right)^2+\frac{2\,A}{15}-\frac{8\,B}{15}-\frac{8\,A\,\sin(e+f\,x)}{15}+\frac{2\,B\,\sin(e+f\,x)}{15}}{a^2\,c^3\,f\,\left(2\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^3\right)}-\frac{\frac{2\,A}{5}+\frac{2\,B}{5}-\frac{2\,A\,\sin(e+f\,x)}{5}-\frac{2\,B\,\sin(e+f\,x)}{5}}{a^2\,c^3\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{16\,A}{15}-\frac{4\,B}{15}\right)}{a^2\,c^3\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x)
```

```
[Out] ((2*A)/15 - (8*B)/15 - (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 - (2*B)/15 - (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x
))/15))/(a^2*c^3*f*(2*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^3)) - ((
2*A)/5 + (2*B)/5 - (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^2*c^3*f*
(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((16*A)/15 - (4*B)/15))/(a^2*c^3*f*(2
*sin(e + f*x) - 2))
```

3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=135

$$\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/35*(5*A-2*B)*sec(f*x+
e)^3/a^2/f/(c^4-c^4*sin(f*x+e))+4/35*(5*A-2*B)*tan(f*x+e)/a^2/c^4/f+4/105*(
5*A-2*B)*tan(f*x+e)^3/a^2/c^4/f
```

Rubi [A]
time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 3852}

$$\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + ((5*A - 2*B
)*Sec[e + f*x]^3)/(35*a^2*f*(c^4 - c^4*Sin[e + f*x])) + (4*(5*A - 2*B)*Tan[
e + f*x])/(35*a^2*c^4*f) + (4*(5*A - 2*B)*Tan[e + f*x]^3)/(105*a^2*c^4*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx &= \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2} \\
&= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3} \\
&= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} \\
&= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} \\
&= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.
time = 0.66, size = 285, normalized size = 2.11

$$
-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2688B+42(25A+4B)\cos(e+fx)-512(5A-2B)\cos(2(e+fx))+225A\cos(3(e+fx))+36B\cos(3(e+fx))-1280A\cos(4(e+fx))+512B\cos(4(e+fx))-75A\cos(5(e+fx))-12B\cos(5(e+fx))-4480A\sin(e+fx)+1792B\sin(e+fx)-600A\sin(2(e+fx))-96B\sin(2(e+fx))-960A\sin(3(e+fx))+384B\sin(3(e+fx))-300A\sin(4(e+fx))-48B\sin(4(e+fx))+320A\sin(5(e+fx))-128B\sin(5(e+fx))\right)}{13440a^2c^4f(-1+\sin(e+fx))^4(1+\sin(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -1/13440*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[2*
(e + f*x)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos[4*
(e + f*x)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e
```

```
 + f*x)] - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e + f*x
)] - 96*B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e + f*x)
] - 300*A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e + f*x)]
 - 128*B*Sin[5*(e + f*x)]))/(a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f
*x])^2)
```

Maple [A]
time = 0.45, size = 233, normalized size = 1.73

method	result
risch	$-\frac{16\left(2iB-8B\,e^{i(fx+e)}+40A\,e^{3i(fx+e)}-16B\,e^{3i(fx+e)}+15iA\,e^{2i(fx+e)}-5iA-28iB\,e^{4i(fx+e)}+70iA\,e^{4i(fx+e)}-6iB\,e^{2i(\cdots}{105\left(e^{i(fx+e)}-i\right)^{7}\left(e^{i(fx+e)}+i\right)^{3}a^{2}fc^{4}}$
derivativedivides	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}\cdots}{a^{2}fc^{4}}$
default	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}\cdots}{a^{2}fc^{4}}$
norman	$-\frac{20A+6B}{35acf}-\frac{4(10A+11B)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}-\frac{2A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{2(30A-47B)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35acf}-\frac{2(7A-4B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}+\frac{2(2\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2/c^4*(-1/2*(-1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/8*A-1/8*B)
/(tan(1/2*f*x+1/2*e)+1)^3-(3/16*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1)-1/7*(2*A+2*
B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(6*A+6*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(10*
A+8*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(10*A+9*B)/(tan(1/2*f*x+1/2*e)-1)^5-(13
/16*A+1/8*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(23/8*A+11/8*B)/(tan(1/2*f*x+1/2*e)
-1)^2-1/3*(55/8*A+35/8*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(135) = 270.
time = 0.34, size = 905, normalized size = 6.70

$$-\frac{2\left(\frac{B\left(\frac{36\sin(fx+e)}{\cos(fx+e)+1}-\frac{132\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{68\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{14\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{84\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{140\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{140\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{105\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-9\right)}{a^{2}c^{4}-\frac{4a^{2}c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{8a^{2}c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{14a^{2}c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{14a^{2}c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{8a^{2}c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{3a^{2}c^{4}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{4a^{2}c^{4}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{a^{2}c^{4}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}}+\frac{5A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{24\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{76\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{28\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{42\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{56\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{28\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{42\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{21\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-6\right)}{a^{2}c^{4}-\frac{4a^{2}c^{4}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{2}c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{8a^{2}c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{14a^{2}c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{14a^{2}c^{4}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{8a^{2}c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{3a^{2}c^{4}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{4a^{2}c^{4}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{a^{2}c^{4}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}}\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(36*sin(f*x + e)/(cos(f*x + e) + 1) - 132*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 68*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 84*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 140*sin(f*x
```

```
+ e)^6/(cos(f*x + e) + 1)^6 + 140*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 105
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2
*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10) + 5*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
24*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 28*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 56*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 42*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9 - 6)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10))/f
```

Fricas [A]
time = 0.35, size = 160, normalized size = 1.19

$$-\frac{16\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-8\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(8\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-12\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}-25\,A+10\,B\right)\sin\left(fx+e\right)-10\,A+25\,B}{105\,\left(a^{2}c^{4}f\cos\left(fx+e\right)^{5}+2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*(
5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*si
n(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*x +
 e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4228 vs. $2(122) = 244$.
time = 22.30, size = 4228, normalized size = 31.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**9/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10
 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**
```

8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)
6 + 1470*a2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/
2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x
/2) - 105*a**2*c**4*f) + 420*A*tan(e/2 + f*x/2)**8/(105*a**2*c**4*f*tan(e/2
 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e
/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan
(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*t
an(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*
tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*A*tan(e/2 + f*x/2)**7/(105*a**2*c
4*f*tan(e/2 + f*x/2)10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2
*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a*
*2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*
a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420
*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 560*A*tan(e/2 + f*x/2)**
6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)*
*9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)
7 - 1470*a2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x
/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*
x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 420*A*tan(e
/2 + f*x/2)**5/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(
e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan
(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*
tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f
*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f)
 + 280*A*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**
2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a*
*2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470
*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 31
5*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*
a**2*c**4*f) - 760*A*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f*tan(e/2 + f*x/2)*
*10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2
)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x
/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f
*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 +
f*x/2) - 105*a**2*c**4*f) + 240*A*tan(e/2 + f*x/2)**2/(105*a**2*c**4*f*tan(
e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*ta
n(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*
tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*
f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4
*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 30*A*tan(e/2 + f*x/2)/(105*a**2*c*
*4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*
c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**
2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a
2*c4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*
a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 60*A/(105*a**2*c**4*f*tan

```
(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*t
an(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f
*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4
*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**
4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 210*B*tan(e/2 + f*x/2)**8/(105*a*
*2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*
a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 147
0*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 -
840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 +
 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 280*B*tan(e/2 + f*x/
2)**7/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x
/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*
x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 +
 f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2
+ f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*B*t
an(e/2 + f*x/2)**6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*
tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f
*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**
4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c*
*4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(135) = 270$.

time = 0.47, size = 295, normalized size = 2.19

$$-\frac{\frac{35\left(9A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+15A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+8A-5B\right)}{a^2c^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{1365A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+210B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-5775A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-105B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+12250A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-175B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-14350A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+910B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+10185A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-756B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3955A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+427B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+760A-31B}{a^2c^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^7}}{840f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/840*(35*(9*A*tan(1/2*f*x + 1/2*e)^2 - 6*B*tan(1/2*f*x + 1/2*e)^2 + 15*A*
tan(1/2*f*x + 1/2*e) - 9*B*tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(tan(
1/2*f*x + 1/2*e) + 1)^3) + (1365*A*tan(1/2*f*x + 1/2*e)^6 + 210*B*tan(1/2*f
*x + 1/2*e)^6 - 5775*A*tan(1/2*f*x + 1/2*e)^5 - 105*B*tan(1/2*f*x + 1/2*e)^
5 + 12250*A*tan(1/2*f*x + 1/2*e)^4 - 175*B*tan(1/2*f*x + 1/2*e)^4 - 14350*A
*tan(1/2*f*x + 1/2*e)^3 + 910*B*tan(1/2*f*x + 1/2*e)^3 + 10185*A*tan(1/2*f*
x + 1/2*e)^2 - 756*B*tan(1/2*f*x + 1/2*e)^2 - 3955*A*tan(1/2*f*x + 1/2*e) +
 427*B*tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(tan(1/2*f*x + 1/2*e)
- 1)^7))/f
```

Mupad [B]

time = 12.75, size = 197, normalized size = 1.46

$$-\frac{\left(\frac{32A}{21}-\frac{64B}{105}-\frac{16A\sin(e+fx)}{21}+\frac{32B\sin(e+fx)}{105}\right)\cos\left(e+fx\right)^4+\left(\frac{8A}{7}+\frac{12B}{35}-\frac{8A\sin(e+fx)}{7}-\frac{12B\sin(e+fx)}{35}+\frac{(4\sin(e+fx)-4)\left(\frac{4A}{7}+\frac{6B}{35}\right)}{2}\right)\cos\left(e+fx\right)^3+\left(\frac{32B}{105}-\frac{16A}{21}+\frac{8A\sin(e+fx)}{7}-\frac{16B\sin(e+fx)}{35}\right)\cos\left(e+fx\right)^2-\frac{4A}{21}+\frac{10B}{21}+\frac{10A\sin(e+fx)}{21}-\frac{4B\sin(e+fx)}{21}}{a^2\,c^4\,f\,\left(4\cos\left(e+fx\right)^3\sin\left(e+fx\right)-4\cos\left(e+fx\right)^3+2\cos\left(e+fx\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x)
```

```
[Out] -((10*B)/21 - (4*A)/21 + (10*A*sin(e + f*x))/21 - (4*B*sin(e + f*x))/21 + c
os(e + f*x)^3*((8*A)/7 + (12*B)/35 - (8*A*sin(e + f*x))/7 - (12*B*sin(e + f
*x))/35 + ((4*sin(e + f*x) - 4)*((4*A)/7 + (6*B)/35))/2) - cos(e + f*x)^2*(
(16*A)/21 - (32*B)/105 - (8*A*sin(e + f*x))/7 + (16*B*sin(e + f*x))/35) + c
os(e + f*x)^4*((32*A)/21 - (64*B)/105 - (16*A*sin(e + f*x))/21 + (32*B*sin(
e + f*x))/105))/(a^2*c^4*f*(4*cos(e + f*x)^3*sin(e + f*x) - 4*cos(e + f*x)^
3 + 2*cos(e + f*x)^5))
```

3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=175

$$\frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{4(2A-B)\tan(e+}{21a^2c^5f}$$

```
[Out] 1/9*(A+B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+1/21*(2*A-B)*sec(f*x+e)
^3/a^2/c^3/f/(c-c*sin(f*x+e))^2+1/21*(2*A-B)*sec(f*x+e)^3/a^2/f/(c^5-c^5*si
n(f*x+e))+4/21*(2*A-B)*tan(f*x+e)/a^2/c^5/f+4/63*(2*A-B)*tan(f*x+e)^3/a^2/c
^5/f
```

Rubi [A]
time = 0.23, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 3852}

$$\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*
Sec[e + f*x]^3)/(21*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + ((2*A - B)*Sec[e +
f*x]^3)/(21*a^2*f*(c^5 - c^5*Sin[e + f*x])) + (4*(2*A - B)*Tan[e + f*x])/(2
1*a^2*c^5*f) + (4*(2*A - B)*Tan[e + f*x]^3)/(63*a^2*c^5*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} +$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} +$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} +$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} +$$

Mathematica [A]
time = 0.76, size = 329, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-10752B+180(31A-5B)\cos(e+fx)-6912(2A-B)\cos(2(e+fx))+310A\cos(3(e+fx))-50B\cos(3(e+fx))-6144A\cos(4(e+fx))+3072B\cos(4(e+fx))-930A\cos(5(e+fx))+150B\cos(5(e+fx))+512A\cos(6(e+fx))-256B\cos(6(e+fx))-18432A\sin(e+fx)+9216B\sin(e+fx)-4185A\sin(2(e+fx))+675B\sin(2(e+fx))-1024A\sin(3(e+fx))+512B\sin(3(e+fx))-1860A\sin(4(e+fx))+300B\sin(4(e+fx))+3072A\sin(5(e+fx))-1536B\sin(5(e+fx))+155A\sin(6(e+fx))-25B\sin(6(e+fx))\right)}{64512a^2c^5f(-1+\sin(e+fx))^5(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f*x
```

```
)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e + f*x
)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*
x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e + f*x
] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e + f*x)]
- 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e + f*x)
] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5*(e + f*
x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*c^5*f*(-1
 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.54, size = 277, normalized size = 1.58

method	result
risch	$\frac{16i\left(72iA\,e^{5i(fx+e)}-36iB\,e^{5i(fx+e)}+42B\,e^{6i(fx+e)}+4iA\,e^{3i(fx+e)}+54A\,e^{4i(fx+e)}-2iB\,e^{3i(fx+e)}-27B\,e^{4i(fx+e)}-12i}{63\left(e^{i(fx+e)}-i\right)^{9}\left(e^{i(fx+e)}+i\right)^{3}f\,a^{2}c^{5}}$
derivativedivides	$\frac{-\frac{-\frac{A}{16}+\frac{B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{7A}{64}-\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a}$
default	$\frac{-\frac{-\frac{A}{16}+\frac{B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{7A}{64}-\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a}$
norman	$\frac{(4A-8B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{(6A-2B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{38A+2B}{63acf}+\frac{8(2A-B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7acf}-\frac{2A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{4(92A-6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2/c^5*(-1/2*(-1/16*A+1/16*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/16*A-1/1
6*B)/(tan(1/2*f*x+1/2*e)+1)^3-(7/64*A-5/64*B)/(tan(1/2*f*x+1/2*e)+1)-1/9*(4
*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^8-1
/7*(34*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(46*A+40*B)/(tan(1/2*f*x+1/2*e)
-1)^6-1/2*(9/2*A+13/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(57/64*A+5/64*B)/(tan(1/2
*f*x+1/2*e)-1)-1/4*(59/2*A+39/2*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/3*(57/4*A+59/
8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/5*(175/4*A+135/4*B)/(tan(1/2*f*x+1/2*e)-1)^
5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(176) = 352.
time = 0.33, size = 1082, normalized size = 6.18

$$-\frac{2\left(\frac{A\left(\frac{51\sin(fx+e)}{\cos(fx+e)+1}-\frac{39\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{235\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{450\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{306\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{294\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{378\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{63\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{273\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{189\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{63\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-19\right)}{a^{2}c^{5}-\frac{6a^{2}c^{5}\sin(fx+e)}{\cos(fx+e)+1}+\frac{12a^{2}c^{5}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{2a^{2}c^{5}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{27a^{2}c^{5}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{36a^{2}c^{5}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{36a^{2}c^{5}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{27a^{2}c^{5}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{2a^{2}c^{5}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{12a^{2}c^{5}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{6a^{2}c^{5}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^{2}c^{5}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}}+\frac{B\left(\frac{6\sin(fx+e)}{\cos(fx+e)+1}-\frac{75\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{128\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{162\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{36\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{42\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{189\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{126\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{63\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-1\right)}{a^{2}c^{5}-\frac{6a^{2}c^{5}\sin(fx+e)}{\cos(fx+e)+1}+\frac{12a^{2}c^{5}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{2a^{2}c^{5}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{27a^{2}c^{5}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{36a^{2}c^{5}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{36a^{2}c^{5}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{27a^{2}c^{5}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{2a^{2}c^{5}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{12a^{2}c^{5}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{6a^{2}c^{5}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^{2}c^{5}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}}\right)}{63f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1
)^9 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 - 19)/(a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
+ 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36
*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2
*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^
5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) + B*(6*sin(f*x + e)/(cos(f*x + e)
+ 1) - 75*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 128*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 - 162*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 36*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 189*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8 + 126*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 63
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1)/(a^2*c^5 - 6*a^2*c^5*sin(f*x +
e)/(cos(f*x + e) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*
a^2*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 36*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2
*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 2*a^2*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 - a^2*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12))/f
```

Fricas [A]
time = 0.36, size = 198, normalized size = 1.13

$$\frac{8\,(2\,A-B)\cos\left(fx+e\right)^{6}-36\,(2\,A-B)\cos\left(fx+e\right)^{4}+15\,(2\,A-B)\cos\left(fx+e\right)^{2}+\left(24\,(2\,A-B)\cos\left(fx+e\right)^{4}-20\,(2\,A-B)\cos\left(fx+e\right)^{2}-14\,A+7\,B\right)\sin\left(fx+e\right)+7\,A-14\,B}{63\,\left(3\,a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}-\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A -
B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x + e
)^2 - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a^2*c^5*f*cos(f*x + e)^5 -
4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^5*f*cos(f*
x + e)^3)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5868 vs. $2(160) = 320$.
time = 45.13, size = 5868, normalized size = 33.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*A*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12
 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)*
*10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/
2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f
*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 +
 f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2
+ f*x/2) - 63*a**2*c**5*f) + 378*A*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*tan
(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*
tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c
**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2
*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**
2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*A*tan(e/2 + f*x/2)**9/(63
*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 +
756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9
- 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/
2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x
/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 126*A*tan(e/2
 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2
 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(
e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*
f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5
*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f)
 + 756*A*tan(e/2 + f*x/2)**7/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**
2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*
a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 22
68*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 +
 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3
 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) -
 63*a**2*c**5*f) - 588*A*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/
2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f
*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e
/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan
(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) - 612*A*tan(e/2 + f*x/2)**5/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
```

```
**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 900*A*tan(e/2 + f*x/2)**
4/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**
11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2
)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*
x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 +
 f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2
+ f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 470*A*ta
n(e/2 + f*x/2)**3/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*ta
n(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f
*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**
5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*
c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2
*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c*
*5*f) - 78*A*tan(e/2 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378
*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 -
126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8
+ 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2
)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/
2) - 63*a**2*c**5*f) + 102*A*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x
/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 +
f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(
e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(176) = 352$.
time = 0.51, size = 355, normalized size = 2.03

$$-\frac{\frac{21\left(21A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-15B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+36A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-24B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+19A-13B\right)}{a^2c^5\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{3591A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8+315B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-19656A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+756B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+56196A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-4200B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-95760A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+11340B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+107730A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-14994B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-79464A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+13356B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+38484A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6768B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-10944A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2196B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1615A-209B}{a^2c^5\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^9}}{2016f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 36
*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c^5
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*tan
(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*x +
1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e)^6 -
 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 107730*A
*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*tan(1/2*
f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f*x + 1/2
```

```
*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*e) + 2196
*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x + 1/2*e) -
1)^9))/f
```

Mupad [B]
time = 12.88, size = 337, normalized size = 1.93

$$\frac{2\,\left(7\,A-14\,B-14\,A\,\sin\left(e+f\,x\right)+7\,B\,\sin\left(e+f\,x\right)+30\,A\,\cos\left(e+f\,x\right)^2-76\,A\,\cos\left(e+f\,x\right)^3-72\,A\,\cos\left(e+f\,x\right)^4+57\,A\,\cos\left(e+f\,x\right)^5+16\,A\,\cos\left(e+f\,x\right)^6-15\,B\,\cos\left(e+f\,x\right)^2-4\,B\,\cos\left(e+f\,x\right)^3+36\,B\,\cos\left(e+f\,x\right)^4+3\,B\,\cos\left(e+f\,x\right)^5-8\,B\,\cos\left(e+f\,x\right)^6-40\,A\,\cos\left(e+f\,x\right)^2\,\sin\left(e+f\,x\right)+76\,A\,\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)+48\,A\,\cos\left(e+f\,x\right)^4\,\sin\left(e+f\,x\right)-19\,A\,\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)+20\,B\,\cos\left(e+f\,x\right)^2\,\sin\left(e+f\,x\right)+4\,B\,\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)-24\,B\,\cos\left(e+f\,x\right)^4\,\sin\left(e+f\,x\right)-B\,\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)\right)}{63\,a^2\,c^5\,f\,\left(8\,\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)-2\,\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)-8\,\cos\left(e+f\,x\right)^3+6\,\cos\left(e+f\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x)
```

```
[Out] (2*(7*A - 14*B - 14*A*sin(e + f*x) + 7*B*sin(e + f*x) + 30*A*cos(e + f*x)^2
 - 76*A*cos(e + f*x)^3 - 72*A*cos(e + f*x)^4 + 57*A*cos(e + f*x)^5 + 16*A*c
os(e + f*x)^6 - 15*B*cos(e + f*x)^2 - 4*B*cos(e + f*x)^3 + 36*B*cos(e + f*x
)^4 + 3*B*cos(e + f*x)^5 - 8*B*cos(e + f*x)^6 - 40*A*cos(e + f*x)^2*sin(e +
 f*x) + 76*A*cos(e + f*x)^3*sin(e + f*x) + 48*A*cos(e + f*x)^4*sin(e + f*x)
 - 19*A*cos(e + f*x)^5*sin(e + f*x) + 20*B*cos(e + f*x)^2*sin(e + f*x) + 4*
B*cos(e + f*x)^3*sin(e + f*x) - 24*B*cos(e + f*x)^4*sin(e + f*x) - B*cos(e
+ f*x)^5*sin(e + f*x)))/(63*a^2*c^5*f*(8*cos(e + f*x)^3*sin(e + f*x) - 2*co
s(e + f*x)^5*sin(e + f*x) - 8*cos(e + f*x)^3 + 6*cos(e + f*x)^5))
```

3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=243

$$-\frac{21(3A-8B)c^5x}{2a^3}-\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f}-\frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f}-\frac{a^5(A-B)c^5\cos^{1}}{5f(a+a\sin(e}$$

```
[Out] -21/2*(3*A-8*B)*c^5*x/a^3-7*(3*A-8*B)*c^5*cos(f*x+e)^3/a^3/f-21/2*(3*A-8*B)
*c^5*cos(f*x+e)*sin(f*x+e)/a^3/f-1/5*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a*sin
(f*x+e))^8+2/15*a^3*(3*A-8*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6-6/5*a^5
*(3*A-8*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^4-42/5*a^5*(3*A-8*B)*c^5
*cos(f*x+e)^5/f/(a^4+a^4*sin(f*x+e))^2
```

Rubi [A]
time = 0.30, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2761, 2715, 8}

$$-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f}+\frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6}-\frac{21c^5(3A-8B)\sin(e+fx)\cos(e+fx)}{2a^3f}-\frac{21c^5x(3A-8B)}{2a^3}-\frac{42a^5c^5(3A-8B)\cos^5(e+fx)}{5f\left(a^4\sin(e+fx)+a^4\right)^2}-\frac{6a^5c^5(3A-8B)\cos^7(e+fx)}{5f\left(a^2\sin(e+fx)+a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-21*(3*A - 8*B)*c^5*x)/(2*a^3) - (7*(3*A - 8*B)*c^5*Cos[e + f*x]^3)/(a^3*f
) - (21*(3*A - 8*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(2*a^3*f) - (a^5*(A - B)
*c^5*Cos[e + f*x]^11)/(5*f*(a + a*Sin[e + f*x])^8) + (2*a^3*(3*A - 8*B)*c^5
*Cos[e + f*x]^9)/(15*f*(a + a*Sin[e + f*x])^6) - (6*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^7)/(5*f*(a^2 + a^2*Sin[e + f*x])^4) - (42*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^5)/(5*f*(a^4 + a^4*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
```

```
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx &= \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^8}\,dx \\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} - \frac{1}{5}\left(a^4(3A-8B)c^5\right)\int \frac{\mathrm{co}}{(a+} \\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+f}{15f(a+a\sin(e+fx))^6} \\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+f}{15f(a+a\sin(e+fx))^6} \\
&= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+f}{15f(a+a\sin(e+fx))^6} \\
&= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} \\
&= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c^5\cos(e+fx}{2a^3f} \\
&= -\frac{21(3A-8B)c^5x}{2a^3} - \frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A}{}
\end{aligned}
$$

Mathematica [A]
time = 1.78, size = 388, normalized size = 1.60

$$
\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(768(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-384(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-128(21A-31B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+64(21A-31B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+128(54A-119B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-630(3A-8B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-15(32A-127B)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-5B\cos(3(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+15(A-8B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sin(2(e+fx))\right)}{60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}(1+\sin(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)*
Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64
*(21*A - 31*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 128*(54*A - 119*B)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 630*(3*A - 8*B)
*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*(32*A - 127*B)*Cos[
e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 5*B*Cos[3*(e + f*x)]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(A - 8*B)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^10*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.51, size = 240, normalized size = 0.99

method	result
derivativedivides	$\frac{2c^5\left(-\frac{\left(\frac{A}{2}-4B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-31B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(16A-64B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+4B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8A-\frac{95B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\right.}{}$
default	$\frac{2c^5\left(-\frac{\left(\frac{A}{2}-4B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-31B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(16A-64B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+4B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8A-\frac{95B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\right.}{}$
risch	$-\frac{63c^5xA}{2a^3}+\frac{84c^5xB}{a^3}-\frac{B\,c^5 e^{3i(fx+e)}}{24a^3f}-\frac{ic^5 e^{2i(fx+e)}A}{8f\,a^3}+\frac{ic^5 e^{2i(fx+e)}B}{f\,a^3}-\frac{4c^5 e^{i(fx+e)}A}{a^3f}+\frac{127c^5 e^{i(fx+e)}B}{8a^3f}$
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^5/a^3*(-((1/2*A-4*B)*tan(1/2*f*x+1/2*e)^5+(8*A-31*B)*tan(1/2*f*x+1/2*
e)^4+(16*A-64*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+4*B)*tan(1/2*f*x+1/2*e)+8*A-9
5/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3-21/2*(3*A-8*B)*arctan(tan(1/2*f*x+1/2*e))
-1/4*(-256*A+256*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(32*A-96*B)/(tan(1/2*f*x+1
/2*e)+1)^2-(32*A-80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(96*A-32*B)/(tan(1/2*f*x+
1/2*e)+1)^3-1/5*(128*A-128*B)/(tan(1/2*f*x+1/2*e)+1)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3572 vs. $2(242) = 484$.
time = 0.69, size = 3572, normalized size = 14.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^5*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(
a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
```

+ a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos (f*x + e) + 1))/a^3) - A*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673* sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f *x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f* x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f* x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 5*B*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(c os(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f* x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f *x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2 0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5 *a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*A*c^5*((10 5*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos (f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^ 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^ 5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*si n(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 60*B*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*s in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin (f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1) ^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos (f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin (f*x + e)/(cos(f*x + e) + 1))/a^3) - 20*A*c^5*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3 *sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1) ^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos (f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin

```
(f*x + e)/(cos(f*x + e) + 1))/a^3) + 20*B*c^5*((95*sin(f*x + e)/(cos(f*x +
e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^5*(20*sin(f*x + e)/(cos(f*x + e)
 + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +...
```

Fricas [A]
time = 0.38, size = 447, normalized size = 1.84

$$-\frac{10\,Bc^5\cos(fx+e)^6+15\,(A-6\,B)c^5\cos(fx+e)^5+10\,(21\,A-74\,B)c^5\cos(fx+e)^4-1260\,(3\,A-8\,B)c^5fx-192\,(A-B)c^5+(315\,(3\,A-8\,B)c^5fx+(2373\,A-6128\,B)c^5)\cos(fx+e)^3+(945\,(3\,A-8\,B)c^5fx-2\,(753\,A-2248\,B)c^5)\cos(fx+e)^2-6\,(105\,(3\,A-8\,B)c^5fx+2\,(323\,A-848\,B)c^5)\cos(fx+e)+(10\,Bc^5\cos(fx+e)^5-5\,(3\,A-20\,B)c^5\cos(fx+e)^4+5\,(39\,A-128\,B)c^5\cos(fx+e)^3-1260\,(3\,A-8\,B)c^5fx+192\,(A-B)c^5+(315\,(3\,A-8\,B)c^5fx-2\,(1089\,A-2744\,B)c^5)\cos(fx+e)^2-6\,(105\,(3\,A-8\,B)c^5fx+2\,(307\,A-832\,B)c^5)\cos(fx+e))\sin(fx+e)}{30\,(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f+(a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/30*(10*B*c^5*cos(f*x + e)^6 + 15*(A - 6*B)*c^5*cos(f*x + e)^5 + 10*(21*A
 - 74*B)*c^5*cos(f*x + e)^4 - 1260*(3*A - 8*B)*c^5*f*x - 192*(A - B)*c^5 +
(315*(3*A - 8*B)*c^5*f*x + (2373*A - 6128*B)*c^5)*cos(f*x + e)^3 + (945*(3*
A - 8*B)*c^5*f*x - 2*(753*A - 2248*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8
*B)*c^5*f*x + 2*(323*A - 848*B)*c^5)*cos(f*x + e) + (10*B*c^5*cos(f*x + e)^
5 - 5*(3*A - 20*B)*c^5*cos(f*x + e)^4 + 5*(39*A - 128*B)*c^5*cos(f*x + e)^3
 - 1260*(3*A - 8*B)*c^5*f*x + 192*(A - B)*c^5 + (315*(3*A - 8*B)*c^5*f*x -
2*(1089*A - 2744*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8*B)*c^5*f*x + 2*(3
07*A - 832*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2
 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. $2(228) = 456$.
time = 48.37, size = 10608, normalized size = 43.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-945*A*c**5*f*x*tan(e/2 + f*x/2)**11/(30*a**3*f*tan(e/2 + f*x/2)
**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 7
50*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3
*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(
e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x
```

```
/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*tan(e/2
 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 +
 f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)
**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 13
80*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3
*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/
2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 23625*A*c**5*f*x
*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)*
*8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 13
80*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*
f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
 + f*x/2) + 30*a**3*f) - 35910*A*c**5*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*ta
n(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 +
 f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)*
*7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 11
40*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f
*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 43470*A*c
**5*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*ta
n(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 +
f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)*
*6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 75
0*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*
tan(e/2 + f*x/2) + 30*a**3*f) - 43470*A*c**5*f*x*tan(e/2 + f*x/2)**5/(30*a*
*3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*ta
n(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 +
f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)*
*5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390
*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 35
910*A*c**5*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a*
*3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan
(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 +
f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)*
*4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*
a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 23625*A*c**5*f*x*tan(e/2 + f*x/2)**3
/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a*
*3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan
(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 +
f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**
3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*
```

```
f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 +
 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**
3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan
(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 +
f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2
 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*tan(e/2 + f*x
/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390
*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*
tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2
 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2
)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 945*A*c**5*f*x/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2
 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2
)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 +
1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**
3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e
/2 + f*x/2) + 30*a**3*f) - 1950*A*c**5*tan(e/2 ...
```

Giac [A]
time = 0.51, size = 376, normalized size = 1.55

$$-\frac{\frac{315\,(3\,Ac^5-8\,Bc^5)(fx+e)}{a^3}+\frac{10\,\left(3\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+48\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-186\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+96\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-384\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+48\,Ac^5-190\,Bc^5\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a^3}+\frac{64\,\left(30\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-75\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+135\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-345\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+255\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-595\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+165\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-395\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+39\,Ac^5-94\,Bc^5\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(315*(3*A*c^5 - 8*B*c^5)*(f*x + e)/a^3 + 10*(3*A*c^5*tan(1/2*f*x + 1/
2*e)^5 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^5 + 48*A*c^5*tan(1/2*f*x + 1/2*e)^4
- 186*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 96*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 384*
B*c^5*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^5*tan(1/2*f*x + 1/2*e) + 24*B*c^5*tan(
1/2*f*x + 1/2*e) + 48*A*c^5 - 190*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a^
3) + 64*(30*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 75*B*c^5*tan(1/2*f*x + 1/2*e)^4
+ 135*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 345*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 255
*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 595*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 165*A*c^
5*tan(1/2*f*x + 1/2*e) - 395*B*c^5*tan(1/2*f*x + 1/2*e) + 39*A*c^5 - 94*B*c
^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 14.75, size = 501, normalized size = 2.06

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(431\,A\,c^5-\frac{3454\,B\,c^5}{3}\right)+\frac{496\,A\,c^5}{5}-\frac{3958\,B\,c^5}{15}+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{10}\left(65\,A\,c^5-168\,B\,c^5\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^9\left(309\,A\,c^5-838\,B\,c^5\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8\left(826\,A\,c^5-\frac{6418\,B\,c^5}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7\left(1418\,A\,c^5-\frac{11636\,B\,c^5}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3\left(1654\,A\,c^5-\frac{13372\,B\,c^5}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5\left(2332\,A\,c^5-\frac{19072\,B\,c^5}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2\left(\frac{4903\,A\,c^5}{5}-\frac{38884\,B\,c^5}{15}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6\left(\frac{11156\,A\,c^5}{5}-\frac{86708\,B\,c^5}{15}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4\left(\frac{11758\,A\,c^5}{5}-\frac{92224\,B\,c^5}{15}\right)}{f\left(a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{11}+5\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{10}+13\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^9+25\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8+38\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7+46\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6+46\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5+38\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4+25\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3+13\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+a^3\right)}-\frac{21\,c^5\,\mathrm{atan}\left(\frac{21\,c^5\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(3\,A-8\,B\right)}{63\,A\,c^5-168\,B\,c^5}\right)\,\left(3\,A-8\,B\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(431*A*c^5 - (3454*B*c^5)/3) + (496*A*c^5)/5 - (3958*
B*c^5)/15 + tan(e/2 + (f*x)/2)^10*(65*A*c^5 - 168*B*c^5) + tan(e/2 + (f*x)/
2)^9*(309*A*c^5 - 838*B*c^5) + tan(e/2 + (f*x)/2)^8*(826*A*c^5 - (6418*B*c^
5)/3) + tan(e/2 + (f*x)/2)^7*(1418*A*c^5 - (11636*B*c^5)/3) + tan(e/2 + (f*
x)/2)^3*(1654*A*c^5 - (13372*B*c^5)/3) + tan(e/2 + (f*x)/2)^5*(2332*A*c^5 -
 (19072*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((4903*A*c^5)/5 - (38884*B*c^5)/15
) + tan(e/2 + (f*x)/2)^6*((11156*A*c^5)/5 - (86708*B*c^5)/15) + tan(e/2 + (
f*x)/2)^4*((11758*A*c^5)/5 - (92224*B*c^5)/15))/(f*(13*a^3*tan(e/2 + (f*x)/
2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*a^3*tan(e/2 + (f*x)/2)^4 + 46*a^3*t
an(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (f*x)/2)^6 + 38*a^3*tan(e/2 + (f*x)/
2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*a^3*tan(e/2 + (f*x)/2)^9 + 5*a^3*ta
n(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x)/2)^11 + a^3 + 5*a^3*tan(e/2 + (f*
x)/2))) - (21*c^5*atan((21*c^5*tan(e/2 + (f*x)/2)*(3*A - 8*B))/(63*A*c^5 -
168*B*c^5))*(3*A - 8*B))/(a^3*f)
```

3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{7(2A-7B)c^4x}{2a^3}-\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f}-\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5}-$$

```
[Out] -7/2*(2*A-7*B)*c^4*x/a^3-7/2*(2*A-7*B)*c^4*cos(f*x+e)/a^3/f-1/5*a^4*(A-B)*c
^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^7+2/15*a^2*(2*A-7*B)*c^4*cos(f*x+e)^7/f/
(a+a*sin(f*x+e))^5-14/15*(2*A-7*B)*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^3-7/
6*(2*A-7*B)*c^4*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A]
time = 0.27, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2938, 2759, 2758, 2761, 8}

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}-\frac{7c^4(2A-7B)\cos(e+fx)}{2a^3f}-\frac{7c^4(2A-7B)\cos^3(e+fx)}{6f(a^3\sin(e+fx)+a^3)}-\frac{7c^4x(2A-7B)}{2a^3}+\frac{2a^2c^4(2A-7B)\cos^7(e+fx)}{15f(a\sin(e+fx)+a)^5}-\frac{14c^4(2A-7B)\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*(2*A - 7*B)*c^4*x)/(2*a^3) - (7*(2*A - 7*B)*c^4*Cos[e + f*x])/(2*a^3*f)
 - (a^4*(A - B)*c^4*Cos[e + f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (2*a^2*(
2*A - 7*B)*c^4*Cos[e + f*x]^7)/(15*f*(a + a*Sin[e + f*x])^5) - (14*(2*A - 7
*B)*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^3) - (7*(2*A - 7*B)*c^4*
Cos[e + f*x]^3)/(6*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
```

```
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} - \frac{1}{5}\left(a^3(2A-7B)c^4\right)\int \frac{c}{(a+}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+f}{15f(a+a\sin(e+fx))^5}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+f}{15f(a+a\sin(e+fx))^5}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+f}{15f(a+a\sin(e+fx))^5}$$
$$= -\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}$$
$$= -\frac{7(2A-7B)c^4x}{2a^3} - \frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-}{5f(a-}$$

Mathematica [A]
time = 1.06, size = 348, normalized size = 1.73

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(384(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-192(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-128(8A-13B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+64(8A-13B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+64(29A-79B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-210(2A-7B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-60(A-7B)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-15B\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sin(2(e+fx))\right)}{60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)*
Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64*
(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)*Sin
[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 210*(2*A - 7*B)*(e
+ f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 60*(A - 7*B)*Cos[e + f*x]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^8*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.44, size = 201, normalized size = 1.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^4/a^3*(-(-1/2*B*tan(1/2*f*x+1/2*e)^3+(A-7*B)*tan(1/2*f*x+1/2*e)^2+1/2
*B*tan(1/2*f*x+1/2*e)+A-7*B)/(1+tan(1/2*f*x+1/2*e)^2)^2-7/2*(2*A-7*B)*arcta
n(tan(1/2*f*x+1/2*e))-1/4*(-128*A+128*B)/(tan(1/2*f*x+1/2*e)+1)^4-(8*A-24*B
)/(tan(1/2*f*x+1/2*e)+1)-1/5*(64*A-64*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(64*A
-32*B)/(tan(1/2*f*x+1/2*e)+1)^3+16*B/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(198) = 396$.
time = 0.56, size = 2604, normalized size = 12.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^4*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5
*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*A*c^4*((105
*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^3) + 24*B*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) - 8*A*c^4*((95*sin(f*x + e)/(cos(f*x + e)
 + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
```

```
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) + 12*B*c^4*((95*sin(f*x + e)/(cos(f*x + e)
 + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^4*(20*sin(f*x + e)/(cos(f*x + e) +
 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 24*A*c^4*(5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 16*B*
c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) + 24*A*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^4*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(198) = 396$.
time = 0.38, size = 408, normalized size = 2.03

$$\frac{15\,Bc^4\cos(fx+e)^5 - 30\,(A-6\,B)c^4\cos(fx+e)^4 + 420\,(2\,A-7\,B)c^4fx + 96\,(A-B)c^4 - (105\,(2\,A-7\,B)c^4fx + (554\,A-1819\,B)c^4)\cos(fx+e)^3 - (315\,(2\,A-7\,B)c^4fx - 2\,(134\,A-619\,B)c^4)\cos(fx+e)^2 + 6\,(35\,(2\,A-7\,B)c^4fx + 2\,(74\,A-249\,B)c^4)\cos(fx+e) - (15\,Bc^4\cos(fx+e)^4 + 15\,(2\,A-11\,B)c^4\cos(fx+e)^3 - 420\,(2\,A-7\,B)c^4fx + 96\,(A-B)c^4 + (105\,(2\,A-7\,B)c^4fx - 2\,(262\,A-827\,B)c^4)\cos(fx+e)^2 - 6\,(35\,(2\,A-7\,B)c^4fx + 2\,(66\,A-241\,B)c^4)\cos(fx+e))\sin(fx+e)}{30\,(a^3f\cos(fx+e)^3 + 3\,a^3f\cos(fx+e)^2 - 2\,a^3f\cos(fx+e) - 4\,a^3f + (a^3f\cos(fx+e)^2 - 2\,a^3f\cos(fx+e) - 4\,a^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/30*(15*B*c^4*cos(f*x + e)^5 - 30*(A - 6*B)*c^4*cos(f*x + e)^4 + 420*(2*A
- 7*B)*c^4*f*x + 96*(A - B)*c^4 - (105*(2*A - 7*B)*c^4*f*x + (554*A - 1819*
B)*c^4)*cos(f*x + e)^3 - (315*(2*A - 7*B)*c^4*f*x - 2*(134*A - 619*B)*c^4)*
cos(f*x + e)^2 + 6*(35*(2*A - 7*B)*c^4*f*x + 2*(74*A - 249*B)*c^4)*cos(f*x
+ e) - (15*B*c^4*cos(f*x + e)^4 + 15*(2*A - 11*B)*c^4*cos(f*x + e)^3 - 420*
(2*A - 7*B)*c^4*f*x + 96*(A - B)*c^4 + (105*(2*A - 7*B)*c^4*f*x - 2*(262*A
- 827*B)*c^4)*cos(f*x + e)^2 - 6*(35*(2*A - 7*B)*c^4*f*x + 2*(66*A - 241*B)
*c^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
 e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. $2(185) = 370$.
time = 30.04, size = 7337, normalized size = 36.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-210*A*c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)*
*9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*
a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*ta
n(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(e
/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)
**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780
*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*t
an(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 +
f*x/2) + 30*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/
2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2
)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 78
0*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*
tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4200*A*c**
4*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e
/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/
2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 6
00*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f
*tan(e/2 + f*x/2) + 30*a**3*f) - 5460*A*c**4*f*x*tan(e/2 + f*x/2)**5/(30*a*
*3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x
/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 +
360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) -
 5460*A*c**4*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a
**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan
```

```
(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*
x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4200*A*c**4*f*x*tan(e/2 + f*x/2
)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*
a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*ta
n(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f
*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 3
0*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)
**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600
*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*t
an(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(
e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**
8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a
**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan
(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*
x/2) + 30*a**3*f) - 210*A*c**4*f*x/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**
3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e
/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/
2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 1
50*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 480*A*c**4*tan(e/2 + f*x/2)**8/(3
0*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*
tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 +
 f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**
3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*
f) - 1980*A*c**4*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a
**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan
(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*
x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5420*A*c**4*tan(e/2 + f*x/2)**6
/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3
*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/
2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2
)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 7060*A*c**4*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 15
0*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*
tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 +
 f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**
2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 10308*A*c**4*tan(e/2 + f*x/2
)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*
a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*ta
n(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)...
```

<u>Giac</u> [A]

time = 0.46, size = 305, normalized size = 1.52

$$-\frac{\frac{105\,(2\,Ac^4-7\,Bc^4)(fx+e)}{a^3}-\frac{30\left(Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^4+14\,Bc^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}+\frac{32\left(15\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+130\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-380\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+80\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-250\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,Ac^4-59\,Bc^4\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(105*(2*A*c^4 - 7*B*c^4)*(f*x + e)/a^3 - 30*(B*c^4*tan(1/2*f*x + 1/2*
e)^3 - 2*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 14*B*c^4*tan(1/2*f*x + 1/2*e)^2 - B
*c^4*tan(1/2*f*x + 1/2*e) - 2*A*c^4 + 14*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 +
1)^2*a^3) + 32*(15*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^4*tan(1/2*f*x + 1/
2*e)^4 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^3
 + 130*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 380*B*c^4*tan(1/2*f*x + 1/2*e)^2 + 80
*A*c^4*tan(1/2*f*x + 1/2*e) - 250*B*c^4*tan(1/2*f*x + 1/2*e) + 19*A*c^4 - 5
9*B*c^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 14.71, size = 419, normalized size = 2.08

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{286\,A\,c^4}{3}-\frac{1007\,B\,c^4}{3}\right)+\frac{334\,A\,c^4}{15}-\frac{1154\,B\,c^4}{15}+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8\,\left(16\,A\,c^4-49\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7\,\left(66\,A\,c^4-243\,B\,c^4\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6\,\left(\frac{542\,A\,c^4}{3}-\frac{1741\,B\,c^4}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5\,\left(\frac{706\,A\,c^4}{3}-\frac{2621\,B\,c^4}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3\,\left(\frac{794\,A\,c^4}{3}-\frac{2875\,B\,c^4}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2\,\left(\frac{1006\,A\,c^4}{5}-\frac{3401\,B\,c^4}{5}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4\,\left(\frac{1718\,A\,c^4}{5}-\frac{5633\,B\,c^4}{5}\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^9+5\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8+12\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7+20\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6+26\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^5+26\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4+20\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3+12\,a^3\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+a^3\right)}-\frac{7\,c^4\,\mathrm{atan}\left(\frac{7\,c^4\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(2\,A-7\,B\right)}{14\,A\,c^4-49\,B\,c^4}\right)\,\left(2\,A-7\,B\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((286*A*c^4)/3 - (1007*B*c^4)/3) + (334*A*c^4)/15 - (
1154*B*c^4)/15 + tan(e/2 + (f*x)/2)^8*(16*A*c^4 - 49*B*c^4) + tan(e/2 + (f*
x)/2)^7*(66*A*c^4 - 243*B*c^4) + tan(e/2 + (f*x)/2)^6*((542*A*c^4)/3 - (174
1*B*c^4)/3) + tan(e/2 + (f*x)/2)^5*((706*A*c^4)/3 - (2621*B*c^4)/3) + tan(e
/2 + (f*x)/2)^3*((794*A*c^4)/3 - (2875*B*c^4)/3) + tan(e/2 + (f*x)/2)^2*((1
006*A*c^4)/5 - (3401*B*c^4)/5) + tan(e/2 + (f*x)/2)^4*((1718*A*c^4)/5 - (56
33*B*c^4)/5))/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*x)/2)^3
 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a^3*tan(e
/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f*x)/2)^8
+ a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (7*c^4*atan
((7*c^4*tan(e/2 + (f*x)/2)*(2*A - 7*B))/(14*A*c^4 - 49*B*c^4))*(2*A - 7*B))
/(a^3*f)
```

3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{(A-6B)c^3x}{a^3}-\frac{(A-6B)c^3\cos(e+fx)}{a^3f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}-\frac{2a^3(A}{3f(a}$$

```
[Out] -(A-6*B)*c^3*x/a^3-(A-6*B)*c^3*cos(f*x+e)/a^3/f-1/5*a^3*(A-B)*c^3*cos(f*x+e
)^7/f/(a+a*sin(f*x+e))^6+2/15*a*(A-6*B)*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
^4-2/3*a^3*(A-6*B)*c^3*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))^2
```

Rubi [A]
time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2759, 2761, 8}

$$-\frac{c^3(A-6B)\cos(e+fx)}{a^3f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{2a^3c^3(A-6B)\cos^3(e+fx)}{3f\left(a^3\sin(e+fx)+a^3\right)^2}-\frac{c^3x(A-6B)}{a^3}+\frac{2ac^3(A-6B)\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(((A - 6*B)*c^3*x)/a^3) - ((A - 6*B)*c^3*Cos[e + f*x])/(a^3*f) - (a^3*(A -
 B)*c^3*Cos[e + f*x]^7)/(5*f*(a + a*Sin[e + f*x])^6) + (2*a*(A - 6*B)*c^3*C
os[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^4) - (2*a^3*(A - 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(a^3 + a^3*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} - \frac{1}{5}\left(a^2(A-6B)c^3\right)\int \frac{\cos^6}{(a+a\,\mathrm{s}}$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}$$
$$= -\frac{(A-6B)c^3\cos(e+fx)}{a^3f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2}{}$$
$$= -\frac{(A-6B)c^3x}{a^3} - \frac{(A-6B)c^3\cos(e+fx)}{a^3f} - \frac{a^3(A-B)c^3\,\mathrm{c}}{5f(a+a\sin}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(153) = 306.
time = 0.72, size = 308, normalized size = 2.01

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-24(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-8(11A-21B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+4(11A-21B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+4(23A-93B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-15(A-6B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+15B\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)(c-c\sin(e+fx))^3}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*x)
/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5)*(c - c*Sin[e + f*x])^3)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.41, size = 157, normalized size = 1.03

method	result
derivativedivides	$\frac{2c^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{}{5(\tan}\right.}{f\,a^3}$
default	$\frac{2c^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{}{5(\tan}\right.}{f\,a^3}$
risch	$-\frac{c^3xA}{a^3}+\frac{6c^3xB}{a^3}+\frac{B\,c^3e^{i(fx+e)}}{2a^3f}+\frac{B\,c^3e^{-i(fx+e)}}{2a^3f}+\frac{\frac{112A\,c^3e^{2i(fx+e)}}{3}-24iA\,c^3e^{3i(fx+e)}+\frac{56iA\,c^3e^{i(fx+e)}}{3}-12A}{}$
norman	$\frac{-\frac{71c^3(A-6B)x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{84c^3(A-6B)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{52A\,c^3-282B\,c^3}{15fa}-\frac{c^3(A-6B)x}{a}-\frac{\left(40A\,c^3-246B\,c^3\right)\tan\left(\frac{fx}{2}+\right.}{3fa}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^3/a^3*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A-6*B)*arctan(tan(1/2*f*x+1/2*e))-
1/4*(-64*A+64*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-8*A-8*B)/(tan(1/2*f*x+1/2*e
)+1)^2-(2*A-6*B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(32*A-32*B)/(tan(1/2*f*x+1/2*e)
+1)^5-1/3*(40*A-24*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1825 vs. 2(154) = 308.
time = 0.55, size = 1825, normalized size = 11.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(3*B*c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
 + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11
*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^3) - A*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a^3) + 3*B*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) - A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*A*c^3*(5*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^3*(5*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^
3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*
c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^3*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(154) = 308$.
time = 0.37, size = 352, normalized size = 2.30

$$\frac{15 Bc^3 \cos(fx+e)^4 + 60(A-6B)c^3 fx + 24(A-B)c^3 - (15(A-6B)c^3 fx + (46A-231B)c^3)\cos(fx+e)^3 - (45(A-6B)c^3 fx - 2(A-66B)c^3)\cos(fx+e)^2 + 6(5(A-6B)c^3 fx + 2(6A-31B)c^3)\cos(fx+e) + (15Bc^3\cos(fx+e)^3 + 60(A-6B)c^3 fx - 24(A-B)c^3 - (15(A-6B)c^3 fx - 2(23A-108B)c^3)\cos(fx+e)^2 + 6(5(A-6B)c^3 fx + 2(4A-29B)c^3)\cos(fx+e))\sin(fx+e)}{15\left(a^3 f\cos(fx+e)^3 + 3a^3 f\cos(fx+e)^2 - 2a^3 f\cos(fx+e) - 4a^3 f + (a^3 f\cos(fx+e)^2 - 2a^3 f\cos(fx+e) - 4a^3 f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*c^3*cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (15
*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*cos(f*x + e)^3 - (45*(A - 6*B)*c^3
*f*x - 2*(A - 66*B)*c^3)*cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*A -
 31*B)*c^3)*cos(f*x + e) + (15*B*c^3*cos(f*x + e)^3 + 60*(A - 6*B)*c^3*f*x
- 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*cos(f*x +
e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*cos(f*x + e))*sin(f*x +
 e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e)
- 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x
 + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.
time = 17.51, size = 4665, normalized size = 30.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*A*c**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f*x*t
an(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x
/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**
3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/
2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2
)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f
*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*A*c**3*f*x*tan(
e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
```

```
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225
*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f
*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**
3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 75*A*c**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3
*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/
2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 15*A*c**3*f*x/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 60*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
 f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*A*c**3*tan(e/2 + f*x/2)**
5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/
2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 460*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 320*A*c**3*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 452*A*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 200*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan
(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*
x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 52*A*c**3/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**
3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*x*tan(e/2 + f*x/2)**6/
(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f
*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2
+ f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
```

```
 15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*
f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + ...
```

Giac [A]
time = 0.47, size = 226, normalized size = 1.48

$$\frac{\frac{30\,Bc^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\,(Ac^3-6\,Bc^3)(fx+e)}{a^3}-\frac{4\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-420\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+50\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-270\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,Ac^3-63\,Bc^3\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(f
*x + e)/a^3 - 4*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*tan(1/2*f*x + 1
/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 5
0*A*c^3*tan(1/2*f*x + 1/2*e) - 270*B*c^3*tan(1/2*f*x + 1/2*e) + 13*A*c^3 -
63*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 14.36, size = 333, normalized size = 2.18

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{40\,A\,c^3}{3}-82\,B\,c^3\right)+\frac{52\,A\,c^3}{15}-\frac{94\,B\,c^3}{5}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(4\,A\,c^3-12\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(8\,A\,c^3-58\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{64\,A\,c^3}{3}-148\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{92\,A\,c^3}{3}-134\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{452\,A\,c^3}{15}-\frac{744\,B\,c^3}{5}\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+5\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+11\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+15\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+15\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+11\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}-\frac{2\,c^3\,\mathrm{atan}\left(\frac{2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A-6\,B\right)}{2\,A\,c^3-12\,B\,c^3}\right)\,\left(A-6\,B\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((40*A*c^3)/3 - 82*B*c^3) + (52*A*c^3)/15 - (94*B*c^3
)/5 + tan(e/2 + (f*x)/2)^6*(4*A*c^3 - 12*B*c^3) + tan(e/2 + (f*x)/2)^5*(8*A
*c^3 - 58*B*c^3) + tan(e/2 + (f*x)/2)^3*((64*A*c^3)/3 - 148*B*c^3) + tan(e/
2 + (f*x)/2)^4*((92*A*c^3)/3 - 134*B*c^3) + tan(e/2 + (f*x)/2)^2*((452*A*c^
3)/15 - (744*B*c^3)/5))/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2 +
(f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 + 5*
a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2 +
 (f*x)/2))) - (2*c^3*atan((2*c^3*tan(e/2 + (f*x)/2)*(A - 6*B))/(2*A*c^3 - 1
2*B*c^3))*(A - 6*B))/(a^3*f)
```

3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=110

$$\frac{Bc^2x}{a^3}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2Bc^2\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] B*c^2*x/a^3-1/5*a^2*(A-B)*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5-2/3*B*c^2*c
os(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*B*c^2*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A]
time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3046, 2938, 2759, 8}

$$\frac{2Bc^2\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bc^2x}{a^3}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*c^2*x)/a^3 - (a^2*(A - B)*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^
5) - (2*B*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*B*c^2*Cos[e
 + f*x])/(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \left(aBc^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+f}$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \cdots$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \cdots$$

$$= \frac{Bc^2x}{a^3} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx}{3f(a+a\sin(e+fx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(110) = 220.
time = 0.46, size = 272, normalized size = 2.47

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-12(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-8(3A-8B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+4(3A-8B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+2(3A-43B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+15B(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)(c-c\sin(e+fx))^2}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^5)*(c - c*Sin[e + f*x])^2)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.36, size = 127, normalized size = 1.15

method	result
derivativedivides	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{4A}{\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,a^3}$
default	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{4A}{\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,a^3}$
risch	$\frac{B\,c^2x}{a^3}-\frac{2\left(-30A\,c^2\mathrm{e}^{2i(fx+e)}+15A\,c^2\mathrm{e}^{4i(fx+e)}+250B\,c^2\mathrm{e}^{2i(fx+e)}-180iB\,c^2\mathrm{e}^{3i(fx+e)}+140iB\,c^2\mathrm{e}^{i(fx+e)}-75B\,c^2\mathrm{e}^{4i(f}\right.}{15f\,a^3\left(\mathrm{e}^{i(fx+e)}+i\right)^5}$
norman	$\frac{\frac{c^2xB}{a}+\frac{8B\,c^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}+\frac{48B\,c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}+\frac{c^2xB\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{6A\,c^2-46B\,c^2}{15fa}+\frac{40B\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3fa}+\frac{64B\,c^2\left(\mathrm{t}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*c^2/a^3*(B*arctan(tan(1/2*f*x+1/2*e))-1/4*(-32*A+32*B)/(tan(1/2*f*x+1/2
*e)+1)^4-(A-B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1
)^5-1/3*(24*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3+4*A/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. 2(112) = 224.
time = 0.52, size = 1232, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*c^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
```

```
os(f*x + e) + 1)^5) - 2*A*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(112) = 224$.
time = 0.56, size = 291, normalized size = 2.65

$$-\frac{60\,Bc^2fx - (15\,Bc^2fx - (3\,A - 43\,B)c^2)\cos\left(fx+e\right)^3 - 12\,(A-B)c^2 - (45\,Bc^2fx - (9\,A + 11\,B)c^2)\cos\left(fx+e\right)^2 + 6\,(5\,Bc^2fx - (A - 11\,B)c^2)\cos\left(fx+e\right) + \left(60\,Bc^2fx + 12\,(A-B)c^2 - (15\,Bc^2fx + (3\,A - 43\,B)c^2)\cos\left(fx+e\right)^2 + 6\,(5\,Bc^2fx + (A + 9\,B)c^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3 + 3\,a^3f\cos\left(fx+e\right)^2 - 2\,a^3f\cos\left(fx+e\right) - 4\,a^3f + \left(a^3f\cos\left(fx+e\right)^2 - 2\,a^3f\cos\left(fx+e\right) - 4\,a^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 12
*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^
2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 - (15
*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9*B)*
c^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos
(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.
time = 9.64, size = 1647, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*
tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*A*c**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
15*B*c**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*
f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*c**2*f*x*tan(
e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**3/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) + 75*B*c**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*c**2*f*x/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) + 30*B*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1
20*B*c**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 340*B*c**2*tan(e/2 + f
*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 200*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(
e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/
2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 46*B*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*si
n(e) + c)**2/(a*sin(e) + a)**3, True))
```

Giac [A]

time = 0.47, size = 159, normalized size = 1.45

$$\frac{\frac{15\,(fx+e)Bc^2}{a^3}-\frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-170\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-100\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Ac^2-23\,Bc^2\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*c^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*c^
2*tan(1/2*f*x + 1/2*e)^4 - 60*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c^2*tan(1
/2*f*x + 1/2*e)^2 - 170*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*c^2*tan(1/2*f*
x + 1/2*e) + 3*A*c^2 - 23*B*c^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 15.08, size = 230, normalized size = 2.09

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{c^2\,\left(120\,B+150\,B\,\left(e+f\,x\right)\right)}{15}-10\,B\,c^2\,\left(e+f\,x\right)\right)+\frac{c^2\,\left(46\,B-6\,A+15\,B\,\left(e+f\,x\right)\right)}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{c^2\,\left(30\,B-30\,A+75\,B\,\left(e+f\,x\right)\right)}{15}-5\,B\,c^2\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{c^2\,\left(340\,B-60\,A+150\,B\,\left(e+f\,x\right)\right)}{15}-10\,B\,c^2\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{c^2\,\left(200\,B+75\,B\,\left(e+f\,x\right)\right)}{15}-5\,B\,c^2\,\left(e+f\,x\right)\right)-B\,c^2\,\left(e+f\,x\right)}{a^3\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^5}+\frac{B\,c^2\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^3*((c^2*(120*B + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f
*x)) + (c^2*(46*B - 6*A + 15*B*(e + f*x)))/15 + tan(e/2 + (f*x)/2)^4*((c^2*
(30*B - 30*A + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) + tan(e/2 + (f*x)/2
)^2*((c^2*(340*B - 60*A + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f*x)) + tan(
e/2 + (f*x)/2)*((c^2*(200*B + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) - B*
c^2*(e + f*x))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5) + (B*c^2*x)/a^3
```

3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{a(A-11B)c\cos(e+fx)}{15f\left(a^2+a^2\sin(e+fx)\right)^2}+\frac{(A+4B)c\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -2/5*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^3+1/15*a*(A-11*B)*c*cos(f*x+e)/f
/(a^2+a^2*sin(f*x+e))^2+1/15*(A+4*B)*c*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3046, 2936, 2829, 2727}

$$\frac{c(A+4B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{ac(A-11B)\cos(e+fx)}{15f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{2c(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*(A - B)*c*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) + (a*(A - 11*B)*c*
Cos[e + f*x])/(15*f*(a^2 + a^2*Sin[e + f*x])^2) + ((A + 4*B)*c*Cos[e + f*x]
)/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
```

```
b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{c\int \frac{aA-6aB+5aB\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{((\cdots}{}$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(\cdots}{15\cdots}$$

Mathematica [A]
time = 0.56, size = 139, normalized size = 1.35

$$\frac{c\left(-15(A+B)\cos\left(e+\frac{fx}{2}\right)+5(A+B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)-25B\sin\left(\frac{fx}{2}\right)-15B\sin\left(2e+\frac{3fx}{2}\right)+A\sin\left(2e+\frac{5fx}{2}\right)+4B\sin\left(2e+\frac{5fx}{2}\right)\right)}{30a^3f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(
f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*
x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 0.33, size = 115, normalized size = 1.12

method	result

derivativedivides	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}\right)}{f\,a^{3}}$
default	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}\right)}{f\,a^{3}}$
risch	$\frac{-\frac{10Bc\,e^{2i(fx+e)}}{3}+2iBc\,e^{3i(fx+e)}-\frac{2iBc\,e^{i(fx+e)}}{3}+2Bc\,e^{4i(fx+e)}-\frac{2iAc\,e^{i(fx+e)}}{3}+\frac{2Ac\,e^{2i(fx+e)}}{3}+\frac{2Ac}{15}+\frac{8Bc}{15}+2iAc\,e^{3i(fx\cdots}}{f\,a^{3}\left(e^{i(fx+e)}+i\right)^{5}}$
norman	$\frac{-\frac{8Ac+2Bc}{15fa}-\frac{2Ac\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(23Ac-3Bc)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-\frac{2(7Ac+7Bc)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{2(11Ac-Bc)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-\cdots}{\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}a^{2}\left(\tan\left(\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*c/a^3*(-1/5*(8*A-8*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-6*A+2*B)/(tan(1/2*
f*x+1/2*e)+1)^2-A/(tan(1/2*f*x+1/2*e)+1)-1/4*(-16*A+16*B)/(tan(1/2*f*x+1/2*
e)+1)^4-1/3*(14*A-10*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(103) = 206$.
time = 0.30, size = 797, normalized size = 7.74

$$-\frac{2\left(\frac{Ac\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{30\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{15\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+7\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{2Bc\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{3Ac\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{5\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}+\frac{3Bc\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{5\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 2*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
```

```
+ 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A]
time = 0.67, size = 203, normalized size = 1.97

$$\frac{(A+4B)c\cos(fx+e)^3-(2A-7B)c\cos(fx+e)^2+3(A-B)c\cos(fx+e)+6(A-B)c-((A+4B)c\cos(fx+e)^2+3(A-B)c\cos(fx+e)+6(A-B)c)\sin(fx+e)}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B)
*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c*c
os(f*x + e) + 6*(A - B)*c)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a
^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(97) = 194$.
time = 5.43, size = 1035, normalized size = 10.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
```

```
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 10*B*c*tan(e
/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f*x/2)/(15*a**3*f*ta
n(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*
x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*
a**3*f) - 2*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(
e) + c)/(a*sin(e) + a)**3, True))
```

Giac [A]
time = 0.52, size = 138, normalized size = 1.34

$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-5\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,Ac+Bc\right)}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 15*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 25*A*c*tan(1/2*f*x + 1/2*e)^2 - 5*B*c*tan(1/2*f
*x + 1/2*e)^2 + 5*A*c*tan(1/2*f*x + 1/2*e) + 5*B*c*tan(1/2*f*x + 1/2*e) + 4
*A*c + B*c)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B]
time = 13.03, size = 172, normalized size = 1.67

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{41\,A\,c}{4}-\frac{B\,c}{4}-\frac{11\,A\,c\,\cos(e+f\,x)}{2}+\frac{B\,c\,\cos(e+f\,x)}{2}+5\,A\,c\,\sin\left(e+f\,x\right)+5\,B\,c\,\sin\left(e+f\,x\right)-\frac{3\,A\,c\,\cos(2\,e+2\,f\,x)}{4}+\frac{3\,B\,c\,\cos(2\,e+2\,f\,x)}{4}-\frac{5\,A\,c\,\sin(2\,e+2\,f\,x)}{4}-\frac{5\,B\,c\,\sin(2\,e+2\,f\,x)}{4}\right)}{15\,a^{3}\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}-\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((41*A*c)/4 - (B*c)/4 - (11*A*c*cos(e + f*x))/2 + (B*
c*cos(e + f*x))/2 + 5*A*c*sin(e + f*x) + 5*B*c*sin(e + f*x) - (3*A*c*cos(2*
e + 2*f*x))/4 + (3*B*c*cos(2*e + 2*f*x))/4 - (5*A*c*sin(2*e + 2*f*x))/4 - (
5*B*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f
*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2
- pi/4 + (5*f*x)/2))/4))
```

3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=102

$$-\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))}+\frac{2(3A+2B)\tan(e+fx)}{15a^3cf}$$

```
[Out] -1/5*(A-B)*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/15*(3*A+2*B)*sec(f*x+e)/c/
f/(a^3+a^3*sin(f*x+e))+2/15*(3*A+2*B)*tan(f*x+e)/a^3/c/f
```

Rubi [A]
time = 0.18, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2938, 2751, 3852, 8}

$$\frac{2(3A+2B)\tan(e+fx)}{15a^3cf}-\frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3\sin(e+fx)+a^3)}-\frac{(A-B)\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/5*((A - B)*Sec[e + f*x])/(a*c*f*(a + a*Sin[e + f*x])^2) - ((3*A + 2*B)*S
ec[e + f*x])/(15*c*f*(a^3 + a^3*Sin[e + f*x])) + (2*(3*A + 2*B)*Tan[e + f*x
])/(15*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{ac}\\ &= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}+\frac{(3A+2B)\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{5a^2c}\\ &= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))}+\\ &= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))}-\\ &= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))}+\end{aligned}$$

Mathematica [A]
time = 0.56, size = 156, normalized size = 1.53

$$\frac{\cos(e+fx)(-80B-5(9A+B)\cos(e+fx)+32(3A+2B)\cos(2(e+fx))+9A\cos(3(e+fx))+B\cos(3(e+fx))-120A\sin(e+fx)-80B\sin(e+fx)-36A\sin(2(e+fx))-4B\sin(2(e+fx))+24A\sin(3(e+fx))+16B\sin(3(e+fx)))}{240a^3cf(-1+\sin(e+fx))(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e +
 f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] - 8
```

```
0*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*Sin[
3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a^3*c*f*(-1 + Sin[e + f*x])*(1
+ Sin[e + f*x])^3)
```

Maple [A]
time = 0.34, size = 145, normalized size = 1.42

method	result
risch	$-\frac{4\left(15 i A\,{\mathrm e}^{2 i (f x+e)}-12 A\,{\mathrm e}^{i (f x+e)}+10 B\,{\mathrm e}^{3 i (f x+e)}-3 i A-8 B\,{\mathrm e}^{i (f x+e)}+10 i B\,{\mathrm e}^{2 i (f x+e)}-2 i B\right)}{15\left({\mathrm e}^{i (f x+e)}+i\right)^{5}\left({\mathrm e}^{i (f x+e)}-i\right) c f\,a^{3}}$
derivativedivides	$\frac{-\frac{-4A+4B}{2\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(2A-2B)}{5\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1}}{c f\,a^{3}}$
default	$\frac{-\frac{-4A+4B}{2\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(2A-2B)}{5\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1}}{c f\,a^{3}}$
norman	$\frac{\frac{12A-2B}{15 c f a}-\frac{2(6A+7B)\left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 c f a}-\frac{2A\left(\tan^{7}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{a c f}+\frac{2(9A-4B)\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{15 c f a}+\frac{2(2A-7B)\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 c f a}-\frac{2(9A+4B)(\ldots}{3\ldots}}{\left(1+\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right) a^{2}\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^3/c*(-1/4*(-4*A+4*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(2*A-2*B)/(tan(1/2*
f*x+1/2*e)+1)^5-1/2*(-5/2*A+3/2*B)/(tan(1/2*f*x+1/2*e)+1)^2-(7/8*A-1/8*B)/(
tan(1/2*f*x+1/2*e)+1)-1/3*(9/2*A-7/2*B)/(tan(1/2*f*x+1/2*e)+1)^3-(1/8*A+1/8
*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(101) = 202$.
time = 0.31, size = 459, normalized size = 4.50

$$\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1\right)}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+2\right)}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a
^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6) - 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
```

```
in(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - a^3*c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [A]
time = 0.34, size = 113, normalized size = 1.11

$$\frac{4\,(3\,A+2\,B)\cos\left(fx+e\right)^2+\left(2\,(3\,A+2\,B)\cos\left(fx+e\right)^2-9\,A-6\,B\right)\sin\left(fx+e\right)-6\,A-9\,B}{15\,\left(a^3cf\cos\left(fx+e\right)^3-2\,a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A -
6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x
+ e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. 2(85) = 170.
time = 5.53, size = 1236, normalized size = 12.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*
a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*
f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*
tan(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
+ f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f*x/2)*
*3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*
a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*
f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 18*A*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(
e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f
*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) -
 15*a**3*c*f) + 12*A/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/
2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75
*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c
*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**3/(15*a**3*c*f*
```

```
tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/
2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a**3*c*f*tan(e/2 + f*x/2)*
*6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75
*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f)
 - 8*B*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(
e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f
*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 2*B/(15*a**3*c*f*t
an(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
+ f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2
) - 15*a**3*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e
) + c)), True))
```

Giac [A]
time = 0.44, size = 175, normalized size = 1.72

$$-\frac{\frac{15\,(A+B)}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+40\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+210\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+50\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+63\,A+7\,B}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A + B)/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 - 15*B*tan(1/2*f*x + 1/2*e)^4 + 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 + 40*B*tan(1/2*f*x
 + 1/2*e)^2 + 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 + 7*B)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 12.43, size = 178, normalized size = 1.75

$$-\frac{2\left(\frac{15\,A\cos(e+f\,x)}{4}-\frac{5\,B}{2}-\frac{5\,B\cos(e+f\,x)}{8}-\frac{15\,A\sin(e+f\,x)}{4}-\frac{5\,B\sin(e+f\,x)}{2}+3\,A\cos\left(2\,e+2\,f\,x\right)-\frac{3\,A\cos(3\,e+3\,f\,x)}{4}+2\,B\cos\left(2\,e+2\,f\,x\right)+\frac{B\cos(3\,e+3\,f\,x)}{8}+3\,A\sin\left(2\,e+2\,f\,x\right)+\frac{3\,A\sin(3\,e+3\,f\,x)}{4}-\frac{B\sin(2\,e+2\,f\,x)}{2}+\frac{B\sin(3\,e+3\,f\,x)}{2}\right)}{15\,a^3\,c\,f\left(\frac{5\cos(e+f\,x)}{4}-\frac{\cos(3\,e+3\,f\,x)}{4}+\sin\left(2\,e+2\,f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*((15*A*cos(e + f*x))/4 - (5*B)/2 - (5*B*cos(e + f*x))/8 - (15*A*sin(e +
 f*x))/4 - (5*B*sin(e + f*x))/2 + 3*A*cos(2*e + 2*f*x) - (3*A*cos(3*e + 3*f
*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*
x) + (3*A*sin(3*e + 3*f*x))/4 - (B*sin(2*e + 2*f*x))/2 + (B*sin(3*e + 3*f*x
))/2))/(15*a^3*c*f*((5*cos(e + f*x))/4 - cos(3*e + 3*f*x)/4 + sin(2*e + 2*f
*x)))
```

3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=90

$$-\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}+\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}$$

```
[Out] -1/5*(A-B)*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+1/5*(4*A+B)*tan(f*x+e)/a
^3/c^2/f+1/15*(4*A+B)*tan(f*x+e)^3/a^3/c^2/f
```

Rubi [A]
time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2938, 3852}

$$\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}-\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3\sin(e+fx)+a^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] -1/5*((A - B)*Sec[e + f*x]^3)/(c^2*f*(a^3 + a^3*Sin[e + f*x])) + ((4*A + B)
*Tan[e + f*x])/(5*a^3*c^2*f) + ((4*A + B)*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{(4A+B)\int \sec^4(e+fx)\,dx}{5a^3c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(4A+B)\text{Subst}\left(\int (1+x^2)\right.}{5a^3c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{(4A+B)\tan(e+fx)}{5a^3c^2f} + \frac{(4}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(90) = 180.
time = 0.68, size = 237, normalized size = 2.63

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(240B+54(A-B)\cos(e+fx)-32(4A+B)\cos(2(e+fx))+18A\cos(3(e+fx))-18B\cos(3(e+fx))-64A\cos(4(e+fx))-16B\cos(4(e+fx))+384A\sin(e+fx)+96B\sin(e+fx)+18A\sin(2(e+fx))-18B\sin(2(e+fx))+128A\sin(3(e+fx))+32B\sin(3(e+fx))+9A\sin(4(e+fx))-9B\sin(4(e+fx)))}{960a^3c^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A*C
os[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*Cos[
4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e + f*x
)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)]
 + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 + Sin[e
 + f*x])^2*(1 + Sin[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(84) = 168.
time = 0.34, size = 185, normalized size = 2.06

method	result
risch	$\frac{4i\left(24iA\,e^{3i(fx+e)}+6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}-8A\,e^{2i(fx+e)}+2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}-4A-B\right)}{15\left(e^{i(fx+e)}+i\right)^5\left(e^{i(fx+e)}-i\right)^3 f\,a^3c^2}$
derivativedivides	$\frac{-\frac{-2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{5A}{2}-2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{11A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{c^2f\,a^3}$

default	$\frac{-\frac{-2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{5A}{2}-2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{11A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}}{c^2 f a^3}$
norman	$\frac{-\frac{6A+4B}{10cfa}-\frac{4(4A+B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cfa}+\frac{A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{(14A+16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10acf}-\frac{(6A+4B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2cfa}-\frac{(2A+8B)(\text{t}}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3/c^2*(-1/4*(-2*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(A-B)/(tan(1/2*f*
x+1/2*e)+1)^5-1/2*(-3/2*A+B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(5/2*A-2*B)/(tan(
1/2*f*x+1/2*e)+1)^3-(11/16*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(1/4*B+1/4*
A)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/4*B+1/4*A)/(tan(1/2*f*x+1/2*e)-1)^2-(5/1
6*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(88) = 176$.
time = 0.30, size = 704, normalized size = 7.82

$$\frac{2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-3\right)}{a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}+\frac{B\left(\frac{6\sin(fx+e)}{\cos(fx+e)+1}+\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{8\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{10\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+3\right)}{a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/(a^3*c^
2 + 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*
c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) + B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3)/(a^3*c^2 + 2*a^3*c^2*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.28

$$-\frac{2\,(4\,A+B)\cos\left(fx+e\right)^{4}-(4\,A+B)\cos\left(fx+e\right)^{2}-\left(2\,(4\,A+B)\cos\left(fx+e\right)^{2}+4\,A+B\right)\sin\left(fx+e\right)-A-4\,B}{15\,\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+a^{3}c^{2}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B)
*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e)^
3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(82) = 164.
time = 11.27, size = 2674, normalized size = 29.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 30*A*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 50*A*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 42*A*tan(e/2 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
```

```
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 18*A*tan(e/2 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*
a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a
**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a*
*3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c
**2*f) + 6*A/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 30*B*tan(e/2
 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 20*B*tan(e/2
 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 10*B*tan(e/2
 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) + 16*B*tan(e/2
 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 18*B*tan(e/2
 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 12*B*tan(e/2
 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*
x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x
/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/
2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 6*B/(15*a**3*c*
*2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**
2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2
*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*
f*tan(e/2 + f*x/2) - 15*a**3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**3*(-c*sin(e) + c)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(88) = 176$.

time = 0.49, size = 235, normalized size = 2.61

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A+7\,B\right)}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-45\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+480\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-60\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+650\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-70\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+400\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-20\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+113\,A-13\,B}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 + 9*B*tan(1/2*f*x + 1/2*e)^2 - 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 - 45*B*tan(1/2*f
*x + 1/2*e)^4 + 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 - 70*B*tan(1/2*f*x + 1/2*e)^2 + 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(tan(1/
2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 12.47, size = 183, normalized size = 2.03

$$\frac{\left(\frac{8\,A}{15}+\frac{2\,B}{15}+\frac{16\,A\,\sin(e+f\,x)}{15}+\frac{4\,B\,\sin(e+f\,x)}{15}\right)\,\cos\left(e+f\,x\right)^2+\frac{2\,A}{15}+\frac{8\,B}{15}+\frac{8\,A\,\sin(e+f\,x)}{15}+\frac{2\,B\,\sin(e+f\,x)}{15}}{a^3\,c^2\,f\,\left(2\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)+2\cos\left(e+f\,x\right)^3\right)}-\frac{\frac{2\,A}{5}-\frac{2\,B}{5}+\frac{2\,A\,\sin(e+f\,x)}{5}-\frac{2\,B\,\sin(e+f\,x)}{5}}{a^3\,c^2\,f\,\left(2\,\sin\left(e+f\,x\right)+2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{16\,A}{15}+\frac{4\,B}{15}\right)}{a^3\,c^2\,f\,\left(2\,\sin\left(e+f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x)
```

```
[Out] ((2*A)/15 + (8*B)/15 + (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 + (2*B)/15 + (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x
))/15))/(a^3*c^2*f*(2*cos(e + f*x)^3*sin(e + f*x) + 2*cos(e + f*x)^3)) - ((
2*A)/5 - (2*B)/5 + (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^3*c^2*f*
(2*sin(e + f*x) + 2)) - (cos(e + f*x)*((16*A)/15 + (4*B)/15))/(a^3*c^2*f*(2
*sin(e + f*x) + 2))
```

3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=84

$$\frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan^5(e+fx)}{5a^3c^3f}$$

```
[Out] 1/5*B*sec(f*x+e)^5/a^3/c^3/f+A*tan(f*x+e)/a^3/c^3/f+2/3*A*tan(f*x+e)^3/a^3/
c^3/f+1/5*A*tan(f*x+e)^5/a^3/c^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2748, 3852}

$$\frac{A\tan^5(e+fx)}{5a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*Tan[e + f*x])/(a^3*c^3*f) + (2*A*Tan[
e + f*x]^3)/(3*a^3*c^3*f) + (A*Tan[e + f*x]^5)/(5*a^3*c^3*f)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\,dx}{a^3c^3} \\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\int \sec^6(e+fx)\,dx}{a^3c^3} \\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}-\frac{A\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(e \cdots\right.}{a^3c^3f} \\
&= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A \cdots}{\cdots}
\end{aligned}
$$

Mathematica [**A**]
time = 0.14, size = 65, normalized size = 0.77

$$
\frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{a^3c^3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3
+ Tan[e + f*x]^5/5))/(a^3*c^3*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(78) = 156$.
time = 0.32, size = 227, normalized size = 2.70

method	result
risch	$\frac{\frac{32B\,e^{5i(fx+e)}}{5}+\frac{32iA\,e^{4i(fx+e)}}{3}+\frac{16iA}{15}+\frac{16iA\,e^{2i(fx+e)}}{3}}{\left(e^{i(fx+e)}+i\right)^5\left(e^{i(fx+e)}-i\right)^5 f\,a^3c^3}$
derivativedivides	$\frac{-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(\frac{A}{2}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{11A}{8}-\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) \cdots}}{f\,c^3a^3}$
default	$\frac{-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(\frac{A}{2}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{11A}{8}-\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) \cdots}}{f\,c^3a^3}$
norman	$\frac{-\frac{2B}{5acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{76A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}-\frac{76A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}+\frac{2A\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^{11}\left(\cdots\right)\right)}{acf} \cdots}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right) \cdots\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3/c^3*(-1/4*(-A+B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-7/8*A+5/8*B)/(tan(1
/2*f*x+1/2*e)+1)^2-1/5*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^5-(1/2*A-3/16*B
)/(tan(1/2*f*x+1/2*e)+1)-1/3*(11/8*A-9/8*B)/(tan(1/2*f*x+1/2*e)+1)^3-1/4*(A
+B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2
*(7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+3/16*B)/(tan(1/2*f*x+1/2*e)-
1)-1/3*(11/8*A+9/8*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.76

$$\frac{\frac{\left(3\,\tan(fx+e)^5+10\,\tan(fx+e)^3+15\,\tan(fx+e)\right)A}{a^3c^3}+\frac{3\,B}{a^3c^3\cos(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*A/(a^3*c^3)
+ 3*B/(a^3*c^3*cos(f*x + e)^5))/f
```

Fricas [A]
time = 0.37, size = 60, normalized size = 0.71

$$\frac{\left(8\,A\cos\left(fx+e\right)^4+4\,A\cos\left(fx+e\right)^2+3\,A\right)\sin\left(fx+e\right)+3\,B}{15\,a^3c^3f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/(
a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(78) = 156$.
time = 7.45, size = 1098, normalized size = 13.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 -
 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 -
```

```
 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 -
 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f*x/2
)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/
2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/
2)**2 - 15*a**3*c**3*f) - 116*A*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*tan(e/2
 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/
2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/
2 + f*x/2)**2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**3/(15*a**3*c**3*f*
tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f
*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f
*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*A*tan(e/2 + f*x/2)/(15*a**3*c**
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*B*tan(e/2 + f*x/2)**8/(15*a
**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*
a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*
a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 60*B*tan(e/2 + f*x/2)**
4/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8
 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**
4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 6*B/(15*a**3*c**
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((
a*sin(e) + a)**3*(-c*sin(e) + c)**3), True))
```

Giac [A]
time = 0.44, size = 134, normalized size = 1.60

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+58\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,B\right)}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{5}a^{3}c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*tan
(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/2*
e)^4 - 20*A*tan(1/2*f*x + 1/2*e)^3 + 15*A*tan(1/2*f*x + 1/2*e) + 3*B)/((tan
(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)
```

Mupad [B]
time = 14.42, size = 126, normalized size = 1.50

$$-\frac{2\left(15\,A\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}+15\,B\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\,A\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}+58\,A\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}+30\,B\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-20\,A\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}+15\,A\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+3\,B\right)}{15\,a^{3}\,c^{3}\,f\,{\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{2}-1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(3*B + 15*A*tan(e/2 + (f*x)/2) - 20*A*tan(e/2 + (f*x)/2)^3 + 58*A*tan(e
/2 + (f*x)/2)^5 - 20*A*tan(e/2 + (f*x)/2)^7 + 15*A*tan(e/2 + (f*x)/2)^9 + 3
0*B*tan(e/2 + (f*x)/2)^4 + 15*B*tan(e/2 + (f*x)/2)^8))/(15*a^3*c^3*f*(tan(e
/2 + (f*x)/2)^2 - 1)^5)
```

3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=121

$$\frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+1/7*(6*A-B)*tan(f*x+e)/a^
3/c^4/f+2/21*(6*A-B)*tan(f*x+e)^3/a^3/c^4/f+1/35*(6*A-B)*tan(f*x+e)^5/a^3/c
^4/f
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2938, 3852}

$$\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + ((6*A - B)*Ta
n[e + f*x])/(7*a^3*c^4*f) + (2*(6*A - B)*Tan[e + f*x]^3)/(21*a^3*c^4*f) + (
(6*A - B)*Tan[e + f*x]^5)/(35*a^3*c^4*f)
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx &= \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^3c^3} \\
&= \frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))} + \frac{(6A-B)\int \sec^6(e+fx)\,dx}{7a^3c^4} \\
&= \frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))} - \frac{(6A-B)\mathrm{Subst}\left(\int (1+2x^2+\right.}{7a^3c^4} \\
&= \frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))} + \frac{(6A-B)\tan(e+fx)}{7a^3c^4f} + \frac{2(6A}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 325 vs. 2(121) = 242.
time = 0.76, size = 325, normalized size = 2.69

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8960B+1500(A+B)\cos(e+fx)-640(6A-B)\cos(2(e+fx))+750A\cos(3(e+fx))+750B\cos(3(e+fx))-3072A\cos(4(e+fx))+512B\cos(4(e+fx))+150A\cos(5(e+fx))+150B\cos(5(e+fx))-768A\cos(6(e+fx))+128B\cos(6(e+fx))-15360A\sin(e+fx)+2560B\sin(e+fx)-375A\sin(2(e+fx))-375B\sin(2(e+fx))-7680A\sin(3(e+fx))+1280B\sin(3(e+fx))-300A\sin(4(e+fx))-300B\sin(4(e+fx))-1536A\sin(5(e+fx))+256B\sin(5(e+fx))-75A\sin(6(e+fx))-75B\sin(6(e+fx))\right)}{53760a^3c^4f(-1+\sin(e+fx))^4(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -1/53760*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e +
f*x)] + 750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(e +
 f*x)] + 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e +
 f*x)] - 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin[e +
f*x] + 2560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e + f*x)
] - 7680*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*(e + f*
x)] - 300*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[5*(e + f
*x)] - 75*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)]))/(a^3*c^4*f*(-1 + Sin
[e + f*x])^4*(1 + Sin[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(113) = 226.
time = 0.50, size = 271, normalized size = 2.24

method	result
risch	$-\frac{16i(120iA\,e^{5i(fx+e)}-20iB\,e^{5i(fx+e)}+70B\,e^{6i(fx+e)}+60iA\,e^{3i(fx+e)}+30A\,e^{4i(fx+e)}-10iB\,e^{3i(fx+e)}-5B\,e^{4i(fx+e)}}{105\left(e^{i(fx+e)}+i\right)^{5}\left(e^{i(fx+e)}-i\right)^{7}f\,a^{3}c^{4}}$
derivativedivides	$\frac{-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{f}$
default	$\frac{-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{f}$
norman	$\frac{(2A-2B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cfa}-\frac{2A+2B}{7cfa}-\frac{152(6A-B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}-\frac{2A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{4B\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cfa}+\frac{(312A-752B)}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3/c^4*(-1/2*(-1/2*A+3/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/2*A+1/2*B
)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(
3/4*A-5/8*B)/(tan(1/2*f*x+1/2*e)+1)^3-(11/32*A-5/32*B)/(tan(1/2*f*x+1/2*e)+
1)-1/7*(A+B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(3*A+3*B)/(tan(1/2*f*x+1/2*e)-1)^
6-1/4*(11/2*A+9/2*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(15/8*A+B)/(tan(1/2*f*x+1
/2*e)-1)^2-(21/32*A+5/32*B)/(tan(1/2*f*x+1/2*e)-1)-1/5*(21/4*A+19/4*B)/(tan
(1/2*f*x+1/2*e)-1)^5-1/3*(33/8*A+11/4*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(119) = 238$.

time = 0.33, size = 1105, normalized size = 9.13

$$-\frac{2\left(\frac{B\left(\frac{30\sin(fx+e)}{\cos(fx+e)+1}-\frac{45\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{80\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{110\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{188\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{266\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{112\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{35\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{70\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{105\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-15\right)}{a^{3}c^{4}-\frac{2a^{3}c^{4}\sin(fx+e)}{\cos(fx+e)+1}-\frac{4a^{3}c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{20a^{3}c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{20a^{3}c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{5a^{3}c^{4}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{10a^{3}c^{4}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{4a^{3}c^{4}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{2a^{3}c^{4}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^{3}c^{4}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}}-\frac{3A\left(\frac{25\sin(fx+e)}{\cos(fx+e)+1}-\frac{55\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{15\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{130\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{26\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}-\frac{182\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{126\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{105\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{35\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}-\frac{35\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{35\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+5\right)}{a^{3}c^{4}-\frac{2a^{3}c^{4}\sin(fx+e)}{\cos(fx+e)+1}-\frac{4a^{3}c^{4}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}c^{4}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}c^{4}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{20a^{3}c^{4}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{20a^{3}c^{4}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}-\frac{5a^{3}c^{4}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}-\frac{10a^{3}c^{4}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{4a^{3}c^{4}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{2a^{3}c^{4}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}-\frac{a^{3}c^{4}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}}\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 45*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 80*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 110*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 188*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 266*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 112*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 35
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 70*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 105*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3*c^4*s
in(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
```

```
+ 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*
a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(
cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) - 3*A
*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 35*s
in(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12))/f
```

Fricas [A]
time = 0.36, size = 159, normalized size = 1.31

$$-\frac{8\,(6\,A-B)\cos\left(fx+e\right)^{6}-4\,(6\,A-B)\cos\left(fx+e\right)^{4}-(6\,A-B)\cos\left(fx+e\right)^{2}+\left(8\,(6\,A-B)\cos\left(fx+e\right)^{4}+4\,(6\,A-B)\cos\left(fx+e\right)^{2}+18\,A-3\,B\right)\sin\left(fx+e\right)-3\,A+18\,B}{105\,\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}c^{4}f\cos\left(fx+e\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B)
*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^2
+ 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f*x
+ e) - a^3*c^4*f*cos(f*x + e)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 6135 vs. 2(109) = 218.
time = 44.45, size = 6135, normalized size = 50.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**11/(105*a**3*c**4*f*tan(e/2 + f*x/2)**1
2 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)
**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x
```

/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**10/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**9/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 630*A*tan(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 756*A*tan(e/2 + f*x/2)**7/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 1092*A*tan(e/2 + f*x/2)**6/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 156*A*tan(e/2 + f*x/2)**5/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 780*A*tan(e/2 + f*x/2)**4/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c

```
**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c
**4*f) - 90*A*tan(e/2 + f*x/2)**3/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 2
10*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10
+ 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**
8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2
)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x
/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*
x/2) - 105*a**3*c**4*f) + 330*A*tan(e/2 + f*x/2)**2/(105*a**3*c**4*f*tan(e/
2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan
(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*
tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4
*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c*
*4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c
**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 150*A*tan(e/2 + f*x/2)/(105*a**
3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*
a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 +
 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(119) = 238.
time = 0.50, size = 355, normalized size = 2.93

$$-\frac{\frac{7\left(165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-75\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+540\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+750\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+480\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-170\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+129\,A-49\,B\right)}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}+\frac{2205\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+525\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-10080\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-1470\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+21945\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+2555\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-26460\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2240\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+18963\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1407\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-7476\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-434\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1383\,A+137\,B}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}}{1680\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/1680*(7*(165*A*tan(1/2*f*x + 1/2*e)^4 - 75*B*tan(1/2*f*x + 1/2*e)^4 + 54
0*A*tan(1/2*f*x + 1/2*e)^3 - 210*B*tan(1/2*f*x + 1/2*e)^3 + 750*A*tan(1/2*f
*x + 1/2*e)^2 - 280*B*tan(1/2*f*x + 1/2*e)^2 + 480*A*tan(1/2*f*x + 1/2*e) -
 170*B*tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e)
+ 1)^5) + (2205*A*tan(1/2*f*x + 1/2*e)^6 + 525*B*tan(1/2*f*x + 1/2*e)^6 - 1
0080*A*tan(1/2*f*x + 1/2*e)^5 - 1470*B*tan(1/2*f*x + 1/2*e)^5 + 21945*A*tan
(1/2*f*x + 1/2*e)^4 + 2555*B*tan(1/2*f*x + 1/2*e)^4 - 26460*A*tan(1/2*f*x +
 1/2*e)^3 - 2240*B*tan(1/2*f*x + 1/2*e)^3 + 18963*A*tan(1/2*f*x + 1/2*e)^2
+ 1407*B*tan(1/2*f*x + 1/2*e)^2 - 7476*A*tan(1/2*f*x + 1/2*e) - 434*B*tan(1
/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [**B**]
time = 13.16, size = 217, normalized size = 1.79

$$\frac{\left(\frac{16\,A}{35}-\frac{8\,B}{105}-\frac{32\,A\,\sin(e+f\,x)}{35}+\frac{16\,B\,\sin(e+f\,x)}{105}\right)\,\cos\left(e+f\,x\right)^4+\left(\frac{4\,A}{35}-\frac{2\,B}{105}-\frac{16\,A\,\sin(e+f\,x)}{35}+\frac{8\,B\,\sin(e+f\,x)}{105}\right)\,\cos\left(e+f\,x\right)^2+\frac{2\,A}{35}-\frac{12\,B}{35}-\frac{12\,A\,\sin(e+f\,x)}{35}+\frac{2\,B\,\sin(e+f\,x)}{35}}{a^3\,c^4\,f\,\left(2\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^5\right)}-\frac{\frac{2\,A}{7}+\frac{2\,B}{7}-\frac{2\,A\,\sin(e+f\,x)}{7}-\frac{2\,B\,\sin(e+f\,x)}{7}}{a^3\,c^4\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{32\,A}{35}-\frac{16\,B}{105}\right)}{a^3\,c^4\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x)
```

```
[Out] ((2*A)/35 - (12*B)/35 - (12*A*sin(e + f*x))/35 + (2*B*sin(e + f*x))/35 + co
s(e + f*x)^2*((4*A)/35 - (2*B)/105 - (16*A*sin(e + f*x))/35 + (8*B*sin(e +
f*x))/105) + cos(e + f*x)^4*((16*A)/35 - (8*B)/105 - (32*A*sin(e + f*x))/35
 + (16*B*sin(e + f*x))/105))/(a^3*c^4*f*(2*cos(e + f*x)^5*sin(e + f*x) - 2*
cos(e + f*x)^5)) - ((2*A)/7 + (2*B)/7 - (2*A*sin(e + f*x))/7 - (2*B*sin(e +
 f*x))/7)/(a^3*c^4*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((32*A)/35 - (16
*B)/105))/(a^3*c^4*f*(2*sin(e + f*x) - 2))
```

3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=162

$$\frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\,(c^5-c^5\sin(e+fx))}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}$$

```
[Out] 1/9*(A+B)*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/63*(7*A-2*B)*sec(f*x+
e)^5/a^3/f/(c^5-c^5*sin(f*x+e))+2/21*(7*A-2*B)*tan(f*x+e)/a^3/c^5/f+4/63*(7
*A-2*B)*tan(f*x+e)^3/a^3/c^5/f+2/105*(7*A-2*B)*tan(f*x+e)^5/a^3/c^5/f
```

Rubi [A]
time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 3852}

$$\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\,(c^5-c^5\sin(e+fx))}+\frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B
)*Sec[e + f*x]^5)/(63*a^3*f*(c^5 - c^5*Sin[e + f*x])) + (2*(7*A - 2*B)*Tan[
e + f*x])/(21*a^3*c^5*f) + (4*(7*A - 2*B)*Tan[e + f*x]^3)/(63*a^3*c^5*f) +
(2*(7*A - 2*B)*Tan[e + f*x]^5)/(105*a^3*c^5*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(162) = 324.
time = 0.91, size = 373, normalized size = 2.30

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-184320B+1125(49A+13B)\cos(e+fx)-20480(7A-2B)\cos(2(e+fx))+23275A\cos(3(e+fx))+6175B\cos(3(e+fx))-114688A\cos(4(e+fx))+32768B\cos(4(e+fx))+1225A\cos(5(e+fx))+325B\cos(5(e+fx))-28672A\cos(6(e+fx))+8192B\cos(6(e+fx))-1225A\cos(7(e+fx))-325B\cos(7(e+fx))-322560A\sin(e+fx)+92160B\sin(e+fx)-24500A\sin(2(e+fx))-6500B\sin(2(e+fx))-136192A\sin(3(e+fx))+38912B\sin(3(e+fx))-19600A\sin(4(e+fx))-5200B\sin(4(e+fx))-7168A\sin(5(e+fx))+2048B\sin(5(e+fx))-4900A\sin(6(e+fx))-1300B\sin(6(e+fx))+7168A\sin(7(e+fx))-2048B\sin(7(e+fx)))}{1290240a^3c^5f(-1+\sin(e+fx))^5(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*(e
 + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A*Co
```

```
s[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 325*B
*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)] - 12
25*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x] + 92
160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*x)] - 1
36192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4*(e + f*
x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*Sin[5*(e +
 f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168*A*Sin[7*(
e + f*x)] - 2048*B*Sin[7*(e + f*x)]))/(1290240*a^3*c^5*f*(-1 + Sin[e + f*x]
)^5*(1 + Sin[e + f*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(152) = 304$.
time = 0.63, size = 321, normalized size = 1.98 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3/c^5*(-1/2*(-9/32*A+7/32*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/4*A+1/4
*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/8*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3
*(13/32*A-11/32*B)/(tan(1/2*f*x+1/2*e)+1)^3-(29/128*A-15/128*B)/(tan(1/2*f*
x+1/2*e)+1)-1/9*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(8*A+8*B)/(tan(1/2*f
*x+1/2*e)-1)^8-1/4*(35/2*A+12*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/7*(35/2*A+33/2*
B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(49/2*A+43/2*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/
2*(51/16*A+21/16*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(49/2*A+77/4*B)/(tan(1/2*f
*x+1/2*e)-1)^5-(99/128*A+15/128*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(147/16*A+81/
16*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(161) = 322$.
time = 0.34, size = 1303, normalized size = 8.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(B*(100*sin(f*x + e)/(cos(f*x + e) + 1) - 340*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 55*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 88*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1608*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1032*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
483*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 588*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 420*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 420*sin(f*x + e)^11/(c
os(f*x + e) + 1)^11 - 315*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 25)/(a^3*
c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(c
```

```
os(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^
3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c
^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^
5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*si
n(f*x + e)^14/(cos(f*x + e) + 1)^14) - 7*A*(5*sin(f*x + e)/(cos(f*x + e) +
1) - 80*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 190*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 269*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 96*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 516*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 69*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 240*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 + 30*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 90*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 + 45*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 10)/(a^3*c^5 -
4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(
f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x + e) +
 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*sin(f*x
+ e)^14/(cos(f*x + e) + 1)^14))/f
```

Fricas [A]
time = 0.38, size = 196, normalized size = 1.21

$$-\frac{32\,(7\,A-2\,B)\cos\left(fx+e\right)^{6}-16\,(7\,A-2\,B)\cos\left(fx+e\right)^{4}-4\,(7\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(16\,(7\,A-2\,B)\cos\left(fx+e\right)^{6}-24\,(7\,A-2\,B)\cos\left(fx+e\right)^{4}-10\,(7\,A-2\,B)\cos\left(fx+e\right)^{2}-49\,A+14\,B\right)\sin\left(fx+e\right)-14\,A+49\,B}{315\,\left(a^{3}c^{5}f\cos\left(fx+e\right)^{7}+2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*(
7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^6 - 24*(7*A - 2*B)
*cos(f*x + e)^4 - 10*(7*A - 2*B)*cos(f*x + e)^2 - 49*A + 14*B)*sin(f*x + e)
 - 14*A + 49*B)/(a^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(
f*x + e) - 2*a^3*c^5*f*cos(f*x + e)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8396 vs. $2(150) = 300$.
time = 78.50, size = 8396, normalized size = 51.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*tan(e/2 + f*x/2)**13/(315*a**3*c**5*f*tan(e/2 + f*x/2)**1
4 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e
/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*t
an(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**
5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1260*A*tan(e/2 + f*x/2)**12/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 42
0*A*tan(e/2 + f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*
c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a
**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8
 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 +
f*x/2) - 315*a**3*c**5*f) - 3360*A*tan(e/2 + f*x/2)**10/(315*a**3*c**5*f*ta
n(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*
f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*
c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*
a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4
- 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**
2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 966*A*tan(e/2 +
f*x/2)**9/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2
+ f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(
e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f
*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c
**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**
3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a
**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a*
*3*c**5*f) + 4956*A*tan(e/2 + f*x/2)**8/(315*a**3*c**5*f*tan(e/2 + f*x/2)**
14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/
2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
 f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(
e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*
tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5
```

```
*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c*
*5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 7224*A*tan(e/2 + f*x/2)**7/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 13
44*A*tan(e/2 + f*x/2)**6/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*
c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a
**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8
 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 +
f*x/2) - 315*a**3*c**5*f) + 3766*A*tan(e/2 + f*x/2)**5/(315*a**3*c**5*f*tan
(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f
*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c
**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a
**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 63
00*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 -
 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2
 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(161) = 322.
time = 0.53, size = 415, normalized size = 2.56

$$-\frac{\frac{21\left(435\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-225\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+1470\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-690\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2060\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-940\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1330\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-590\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+353\,A-163\,B\right)}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}+\frac{31185\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+4725\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-185220\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-11340\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+546840\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+15120\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-961380\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3780\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+1101618\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-24318\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-828492\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+33852\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+404208\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-19368\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-116172\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6732\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+16373\,A-223\,B}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{9}}}{20160\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/20160*(21*(435*A*tan(1/2*f*x + 1/2*e)^4 - 225*B*tan(1/2*f*x + 1/2*e)^4 +
 1470*A*tan(1/2*f*x + 1/2*e)^3 - 690*B*tan(1/2*f*x + 1/2*e)^3 + 2060*A*tan(
1/2*f*x + 1/2*e)^2 - 940*B*tan(1/2*f*x + 1/2*e)^2 + 1330*A*tan(1/2*f*x + 1/
2*e) - 590*B*tan(1/2*f*x + 1/2*e) + 353*A - 163*B)/(a^3*c^5*(tan(1/2*f*x +
1/2*e) + 1)^5) + (31185*A*tan(1/2*f*x + 1/2*e)^8 + 4725*B*tan(1/2*f*x + 1/2
*e)^8 - 185220*A*tan(1/2*f*x + 1/2*e)^7 - 11340*B*tan(1/2*f*x + 1/2*e)^7 +
546840*A*tan(1/2*f*x + 1/2*e)^6 + 15120*B*tan(1/2*f*x + 1/2*e)^6 - 961380*A
*tan(1/2*f*x + 1/2*e)^5 + 3780*B*tan(1/2*f*x + 1/2*e)^5 + 1101618*A*tan(1/2
*f*x + 1/2*e)^4 - 24318*B*tan(1/2*f*x + 1/2*e)^4 - 828492*A*tan(1/2*f*x + 1
/2*e)^3 + 33852*B*tan(1/2*f*x + 1/2*e)^3 + 404208*A*tan(1/2*f*x + 1/2*e)^2
```

```
- 19368*B*tan(1/2*f*x + 1/2*e)^2 - 116172*A*tan(1/2*f*x + 1/2*e) + 6732*B*t
an(1/2*f*x + 1/2*e) + 16373*A - 223*B)/(a^3*c^5*(tan(1/2*f*x + 1/2*e) - 1)^
9))/f
```

Mupad [B]
time = 13.14, size = 231, normalized size = 1.43

$$\frac{\left(\frac{128\,B}{315}-\frac{64\,A}{45}+\frac{32\,A\,\sin(e+f\,x)}{45}-\frac{64\,B\,\sin(e+f\,x)}{315}\right)\cos\left(e+f\,x\right)^6+\left(\frac{8\,A\,\sin(e+f\,x)}{9}-\frac{20\,B}{63}-\frac{8\,A}{9}+\frac{20\,B\,\sin(e+f\,x)}{63}-\frac{(4\,\sin(e+f\,x)-4)\left(\frac{4\,A}{9}+\frac{10\,B}{63}\right)}{2}\right)\cos\left(e+f\,x\right)^5+\left(\frac{32\,A}{45}-\frac{64\,B}{315}-\frac{16\,A\,\sin(e+f\,x)}{15}+\frac{32\,B\,\sin(e+f\,x)}{105}\right)\cos\left(e+f\,x\right)^4+\left(\frac{8\,A}{45}-\frac{16\,B}{315}-\frac{4\,A\,\sin(e+f\,x)}{9}+\frac{8\,B\,\sin(e+f\,x)}{63}\right)\cos\left(e+f\,x\right)^2+\frac{4\,A}{45}-\frac{14\,B}{45}-\frac{14\,A\,\sin(e+f\,x)}{45}+\frac{4\,B\,\sin(e+f\,x)}{45}}{a^3\,c^5\,f\,\left(4\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)-4\cos\left(e+f\,x\right)^5+2\cos\left(e+f\,x\right)^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x)
```

```
[Out] ((4*A)/45 - (14*B)/45 - (14*A*sin(e + f*x))/45 + (4*B*sin(e + f*x))/45 - co
s(e + f*x)^5*((8*A)/9 + (20*B)/63 - (8*A*sin(e + f*x))/9 - (20*B*sin(e + f*
x))/63 + ((4*sin(e + f*x) - 4)*((4*A)/9 + (10*B)/63))/2) + cos(e + f*x)^2*(
(8*A)/45 - (16*B)/315 - (4*A*sin(e + f*x))/9 + (8*B*sin(e + f*x))/63) + cos
(e + f*x)^4*((32*A)/45 - (64*B)/315 - (16*A*sin(e + f*x))/15 + (32*B*sin(e
+ f*x))/105) - cos(e + f*x)^6*((64*A)/45 - (128*B)/315 - (32*A*sin(e + f*x)
)/45 + (64*B*sin(e + f*x))/315))/(a^3*c^5*f*(4*cos(e + f*x)^5*sin(e + f*x)
- 4*cos(e + f*x)^5 + 2*cos(e + f*x)^7))
```

3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=205

$$\frac{(A+B)\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\,(c^3-c^3\sin(e+fx))^2}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\,(c^6-c^6\sin(e+fx))}+\frac{2(8A-3B)\tan(e}{33a^3c^6f}$$

```
[Out] 1/11*(A+B)*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+1/99*(8*A-3*B)*sec(f*x
+e)^5/a^3/f/(c^3-c^3*sin(f*x+e))^2+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^6-c
^6*sin(f*x+e))+2/33*(8*A-3*B)*tan(f*x+e)/a^3/c^6/f+4/99*(8*A-3*B)*tan(f*x+e
)^3/a^3/c^6/f+2/165*(8*A-3*B)*tan(f*x+e)^5/a^3/c^6/f
```

Rubi [A]
time = 0.24, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2938, 2751, 3852}

$$\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\,(c^6-c^6\sin(e+fx))}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\,(c^3-c^3\sin(e+fx))^2}+\frac{(A+B)\sec^5(e+fx)}{11a^3f\,(c^2-c^2\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + ((8*A - 3*
B)*Sec[e + f*x]^5)/(99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + ((8*A - 3*B)*Sec
[e + f*x]^5)/(99*a^3*f*(c^6 - c^6*Sin[e + f*x])) + (2*(8*A - 3*B)*Tan[e + f
*x])/(33*a^3*c^6*f) + (4*(8*A - 3*B)*Tan[e + f*x]^3)/(99*a^3*c^6*f) + (2*(8
*A - 3*B)*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,d}{11a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}$$

Mathematica [A]
time = 2.25, size = 401, normalized size = 1.96

$$\frac{1013760B-3850(107A-3B)\cos(e+fx)+135168(8A-3B)\cos(2(e+fx))-127330A\cos(3(e+fx))+3570B\cos(3(e+fx))+819200A\cos(4(e+fx))-307200B\cos(4(e+fx))+37450A\cos(5(e+fx))-1050B\cos(5(e+fx))+163840A\cos(6(e+fx))-61440B\cos(6(e+fx))+22470A\cos(7(e+fx))-630B\cos(7(e+fx))-16384A\cos(8(e+fx))+6144B\cos(8(e+fx))+1802240A\sin(e+fx)-675840B\sin(e+fx)+247170A\sin(2(e+fx))-6930B\sin(2(e+fx))+557056A\sin(3(e+fx))-208896B\sin(3(e+fx))+187250A\sin(4(e+fx))-5250B\sin(4(e+fx))-163840A\sin(5(e+fx))+61440B\sin(5(e+fx))+37450A\sin(6(e+fx))-1050B\sin(6(e+fx))-98304A\sin(7(e+fx))+36864B\sin(7(e+fx))-3745A\sin(8(e+fx))+105B\sin(8(e+fx))}{8110080a^3c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^6),x]
```

```
[Out] (1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e
+ f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*Co
s[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] - 105
0*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f*x)]
 + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(e + f
*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Sin[e +
f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*A*Sin[3
*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)] - 5250*
B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e + f*x)] +
 37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin[7*(e + f*
x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B*Sin[8*(e +
 f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 0.45, size = 365, normalized size = 1.78

method	result
risch	$\frac{32i\left(-102iB\,e^{5i(fx+e)}+272iA\,e^{5i(fx+e)}+495B\,e^{8i(fx+e)}-80iA\,e^{3i(fx+e)}+528A\,e^{6i(fx+e)}-48iA\,e^{i(fx+e)}-198B\,e^{6i(f\ldots}\right.}{495(\ldots}$
derivativedivides	$-\frac{-\frac{5A}{32}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{8}+\frac{B}{8}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{37A}{256}-\frac{21B}{256}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{7A}{32}-\frac{3B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\ldots\right.}$
default	$-\frac{-\frac{5A}{32}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{8}+\frac{B}{8}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{37A}{256}-\frac{21B}{256}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{7A}{32}-\frac{3B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3/c^6*(-1/2*(-5/32*A+1/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/8*A+1/8*
B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)^5-(3
7/256*A-21/256*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(7/32*A-3/16*B)/(tan(1/2*f*x+1
/2*e)+1)^3-1/11*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/10*(20*A+20*B)/(tan(1
/2*f*x+1/2*e)-1)^10-1/9*(53*A+51*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(92*A+84*B
)/(tan(1/2*f*x+1/2*e)-1)^8-1/4*(169/4*A+99/4*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/
6*(217/2*A+84*B)/(tan(1/2*f*x+1/2*e)-1)^6-(219/256*A+21/256*B)/(tan(1/2*f*x
+1/2*e)-1)-1/7*(231/2*A+98*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(303/64*A+99/64*
B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(623/8*A+427/8*B)/(tan(1/2*f*x+1/2*e)-1)^5-
1/3*(1095/64*A+507/64*B)/(tan(1/2*f*x+1/2*e)-1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1505 vs. $2(205) = 410$.
time = 0.37, size = 1505, normalized size = 7.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/495*(A*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f
*x + e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 -
1815*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x
+ e) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/(a^3*c^6 -
6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c
^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*
c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(
f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16) + 3*B*(30
*sin(f*x + e)/(cos(f*x + e) + 1) - 215*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 245*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 434*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 880*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1815*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 + 330*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 99*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 264*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 - 495*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 330*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - 165*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 5)/(a^3*c
^6 - 6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*
a^3*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^
3*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^
3*c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(c
os(f*x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10
*a^3*c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16))/f
```

Fricas [A]
time = 0.38, size = 234, normalized size = 1.14

$$\frac{16\,(8\,A-3\,B)\cos\left(fx+e\right)^{8}-72\,(8\,A-3\,B)\cos\left(fx+e\right)^{6}+30\,(8\,A-3\,B)\cos\left(fx+e\right)^{4}+7\,(8\,A-3\,B)\cos\left(fx+e\right)^{2}+\left(48\,(8\,A-3\,B)\cos\left(fx+e\right)^{6}-40\,(8\,A-3\,B)\cos\left(fx+e\right)^{4}-14\,(8\,A-3\,B)\cos\left(fx+e\right)^{2}-72\,A+27\,B\right)\sin\left(fx+e\right)+27\,A-72\,B}{495\,\left(3\,a^{3}c^{6}f\cos\left(fx+e\right)^{7}-4\,a^{3}c^{6}f\cos\left(fx+e\right)^{5}-\left(a^{3}c^{6}f\cos\left(fx+e\right)^{7}-4\,a^{3}c^{6}f\cos\left(fx+e\right)^{5}\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/495*(16*(8*A - 3*B)*cos(f*x + e)^8 - 72*(8*A - 3*B)*cos(f*x + e)^6 + 30*(
8*A - 3*B)*cos(f*x + e)^4 + 7*(8*A - 3*B)*cos(f*x + e)^2 + (48*(8*A - 3*B)*
cos(f*x + e)^6 - 40*(8*A - 3*B)*cos(f*x + e)^4 - 14*(8*A - 3*B)*cos(f*x + e
)^2 - 72*A + 27*B)*sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*cos(f*x + e)^7
- 4*a^3*c^6*f*cos(f*x + e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(
f*x + e)^5)*sin(f*x + e))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11011 vs. $2(187) = 374$.
time = 136.24, size = 11011, normalized size = 53.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-990*A*tan(e/2 + f*x/2)**15/(495*a**3*c**6*f*tan(e/2 + f*x/2)**1
6 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/
2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2
+ f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*t
an(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c*
*6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a*
*3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 495
0*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 +
2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 2970*A*tan(e/2 + f*x
/2)**14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 +
f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e
/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*
f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**
3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 3267
0*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 +
 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)*
*3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/
2) - 495*a**3*c**6*f) - 3630*A*tan(e/2 + f*x/2)**13/(495*a**3*c**6*f*tan(e/
2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*t
an(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c*
*6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*
a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 +
54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)*
*6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*
x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 +
```

```
 f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 4950*A*
tan(e/2 + f*x/2)**12/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6
*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3
*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 168
30*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**1
0 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x
/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2
+ f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(
e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*t
an(e/2 + f*x/2) - 495*a**3*c**6*f) + 9834*A*tan(e/2 + f*x/2)**11/(495*a**3*
c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 -
24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)
**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 +
 f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(
e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f
*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**
6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6
*f) + 66*A*tan(e/2 + f*x/2)**10/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 297
0*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14
+ 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2
)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2
+ f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*ta
n(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6
*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*
c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a*
*3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 23430*A*tan(e/2 + f*x/2)**9
/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)*
*15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*
x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e
/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*
f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*
c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*
a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 49
50*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 49
5*a**3*c**6*f) + 17490*A*tan(e/2 + f*x/2)**8/(495*a**3*c**6*f*tan(e/2 + f*x
/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2
+ f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*ta
n(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c*
*6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a
**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16
830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(205) = 410.

time = 0.48, size = 475, normalized size = 2.32

$$-\frac{\frac{33\left(555 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-315 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+1920 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{3}-1020 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{3}+2710 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-1410 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+1760 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)-900 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+463 A-243 B\right)}{a^{3} c^{6}\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)^{5}}+\frac{108405 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{10}+10395 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{10}-784080 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{9}-5940 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{9}+2901195 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{8}-79695 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{8}-6652800 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{7}+388080 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{7}+10407474 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{6}-816354 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{6}-11435424 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{5}+1114344 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{5}+8949270 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-990990 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-4899840 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{3}+609840 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{3}+1816265 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-235785 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-411664 A \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+56364 B \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+47279 A-4179 B}{a^{3} c^{6}\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)^{11}}}{63360 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/63360*(33*(555*A*tan(1/2*f*x + 1/2*e)^4 - 315*B*tan(1/2*f*x + 1/2*e)^4 +
 1920*A*tan(1/2*f*x + 1/2*e)^3 - 1020*B*tan(1/2*f*x + 1/2*e)^3 + 2710*A*tan
(1/2*f*x + 1/2*e)^2 - 1410*B*tan(1/2*f*x + 1/2*e)^2 + 1760*A*tan(1/2*f*x +
1/2*e) - 900*B*tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(tan(1/2*f*x
+ 1/2*e) + 1)^5) + (108405*A*tan(1/2*f*x + 1/2*e)^10 + 10395*B*tan(1/2*f*x
+ 1/2*e)^10 - 784080*A*tan(1/2*f*x + 1/2*e)^9 - 5940*B*tan(1/2*f*x + 1/2*e)
^9 + 2901195*A*tan(1/2*f*x + 1/2*e)^8 - 79695*B*tan(1/2*f*x + 1/2*e)^8 - 66
52800*A*tan(1/2*f*x + 1/2*e)^7 + 388080*B*tan(1/2*f*x + 1/2*e)^7 + 10407474
*A*tan(1/2*f*x + 1/2*e)^6 - 816354*B*tan(1/2*f*x + 1/2*e)^6 - 11435424*A*ta
n(1/2*f*x + 1/2*e)^5 + 1114344*B*tan(1/2*f*x + 1/2*e)^5 + 8949270*A*tan(1/2
*f*x + 1/2*e)^4 - 990990*B*tan(1/2*f*x + 1/2*e)^4 - 4899840*A*tan(1/2*f*x +
 1/2*e)^3 + 609840*B*tan(1/2*f*x + 1/2*e)^3 + 1816265*A*tan(1/2*f*x + 1/2*e
)^2 - 235785*B*tan(1/2*f*x + 1/2*e)^2 - 411664*A*tan(1/2*f*x + 1/2*e) + 563
64*B*tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(tan(1/2*f*x + 1/2*e
) - 1)^11))/f
```

Mupad [B]
time = 14.52, size = 474, normalized size = 2.31

$$\frac{2\left(\frac{165 B \sin(e+f x)}{4}-\frac{6875 A \cos(e+f x)}{64}-\frac{825 B \cos(e+f x)}{64}-110 A \sin(e+f x)-\frac{495 B}{8}-66 A \cos(2 e+2 f x)-\frac{2125 A \cos(3 e+3 f x)}{64}-50 A \cos(4 e+4 f x)+\frac{625 A \cos(5 e+5 f x)}{64}-10 A \cos(6 e+6 f x)+\frac{375 A \cos(7 e+7 f x)}{64}+A \cos(8 e+8 f x)+\frac{99 B \cos(2 e+2 f x)}{4}-\frac{255 B \cos(3 e+3 f x)}{64}+\frac{75 B \cos(4 e+4 f x)}{4}+\frac{75 B \cos(5 e+5 f x)}{64}+\frac{15 B \cos(6 e+6 f x)}{4}+\frac{45 B \cos(7 e+7 f x)}{64}-\frac{3 B \cos(8 e+8 f x)}{8}+\frac{4125 A \sin(2 e+2 f x)}{64}-34 A \sin(3 e+3 f x)+\frac{3125 A \sin(4 e+4 f x)}{64}+10 A \sin(5 e+5 f x)+\frac{625 A \sin(6 e+6 f x)}{64}+6 A \sin(7 e+7 f x)-\frac{125 A \sin(8 e+8 f x)}{128}+\frac{495 B \sin(2 e+2 f x)}{64}+\frac{51 B \sin(3 e+3 f x)}{4}+\frac{375 B \sin(4 e+4 f x)}{64}-\frac{15 B \sin(5 e+5 f x)}{4}+\frac{75 B \sin(6 e+6 f x)}{64}-\frac{9 B \sin(7 e+7 f x)}{4}-\frac{15 B \sin(8 e+8 f x)}{128}\right)}{495 a^{3} c^{6} f\left(\frac{5 \cos(5 e+5 f x)}{32}-\frac{17 \cos(3 e+3 f x)}{32}-\frac{55 \cos(e+f x)}{32}+\frac{3 \cos(7 e+7 f x)}{32}+\frac{33 \sin(2 e+2 f x)}{32}+\frac{25 \sin(4 e+4 f x)}{32}+\frac{5 \sin(6 e+6 f x)}{32}-\frac{\sin(8 e+8 f x)}{64}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x)
```

```
[Out] (2*((165*B*sin(e + f*x))/4 - (6875*A*cos(e + f*x))/64 - (825*B*cos(e + f*x)
)/64 - 110*A*sin(e + f*x) - (495*B)/8 - 66*A*cos(2*e + 2*f*x) - (2125*A*cos
(3*e + 3*f*x))/64 - 50*A*cos(4*e + 4*f*x) + (625*A*cos(5*e + 5*f*x))/64 - 1
0*A*cos(6*e + 6*f*x) + (375*A*cos(7*e + 7*f*x))/64 + A*cos(8*e + 8*f*x) + (
99*B*cos(2*e + 2*f*x))/4 - (255*B*cos(3*e + 3*f*x))/64 + (75*B*cos(4*e + 4*
f*x))/4 + (75*B*cos(5*e + 5*f*x))/64 + (15*B*cos(6*e + 6*f*x))/4 + (45*B*co
s(7*e + 7*f*x))/64 - (3*B*cos(8*e + 8*f*x))/8 + (4125*A*sin(2*e + 2*f*x))/6
4 - 34*A*sin(3*e + 3*f*x) + (3125*A*sin(4*e + 4*f*x))/64 + 10*A*sin(5*e + 5
*f*x) + (625*A*sin(6*e + 6*f*x))/64 + 6*A*sin(7*e + 7*f*x) - (125*A*sin(8*e
 + 8*f*x))/128 + (495*B*sin(2*e + 2*f*x))/64 + (51*B*sin(3*e + 3*f*x))/4 +
(375*B*sin(4*e + 4*f*x))/64 - (15*B*sin(5*e + 5*f*x))/4 + (75*B*sin(6*e + 6
*f*x))/64 - (9*B*sin(7*e + 7*f*x))/4 - (15*B*sin(8*e + 8*f*x))/128))/(495*a
^3*c^6*f*((5*cos(5*e + 5*f*x))/32 - (17*cos(3*e + 3*f*x))/32 - (55*cos(e +
f*x))/32 + (3*cos(7*e + 7*f*x))/32 + (33*sin(2*e + 2*f*x))/32 + (25*sin(4*e
 + 4*f*x))/32 + (5*sin(6*e + 6*f*x))/32 - sin(8*e + 8*f*x)/64))
```

3.81 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx$

Optimal. Leaf size=198

$$\frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(}}{231f}$$

```
[Out] 256/3465*a*(11*A-5*B)*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/99*a*(11*
A-5*B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f-2/11*a*B*c*cos(f*x+e)^3*(c
-c*sin(f*x+e))^(5/2)/f+64/1155*a*(11*A-5*B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x
+e))^(1/2)+8/231*a*(11*A-5*B)*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.33, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{256ac^5(11A-5B)\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4(11A-5B)\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(11A-5B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f}+\frac{2ac^2(11A-5B)\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f}-\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*(11*A - 5*B)*c^5*Cos[e + f*x]^3)/(3465*f*(c - c*Sin[e + f*x])^(3/2))
 + (64*a*(11*A - 5*B)*c^4*Cos[e + f*x]^3)/(1155*f*Sqrt[c - c*Sin[e + f*x]])
 + (8*a*(11*A - 5*B)*c^3*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(231*f) +
 (2*a*(11*A - 5*B)*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(99*f) -
(2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=(ac)\int \cos^2(e+fx)(A+B\sin(e+fx$$

$$=-\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx)}{11f}$$

$$=\frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin}{99f}$$

$$=\frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin}}{231f}$$

$$=\frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8a(}{}$$

$$=\frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64}{}$$

Mathematica [A]
time = 1.88, size = 149, normalized size = 0.75

$$-\frac{ac^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\,(-35332A+27085B+60(121A-202B)\cos(2(e+fx))+315B\cos(4(e+fx))+30558A\sin(e+fx)-31530B\sin(e+fx)-770A\sin(3(e+fx))+2870B\sin(3(e+fx)))}{13860f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/13860*(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e +
f*x]]*(-35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*Cos
[4*(e + f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[3*(
e + f*x)] + 2870*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]))
```

Maple [A]
time = 6.21, size = 119, normalized size = 0.60

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^2 a((-385A+1435B)\sin(fx+e)(\cos^2(fx+e))+(3916A-4300B)\sin(fx+e)+315B(\cos^4(fx+e))+(18}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3465*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*((-385*A+1435*B)*sin(f*x+e)*co
s(f*x+e)^2+(3916*A-4300*B)*sin(f*x+e)+315*B*cos(f*x+e)^4+(1815*A-3345*B)*co
s(f*x+e)^2-5324*A+4940*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [A]
time = 0.36, size = 302, normalized size = 1.53

$$\frac{2\,\left(315\,Bac^3\cos\left(fx+e\right)^6-35\,(11\,A-32\,B)ac^3\cos\left(fx+e\right)^5+5\,(209\,A-221\,B)ac^3\cos\left(fx+e\right)^4+2\,(1243\,A-1195\,B)ac^3\cos\left(fx+e\right)^3-32\,(11\,A-5\,B)ac^3\cos\left(fx+e\right)^2+128\,(11\,A-5\,B)ac^3\cos\left(fx+e\right)+256\,(11\,A-5\,B)ac^3-\left(315\,Bac^3\cos\left(fx+e\right)^5+35\,(11\,A-23\,B)ac^3\cos\left(fx+e\right)^4+10\,(143\,A-191\,B)ac^3\cos\left(fx+e\right)^3-96\,(11\,A-5\,B)ac^3\cos\left(fx+e\right)^2-128\,(11\,A-5\,B)ac^3\cos\left(fx+e\right)-256\,(11\,A-5\,B)ac^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3465\,\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5
+ 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(f*
x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3*co
```

```
s(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B*a*c^3*cos(f*x + e)^5 + 35*(11*
A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)^3 -
96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x + e)
- 256*(11*A - 5*B)*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e) - f*sin(f*x + e) + f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [**A**]
time = 0.70, size = 312, normalized size = 1.58

$$-\frac{\sqrt{2}\left(6930\,Bac^3\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-315\,Bac^3\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+48510\left(2\,Aac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-693\left(16\,Aac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5\,Bac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+495\left(10\,Aac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-9\,Bac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)-385\left(2\,Aac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5\,Bac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\right)\sqrt{c}}{55440\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] -1/55440*sqrt(2)*(6930*B*a*c^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 315*B*a*c^3*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 48510*(2*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 693*(16*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*
a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) +
 495*(10*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 9*B*a*c^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 385*(2*A*a*c^3*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2), x
)
```

3.82 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=157

$$\frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}$$

```
[Out] 64/315*a*(3*A-B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/9*a*B*c*cos(f*
x+e)^3*(c-c*sin(f*x+e))^(3/2)/f+16/105*a*(3*A-B)*c^3*cos(f*x+e)^3/f/(c-c*si
n(f*x+e))^(1/2)+2/21*a*(3*A-B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.28, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{64ac^4(3A-B)\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3(3A-B)\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2(3A-B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*(3*A - B)*c^4*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (1
6*a*(3*A - B)*c^3*Cos[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*(
3*A - B)*c^2*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(21*f) - (2*a*B*c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx$$
$$= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx)}{9f}$$
$$= \frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin}}{21f}$$
$$= \frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} + \frac{2a(3A}{}$$
$$= \frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}} + \frac{16a(3}{105f}$$

Mathematica [A]
time = 0.98, size = 123, normalized size = 0.78

$$-\frac{ac^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\,(-942A+664B+30(3A-8B)\cos(2(e+fx))+(648A-741B)\sin(e+fx)+35B\sin(3(e+fx)))}{630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/630*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*
x]]*(-942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*Sin
```

```
[e + f*x] + 35*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
))
```

Maple [A]
time = 5.45, size = 103, normalized size = 0.66

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^2 a(-35B(\cos^2(fx+e))\sin(fx+e)+(-162A+194B)\sin(fx+e)+(-45A+120B)(\cos^2(fx+e))+25}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(-35*B*cos(f*x+e)^2*sin(f*x+e)
+(-162*A+194*B)*sin(f*x+e)+(-45*A+120*B)*cos(f*x+e)^2+258*A-226*B)/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [A]
time = 0.37, size = 256, normalized size = 1.63

$$\frac{2\left(35\,Bac^2\cos\left(fx+e\right)^5+5\left(9\,A-10\,B\right)ac^2\cos\left(fx+e\right)^4+\left(117\,A-109\,B\right)ac^2\cos\left(fx+e\right)^3-8\left(3\,A-B\right)ac^2\cos\left(fx+e\right)^2+32\left(3\,A-B\right)ac^2\cos\left(fx+e\right)+64\left(3\,A-B\right)ac^2+\left(35\,Bac^2\cos\left(fx+e\right)^4-5\left(9\,A-17\,B\right)ac^2\cos\left(fx+e\right)^3+24\left(3\,A-B\right)ac^2\cos\left(fx+e\right)^2+32\left(3\,A-B\right)ac^2\cos\left(fx+e\right)+64\left(3\,A-B\right)ac^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{315\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (1
17*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 32*
(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x + e
)^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x + e)
^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int Ac^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-Ac^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)\right)dx+\int\left(-Ac^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx+\int Ac^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)dx+\int Bc^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int\left(-Bc^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx+\int\left(-Bc^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)\right)dx+\int Bc^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c**2*sqrt(-c
*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c**2*sqrt(-c*sin(e + f*x)
 + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e
 + f*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x)
 + Integral(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integra
l(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*c**2*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x))
```

Giac [A]
time = 0.70, size = 276, normalized size = 1.76

$$-\frac{\sqrt{2}\left(35\,Bac^{2}\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+630\left(5\,Aac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,Bac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+210\left(Aac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Bac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)-126\left(3\,Aac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+45\left(2\,Aac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,Bac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\right)\sqrt{c}}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(35*B*a*c^2*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 630*(5*A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
2*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 210*(A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 126*(3*A*a*c^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*A*a*c^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) - 3*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi
 + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2), x
)
```

3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] 8/105*a*(7*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/35*a*(7*A-B)*c^
2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)-2/7*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x
+e))^(1/2)/f
```

Rubi [A]
time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{8ac^3(7A-B)\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2(7A-B)\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a*(7*A - B)*c^3*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (2*
a*(7*A - B)*c^2*Cos[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*B*c*
Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=(ac)\int \cos^2(e+fx)(A+B\sin(e+fx$$

$$=-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+f}}{7f}$$

$$=\frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\,c}{}$$

$$=\frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2a(7A}{35f\sqrt{}}$$

Mathematica [A]
time = 0.66, size = 104, normalized size = 0.90

$$\frac{ac\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(98A-59B+15B\cos(2(e+fx))+(-42A+66B)\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e +
 f*x)] + (-42*A + 66*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.25, size = 81, normalized size = 0.70

method	result	size
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^2 a(\sin(fx+e)(21A-33B)-15B(\cos^2(fx+e))-49A+37B)}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(sin(f*x+e)*(21*A-33*B)-15*B*co
s(f*x+e)^2-49*A+37*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [A]
time = 0.36, size = 195, normalized size = 1.68

$$-\frac{2\,\left(15\,Bac\cos\left(fx+e\right)^{4}-3\,\left(7\,A-6\,B\right)ac\cos\left(fx+e\right)^{3}+\left(7\,A-B\right)ac\cos\left(fx+e\right)^{2}-4\,\left(7\,A-B\right)ac\cos\left(fx+e\right)-8\,\left(7\,A-B\right)ac-\left(15\,Bac\cos\left(fx+e\right)^{3}+3\,\left(7\,A-B\right)ac\cos\left(fx+e\right)^{2}+4\,\left(7\,A-B\right)ac\cos\left(fx+e\right)+8\,\left(7\,A-B\right)ac\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{105\,\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A -
 B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c - (
15*B*a*c*cos(f*x + e)^3 + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a*c*
cos(f*x + e) + 8*(7*A - B)*a*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)\right)dx+\int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x), x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3,
x))
```

Giac [A]
time = 0.58, size = 205, normalized size = 1.77

$$\frac{\sqrt{2}\left(15\,Bac\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-105\left(4\,Aac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Bac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-35\left(2\,Aac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Bac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)+21\left(2\,Aac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Bac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\right)\sqrt{c}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a*c*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) - 105*(4*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) - 35*(2
*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a*c*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - B*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi
 + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2), x
)
```

3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=73

$$\frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2/15*a*(5*A+B)*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/5*a*B*c*cos(f*x+
e)^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3046, 2935, 2752}

$$\frac{2ac^2(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(5*A + B)*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2*a
*B*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}} + \frac{1}{5}(a(5A+$$
$$= \frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}} - \frac{2aBc}{5f\sqrt{c-}}$$

Mathematica [A]
time = 0.28, size = 87, normalized size = 1.19

$$\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(5A-2B+3B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(5*A - 2*B + 3*B*Sin[e + f*x])
*Sqrt[c - c*Sin[e + f*x]])/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.18, size = 63, normalized size = 0.86

method	result	size
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^2a(3B\sin(fx+e)+5A-2B)}{15\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(69) = 138$.
time = 0.35, size = 139, normalized size = 1.90

$$-\frac{2\,(3\,Ba\cos\left(fx+e\right)^{3}+(5\,A+4\,B)a\cos\left(fx+e\right)^{2}-(5\,A+B)a\cos\left(fx+e\right)-2\,(5\,A+B)a+(3\,Ba\cos\left(fx+e\right)^{2}-(5\,A+B)a\cos\left(fx+e\right)-2\,(5\,A+B)a)\sin\left(fx+e\right))\sqrt{-c\sin\left(fx+e\right)+c}}{15\,(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2/15*(3*B*a*cos(f*x + e)^3 + (5*A + 4*B)*a*cos(f*x + e)^2 - (5*A + B)*a*co
s(f*x + e) - 2*(5*A + B)*a + (3*B*a*cos(f*x + e)^2 - (5*A + B)*a*cos(f*x +
e) - 2*(5*A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
 - f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))
```

Giac [A]
time = 0.54, size = 125, normalized size = 1.71

$$-\frac{\sqrt{2}\,(30\,Aa\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right))+3\,Ba\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)\mathrm{sgn}(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right))+5\,(2\,Aa\mathrm{sgn}(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right))+Ba\mathrm{sgn}(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)))\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right))\sqrt{c}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/30*sqrt(2)*(30*A*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3*B*a*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/2*
```

```
f*x + 1/2*e)) + 5*(2*A*a*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(c)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x)) \sqrt{c - c \sin(e + f x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2), x
)
```

3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2\sqrt{2}\,a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

```
[Out] 2*a*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^
(1/2)/f/c^(1/2)-2/3*a*(3*A+5*B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)+2/3*a*B
*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.23, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$,
Rules used = {3046, 2937, 2830, 2728, 212}

$$-\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2\sqrt{2}\,a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin
[e + f*x]])])/(Sqrt[c]*f) - (2*a*(3*A + 5*B)*Cos[e + f*x])/(3*f*Sqrt[c - c*
Sin[e + f*x]]) + (2*a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2937

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*Cos[e + f*x]*((
a + b*Sin[e + f*x])^(m + 2)/(b^2*f*(m + 3))), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$= \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} - \frac{(2a)\int \frac{-\frac{3Ac}{2}-\frac{Bc}{2}+(-}{\sqrt{c-c}}}{3}$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(}}{3cf}$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(}}{3cf}$$

$$= \frac{2\sqrt{2}\,a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a(3}{3f}$$

Mathematica [A]
time = 0.88, size = 166, normalized size = 1.36

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6\sqrt{2}\,(A+B)\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\,\sqrt{c}}\right)\sqrt{-c(1+\sin(e+fx))}+\sqrt{c}\,(6A+9B-B\cos(2(e+fx))+2(3A+5B)\sin(e+fx))\right)}{3\sqrt{c}\,f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] -1/3*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*Sqrt[2]*(A + B)*ArcTan[Sqr
t[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x]))]
 + Sqrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x])))/
(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 8.15, size = 159, normalized size = 1.30

method	result
default	$-\frac{2(\sin(fx+e)-1)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, a\left(3c^{\frac{3}{2}}\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)A+3c^{\frac{3}{2}}\sqrt{2}\, \operatorname{arctan}}{3c^{2}\cos(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(3*c^(3/2)*2^(1/2)*arctanh(1
/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+3*c^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-B*(c*(1+sin(f*x+e)))^(3/2)-3*
A*c*(c*(1+sin(f*x+e)))^(1/2)-3*B*c*(c*(1+sin(f*x+e)))^(1/2))/c^2/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(111) = 222.
time = 0.38, size = 274, normalized size = 2.25

$$\frac{\frac{3\sqrt{2}\left((A+B)ac\cos(fx+e)-(A+B)ac\sin(fx+e)+(A+B)ac\right)\log\left(-\frac{\cos(fx+e)^{2}+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}+2\left(Ba\cos\left(fx+e\right)^{2}-(3A+4B)a\cos\left(fx+e\right)-(3A+5B)a-\left(Ba\cos\left(fx+e\right)+(3A+5B)a\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A +
B)*a*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos
(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2))/sqrt(c) + 2*(B*a*cos(f*x + e)^2 - (3*A + 4*B)*a*cos(f*x + e) - (
3*A + 5*B)*a - (B*a*cos(f*x + e) + (3*A + 5*B)*a)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt(
-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c
), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(111) = 222.
time = 0.47, size = 314, normalized size = 2.57

$$\frac{\frac{3\sqrt{2}\left(Aa\sqrt{c}+Ba\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\sqrt{2}\left(3Aa\sqrt{c}+5Ba\sqrt{c}-\frac{6Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{9Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/3*(3*sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) - 4*sqrt(2)*(3*A*a*sqrt(c) + 5*B*a*sqrt(c) - 6*A*a*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 6*
B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 3*A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2 + 9*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c*((cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\ (a+a\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2),
 x)
```

3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*(A+5*B)*arctanh(1/2*cos(f
*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)+2*a*B*cos(f
*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2936, 2830, 2728, 212}

$$-\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((a*(A + 5*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[2]*c^(3/2)*f)) + (a*(A + B)*Cos[e + f*x])/(f*(c - c*Sin[e + f
*x])^(3/2)) + (2*a*B*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{a\int \frac{-Ac-3Bc-2Bc\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(a(A+\ldots}{\ldots}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+\ldots}{\ldots}$$

$$= -\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+\ldots}{f(c-c\ldots}$$

Mathematica [A]
time = 1.13, size = 157, normalized size = 1.37

$$\frac{a\sec(e+fx)\left(\sqrt{2}(A+5B)\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{-c(1+\sin(e+fx))}+2\sqrt{c}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2(A+3B-2B\sin(e+fx))\right)}{2c^{3/2}f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sq
rt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin[e
 + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B - 2
*B*Sin[e + f*x])))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.
time = 6.36, size = 227, normalized size = 1.97

method	result
default	$\frac{a\left(A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+5B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/c^(5/2)*a*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/
2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)-5*B*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+2*(c*(1+sin(f*x+e)))^(
1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B)*(c*(1+sin(f*x+e)))^(1/
2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(108) = 216$.
time = 0.37, size = 342, normalized size = 2.97

$$\frac{\frac{\sqrt{2}\left((A+5B)ac\cos(fx+e)^2-(A+5B)ac\cos(fx+e)-2(A+5B)ac+((A+5B)ac\cos(fx+e)+2(A+5B)ac)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-4\left(2Ba\cos\left(fx+e\right)^2+(A+3B)a\cos\left(fx+e\right)+(A+B)a-(2Ba\cos\left(fx+e\right)-(A+B)a)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{4\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2c^2f+(c^2f\cos\left(fx+e\right)+2c^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) - 2
*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x + e
))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x
+ e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
- 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) + (A + B
)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x
+ e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e
+ f*x) + c)), x) + Integral(A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c
*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x)
+ Integral(B*sin(e + f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c
*sqrt(-c*sin(e + f*x) + c)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(108) = 216.
time = 0.51, size = 409, normalized size = 3.56

$$-\frac{\frac{2\sqrt{2}\left(Aa\sqrt{c}+5Ba\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\frac{Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(Aa\sqrt{c}+Ba\sqrt{c}-\frac{28Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{5Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -1/8*(2*sqrt(2)*(A*a*sqrt(c) + 5*B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) + sqrt(2)*(A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4*pi
```

```
+ 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) - 28*B*a*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2 - 5*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - (cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2),
 x)
```

3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}+\frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*(A+9*B)*cos(f*x+e)/c/
f/(c-c*sin(f*x+e))^(3/2)-1/16*a*(A-7*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1
/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2936, 2829, 2728, 212}

$$-\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -1/8*(a*(A - 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(Sqrt[2]*c^(5/2)*f) + (a*(A + B)*Cos[e + f*x])/(2*f*(c - c*Sin[e
 + f*x])^(5/2)) - (a*(A + 9*B)*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
```

```
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} + \frac{a\int \frac{-Ac-5Bc-4Bc\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} - $$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + $$

$$= -\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f} + \frac{a(A-}{2f(c-}$$

Mathematica [A]

time = 1.53, size = 199, normalized size = 1.58

$$-\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(\sqrt{2}\,(A-7B)\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\,\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin(e+fx))}+\frac{2\sqrt{c}\,\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(3A-5B+(A+9B)\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}\right)}{16c^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/16*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[S
qrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1 +
Sin[e + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A - 5
*B + (A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5))/(c^
(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(107) = 214$.
time = 8.11, size = 268, normalized size = 2.13

method	result
default	$\frac{a\left(-2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}(A-7B)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16*a*(-2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^2*(A-7*B)-2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2*(A-7*B)*cos(f*x+e)^2+2*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)
*2^(1/2)/c^(1/2))*c^2-4*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-2*A*(c+c*sin(f*x+e
))^(3/2)*c^(1/2)-14*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^2+28*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-18*B*(c+c*sin(f*x+e))^(3/2)*
c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(9/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*si
n(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(113) = 226$.
time = 0.38, size = 422, normalized size = 3.35

$$-\frac{\sqrt{2}\left((A-7B)a\cos\left(fx+e\right)^{3}+3(A-7B)a\cos\left(fx+e\right)^{2}-2(A-7B)a\cos\left(fx+e\right)-4(A-7B)a-\left((A-7B)a\cos\left(fx+e\right)^{2}-2(A-7B)a\cos\left(fx+e\right)-4(A-7B)a\right)\sin\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)-2c\right)\sin\left(fx+e\right)+2c}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)-4\left((A+9B)a\cos\left(fx+e\right)^{2}-(3A-5B)a\cos\left(fx+e\right)-4(A+B)a-\left((A+9B)a\cos\left(fx+e\right)+4(A+B)a\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{32\left(c^{3}f\cos\left(fx+e\right)^{3}+3c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)-4c^{3}f-\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)-4c^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) - 4*(A +
B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f
)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(113) = 226$.
time = 0.57, size = 440, normalized size = 3.49

$$-\frac{\frac{4\sqrt{2}\left(Aa-7Ba\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa\sqrt{c}+Ba\sqrt{c}+\frac{16Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}+\frac{42Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}{c^{3}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\frac{16\sqrt{2}Bac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{\sqrt{2}Aac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}+\frac{\sqrt{2}Bac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}}{c^{6}}}{128f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/128*(4*sqrt(2)*(A*a - 7*B*a)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) + sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) + 16*B*a*sqrt(c)*(cos(-1/4*pi
```

```
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 6*A*a*sqrt(c
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^2 + 42*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (1
6*sqrt(2)*B*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + sqrt(2)*A*a*c^(7
/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + sqrt(2)*B*a*c^(7/2)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2)/c^6)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2),
 x)
```

3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}+\frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\ 3$$

```
[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*(A+13*B)*cos(f*x+e)/
c/f/(c-c*sin(f*x+e))^(5/2)-1/32*a*(A-3*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))
^(3/2)-1/64*a*(A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e
))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3046, 2936, 2829, 2729, 2728, 212}

$$-\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}-\frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] -1/32*(a*(A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(Sqrt[2]*c^(7/2)*f) + (a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[
e + f*x])^(7/2)) - (a*(A + 13*B)*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A - 3*B)*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} + \frac{a\int \frac{-Ac-7Bc-6Bc\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c^2} \\
&= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{(}{} \\
&= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{}{3} \\
&= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{}{3} \\
&= -\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} + \frac{a(A}{3f(c}
\end{aligned}
$$

Mathematica [A]
time = 2.25, size = 217, normalized size = 1.33

$$
-\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(3\sqrt{2}(A-3B)\tan^{-1}\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin(e+fx))}+\frac{\sqrt{c}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(47A-13B+3(A-3B)\cos(2(e+fx))+4(5A+17B)\sin(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}\right)}{192c^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -1/192*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*Sqrt[2]*(A - 3*B)*ArcTa
n[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1
 + Sin[e + f*x]))] + (Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*A -
 13*B + 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7))/(c^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(140) = 280.
time = 10.47, size = 352, normalized size = 2.16

method	result

default	$-\frac{a\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{4}(A-3B)\sin(fx+e)\left(\cos^{2}(fx+e)\right)-12\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+\right.}}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/192/c^(15/2)*a*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^4*(A-3*B)*sin(f*x+e)*cos(f*x+e)^2-12*arctanh(1/2*(c+c*sin(f*x+e))^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*sin(f*x+e)-9*arctanh(1/2*(c+c*sin
(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*cos(f*x+e)^2+6*A*(c+c*s
in(f*x+e))^(5/2)*c^(3/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-24*A*(c+c*sin(
f*x+e))^(1/2)*c^(7/2)-18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-32*B*(c+c*sin(f*x
+e))^(3/2)*c^(5/2)+72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+12*A*2^(1/2)*arctanh
(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-36*B*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f
*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(148) = 296$.
time = 0.37, size = 524, normalized size = 3.21

$$-\frac{3\sqrt{2}\left((A-3B)a\cos\left(fx+e\right)^{4}-3(A-3B)a\cos\left(fx+e\right)^{3}-8(A-3B)a\cos\left(fx+e\right)^{2}+4(A-3B)a\cos\left(fx+e\right)+8(A-3B)a+\left((A-3B)a\cos\left(fx+e\right)^{3}+4(A-3B)a\cos\left(fx+e\right)^{2}-4(A-3B)a\cos\left(fx+e\right)-8(A-3B)a\right)\sin\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(3(A-3B)a\cos\left(fx+e\right)^{3}-(7A+43B)a\cos\left(fx+e\right)^{2}+2(11A-B)a\cos\left(fx+e\right)+32(A+B)a+\left(3(A-3B)a\cos\left(fx+e\right)^{2}+2(5A+17B)a\cos\left(fx+e\right)+32(A+B)a\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{384\left(c^{4}f\cos\left(fx+e\right)^{4}-3c^{4}f\cos\left(fx+e\right)^{3}-8c^{4}f\cos\left(fx+e\right)^{2}+4c^{4}f\cos\left(fx+e\right)+8c^{4}f+\left(c^{4}f\cos\left(fx+e\right)^{3}+4c^{4}f\cos\left(fx+e\right)^{2}-4c^{4}f\cos\left(fx+e\right)-8c^{4}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((A - 3*B)*a*cos(f*x + e)^4 - 3*(A - 3*B)*a*cos(f*x + e)^
3 - 8*(A - 3*B)*a*cos(f*x + e)^2 + 4*(A - 3*B)*a*cos(f*x + e) + 8*(A - 3*B)
*a + ((A - 3*B)*a*cos(f*x + e)^3 + 4*(A - 3*B)*a*cos(f*x + e)^2 - 4*(A - 3*
```

```
B)*a*cos(f*x + e) - 8*(A - 3*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x +
e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x
+ e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
3*(A - 3*B)*a*cos(f*x + e)^3 - (7*A + 43*B)*a*cos(f*x + e)^2 + 2*(11*A - B)
*a*cos(f*x + e) + 32*(A + B)*a + (3*(A - 3*B)*a*cos(f*x + e)^2 + 2*(5*A + 1
7*B)*a*cos(f*x + e) + 32*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 +
 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x +
 e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(148) = 296.
time = 0.61, size = 659, normalized size = 4.04

$$-\frac{\frac{12\sqrt{2}\left(Aa\sqrt{c}-3Ba\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(Aa\sqrt{c}+Ba\sqrt{c}-\frac{3Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{3Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{22Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{66Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}{c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\frac{3Aac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3Bac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3Aac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{9Bac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{Aac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{Bac^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{c^{12}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{1536f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] -1/1536*(12*sqrt(2)*(A*a*sqrt(c) - 3*B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) - 3*A*a*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) +
 9*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 3*A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 22*A*a*sqrt(c)*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 6
6*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3/(c^4*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3
*A*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 3*B*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
```

```
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 9*B*a*c^(17/2)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - A*a*
c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)^3 - B*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/
f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
 x)
```

3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=210

$$\frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(1}{}$$

```
[Out] 256/15015*a^2*(13*A-3*B)*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/3003*
a^2*(13*A-3*B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^2*B*c^2*cos
(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/f+8/429*a^2*(13*A-3*B)*c^4*cos(f*x+e)^5/f/
(c-c*sin(f*x+e))^(1/2)+2/143*a^2*(13*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e
))^(1/2)/f
```

Rubi [A]
time = 0.37, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{256a^2c^6(13A-3B)\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5(13A-3B)\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(13A-3B)\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3(13A-3B)\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^2*(13*A - 3*B)*c^6*Cos[e + f*x]^5)/(15015*f*(c - c*Sin[e + f*x])^(5/
2)) + (64*a^2*(13*A - 3*B)*c^5*Cos[e + f*x]^5)/(3003*f*(c - c*Sin[e + f*x])
^(3/2)) + (8*a^2*(13*A - 3*B)*c^4*Cos[e + f*x]^5)/(429*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*a^2*(13*A - 3*B)*c^3*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/
(143*f) - (2*a^2*B*c^2*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
```

```
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+\ldots \\
&= -\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+f\ldots}{13f} \\
&= \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\ldots}}{143f} \\
&= \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(\ldots}{\ldots} \\
&= \frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2\ldots}{4\ldots} \\
&= \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64\ldots}{\ldots}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1355 vs. $2(210) = 420$.
time = 6.45, size = 1355, normalized size = 6.45

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(
7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x))/2]
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((A - 4*
B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/
(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e
+ f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[
e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos[(13*(e + f*x)
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((7*
A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
)/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2)*Sin[(5*(e + f*x))/2])/(160*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((A - 4*B)*(
a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(112
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4) + (A*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e +
 f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4) - ((2*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*(a + a*Sin[e + f*x])^
2*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A]
time = 6.19, size = 121, normalized size = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^3a^2\left((-1365A+4935B)\sin(fx+e)\left(\cos^2(fx+e)\right)+(11180A-11820B)\sin(fx+e)+1155B\left(\cos^4(fx+\right.\right.}{15015\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15015*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*((-1365*A+4935*B)*sin(f*x+e
)*cos(f*x+e)^2+(11180*A-11820*B)*sin(f*x+e)+1155*B*cos(f*x+e)^4+(5915*A-106
05*B)*cos(f*x+e)^2-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [A]
time = 0.39, size = 375, normalized size = 1.79

$$\frac{2\left(1155\,Ba^{2}c^{3}\cos\left(fx+e\right)^{7}+105\,(13\,A-14\,B)a^{2}c^{3}\cos\left(fx+e\right)^{6}+35\,(91\,A-87\,B)a^{2}c^{3}\cos\left(fx+e\right)^{5}-20\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)^{4}+32\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)^{3}-64\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)^{2}+256\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)+512\,(13\,A-3\,B)a^{2}c^{3}+\left(1155\,Ba^{2}c^{3}\cos\left(fx+e\right)^{6}-105\,(13\,A-25\,B)a^{2}c^{3}\cos\left(fx+e\right)^{5}+140\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)^{4}+160\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)^{3}+192\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)^{2}+256\,(13\,A-3\,B)a^{2}c^{3}\cos\left(fx+e\right)+512\,(13\,A-3\,B)a^{2}c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15015\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/15015*(1155*B*a^2*c^3*cos(f*x + e)^7 + 105*(13*A - 14*B)*a^2*c^3*cos(f*x
+ e)^6 + 35*(91*A - 87*B)*a^2*c^3*cos(f*x + e)^5 - 20*(13*A - 3*B)*a^2*c^3*
cos(f*x + e)^4 + 32*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^3 - 64*(13*A - 3*B)*a
^2*c^3*cos(f*x + e)^2 + 256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A -
 3*B)*a^2*c^3 + (1155*B*a^2*c^3*cos(f*x + e)^6 - 105*(13*A - 25*B)*a^2*c^3*
cos(f*x + e)^5 + 140*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^4 + 160*(13*A - 3*B)
*a^2*c^3*cos(f*x + e)^3 + 192*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^2 + 256*(13
*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A - 3*B)*a^2*c^3)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [A]
time = 0.78, size = 392, normalized size = 1.87

$$-\frac{\sqrt{2}\left(10010\,Aa^2c^3\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-1155\,Ba^2c^3\cos\left(-\frac{13}{4}\pi+\frac{13}{2}fx+\frac{13}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+60060\left(7\,Aa^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,Ba^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+15015\left(4\,Aa^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)-3003\left(22\,Aa^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\,Ba^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+4290\left(Aa^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\,Ba^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)-1365\left(2\,Aa^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,Ba^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\right)\sqrt{c}}{480480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] -1/480480*sqrt(2)*(10010*A*a^2*c^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 1155*B*a^2*c^3*cos(-13/4*pi + 13/2*f*x + 13/2*
e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 60060*(7*A*a^2*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(4*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) + B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/
2*f*x + 3/2*e) - 3003*(22*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7
*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2
*e) + 4290*(A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^2*c^3*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 1365*(2*
A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
 x)
```

3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=167

$$\frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5}{}$$

```
[Out] 64/3465*a^2*(11*A-B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/693*a^2*(
11*A-B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/99*a^2*(11*A-B)*c^3*cos
(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)-2/11*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x
+e))^(1/2)/f
```

Rubi [A]
time = 0.29, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{64a^2c^5(11A-B)\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2c^4(11A-B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3(11A-B)\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^2*(11*A - B)*c^5*Cos[e + f*x]^5)/(3465*f*(c - c*Sin[e + f*x])^(5/2))
+ (16*a^2*(11*A - B)*c^4*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^(3/2))
 + (2*a^2*(11*A - B)*c^3*Cos[e + f*x]^5)/(99*f*Sqrt[c - c*Sin[e + f*x]]) -
(2*a^2*B*c^2*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+$$
$$= -\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+}}{11f}$$
$$= \frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}} - \frac{2a^2}{}$$
$$= \frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2}{9}$$
$$= \frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}} + \frac{16a}{6}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1173 vs. $2(167) = 334$.
time = 6.36, size = 1173, normalized size = 7.02

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((6*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c
- c*Sin[e + f*x])^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*Cos[(5*(e + f*x))/2]*(a
 + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(80*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((2*A + 3*B)
*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(1
12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + ((2*A - B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^4) - (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e +
 f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((6*A - B)*Sin[(e + f*x
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A
+ B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e + f*x))/2]
)/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4) + ((8*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5
/2)*Sin[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A + 3*B)*(a + a*Sin[e + f*x])^2
*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - B)
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(1
44*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + (B*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(
e + f*x))/2])/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)
```

Maple [A]
time = 5.74, size = 105, normalized size = 0.63

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^3a^2(-315B(\cos^2(fx+e))\sin(fx+e)+(-1210A+1370B)\sin(fx+e)+(-385A+980B)(\cos^2(fx+e}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/3465*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(-315*B*cos(f*x+e)^2*sin(f*
x+e)+(-1210*A+1370*B)*sin(f*x+e)+(-385*A+980*B)*cos(f*x+e)^2+1562*A-1402*B)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(159) = 318$.
time = 0.36, size = 328, normalized size = 1.96

$$-\frac{2\left(315\,Ba^2c^2\cos\left(fx+e\right)^6-35\,(11\,A-10\,B)a^2c^2\cos\left(fx+e\right)^5+5\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^4-8\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^3+16\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^2-64\,(11\,A-B)a^2c^2\cos\left(fx+e\right)-128\,(11\,A-B)a^2c^2-\left(315\,Ba^2c^2\cos\left(fx+e\right)^5+35\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^4+40\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^3+48\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^2+64\,(11\,A-B)a^2c^2\cos\left(fx+e\right)+128\,(11\,A-B)a^2c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3465\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*c^2*cos(f*x + e)^6 - 35*(11*A - 10*B)*a^2*c^2*cos(f*x +
e)^5 + 5*(11*A - B)*a^2*c^2*cos(f*x + e)^4 - 8*(11*A - B)*a^2*c^2*cos(f*x +
 e)^3 + 16*(11*A - B)*a^2*c^2*cos(f*x + e)^2 - 64*(11*A - B)*a^2*c^2*cos(f*
x + e) - 128*(11*A - B)*a^2*c^2 - (315*B*a^2*c^2*cos(f*x + e)^5 + 35*(11*A
- B)*a^2*c^2*cos(f*x + e)^4 + 40*(11*A - B)*a^2*c^2*cos(f*x + e)^3 + 48*(11
*A - B)*a^2*c^2*cos(f*x + e)^2 + 64*(11*A - B)*a^2*c^2*cos(f*x + e) + 128*(
11*A - B)*a^2*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
- f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int Ac^2\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-2Ac^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)\right)dx+\int Ac^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^4\left(e+fx\right)dx+\int Bc^2\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int\left(-2Bc^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)dx+\int Bc^2\sqrt{-c\sin\left(e+fx\right)+c}\sin^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-2*A*c**2*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x)**4, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c
)*sin(e + f*x), x) + Integral(-2*B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f
*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**5, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(159) = 318$.
time = 0.66, size = 357, normalized size = 2.14

$$-\frac{\sqrt{2}\left(315\,Ba^2c^2\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6930\left(6\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Ba^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+2310\left(4\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)-693\left(8\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,Ba^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)-495\left(2\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+385\left(2\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Ba^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\right)\sqrt{c}}{55440\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -1/55440*sqrt(2)*(315*B*a^2*c^2*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 6930*(6*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 2310*(4*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^
2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) -
 693*(8*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 495*(2*A*a^2
*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) + 385*(2*A*a^2*c^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
)*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\sin(e + f x))\,(a + a\sin(e + f x))^2\,(c - c\sin(e + f x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),
 x)
```

3.91 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 8/315*a^2*(9*A+B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/63*a^2*(9*A+B
)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/9*a^2*B*c^2*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^2*(9*A + B)*c^4*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (
2*a^2*(9*A + B)*c^3*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) - (2*
a^2*B*c^2*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+}{\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{9}\left(a^2(9A+\right.$$

$$= \frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} - \frac{2a^2B}{9f\sqrt{c}}$$

$$= \frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{2a^2(9A}{63f(c}$$

Mathematica [A]
time = 2.83, size = 106, normalized size = 0.88

$$\frac{a^2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(162A-87B+35B\cos(2(e+fx))+(-90A+130B)\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*(
e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A]
time = 6.94, size = 83, normalized size = 0.69

method	result	size
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^3a^2(\sin(fx+e)(45A-65B)-35B(\cos^2(fx+e))-81A+61B)}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/315*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(sin(f*x+e)*(45*A-65*B)-35*B*
cos(f*x+e)^2-81*A+61*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(114) = 228$.
time = 0.37, size = 241, normalized size = 2.01

$$-\frac{2\left(35\,Ba^2c\cos\left(fx+e\right)^5+5\left(9\,A+8\,B\right)a^2c\cos\left(fx+e\right)^4-\left(9\,A+B\right)a^2c\cos\left(fx+e\right)^3+2\left(9\,A+B\right)a^2c\cos\left(fx+e\right)^2-8\left(9\,A+B\right)a^2c\cos\left(fx+e\right)-16\left(9\,A+B\right)a^2c+\left(35\,Ba^2c\cos\left(fx+e\right)^4-5\left(9\,A+B\right)a^2c\cos\left(fx+e\right)^3-6\left(9\,A+B\right)a^2c\cos\left(fx+e\right)^2-8\left(9\,A+B\right)a^2c\cos\left(fx+e\right)-16\left(9\,A+B\right)a^2c\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{315\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (9
*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A +
B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4 - 5
*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A
 + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)\right)dx+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)dx+\int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)dx+\int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^3\left(e+fx\right)\right)dx+\int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^4\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*c*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**
3, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(
B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-B*c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(-B*c*sqrt(-c*sin(e + f*x) +
 c)*sin(e + f*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(114) = 228.
time = 0.66, size = 251, normalized size = 2.09

$$-\frac{\sqrt{2}\left(1890\,Aa^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-35\,Ba^{2}c\cos\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+210\left(3\,Aa^{2}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Ba^{2}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\cos\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)-126\left(Aa^{2}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Ba^{2}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\cos\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)-45\left(2\,Aa^{2}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Ba^{2}c\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\cos\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)\right)\sqrt{c}}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/2520*sqrt(2)*(1890*A*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 35*B*a^2*c*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 210*(3*A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x +
 3/2*e) - 126*(A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 45*(2*A*a^2
*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),
 x)
```

3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+ ...$

Optimal. Leaf size=81

$$\frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 2/35*a^2*(7*A+3*B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/7*a^2*B*c^2*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.21, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3046, 2935, 2752}

$$\frac{2a^2c^3(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(7*A + 3*B)*c^3*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) -
(2*a^2*B*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+f}{(c-c\sin(e+fx))^{3/2}} \\ &= -\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{1}{7}\left(a^2(7A+\right. \\ &= \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2B}{7f(c-}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 89, normalized size = 1.10

$$\frac{2a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(7A-2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(7*A - 2*B + 5*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A]
time = 4.94, size = 65, normalized size = 0.80

method	result	size
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^3a^2(5B\sin(fx+e)+7A-2B)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/35*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(77) = 154$.
time = 0.36, size = 204, normalized size = 2.52

$$\frac{2\left(5\,Ba^{2}\cos\left(fx+e\right)^{4}-\left(7\,A+8\,B\right)a^{2}\cos\left(fx+e\right)^{3}-\left(21\,A+19\,B\right)a^{2}\cos\left(fx+e\right)^{2}+2\left(7\,A+3\,B\right)a^{2}\cos\left(fx+e\right)+4\left(7\,A+3\,B\right)a^{2}-\left(5\,Ba^{2}\cos\left(fx+e\right)^{3}+\left(7\,A+13\,B\right)a^{2}\cos\left(fx+e\right)^{2}-2\left(7\,A+3\,B\right)a^{2}\cos\left(fx+e\right)-4\left(7\,A+3\,B\right)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{35\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/35*(5*B*a^2*cos(f*x + e)^4 - (7*A + 8*B)*a^2*cos(f*x + e)^3 - (21*A + 19*
B)*a^2*cos(f*x + e)^2 + 2*(7*A + 3*B)*a^2*cos(f*x + e) + 4*(7*A + 3*B)*a^2
- (5*B*a^2*cos(f*x + e)^3 + (7*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(7*A + 3*B)
*a^2*cos(f*x + e) - 4*(7*A + 3*B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 2A\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int 2B\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Int
egral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(77) = 154$.
time = 0.58, size = 211, normalized size = 2.60

$$-\frac{\sqrt{2}\left(5\,Ba^{2}\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+35\left(4\,Aa^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+35\left(2\,Aa^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+7\left(2\,Aa^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\right)\sqrt{c}}{140\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/140*sqrt(2)*(5*B*a^2*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 35*(4*A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(2*
A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 7*(2*A*a^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 3*B*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi
 + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),
 x)
```

3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=161

$$\frac{4\sqrt{2}\,a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] -2/5*a^2*B*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/3*a^2*(A+B)*c*cos(f*
x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*
2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-4*a^2*(A+B)*cos(f*x+e)/f/
(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2939, 2758, 2728, 212}

$$-\frac{2a^2c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}\,a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (4*Sqrt[2]*a^2*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^2*B*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[
e + f*x])^(5/2)) - (2*a^2*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x
])^(3/2)) - (4*a^2*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(a^2(A+B)c^2\right)\int \frac{\cos^4(e}{(c-c\sin(e}$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + ($$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} -$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{4\sqrt{2}\,a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a}{5f(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.80, size = 175, normalized size = 1.09

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left((120+120i)\sqrt[4]{-1}\,(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(70A+79B-3B\cos(2(e+fx))+2(5A+11B)\sin(e+fx))\right)}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] -1/15*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*((120
 + 120*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*
x)/4])] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(70*A + 79*B - 3*B*Cos[2*(e
 + f*x)] + 2*(5*A + 11*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 6.42, size = 197, normalized size = 1.22

method	result
default	$\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^2\left(-30c^{\frac{5}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A-30c^{\frac{5}{2}}\sqrt{2}\,\mathrm{a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(-30*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-30*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+3*B*(c*(1+sin(f*x+e)))^(
5/2)+5*A*(c*(1+sin(f*x+e)))^(3/2)*c+5*B*(c*(1+sin(f*x+e)))^(3/2)*c+30*A*c^2
*(c*(1+sin(f*x+e)))^(1/2)+30*B*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(150) = 300$.

time = 0.37, size = 332, normalized size = 2.06

$$\frac{2\left(\frac{15\sqrt{2}\left((A+B)a^2c\cos(fx+e)-(A+B)a^2c\sin(fx+e)+(A+B)a^2c\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}+\left(3Ba^2\cos(fx+e)^3+(5A+14B)a^2\cos(fx+e)^2-(35A+41B)a^2\cos(fx+e)-4(10A+13B)a^2+\left(3Ba^2\cos(fx+e)^2-(5A+11B)a^2\cos(fx+e)-4(10A+13B)a^2\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}\right)}{15\left(cf\cos(fx+e)-cf\sin(fx+e)+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*sqrt(2)*((A + B)*a^2*c*cos(f*x + e) - (A + B)*a^2*c*sin(f*x + e) +
 (A + B)*a^2*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*
sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c)
 + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2))/sqrt(c) + (3*B*a^2*cos(f*x + e)^3 + (5*A + 14*B)*a^2*cos
(f*x + e)^2 - (35*A + 41*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2 + (3*B*a
^2*cos(f*x + e)^2 - (5*A + 11*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2)*si
n(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e)
 + c*f)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{2A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{2B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sin(e + f*x)/
sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)**2/sqrt(-c*sin(e +
f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Int
egral(2*B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e
+ f*x)**3/sqrt(-c*sin(e + f*x) + c), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(150) = 300$.

time = 0.51, size = 502, normalized size = 3.12

$$\frac{2\left(\frac{15\sqrt{2}\left(Aa^2\sqrt{c}+Ba^2\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{\mathrm{csgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{8\sqrt{2}\left(10Aa^2\sqrt{c}+13Ba^2\sqrt{c}-\frac{35Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{35Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{55Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{85Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{45Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{45Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{15Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}+\frac{30Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 2/15*(15*sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) - 8*sqrt(2)*(10*A*a^2*sqrt(c) + 13*B*a^2*sqrt(c) - 35*A*
a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) - 35*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) + 55*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 85*B*a^2*sqrt(c)*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
45*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^3 - 45*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*a^2*sqrt(c)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 30*B*a^2*
sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4)/(c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^2}{\sqrt{c - c \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{2}\,a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+ \ldots}{6f(c-c\sin(e+fx))^3 \ldots}$$

```
[Out] 1/2*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)+1/6*a^2*(3*A+7*B)*c
os(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-a^2*(3*A+7*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+a^2*(3*A+7*B)*cos(
f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2938, 2758, 2728, 212}

$$-\frac{\sqrt{2}\,a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((Sqrt[2]*a^2*(3*A + 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
 c*Sin[e + f*x]])])/(c^(3/2)*f)) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(2*f*(c
 - c*Sin[e + f*x])^(7/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x]^3)/(6*f*(c - c*Si
n[e + f*x])^(3/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e +
 f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{1}{4}(a^2(3A+7B)c)\int \frac{\cos^4}{(c-c\sin}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} +$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} +$$

$$= -\frac{\sqrt{2}\,a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 355, normalized size = 2.02

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(6(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+(6+6i)\sqrt[4]{-1}(3A+7B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3(2A+7B)\cos\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-B\cos\left(\frac{3}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+12(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(2A+7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{3}{2}(e+fx)\right)\right)}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*Ar
cTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c -
c*Sin[e + f*x])^(3/2))
```

Maple [A]
time = 6.72, size = 282, normalized size = 1.60

method	result

default	$-\frac{a^{2}\left(\sin(fx+e)\left(-9A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}+6A\sqrt{c+c\sin\left(fx+e\right)}\,c^{\frac{3}{2}}-21B\sqrt{2}\,\operatorname{arc}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3/c^(7/2)*a^2*(sin(f*x+e)*(-9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/
2)*2^(1/2)/c^(1/2))*c^2+6*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-21*B*2^(1/2)*arc
tanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+2*B*(c+c*sin(f*x+e))^(
3/2)*c^(1/2)+18*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2))+9*A*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*A*(c+c*sin(f*x+e))^(1/2)*c^(3
/2)+21*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-2*
B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-24*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2))*(c*(1
+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(165) = 330.
time = 0.38, size = 411, normalized size = 2.34

$$\frac{\frac{3\sqrt{2}\left((3A+7B)a^{2}c\cos(fx+e)^{2}-(3A+7B)a^{2}c\cos(fx+e)-2(3A+7B)a^{2}c+\left((3A+7B)a^{2}c\cos(fx+e)+2(3A+7B)a^{2}c\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^{2}+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-4\left(Ba^{2}\cos\left(fx+e\right)^{3}+(3A+10B)a^{2}\cos\left(fx+e\right)^{2}+6(A+2B)a^{2}\cos\left(fx+e\right)+3(A+B)a^{2}+\left(Ba^{2}\cos\left(fx+e\right)^{2}-3(A+3B)a^{2}\cos\left(fx+e\right)+3(A+B)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)-2c^{2}f+\left(c^{2}f\cos\left(fx+e\right)+2c^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((3*A + 7*B)*a^2*c*cos(f*x + e)^2 - (3*A + 7*B)*a^2*c*cos(f*
x + e) - 2*(3*A + 7*B)*a^2*c + ((3*A + 7*B)*a^2*c*cos(f*x + e) + 2*(3*A + 7
*B)*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x
+ e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1
```

```
)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(B*a^2*cos(f*x + e)^3 + (3*A + 10*B
)*a^2*cos(f*x + e)^2 + 6*(A + 2*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2 + (B*a^
2*cos(f*x + e)^2 - 3*(A + 3*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2)*sin(f*x +
e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) -
 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{2A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{A\sin^{2}(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{2B\sin^{2}(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin^{3}(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin
(e + f*x) + c)), x) + Integral(2*A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) +
c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(A*sin(e + f*x
)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) +
c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(2*B*sin(e + f*x)**2/(-c*sqr
t(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + In
tegral(B*sin(e + f*x)**3/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqr
t(-c*sin(e + f*x) + c)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(165) =$ 330.
time = 0.59, size = 592, normalized size = 3.36

$$-\frac{\frac{6\sqrt{2}\left(3Aa^{2}\sqrt{c}+7Ba^{2}\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(\frac{Aa^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{Ba^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(Aa^{2}\sqrt{c}+Ba^{2}\sqrt{c}+\frac{6Aa^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{14Ba^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{c^{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{16\sqrt{2}\left(3Aa^{2}\sqrt{c}+11Ba^{2}\sqrt{c}-\frac{6Aa^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{18Ba^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3Aa^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}+\frac{15Ba^{2}\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{2}}\right)}{c^{2}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(3*A*a^2*sqrt(c) + 7*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a^2*sqrt(c)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c
) + 6*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) + 14*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/
(c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
```

```
))) - 16*sqrt(2)*(3*A*a^2*sqrt(c) + 11*B*a^2*sqrt(c) - 6*A*a^2*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 18*
B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 3*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^2}{\left(c-c\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2
), x)
```

3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=175

$$\frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}-\frac{3a}{8c}$$

```
[Out] 1/4*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)-1/8*a^2*(A+9*B)*cos
(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)+3/8*a^2*(A+9*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-3/8*a^2*(A+9*B)*co
s(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2938, 2759, 2758, 2728, 212}

$$\frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (3*a^2*(A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(4*Sqrt[2]*c^(5/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(4*f*(c
- c*Sin[e + f*x])^(9/2)) - (a^2*(A + 9*B)*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e
 + f*x])^(5/2)) - (3*a^2*(A + 9*B)*Cos[e + f*x])/(8*c^2*f*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (a^2c^2)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{1}{8}(a^2(A+9B)c)\int \frac{\cos^4(}{(c-c\sin(}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} + \frac{(3}{}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3}{8}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3}{8}$$
$$= \frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} + \frac{a^2(A+}{4f(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.80, size = 344, normalized size = 1.97

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-(5A+13B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-(3+3i)\sqrt[4]{-1}(A+9B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-8B\cos\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-2(5A+13B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)-8B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (
3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*(5*A
+ 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 8*B*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(152) = 304$.
time = 10.19, size = 386, normalized size = 2.21

method	result

default	$-\frac{a^{2}\left(3A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{2}\left(fx+e\right)\right)c^{2}+27B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right.\right.}}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^2*c^2+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)
*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-6*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2-16*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3
/2)*sin(f*x+e)^2-54*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)*c^2+10*A*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*A*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+26*B*(c*(1+sin(f*x
+e)))^(3/2)*c^(1/2)+32*B*c^(3/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)+27*B*2
^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*A*(c*(1
+sin(f*x+e)))^(1/2)*c^(3/2)-60*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+si
n(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(160) = 320$.
time = 0.49, size = 479, normalized size = 2.74

$$\frac{3\sqrt{2}\left(\left(A+9B\right)a^{2}\cos\left(fx+e\right)^{3}+3\left(A+9B\right)a^{2}\cos\left(fx+e\right)^{2}-2\left(A+9B\right)a^{2}\cos\left(fx+e\right)-4\left(A+9B\right)a^{2}-\left(\left(A+9B\right)a^{2}\cos\left(fx+e\right)^{2}-2\left(A+9B\right)a^{2}\cos\left(fx+e\right)-4\left(A+9B\right)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(8Ba^{2}\cos\left(fx+e\right)^{3}-\left(5A+21B\right)a^{2}\cos\left(fx+e\right)^{2}-\left(A+25B\right)a^{2}\cos\left(fx+e\right)+4\left(A+B\right)a^{2}+\left(8Ba^{2}\cos\left(fx+e\right)^{2}+\left(5A+29B\right)a^{2}\cos\left(fx+e\right)+4\left(A+B\right)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{16\left(c^{3}f\cos\left(fx+e\right)^{3}+3c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)-4c^{3}f-\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)-4c^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x + e
)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos(f
```

```
*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e))*s
qrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)
*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2
*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*cos(f*x
+ e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos(f*x + e
)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*
sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*
c^3*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(160) = 320.
time = 0.59, size = 630, normalized size = 3.60

$$-\frac{\frac{256 \sqrt{2} B a^{2}}{c^{\frac{5}{2}} \left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1}-1\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}-\frac{12 \sqrt{2}\left(A a^{2} \sqrt{c}+9 B a^{2} \sqrt{c}\right) \log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1}\right)}{c^{3} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}+\frac{\sqrt{2}\left(A a^{2} \sqrt{c}+B a^{2} \sqrt{c}+\frac{8 A a^{2} \sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1}+\frac{24 B a^{2} \sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1}+\frac{18 A a^{2} \sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)^{2}}+\frac{162 B a^{2} \sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)^{2}}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)^{2}}{c^{3}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)^{2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}-\frac{\frac{8 \sqrt{2} A a^{2} c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1}+\frac{24 \sqrt{2} B a^{2} c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1}+\frac{\sqrt{2} A a^{2} c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)^{2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)^{2}}+\frac{\sqrt{2} B a^{2} c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)-1\right)^{2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} f x+\frac{1}{2} e\right)+1\right)^{2}}}{c^{6}}}{64 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -1/64*(256*sqrt(2)*B*a^2/(c^(5/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 - 12*sqrt(2)*(A*a^2*sqrt(c) + 9*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c) + 8*A*a^2*sqrt
(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1) + 24*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 18*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 162*B*a^2*sqrt(c)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (8*sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) + 24*sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
```

```
/2*e) + 1) + sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 +
 sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/c^6)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\sin\left(e + f\,x\right))\,\left(a + a\,\sin\left(e + f\,x\right)\right)^2}{\left(c - c\,\sin\left(e + f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2
), x)
```

3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=175

$$\frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}+\frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}}-1$$

```
[Out] 1/6*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)+1/24*a^2*(A-11*B)*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-1/16*a^2*(A-11*B)*cos(f*x+e)/c^2/f/(c
-c*sin(f*x+e))^(3/2)+1/32*a^2*(A-11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/
2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2938, 2759, 2728, 212}

$$\frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}-\frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*(A - 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(16*Sqrt[2]*c^(7/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(6*f*(c
- c*Sin[e + f*x])^(11/2)) + (a^2*(A - 11*B)*Cos[e + f*x]^3)/(24*f*(c - c*Si
n[e + f*x])^(7/2)) - (a^2*(A - 11*B)*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e +
 f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{1}{12}\left(a^2(A-11B)c\right)\int \frac{\cos}{(c-c\sin}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} -$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} -$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} -$$

$$= \frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A-}{6f(c-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.22, size = 342, normalized size = 1.95

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(7A+19B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+3(A+21B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-(3+3i)\sqrt[4]{-1}(A-11B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6+64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-8(7A+19B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)+6(A+21B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{48f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(7*A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
+ 3*(A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/
4)*(A - 11*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(7*A + 1
9*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(A + 21*B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*
x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(
7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(152) = 304$.
time = 9.52, size = 354, normalized size = 2.02

method	result

default	$\frac{a^2\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^3(A-11B)\sin(fx+e)\left(\cos^2(fx+e)\right)-12\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(fx\cdots}}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/96/c^(13/2)*a^2*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^3*(A-11*B)*sin(f*x+e)*cos(f*x+e)^2-12*arctanh(1/2*(c+c*sin(f*x+e))^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3*(A-11*B)*sin(f*x+e)-9*arctanh(1/2*(c+c*s
in(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3*(A-11*B)*cos(f*x+e)^2+6*A*(c+
c*sin(f*x+e))^(5/2)*c^(1/2)+32*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-24*A*(c+c*s
in(f*x+e))^(1/2)*c^(5/2)+126*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-352*B*(c+c*si
n(f*x+e))^(3/2)*c^(3/2)+264*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+12*A*2^(1/2)*a
rctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-132*B*2^(1/2)*arctan
h(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)
/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(160) = 320$.
time = 0.59, size = 555, normalized size = 3.17

$$-\frac{3\sqrt{2}\left((A-11B)a^2\cos(fx+e)^4-3(A-11B)a^2\cos(fx+e)^3-8(A-11B)a^2\cos(fx+e)^2+4(A-11B)a^2\cos(fx+e)+8(A-11B)a^2+\left((A-11B)a^2\cos(fx+e)^3+4(A-11B)a^2\cos(fx+e)^2-4(A-11B)a^2\cos(fx+e)-8(A-11B)a^2\right)\sin(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(3(A+21B)a^2\cos(fx+e)^3+(25A+13B)a^2\cos(fx+e)^2-2(5A+41B)a^2\cos(fx+e)-32(A+B)a^2+\left(3(A+21B)a^2\cos(fx+e)^2-2(11A-25B)a^2\cos(fx+e)-32(A+B)a^2\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{192\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+4c^4f\cos(fx+e)+8c^4f+\left(c^4f\cos(fx+e)^3+4c^4f\cos(fx+e)^2-4c^4f\cos(fx+e)-8c^4f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*x
 + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e) +
 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*cos(f
```

```
*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x + e))
*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(
c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
 - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A + 13*B)*
a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*a^2 + (3*
(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e) - 32*(A +
B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*
c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^
4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e)
 - 8*c^4*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(160) = 320.
time = 0.63, size = 689, normalized size = 3.94

$$\frac{\frac{12\sqrt{2}\left(Aa^2\sqrt{c}-11\,Ba^2\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa^2\sqrt{c}+Ba^2\sqrt{c}+\frac{3\,Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{15\,Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3\,Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{93\,Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{22\,Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{242\,Ba^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}{c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\frac{3\,Aa^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{93\,Ba^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3\,Aa^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{15\,Ba^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{Aa^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{Ba^2c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{c^{12}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/768*(12*sqrt(2)*(A*a^2*sqrt(c) - 11*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c) + 3*A*a^2
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1) + 15*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) - 3*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 93*B*a^2*sqrt(c)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 22*A
*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^3 + 242*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))) + sqrt(2)*(3*A*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 93*B*a^2*c^(17/2)*(cos(-1/4*pi + 1/2
```

```
*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*A*a^2*c^(17/2)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^2 - 15*B*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^2 - A*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - B*a^2*c^(17/2)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^2}{\left(c-c\,\sin(e+f\,x)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2
), x)
```

3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=222

$$\frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)+1/48*a^2*(3*A-13*B
)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/64*a^2*(3*A-13*B)*cos(f*x+e)/c^2/
f/(c-c*sin(f*x+e))^(5/2)+1/256*a^2*(3*A-13*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x
+e))^(3/2)+1/512*a^2*(3*A-13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c
*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2938, 2759, 2729, 2728, 212}

$$\frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (a^2*(3*A - 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(256*Sqrt[2]*c^(9/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(8*f*
(c - c*Sin[e + f*x])^(13/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x]^3)/(48*f*(c -
 c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 13*B)*Cos[e + f*x])/(64*c^2*f*(c - c*
Sin[e + f*x])^(5/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x])/(256*c^3*f*(c - c*Si
n[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{1}{16}\left(a^2(3A-13B)c\right)\int \frac{\text{co}}{(c-c}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} -$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} -$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} -$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} -$$

$$= \frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(A}{8f(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.67, size = 357, normalized size = 1.61

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(2013A\cos\left(\frac{1}{2}(e+fx)\right)+1517B\cos\left(\frac{1}{2}(e+fx)\right)-999A\cos\left(\frac{3}{2}(e+fx)\right)-791B\cos\left(\frac{3}{2}(e+fx)\right)-69A\cos\left(\frac{5}{2}(e+fx)\right)-725B\cos\left(\frac{5}{2}(e+fx)\right)-9A\cos\left(\frac{7}{2}(e+fx)\right)+39B\cos\left(\frac{7}{2}(e+fx)\right)-(24+24i)\sqrt[4]{-1}(3A-13B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8+2013A\sin\left(\frac{1}{2}(e+fx)\right)+1517B\sin\left(\frac{1}{2}(e+fx)\right)+999A\sin\left(\frac{3}{2}(e+fx)\right)+791B\sin\left(\frac{3}{2}(e+fx)\right)-69A\sin\left(\frac{5}{2}(e+fx)\right)-725B\sin\left(\frac{5}{2}(e+fx)\right)+9A\sin\left(\frac{7}{2}(e+fx)\right)-39B\sin\left(\frac{7}{2}(e+fx)\right)\right)}{6144f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Cos
[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 791*
B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f*x)
)/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*I)*(
-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] + 1517*
B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e + f*x))/2
] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Sin[(7*(e
+ f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(195) = 390.
time = 9.96, size = 440, normalized size = 1.98

method	result
default	$\frac{a^2\left(-12\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)\sin(fx+e)(\cos^2(fx+e))+24\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(}{2\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/1536/c^(19/2)*a^2*(-12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)*c^5*(3*A-13*B)*sin(f*x+e)*cos(f*x+e)^2+24*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*sin(f*x+e)-3*arctanh(1/
2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)
^4+24*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-
13*B)*cos(f*x+e)^2+18*A*(c+c*sin(f*x+e))^(7/2)*c^(3/2)-132*A*(c+c*sin(f*x+e
))^(5/2)*c^(5/2)-264*A*(c+c*sin(f*x+e))^(3/2)*c^(7/2)+144*A*(c+c*sin(f*x+e)
)^(1/2)*c^(9/2)-78*B*(c+c*sin(f*x+e))^(7/2)*c^(3/2)-452*B*(c+c*sin(f*x+e))^
(5/2)*c^(5/2)+1144*B*(c+c*sin(f*x+e))^(3/2)*c^(7/2)-624*B*(c+c*sin(f*x+e))^
(1/2)*c^(9/2)-72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^5+312*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c
^5)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(9/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(205) = 410$.
time = 0.87, size = 694, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/3072*(3*sqrt(2)*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*co
s(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*cos(
f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3*A
- 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*A -
13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B
)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*cos(f*x +
 e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2*cos(f*x +
 e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3*(3*A - 13*
B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 12*(19*A + 3*
B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^
3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(
f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos
(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(205) = 410.
time = 0.64, size = 781, normalized size = 3.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] 1/24576*(24*sqrt(2)*(3*A*a^2*sqrt(c) - 13*B*a^2*sqrt(c))*log(-(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^5*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*a^2*sqrt(c) + 3*B*a^2*sqrt(c) +
 32*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 24*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 72*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 96*B*a^2*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
```

```
 + 1)^3 + 150*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 650*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^4/(c^5*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - (96*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) + 24*sqrt(2)*A*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2 - 72*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
32*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 3*sqrt(2
)*A*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 3*sqrt(2)*B*a^2*c
^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/c^20)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2
), x)
```

3.98 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=210

$$\frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 256/45045*a^3*(15*A-B)*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/6435*a^
3*(15*A-B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+8/715*a^3*(15*A-B)*c^5
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)+2/195*a^3*(15*A-B)*c^4*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^(1/2)-2/15*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(1/2)
/f
```

Rubi [A]
time = 0.37, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{256a^3c^7(15A-B)\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3c^6(15A-B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5(15A-B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3c^4(15A-B)\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^3*(15*A - B)*c^7*Cos[e + f*x]^7)/(45045*f*(c - c*Sin[e + f*x])^(7/2)
) + (64*a^3*(15*A - B)*c^6*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^(5/
2)) + (8*a^3*(15*A - B)*c^5*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^(3/
2)) + (2*a^3*(15*A - B)*c^4*Cos[e + f*x]^7)/(195*f*Sqrt[c - c*Sin[e + f*x]]
) - (2*a^3*B*c^3*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
```

```
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+$$
$$= -\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+}}{15f}$$
$$= \frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}} - \frac{2a^3I}{}$$
$$= \frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(}{19}$$
$$= \frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}} + \frac{8a^3}{71}$$
$$= \frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}} + \frac{64}{6}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1569 vs. $2(210) = 420$.
time = 6.54, size = 1569, normalized size = 7.47

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(
7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e + f*
x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (3
*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x
])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)*Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(704*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(6*A + B)*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(19
2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6) + (3*(10*A - 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/
2)*Sin[(5*(e + f*x))/2])/(320*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(4*A + 3*B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(448*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((12*A -
 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2
])/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6) + ((2*A + 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^
(7/2)*Sin[(11*(e + f*x))/2])/(704*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(832*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*(a +
 a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(15*(e + f*x))/2])/(960*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^6)
```

Maple [A]
time = 6.13, size = 121, normalized size = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^4a^3((-3465A+12243B)\sin(fx+e)(\cos^2(fx+e))+(24780A-25676B)\sin(fx+e)+3003B(\cos^4(fx}{45045\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/45045*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*((-3465*A+12243*B)*sin(f*x+
e)*cos(f*x+e)^2+(24780*A-25676*B)*sin(f*x+e)+3003*B*cos(f*x+e)^4+(14175*A-2
4969*B)*cos(f*x+e)^2-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(200) = 400$.
time = 0.37, size = 424, normalized size = 2.02

$$-\frac{2\left(3003\,Ba^3c^3\cos\left(fx+e\right)^8-231\left(15\,A-14\,B\right)a^3c^3\cos\left(fx+e\right)^7+21\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^6-28\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^5+40\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^4-64\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^3+128\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^2-512\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)-1024\left(15\,A-B\right)a^3c^3-\left(3003\,Ba^3c^3\cos\left(fx+e\right)^7+231\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^6+252\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^5+280\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^4+320\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^3+384\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)^2+512\left(15\,A-B\right)a^3c^3\cos\left(fx+e\right)+1024\left(15\,A-B\right)a^3c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{45045\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -2/45045*(3003*B*a^3*c^3*cos(f*x + e)^8 - 231*(15*A - 14*B)*a^3*c^3*cos(f*x
 + e)^7 + 21*(15*A - B)*a^3*c^3*cos(f*x + e)^6 - 28*(15*A - B)*a^3*c^3*cos(
f*x + e)^5 + 40*(15*A - B)*a^3*c^3*cos(f*x + e)^4 - 64*(15*A - B)*a^3*c^3*c
os(f*x + e)^3 + 128*(15*A - B)*a^3*c^3*cos(f*x + e)^2 - 512*(15*A - B)*a^3*
c^3*cos(f*x + e) - 1024*(15*A - B)*a^3*c^3 - (3003*B*a^3*c^3*cos(f*x + e)^7
 + 231*(15*A - B)*a^3*c^3*cos(f*x + e)^6 + 252*(15*A - B)*a^3*c^3*cos(f*x +
 e)^5 + 280*(15*A - B)*a^3*c^3*cos(f*x + e)^4 + 320*(15*A - B)*a^3*c^3*cos(
f*x + e)^3 + 384*(15*A - B)*a^3*c^3*cos(f*x + e)^2 + 512*(15*A - B)*a^3*c^3
*cos(f*x + e) + 1024*(15*A - B)*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7315 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(200) = 400.
time = 0.73, size = 481, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/2882880*sqrt(2)*(3003*B*a^3*c^3*cos(-15/4*pi + 15/2*f*x + 15/2*e)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) - 225225*(8*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e) - 75075*(6*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) +
 B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2
*e) + 27027*(10*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^3*c^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) + 1930
5*(4*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 5005*(12*A*a^3*
c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a^3*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) - 4095*(2*A*a^3*c^3*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))*cos(-11/4*pi + 11/2*f*x + 11/2*e) + 3465*(2*A*a^3*c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-13
/4*pi + 13/2*f*x + 13/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\,\sin(e + f\,x))\,\left(a + a\,\sin(e + f\,x)\right)^3\,\left(c - c\,\sin(e + f\,x)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),
 x)
```

3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=161

$$\frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3 c}{13f\sqrt{c-}}$$

```
[Out] 64/9009*a^3*(13*A+B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/1287*a^3*
(13*A+B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/143*a^3*(13*A+B)*c^4*c
os(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{64a^3c^6(13A+B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5(13A+B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4(13A+B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^3*(13*A + B)*c^6*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^(7/2))
+ (16*a^3*(13*A + B)*c^5*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^(5/2)
) + (2*a^3*(13*A + B)*c^4*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)
) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+ \ldots}{\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}} + \frac{1}{13}\left(a^3(13 \ldots\right.$$
$$= \frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} - \frac{2a^3 \ldots}{13f\sqrt{\ldots}}$$
$$= \frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3(\ldots}{143 \ldots}$$
$$= \frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}} + \frac{16a^3 \ldots}{128 \ldots}$$

Mathematica [A]
time = 5.92, size = 143, normalized size = 0.89

$$-\frac{a^3c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,(-9490A+6200B+126(13A-32B)\cos(2(e+fx))+(9464A-9667B)\sin(e+fx)+693B\sin(3(e+fx)))}{18018f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

[Out] -1/18018*(a^3*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]*(-9490*A + 6200*B + 126*(13*A - 32*B)*Cos[2*(e + f*x)] + (9464*A - 9667*B)*Sin[e + f*x] + 693*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)

Maple [A]
time = 6.06, size = 105, normalized size = 0.65

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^4a^3(-693B(\cos^2(fx+e))\sin(fx+e)+(-2366A+2590B)\sin(fx+e)+(-819A+2016B)(\cos^2(fx$ $9009\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/9009*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(-693*B*cos(f*x+e)^2*sin(f*x+e)+(-2366*A+2590*B)*sin(f*x+e)+(-819*A+2016*B)*cos(f*x+e)^2+2782*A-2558*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(153) = 306.
time = 0.37, size = 351, normalized size = 2.18

$$-\frac{2\left(693\,Ba^3c^2\cos(fx+e)^7+63\,(13\,A+12\,B)a^3c^2\cos(fx+e)^6-7\,(13\,A+B)a^3c^2\cos(fx+e)^5+10\,(13\,A+B)a^3c^2\cos(fx+e)^4-16\,(13\,A+B)a^3c^2\cos(fx+e)^3+32\,(13\,A+B)a^3c^2\cos(fx+e)^2-128\,(13\,A+B)a^3c^2\cos(fx+e)-256\,(13\,A+B)a^3c^2+\left(693\,Ba^3c^2\cos(fx+e)^6-63\,(13\,A+B)a^3c^2\cos(fx+e)^5-70\,(13\,A+B)a^3c^2\cos(fx+e)^4-80\,(13\,A+B)a^3c^2\cos(fx+e)^3-96\,(13\,A+B)a^3c^2\cos(fx+e)^2-128\,(13\,A+B)a^3c^2\cos(fx+e)-256\,(13\,A+B)a^3c^2\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{9009\,(f\cos(fx+e)-f\sin(fx+e)+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -2/9009*(693*B*a^3*c^2*cos(f*x + e)^7 + 63*(13*A + 12*B)*a^3*c^2*cos(f*x + e)^6 - 7*(13*A + B)*a^3*c^2*cos(f*x + e)^5 + 10*(13*A + B)*a^3*c^2*cos(f*x + e)^4 - 16*(13*A + B)*a^3*c^2*cos(f*x + e)^3 + 32*(13*A + B)*a^3*c^2*cos(f

```
*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^2 +
(693*B*a^3*c^2*cos(f*x + e)^6 - 63*(13*A + B)*a^3*c^2*cos(f*x + e)^5 - 70*(
13*A + B)*a^3*c^2*cos(f*x + e)^4 - 80*(13*A + B)*a^3*c^2*cos(f*x + e)^3 - 9
6*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) -
 256*(13*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x
 + e) - f*sin(f*x + e) + f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(153) = 306.
time = 0.75, size = 391, normalized size = 2.43

$$-\frac{\sqrt{2}\left(180180\,Aa^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+693\,Ba^3c^2\cos\left(-\frac{13}{4}\pi+\frac{13}{2}fx+\frac{13}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+15015\left(4\,Aa^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)-9009\left(2\,Aa^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Ba^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)-2574\left(5\,Aa^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Ba^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+2002\left(Aa^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,Ba^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)+819\left(2\,Aa^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\right)\sqrt{c}}{288288\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -1/288288*sqrt(2)*(180180*A*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 693*B*a^3*c^2*cos(-13/4*pi + 13/2*f*x + 13/2*
e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15015*(4*A*a^3*c^2*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-
3/4*pi + 3/2*f*x + 3/2*e) - 9009*(2*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)) - B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f
*x + 5/2*e) - 2574*(5*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a
^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e)
+ 2002*(A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^3*c^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) + 819*(2*A*a^3
*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),
 x)
```

3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 8/693*a^3*(11*A+3*B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/99*a^3*(11
*A+3*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)-2/11*a^3*B*c^3*cos(f*x+e)
^7/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2935, 2753, 2752}

$$\frac{8a^3c^5(11A+3B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4(11A+3B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^3*(11*A + 3*B)*c^5*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2))
+ (2*a^3*(11*A + 3*B)*c^4*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2))
 - (2*a^3*B*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+\cdots}{(c-c\sin(e+fx))^{3/2}}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{1}{11}\left(a^3\cdots\right.$$

$$= \frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2\cdots}{11\cdots}$$

$$= \frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3\cdots}{9\cdots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1157 vs. $2(124) = 248$.
time = 6.35, size = 1157, normalized size = 9.33

$$\frac{(6A+B)\cos\left(\frac{1}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{(8A+3B)\cos\left(\frac{3}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{B\cos\left(\frac{5}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{16f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{(6A+B)\cos\left(\frac{7}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{112f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{(2A+3B)\cos\left(\frac{9}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{144f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}+\frac{B\cos\left(\frac{11}{2}(e+fx)\right)(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}{176f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}+\frac{(6A+B)\sin\left(\frac{1}{2}(e+fx)\right)\cdots}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\cdots\right.}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(C
```

```
os[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Cos[(7*(e
 + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
 - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e +
f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3
*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((8*A + 3*B)*(
a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e + f*x))/2])/(24*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(5*(e +
f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6) + ((6*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(3/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 3*B)*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(11*(e + f*x))/2])/(1
76*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6)
```

Maple [A]
time = 6.02, size = 83, normalized size = 0.67

method	result	size
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^4a^3(\sin(fx+e)(77A-105B)-63B(\cos^2(fx+e))-121A+93B)}{693\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/693*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(sin(f*x+e)*(77*A-105*B)-63*B
*cos(f*x+e)^2-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(118) = 236$.
time = 0.36, size = 302, normalized size = 2.44

$$\frac{2\left(63\,Ba^{3}c\cos\left(fx+e\right)^{6}-7\left(11\,A+12\,B\right)a^{3}c\cos\left(fx+e\right)^{5}-\left(187\,A+177\,B\right)a^{3}c\cos\left(fx+e\right)^{4}+2\left(11\,A+3\,B\right)a^{3}c\cos\left(fx+e\right)^{3}-4\left(11\,A+3\,B\right)a^{3}c\cos\left(fx+e\right)^{2}+16\left(11\,A+3\,B\right)a^{3}c\cos\left(fx+e\right)+32\left(11\,A+3\,B\right)a^{3}c-\left(63\,Ba^{3}c\cos\left(fx+e\right)^{5}+7\left(11\,A+21\,B\right)a^{3}c\cos\left(fx+e\right)^{4}-10\left(11\,A+3\,B\right)a^{3}c\cos\left(fx+e\right)^{3}-12\left(11\,A+3\,B\right)a^{3}c\cos\left(fx+e\right)^{2}-16\left(11\,A+3\,B\right)a^{3}c\cos\left(fx+e\right)-32\left(11\,A+3\,B\right)a^{3}c\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{693\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 2/693*(63*B*a^3*c*cos(f*x + e)^6 - 7*(11*A + 12*B)*a^3*c*cos(f*x + e)^5 - (
187*A + 177*B)*a^3*c*cos(f*x + e)^4 + 2*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 -
 4*(11*A + 3*B)*a^3*c*cos(f*x + e)^2 + 16*(11*A + 3*B)*a^3*c*cos(f*x + e) +
 32*(11*A + 3*B)*a^3*c - (63*B*a^3*c*cos(f*x + e)^5 + 7*(11*A + 21*B)*a^3*c
*cos(f*x + e)^4 - 10*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 - 12*(11*A + 3*B)*a^
3*c*cos(f*x + e)^2 - 16*(11*A + 3*B)*a^3*c*cos(f*x + e) - 32*(11*A + 3*B)*a
^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x +
 e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 2Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int\left(-2Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)\right)dx+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin^{4}\left(e+fx\right)\right)dx+\int Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int 2Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int\left(-2Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^{4}\left(e+fx\right)\right)dx+\int\left(-Bc\sqrt{-c\sin\left(e+fx\right)+c}\sin^{5}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x), x) + Integral(-2*A*c*sqrt(-c*sin(e + f*x) +
c)*sin(e + f*x)**3, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*
x)**4, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integ
ral(2*B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-2*B*c*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x) + Integral(-B*c*sqrt(-c*sin(e
+ f*x) + c)*sin(e + f*x)**5, x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(118) = 236$.
time = 0.68, size = 310, normalized size = 2.50

$$-\frac{\sqrt{2}\left(693\,Ba^{3}c\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-63\,Ba^{3}c\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+1386\left(6\,Aa^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+462\left(8\,Aa^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)-99\left(6\,Aa^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)-77\left(2\,Aa^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba^{3}c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\right)\sqrt{c}}{11088\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/11088*sqrt(2)*(693*B*a^3*c*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 63*B*a^3*c*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) + 1386*(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 462*(8*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*
c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 99*
(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 77*(2*A*a^3*c*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^3\,\left(c-c\,\sin(e+f\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),
 x)
```

3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

Optimal. Leaf size=81

$$\frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 2/63*a^3*(9*A+5*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/9*a^3*B*c^3*
cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A]
time = 0.21, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3046, 2935, 2752}

$$\frac{2a^3c^4(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(9*A + 5*B)*c^4*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) -
(2*a^3*B*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+f}{(c-c\sin(e+fx))^{5/2}}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}} + \frac{1}{9}\left(a^3(9A+\right.$$

$$= \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3B}{9f(c-}$$

Mathematica [A]
time = 0.66, size = 89, normalized size = 1.10

$$\frac{2a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(9A-2B+7B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(9*A - 2*B + 7*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.57, size = 65, normalized size = 0.80

method	result	size
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^4a^3(7B\sin(fx+e)+9A-2B)}{63\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/63*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(77) = 154$.
time = 0.37, size = 245, normalized size = 3.02

$$\frac{2\left(7 B a^{3} \cos\left(fx+e\right)^{5}+\left(9 A+26 B\right) a^{3} \cos\left(fx+e\right)^{4}-\left(27 A+29 B\right) a^{3} \cos\left(fx+e\right)^{3}-4\left(18 A+17 B\right) a^{3} \cos\left(fx+e\right)^{2}+4\left(9 A+5 B\right) a^{3} \cos\left(fx+e\right)+8\left(9 A+5 B\right) a^{3}+\left(7 B a^{3} \cos\left(fx+e\right)^{4}-\left(9 A+19 B\right) a^{3} \cos\left(fx+e\right)^{3}-12\left(3 A+4 B\right) a^{3} \cos\left(fx+e\right)^{2}+4\left(9 A+5 B\right) a^{3} \cos\left(fx+e\right)+8\left(9 A+5 B\right) a^{3}\right) \sin\left(fx+e\right)\right)\sqrt{-c \sin\left(fx+e\right)+c}}{63\left(f \cos\left(fx+e\right)-f \sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/63*(7*B*a^3*cos(f*x + e)^5 + (9*A + 26*B)*a^3*cos(f*x + e)^4 - (27*A + 29
*B)*a^3*cos(f*x + e)^3 - 4*(18*A + 17*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)
*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3 + (7*B*a^3*cos(f*x + e)^4 - (9*A + 19
*B)*a^3*cos(f*x + e)^3 - 12*(3*A + 4*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)*
a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 3A\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int 3A\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int 3B\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)dx+\int 3B\sqrt{-c\sin\left(e+fx\right)+c}\sin^{3}\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\sin^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(3*A*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x)**2, x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x)
+ Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*B*sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(3*B*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x)**3, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e +
f*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(77) = 154$.
time = 0.67, size = 267, normalized size = 3.30

$$-\frac{\sqrt{2}\left(7 B a^{3} \cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+126\left(5 A a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2 B a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+42\left(9 A a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5 B a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+126\left(A a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+9\left(2 A a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5 B a^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\right)\sqrt{c}}{504 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/504*sqrt(2)*(7*B*a^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 126*(5*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a^
3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 42*
(9*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(A*a^3*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4
*pi + 5/2*f*x + 5/2*e) + 9*(2*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5
*B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e))
*sqrt(c)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
 x)
```

3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=200

$$\frac{8\sqrt{2}\,a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] -2/7*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/5*a^3*(A+B)*c^2*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*(A+B)*c*cos(f*x+e)^3/f/(c-c*sin(f
*x+e))^(3/2)+8*a^3*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*
x+e))^(1/2))*2^(1/2)/f/c^(1/2)-8*a^3*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.36, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2939, 2758, 2728, 212}

$$-\frac{2a^3c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}\,a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (8*Sqrt[2]*a^3*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[
e + f*x])^(7/2)) - (2*a^3*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f
*x])^(5/2)) - (4*a^3*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (8*a^3*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} + \left(a^3(A+B)c^3\right)\int \frac{\cos^6(e}{(c-c\sin(}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{8\sqrt{2}\,a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{}{7f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.89, size = 193, normalized size = 0.96

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left((6720+6720i)\sqrt[4]{-1}(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2086A-2236B+6(7A+22B)\cos(2(e+fx))-(448A+673B)\sin(e+fx)+15B\sin(3(e+fx)))\right)}{420f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] -1/420*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((67
20 + 6720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e +
 f*x)/4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B + 6*
(7*A + 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin[3*(
e + f*x)])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin[e +
f*x]])
```

Maple [A]
time = 8.62, size = 233, normalized size = 1.16

method	result

default	$\frac{2(\sin(fx+e)-1)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, a^{3}\left(-420c^{\frac{7}{2}}\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, \sqrt{2}}{2\sqrt{c}}\right)A-420c^{\frac{7}{2}}\sqrt{2}\, a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/105*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(-420*c^(7/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-420*c^(7/2)*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+15*B*(c*(1+sin(f*x+e)
))^(7/2)+21*A*(c*(1+sin(f*x+e)))^(5/2)*c+21*B*(c*(1+sin(f*x+e)))^(5/2)*c+70
*A*(c*(1+sin(f*x+e)))^(3/2)*c^2+70*B*(c*(1+sin(f*x+e)))^(3/2)*c^2+420*A*c^3
*(c*(1+sin(f*x+e)))^(1/2)+420*B*c^3*(c*(1+sin(f*x+e)))^(1/2))/c^4/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(187) = 374.
time = 0.37, size = 377, normalized size = 1.88

$$\frac{2\left(\frac{210\sqrt{2}\left((A+B)a^3c\cos(fx+e)-(A+B)a^3c\sin(fx+e)+(A+B)a^3c\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}-\left(15\,Ba^3\cos\left(fx+e\right)^4-3\left(7\,A+22\,B\right)a^3\cos\left(fx+e\right)^3-\left(133\,A+253\,B\right)a^3\cos\left(fx+e\right)^2+4\left(133\,A+148\,B\right)a^3\cos\left(fx+e\right)+4\left(161\,A+191\,B\right)a^3-\left(15\,Ba^3\cos\left(fx+e\right)^3+3\left(7\,A+27\,B\right)a^3\cos\left(fx+e\right)^2-4\left(28\,A+43\,B\right)a^3\cos\left(fx+e\right)-4\left(161\,A+191\,B\right)a^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}\right)}{105\left(cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(210*sqrt(2)*((A + B)*a^3*c*cos(f*x + e) - (A + B)*a^3*c*sin(f*x + e)
 + (A + B)*a^3*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(
c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(c) - (15*B*a^3*cos(f*x + e)^4 - 3*(7*A + 22*B)*a^
```

```
3*cos(f*x + e)^3 - (133*A + 253*B)*a^3*cos(f*x + e)^2 + 4*(133*A + 148*B)*a
^3*cos(f*x + e) + 4*(161*A + 191*B)*a^3 - (15*B*a^3*cos(f*x + e)^3 + 3*(7*A
 + 27*B)*a^3*cos(f*x + e)^2 - 4*(28*A + 43*B)*a^3*cos(f*x + e) - 4*(161*A +
 191*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c
*f*sin(f*x + e) + c*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{3A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{3A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{A\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{3B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{3B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx+\int\frac{B\sin^4(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)/
sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)**2/sqrt(-c*sin(e
+ f*x) + c), x) + Integral(A*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x)
+ Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(e + f*x)**3/sq
rt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**4/sqrt(-c*sin(e + f*
x) + c), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(187) = 374.
time = 0.52, size = 677, normalized size = 3.38

$$\frac{4\left(\frac{105\left(\sqrt{2}Aa^3\sqrt{c}+\sqrt{2}Ba^3\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\sqrt{2}\left(161Aa^3\sqrt{c}+191Ba^3\sqrt{c}-\frac{812Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{812Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{2121Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{2751Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{3080Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}-\frac{3080Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{2555Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}+\frac{3605Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}-\frac{1260Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}-\frac{1260Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}+\frac{315Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^6}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^6}+\frac{525Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^6}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^6}\right)}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^7\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 4/105*(105*(sqrt(2)*A*a^3*sqrt(c) + sqrt(2)*B*a^3*sqrt(c))*log(-(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*(161*A*a^3*sqrt(c) + 191*B*a^3*sqr
t(c) - 812*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) - 812*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 2121*A*a^3*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 2751*B
*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2 - 3080*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 3080*B*a^3*sqrt(c)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 2555*A*a^
3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4 + 3605*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(c
```

```
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1260*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 1260*B*a^3*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^5 + 315*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^6/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^6 + 525*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^6/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^6)/(c*((cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^7*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{2\sqrt{2}\,a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3(5A+9B)c\cos^5(e$$

```
[Out] 1/2*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(9/2)+1/10*a^3*(5*A+9*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+1/3*a^3*(5*A+9*B)*cos(f*x+e)^3/f/(c
-c*sin(f*x+e))^(3/2)-2*a^3*(5*A+9*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)
/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+2*a^3*(5*A+9*B)*cos(f*x+e)/c/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2938, 2758, 2728, 212}

$$-\frac{2\sqrt{2}\,a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3c(5A+9B)\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}+\frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (-2*Sqrt[2]*a^3*(5*A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c
- c*Sin[e + f*x]])])/(c^(3/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(2*f*(c
 - c*Sin[e + f*x])^(9/2)) + (a^3*(5*A + 9*B)*c*Cos[e + f*x]^5)/(10*f*(c - c
*Sin[e + f*x])^(5/2)) + (a^3*(5*A + 9*B)*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e
+ f*x])^(3/2)) + (2*a^3*(5*A + 9*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{1}{4}\left(a^3(5A+9B)c^2\right)\int \frac{\text{co}}{(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}$$

$$= -\frac{2\sqrt{2}\,a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.11, size = 444, normalized size = 2.04

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A + 9
*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 5*(2*A + 9*B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2] - 3*B*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^2*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A]
time = 7.24, size = 354, normalized size = 1.62

method	result
default	$\frac{2a^{3}\left(\sin(fx+e)\left(-5A(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}-60A\sqrt{c+c\sin\left(fx+e\right)}\,c^{\frac{5}{2}}-3B(c+c\sin(fx+e))^{\frac{5}{2}}\sqrt{c}-15B(c+c\sin(fx+e))^{\frac{3}{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*a^3*(sin(f*x+e)*(-5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-60*A*(c+c*sin(f*x
+e))^(1/2)*c^(5/2)-3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-15*B*(c+c*sin(f*x+e))
^(3/2)*c^(3/2)-120*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+75*A*2^(1/2)*arctanh(1/
2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+135*B*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)+5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2
)+90*A*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)+15
*B*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+150*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-75*A
*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-135*B*2^(1
/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+
e)))^(1/2)/c^(9/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(205) = 410.
time = 0.37, size = 458, normalized size = 2.10

$$\frac{\frac{15\sqrt{2}\left((5A+9B)a^{3}c\cos(fx+e)^{2}-(5A+9B)a^{3}c\cos(fx+e)-2(5A+9B)a^{3}c+\left((5A+9B)a^{3}c\cos(fx+e)+2(5A+9B)a^{3}c\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^{2}+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{c}}+2\left(3Ba^{3}\cos(fx+e)^{4}-(5A+18B)a^{3}\cos(fx+e)^{3}-(65A+141B)a^{3}\cos(fx+e)^{2}-30(3A+5B)a^{3}\cos(fx+e)-30(A+B)a^{3}-\left(3Ba^{3}\cos(fx+e)^{3}+(5A+21B)a^{3}\cos(fx+e)^{2}-60(A+2B)a^{3}\cos(fx+e)+30(A+B)a^{3}\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{15\left(c^{2}f\cos(fx+e)^{2}-c^{2}f\cos(fx+e)-2c^{2}f+\left(c^{2}f\cos(fx+e)+2c^{2}f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((5*A + 9*B)*a^3*c*cos(f*x + e)^2 - (5*A + 9*B)*a^3*c*cos(
f*x + e) - 2*(5*A + 9*B)*a^3*c + ((5*A + 9*B)*a^3*c*cos(f*x + e) + 2*(5*A +
```

```
 9*B)*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*
x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) +
 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2))/sqrt(c) + 2*(3*B*a^3*cos(f*x + e)^4 - (5*A +
18*B)*a^3*cos(f*x + e)^3 - (65*A + 141*B)*a^3*cos(f*x + e)^2 - 30*(3*A + 5*
B)*a^3*cos(f*x + e) - 30*(A + B)*a^3 - (3*B*a^3*cos(f*x + e)^3 + (5*A + 21*
B)*a^3*cos(f*x + e)^2 - 60*(A + 2*B)*a^3*cos(f*x + e) + 30*(A + B)*a^3)*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x
 + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(205) = 410.
time = 0.60, size = 764, normalized size = 3.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -1/30*(30*sqrt(2)*(5*A*a^3*sqrt(c) + 9*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) + 15*sqrt(2)*(A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a^3*sqrt(c)*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqr
t(c) + 10*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 18*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) - 16*sqrt(2)*(35*A*a^3*sqrt(c) + 81*B*a^3*sqrt(c) - 130*A*a^3*sqrt(
c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
) - 270*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 200*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 480*B*a^3*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 150*A*
```

```
a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^3 - 330*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 135*B*a^3*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^4)/(c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2
), x)
```

3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{a^3(3A+11B)c\cos^5(e+ }{8f(c-c\sin(e+fx))^{7/}}$$

```
[Out] 1/4*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^3*(3*A+11*B)
*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)-5/24*a^3*(3*A+11*B)*cos(f*x+e)^3/c
/f/(c-c*sin(f*x+e))^(3/2)+5/4*a^3*(3*A+11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)
*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-5/4*a^3*(3*A+11*B)*cos(f
*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2938, 2759, 2758, 2728, 212}

$$\frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^3c(3A+11B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}}-\frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (5*a^3*(3*A + 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[
e + f*x]])])/(2*Sqrt[2]*c^(5/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(4*f*
(c - c*Sin[e + f*x])^(11/2)) - (a^3*(3*A + 11*B)*c*Cos[e + f*x]^5)/(8*f*(c
- c*Sin[e + f*x])^(7/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x]^3)/(24*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x])/(4*c^2*f*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (a^3c^3)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}(a^3(3A+11B)c^2)\int \frac{c}{(c-}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f} + \frac{a^3}{4f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.49, size = 434, normalized size = 1.93

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(12*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 3*(9*A + 17*B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(3*A + 11*B)*ArcTan[(1/2 + I/
2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - 6*(2*A + 11*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4 + 2*B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 24*(
A + B)*Sin[(e + f*x)/2] - 6*(9*A + 17*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^2*Sin[(e + f*x)/2] - 6*(2*A + 11*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^4*Sin[(e + f*x)/2] - 2*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(3*
(e + f*x))/2]))/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
 f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(198) = 396.
time = 9.44, size = 434, normalized size = 1.93

method	result
default	$-\frac{a^{3}\left(\sin(fx+e)\left(-90A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}+48A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-330B\sqrt{2}\,\operatorname{ar}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/12/c^(9/2)*a^3*(sin(f*x+e)*(-90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(
1/2)*2^(1/2)/c^(1/2))*c^2+48*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-330*B*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+16*B*(c+c*sin(f*x+
e))^(3/2)*c^(1/2)+240*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2))+(-45*A*2^(1/2)*arct
anh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+24*A*(c+c*sin(f*x+e))^(
1/2)*c^(3/2)-165*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2+8*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+120*B*(c+c*sin(f*x+e))^(1/2)*c^(
3/2))*cos(f*x+e)^2+90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/
c^(1/2))*c^2+54*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-132*A*(c+c*sin(f*x+e))^(1/
2)*c^(3/2)+330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*c^2+86*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-420*B*(c+c*sin(f*x+e))^(1/2)*c^(3
/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1
/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(208) = 416.
time = 0.38, size = 537, normalized size = 2.39

$$\frac{15\sqrt{2}\left((3A+11B)a^{3}\cos(fx+e)^{3}+3(3A+11B)a^{3}\cos(fx+e)^{2}-2(3A+11B)a^{3}\cos(fx+e)-4(3A+11B)a^{3}-\left((3A+11B)a^{3}\cos(fx+e)^{2}-2(3A+11B)a^{3}\cos(fx+e)-4(3A+11B)a^{3}\right)\sin(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(4Ba^{3}\cos(fx+e)^{4}-4(3A+14B)a^{3}\cos(fx+e)^{3}+3(13A+37B)a^{3}\cos(fx+e)^{2}+3(13A+53B)a^{3}\cos(fx+e)-12(A+B)a^{3}-\left(4Ba^{3}\cos(fx+e)^{3}+12(A+5B)a^{3}\cos(fx+e)^{2}+3(17A+57B)a^{3}\cos(fx+e)+12(A+B)a^{3}\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{24\left(c^{3}f\cos(fx+e)^{3}+3c^{3}f\cos(fx+e)^{2}-2c^{3}f\cos(fx+e)-4c^{3}f-\left(c^{3}f\cos(fx+e)^{2}-2c^{3}f\cos(fx+e)-4c^{3}f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(15*sqrt(2)*((3*A + 11*B)*a^3*cos(f*x + e)^3 + 3*(3*A + 11*B)*a^3*cos(
f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B)*a^3 - ((3*A +
 11*B)*a^3*cos(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B
)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*B*a^3*cos(f*x + e)^4 - 4*(
3*A + 14*B)*a^3*cos(f*x + e)^3 + 3*(13*A + 37*B)*a^3*cos(f*x + e)^2 + 3*(13
*A + 53*B)*a^3*cos(f*x + e) - 12*(A + B)*a^3 - (4*B*a^3*cos(f*x + e)^3 + 12
*(A + 5*B)*a^3*cos(f*x + e)^2 + 3*(17*A + 57*B)*a^3*cos(f*x + e) + 12*(A +
B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(208) = 416.
time = 0.63, size = 815, normalized size = 3.62

$$\frac{\frac{60\sqrt{2}\left(3Aa^3\sqrt{c}+11Ba^3\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(Aa^3\sqrt{c}+Ba^3\sqrt{c}+\frac{16Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{32Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{90Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{330Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\left(\frac{16\sqrt{2}Aa^3c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{32\sqrt{2}Ba^3c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{\sqrt{2}Aa^3c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{\sqrt{2}Ba^3c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c^6}-\frac{128\sqrt{2}\left(3Aa^3\sqrt{c}+17Ba^3\sqrt{c}-\frac{6Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{30Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{21Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{c^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/96*(60*sqrt(2)*(3*A*a^3*sqrt(c) + 11*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqrt(c) + 16*A
*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 32*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 90*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 330*B*a^3*sqrt(c)*(co
```

```
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 3*(16*sqrt(2)*A*a^3*c^(7/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 32*sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2 + sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/
c^6 - 128*sqrt(2)*(3*A*a^3*sqrt(c) + 17*B*a^3*sqrt(c) - 6*A*a^3*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 30
*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) + 3*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 21*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^3*((cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^3}{\left(c-c\,\sin(e+f\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2
), x)
```

3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,c^{7/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{a^3(A+13B)c\cos^5(e+f}{24f(c-c\sin(e+fx))^{9/}}$$

```
[Out] 1/6*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(13/2)-1/24*a^3*(A+13*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)+5/48*a^3*(A+13*B)*cos(f*x+e)^3/c/f/
(c-c*sin(f*x+e))^(5/2)-5/16*a^3*(A+13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(
1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)+5/16*a^3*(A+13*B)*cos(f*x+e)
/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2938, 2759, 2758, 2728, 212}

$$-\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,c^{7/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}-\frac{a^3c(A+13B)\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}}+\frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (-5*a^3*(A + 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(8*Sqrt[2]*c^(7/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(6*f*(
c - c*Sin[e + f*x])^(13/2)) - (a^3*(A + 13*B)*c*Cos[e + f*x]^5)/(24*f*(c -
c*Sin[e + f*x])^(9/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x]^3)/(48*c*f*(c - c*S
in[e + f*x])^(5/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x])/(16*c^3*f*Sqrt[c - c*
Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{1}{12}\left(a^3(A+13B)c^2\right)\int \frac{\cdots}{(c-\cdots}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \cdots$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \cdots$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \cdots$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \cdots$$

$$= -\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3\cdots}{6\cdots}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 2.08, size = 422, normalized size = 1.94

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-
1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A + 25*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A + 47*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(24*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(190) = 380$.
time = 9.84, size = 524, normalized size = 2.41

method	result
default	$\frac{a^3\left(15A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^3(fx+e))c^3+195B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*sin(f*x+e)^3*c^3+195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3-45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-96*B*(c*(1+sin(f*x+e)))^(1/2
)*c^(5/2)*sin(f*x+e)^3-585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*A
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^(1/2)*c^
(5/2)*sin(f*x+e)^2+585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*A*2^(1
/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3-928*B*(c*(1+s
in(f*x+e)))^(3/2)*c^(3/2)-288*B*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)
-195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3+12
0*A*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)+888*B*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)
)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(200) = 400$.
time = 0.38, size = 590, normalized size = 2.72

$$\frac{15\sqrt{2}\left((A+13B)a^3\cos(fx+e)^4-3(A+13B)a^3\cos(fx+e)^3-8(A+13B)a^3\cos(fx+e)^2+4(A+13B)a^3\cos(fx+e)+8(A+13B)a^3+\left((A+13B)a^3\cos(fx+e)^3+4(A+13B)a^3\cos(fx+e)^2-4(A+13B)a^3\cos(fx+e)-8(A+13B)a^3\right)\sin(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(48Ba^3\cos(fx+e)^4+3(11A+95B)a^3\cos(fx+e)^3+(19A-137B)a^3\cos(fx+e)^2-2(23A+203B)a^3\cos(fx+e)-32(A+B)a^3-\left(48Ba^3\cos(fx+e)^3-3(11A+79B)a^3\cos(fx+e)^2-2(7A+187B)a^3\cos(fx+e)+32(A+B)a^3\right)\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}}{96\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+4c^4f\cos(fx+e)+8c^4f+\left(c^4f\cos(fx+e)^3+4c^4f\cos(fx+e)^2-4c^4f\cos(fx+e)-8c^4f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x
+ e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e) +
8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos(f*
x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)*a^3*cos
(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)*a^3*cos(
f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A + 79*B)*a^3*
cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x +
 e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*co
s(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin
(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(200) = 400.
time = 0.66, size = 753, normalized size = 3.47

$$\frac{\frac{1536\sqrt{2}Ba^3}{c^{\frac{7}{2}}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{60\sqrt{2}\left(Aa^3\sqrt{c}+13Ba^3\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa^3\sqrt{c}+Ba^3\sqrt{c}+\frac{9Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{21Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{45Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{237Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{110Aa^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{1430Ba^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}{c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\frac{45Aa^3c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{237Ba^3c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9Aa^3c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{21Ba^3c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{Aa^3c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{Ba^3c^{\frac{17}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{c^{12}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{384f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/384*(1536*sqrt(2)*B*a^3/(c^(7/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
) - 60*sqrt(2)*(A*a^3*sqrt(c) + 13*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqrt(c) + 9*A*a^3*sq
```

```
rt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1) + 21*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 45*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 237*B*a^3*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 110*A
*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^3 + 1430*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))) - sqrt(2)*(45*A*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 237*B*a^3*c^(17/2)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*a^3*c^(17/
2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2 + 21*B*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2 + A*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + B*a^3*c^(17/2)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2
), x)
```

3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,c^{9/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{a^3(A-15B)c\cos^5(e+f}{48f(c-c\sin(e+fx))^{11}}$$

```
[Out] 1/8*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(15/2)+1/48*a^3*(A-15*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/192*a^3*(A-15*B)*cos(f*x+e)^3/c/
f/(c-c*sin(f*x+e))^(7/2)+5/128*a^3*(A-15*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e
))^(3/2)-5/256*a^3*(A-15*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin
(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2938, 2759, 2728, 212}

$$-\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,c^{9/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3c(A-15B)\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (-5*a^3*(A - 15*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(128*Sqrt[2]*c^(9/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(8*f
*(c - c*Sin[e + f*x])^(15/2)) + (a^3*(A - 15*B)*c*Cos[e + f*x]^5)/(48*f*(c
- c*Sin[e + f*x])^(11/2)) - (5*a^3*(A - 15*B)*Cos[e + f*x]^3)/(192*c*f*(c -
 c*Sin[e + f*x])^(7/2)) + (5*a^3*(A - 15*B)*Cos[e + f*x])/(128*c^3*f*(c - c
*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{1}{16}\left(a^3(A-15B)c^2\right)\int \frac{\cdots}{(c-\cdots}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \cdots$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \cdots$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \cdots$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} \cdots$$

$$= -\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3\cdots}{8\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.95, size = 355, normalized size = 1.64

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(1765A\cos\left(\frac{1}{2}(e+fx)\right)+405B\cos\left(\frac{1}{2}(e+fx)\right)-895A\cos\left(\frac{3}{2}(e+fx)\right)-2703B\cos\left(\frac{3}{2}(e+fx)\right)-397A\cos\left(\frac{5}{2}(e+fx)\right)+579B\cos\left(\frac{5}{2}(e+fx)\right)+15A\cos\left(\frac{7}{2}(e+fx)\right)+543B\cos\left(\frac{7}{2}(e+fx)\right)+(120+120i)\sqrt[4]{-1}(A-15B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8+1765A\sin\left(\frac{1}{2}(e+fx)\right)+405B\sin\left(\frac{1}{2}(e+fx)\right)+895A\sin\left(\frac{3}{2}(e+fx)\right)+2703B\sin\left(\frac{3}{2}(e+fx)\right)-397A\sin\left(\frac{5}{2}(e+fx)\right)+579B\sin\left(\frac{5}{2}(e+fx)\right)-15A\sin\left(\frac{7}{2}(e+fx)\right)-543B\sin\left(\frac{7}{2}(e+fx)\right)\right)}{3072f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Cos
[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 2703*
B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e + f*x
))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 + 120
*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2] + 40
5*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e + f*x)
)/2] - 397*A*Sin[(5*(e + f*x))/2] + 579*B*Sin[(5*(e + f*x))/2] - 15*A*Sin[(
7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2]))/(3072*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(190) = 380$.
time = 9.53, size = 432, normalized size = 1.99

method	result
default	$-\frac{a^3\left(-60\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^4(A-15B)\sin(fx+e)\left(\cos^2(fx+e)\right)+120\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin(}{2\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/768/c^(17/2)*a^3*(-60*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)*c^4*(A-15*B)*sin(f*x+e)*cos(f*x+e)^2+120*arctanh(1/2*(c+c*sin(f*x
+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*sin(f*x+e)-15*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)^4+1
20*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)
*cos(f*x+e)^2+30*A*(c+c*sin(f*x+e))^(7/2)*c^(1/2)+292*A*(c+c*sin(f*x+e))^(5
/2)*c^(3/2)-440*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)+240*A*(c+c*sin(f*x+e))^(1/
2)*c^(7/2)+1086*B*(c+c*sin(f*x+e))^(7/2)*c^(1/2)-4380*B*(c+c*sin(f*x+e))^(5
/2)*c^(3/2)+6600*B*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-3600*B*(c+c*sin(f*x+e))^(
1/2)*c^(7/2)-120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^4+1800*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
c^4)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^
(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(9/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(200) = 400$.
time = 0.39, size = 673, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/1536*(15*sqrt(2)*((A - 15*B)*a^3*cos(f*x + e)^5 + 5*(A - 15*B)*a^3*cos(f
*x + e)^4 - 8*(A - 15*B)*a^3*cos(f*x + e)^3 - 20*(A - 15*B)*a^3*cos(f*x + e
)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3 - ((A - 15*B)*a^3*c
os(f*x + e)^4 - 4*(A - 15*B)*a^3*cos(f*x + e)^3 - 12*(A - 15*B)*a^3*cos(f*x
 + e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(5*A + 181*B)*a^3*cos(f*x + e)^4 - (191*A - 561
*B)*a^3*cos(f*x + e)^3 - 2*(169*A + 537*B)*a^3*cos(f*x + e)^2 + 12*(21*A -
59*B)*a^3*cos(f*x + e) + 384*(A + B)*a^3 - (3*(5*A + 181*B)*a^3*cos(f*x + e
)^3 + 2*(103*A - 9*B)*a^3*cos(f*x + e)^2 - 12*(11*A + 91*B)*a^3*cos(f*x + e
) - 384*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*
x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f
*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5
*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5
*f)*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(200) = 400.
time = 0.69, size = 974, normalized size = 4.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] -1/12288*(120*sqrt(2)*(A*a^3 - 15*B*a^3)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(9/2)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^3*sqrt(c) + 3*B*a^3*sqrt(c) + 16*A*a^3*sq
rt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1) + 48*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 24*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 312*B*a^3*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 48*A*
```

```
a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^3 + 1392*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 250*A*a^3*sqrt(c)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 3750*B*a^3*
sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4/(c^5*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (48*sqrt(2)*A*a^
3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1392*sqrt(2)*B*a^3*c^(31/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 24*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^2 - 312*sqrt(2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)^2 - 16*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^3 - 48*sqrt(2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^3 - 3*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 3*
sqrt(2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/c^20)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2
), x)
```

3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{a^3(3A-17B)c\cos^5(e+}{80f(c-c\sin(e+fx))^{1}}$$

```
[Out] 1/10*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(17/2)+1/80*a^3*(3*A-17*
B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-1/96*a^3*(3*A-17*B)*cos(f*x+e)^
3/c/f/(c-c*sin(f*x+e))^(9/2)+1/128*a^3*(3*A-17*B)*cos(f*x+e)/c^3/f/(c-c*sin
(f*x+e))^(5/2)-1/512*a^3*(3*A-17*B)*cos(f*x+e)/c^4/f/(c-c*sin(f*x+e))^(3/2)
-1/1024*a^3*(3*A-17*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+
e))^(1/2))/c^(11/2)/f*2^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2938, 2759, 2729, 2728, 212}

$$-\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}+\frac{a^3c(3A-17B)\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}-\frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/
2),x]
```

```
[Out] -1/512*(a^3*(3*A - 17*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c
*Sin[e + f*x]])])/(Sqrt[2]*c^(11/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(
10*f*(c - c*Sin[e + f*x])^(17/2)) + (a^3*(3*A - 17*B)*c*Cos[e + f*x]^5)/(80
*f*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x]^3)/(96*c*f
*(c - c*Sin[e + f*x])^(9/2)) + (a^3*(3*A - 17*B)*Cos[e + f*x])/(128*c^3*f*(
c - c*Sin[e + f*x])^(5/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x])/(512*c^4*f*(c
- c*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{1}{20}\left(a^3(3A-17B)c^2\right)\int \frac{}{(c-}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}$$

$$= -\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f} + \frac{a^3}{10}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.67, size = 409, normalized size = 1.54

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(56370A\cos\left(\frac{1}{2}(e+fx)\right)+38970B\cos\left(\frac{1}{2}(e+fx)\right)-31140A\cos\left(\frac{3}{2}(e+fx)\right)-38580B\cos\left(\frac{3}{2}(e+fx)\right)-10404A\cos\left(\frac{5}{2}(e+fx)\right)-12724B\cos\left(\frac{5}{2}(e+fx)\right)+435A\cos\left(\frac{7}{2}(e+fx)\right)+7775B\cos\left(\frac{7}{2}(e+fx)\right)-45A\cos\left(\frac{9}{2}(e+fx)\right)+255B\cos\left(\frac{9}{2}(e+fx)\right)+(240+240i)\sqrt[4]{-1}(3A-17B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}+56370A\sin\left(\frac{1}{2}(e+fx)\right)+38970B\sin\left(\frac{1}{2}(e+fx)\right)+31140A\sin\left(\frac{3}{2}(e+fx)\right)+38580B\sin\left(\frac{3}{2}(e+fx)\right)-10404A\sin\left(\frac{5}{2}(e+fx)\right)-12724B\sin\left(\frac{5}{2}(e+fx)\right)-435A\sin\left(\frac{7}{2}(e+fx)\right)-7775B\sin\left(\frac{7}{2}(e+fx)\right)-45A\sin\left(\frac{9}{2}(e+fx)\right)+255B\sin\left(\frac{9}{2}(e+fx)\right)\right)}{122880f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(56370*A*Co
s[(e + f*x)/2] + 38970*B*Cos[(e + f*x)/2] - 31140*A*Cos[(3*(e + f*x))/2] -
38580*B*Cos[(3*(e + f*x))/2] - 10404*A*Cos[(5*(e + f*x))/2] - 12724*B*Cos[(
5*(e + f*x))/2] + 435*A*Cos[(7*(e + f*x))/2] + 7775*B*Cos[(7*(e + f*x))/2]
- 45*A*Cos[(9*(e + f*x))/2] + 255*B*Cos[(9*(e + f*x))/2] + (240 + 240*I)*(-
1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10 + 56370*A*Sin[(e + f*x)/2] + 3897
0*B*Sin[(e + f*x)/2] + 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(3*(e + f
*x))/2] - 10404*A*Sin[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))/2] - 435
*A*Sin[(7*(e + f*x))/2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[(9*(e + f*
```

```
x))/2] + 255*B*Sin[(9*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^6*(c - c*Sin[e + f*x])^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(235) = 470$.
time = 10.56, size = 526, normalized size = 1.98

method	result
default	$a^3 \left(15 \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\, c^6 (3A-17B)\sin(fx+e)\left(\cos^4(fx+e)\right)-180\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx}{2\sqrt{c}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/15360*a^3*(15*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)
*c^6*(3*A-17*B)*sin(f*x+e)*cos(f*x+e)^4-180*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^6*(3*A-17*B)*cos(f*x+e)^2*sin(f*x+e)+240*arc
tanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^6*(3*A-17*B)*sin
(f*x+e)-75*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^6*
(3*A-17*B)*cos(f*x+e)^4+300*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*2^(1/2)*c^6*(3*A-17*B)*cos(f*x+e)^2-90*A*(c+c*sin(f*x+e))^(9/2)*c^(3/2
)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x+e))^(5/2)*c^(7/2
)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f*x+e))^(1/2)*c^(11
/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin(f*x+e))^(7/2)*c^(5
/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c*sin(f*x+e))^(3/2)*c
^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/c^(23/2)/(si
n(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(11/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(247) = 494$.

time = 0.39, size = 806, normalized size = 3.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3*
cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3*c
os(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3*co
s(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6*(3*
A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32*(3*A
 - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(3*A -
17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c
*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x
+ e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B)*a^3*cos(
f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 389*B)*a^3*co
s(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*A + 43*B)*a^
3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 8
0*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*cos(f*x + e)^2 -
 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin(f*x + e))*sqrt
(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*
c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 -
16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x
+ e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f
*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8856 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. 2(247) = 494.
time = 0.76, size = 1037, normalized size = 3.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] -1/245760*(120*sqrt(2)*(3*A*a^3*sqrt(c) - 17*B*a^3*sqrt(c))*log(-(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^6*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(6*A*a^3*sqrt(c) + 6*B*a^3*sqrt(c
) + 15*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1) + 75*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 30*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 290*B*a^3*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 - 120*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 360*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 60*A*a^3*sqrt(c)*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4
 - 900*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^4 + 822*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 4658*B*a^3*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5/(c^6*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(60*A*a^3*c^(49/2)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 900
*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 120*A*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 360*B*a^3*c^(49/2)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 30*A*a^3*c
^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^3 + 290*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 75*B*a^3*c^(49/
2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^4 + 6*A*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^5 + 6*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5)/(c^30*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/
2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/
2), x)
```

3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=200

$$-\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}-\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{12(7A-9B)c^2\cos(e+fx)}{35a}$$

```
[Out] -12/35*(7*A-9*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-1/7*(7*A-9*B)*c*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(9/
2)/a/c/f-128/35*(7*A-9*B)*c^4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-32/35*(
7*A-9*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]

time = 0.26, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2726, 2725}

$$-\frac{128c^4(7A-9B)\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}-\frac{32c^3(7A-9B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{12c^2(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}-\frac{c(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-128*(7*A - 9*B)*c^4*Cos[e + f*x])/(35*a*f*Sqrt[c - c*Sin[e + f*x]]) - (32
*(7*A - 9*B)*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(35*a*f) - (12*(7*A
 - 9*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(35*a*f) - ((7*A - 9*B
)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*a*f) - ((A - B)*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(9/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
```

```
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{ac} \\
&= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} - \frac{(7A-9B}{} \\
&= -\frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} - \frac{(A-}{} \\
&= -\frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} - \frac{(7}{} \\
&= -\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af} - \frac{12(}{} \\
&= -\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}} - \frac{32(7A-9B)c^3\cos(e+}{3}
\end{aligned}
$$

Mathematica [A]
time = 3.51, size = 157, normalized size = 0.78

$$
-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(4900A-6125B+196(A-2B)\cos(2(e+fx))+5B\cos(4(e+fx))+2450A\sin(e+fx)-3430B\sin(e+fx)-14A\sin(3(e+fx))+58B\sin(3(e+fx)))}{140af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -1/140*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(4900*A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] +
```

```
2450*A*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B*Si
n[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*
x]))
```

Maple [A]
time = 4.51, size = 111, normalized size = 0.56

method	result
default	$\frac{2c^4(\sin(fx+e)-1)((-7A+29B)\sin(fx+e)(\cos^2(fx+e))+(308A-436B)\sin(fx+e)+5B(\cos^4(fx+e))+(49A-103B)(\cos^2(fx+e))+5}{35a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/35*c^4/a*(sin(f*x+e)-1)*((-7*A+29*B)*sin(f*x+e)*cos(f*x+e)^2+(308*A-436*B
)*sin(f*x+e)+5*B*cos(f*x+e)^4+(49*A-103*B)*cos(f*x+e)^2+588*A-716*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(192) = 384.
time = 0.57, size = 518, normalized size = 2.59

$$\frac{2\left(\frac{7\left(91c^{\frac{7}{2}}+\frac{86c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{336c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{266c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{490c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{266c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{336c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{86c^{\frac{7}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{91c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)A}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}-\frac{2\left(407c^{\frac{7}{2}}+\frac{407c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{1442c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{1337c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{2030c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{1337c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{1442c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{407c^{\frac{7}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{407c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)B}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}\right)}{35f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7
/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*x + e)
 + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(7/2) +
407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 20
30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Fricas [A]
time = 0.34, size = 121, normalized size = 0.60

$$-\frac{2\,\left(5\,Bc^{3}\cos\left(fx+e\right)^{4}+\left(49\,A-103\,B\right)c^{3}\cos\left(fx+e\right)^{2}+4\,\left(147\,A-179\,B\right)c^{3}-\left(\left(7\,A-29\,B\right)c^{3}\cos\left(fx+e\right)^{2}-4\,\left(77\,A-109\,B\right)c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{35\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147*
A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. 2(192) = 384.
time = 0.74, size = 821, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 16/35*sqrt(2)*sqrt(c)*(35*(A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c^
3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (77*A*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 109*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 504*
A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 728*B*c^3*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 1337*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 2009*B*c
^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1680*A*c^3*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f
```

```
*x + 1/2*e) + 1)^3 + 2800*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 10
15*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1015*B*c^3*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^4 - 280*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5
 + 280*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 + 35*A*c^3*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^6 - 35*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^
6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
6)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) - 1)^7))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),
 x)
```

3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=159

$$-\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}}-\frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}-\frac{(5A-7B)c\cos(e+fx)(c}{5af}$$

```
[Out] -1/5*(5*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-(A-B)*sec(f*x+e)*(c-
c*sin(f*x+e))^(7/2)/a/c/f-32/15*(5*A-7*B)*c^3*cos(f*x+e)/a/f/(c-c*sin(f*x+e
))^(1/2)-8/15*(5*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]

time = 0.23, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2726, 2725}

$$-\frac{32c^3(5A-7B)\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}}-\frac{8c^2(5A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}-\frac{c(5A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-32*(5*A - 7*B)*c^3*Cos[e + f*x])/(15*a*f*Sqrt[c - c*Sin[e + f*x]]) - (8*(
5*A - 7*B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a*f) - ((5*A - 7*
B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) - ((A - B)*Sec[e + f*
x]*(c - c*Sin[e + f*x])^(7/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
```

```
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,d}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf} - \frac{(5A-7B)}{}$$

$$= -\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} - \frac{(A-B}{}$$

$$= -\frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} - \frac{(5A-}{}$$

$$= -\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx}{15a}$$

Mathematica [A]
time = 1.12, size = 134, normalized size = 0.84

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(450A-600B+2(5A-16B)\cos(2(e+fx))+25(8A-13B)\sin(e+fx)+3B\sin(3(e+fx)))}{30af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
450*A - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e + f
*x] + 3*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
+ Sin[e + f*x]))
```

Maple [A]
time = 4.34, size = 95, normalized size = 0.60

method	result	size
default	$-\frac{2c^{3}(\sin(fx+e)-1)(-3B(\cos^{2}(fx+e))\sin(fx+e)+(-50A+82B)\sin(fx+e)+(-5A+16B)(\cos^{2}(fx+e))-110A+142B)}{15a\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\, f}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15*c^3/a*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2*sin(f*x+e)+(-50*A+82*B)*sin(f
*x+e)+(-5*A+16*B)*cos(f*x+e)^2-110*A+142*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(153) = 306$.
time = 0.54, size = 418, normalized size = 2.63

$$\frac{2\left(\frac{5\left(23\,c^{\frac{5}{2}}+\frac{20\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{65\,c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{40\,c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{65\,c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{20\,c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{23\,c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}\right)A}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}-\frac{2\left(79\,c^{\frac{5}{2}}+\frac{79\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{205\,c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{170\,c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{205\,c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{79\,c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{79\,c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}\right)B}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/
2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(5/2)) - 2*(79*c^(5/2) + 79*c^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) + 205*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 170*c^(5/
2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 205*c^(5/2)*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 79*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 79*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Fricas [A]
time = 0.35, size = 100, normalized size = 0.63

$$-\frac{2\left((5\,A-16\,B)c^{2}\cos\left(fx+e\right)^{2}+2\,(55\,A-71\,B)c^{2}+\left(3\,Bc^{2}\cos\left(fx+e\right)^{2}+2\,(25\,A-41\,B)c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*cos
(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/
(a*f*cos(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(153) = 306.
time = 0.68, size = 605, normalized size = 3.81

$$\frac{8\sqrt{2}\sqrt{c}\left(\frac{15\left(Ac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bc^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{a\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)}-\frac{25Ac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-41Bc^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{110Ac^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{190Bc^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{160Ac^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{320Bc^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{90Ac^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{90Bc^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{15Ac^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}-\frac{15Bc^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}}{a\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^5}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 8/15*sqrt(2)*sqrt(c)*(15*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (25*A*c^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - 41*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 110*A*
c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 190*B*c^2*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 160*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 320*B*c^2*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^3 + 90*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*c^2*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 1)^5))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),
 x)
```

3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e$$

[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/c/f-4/3*(3*A-5*B)*c^2*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-1/3*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f

Rubi [A]
time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2726, 2725}

$$-\frac{4c^2(3A-5B)\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{c(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]), x]
```

```
[Out] (-4*(3*A - 5*B)*c^2*Cos[e + f*x])/(3*a*f*Sqrt[c - c*Sin[e + f*x]]) - ((3*A - 5*B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) - ((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n), Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1))), x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
```

```
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf} - \frac{(3A-5B}{}$$

$$= -\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} - \frac{(A-B}{}$$

$$= -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)}{3af}$$

Mathematica [A]
time = 0.42, size = 113, normalized size = 0.96

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-18A+27B+B\cos(2(e+fx))+(-6A+14B)\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{3af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)]
 + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

Maple [A]
time = 4.13, size = 73, normalized size = 0.62

method	result	size

default	$\frac{2c^2(\sin(fx+e)-1)(\sin(fx+e)(3A-7B)-B(\cos^2(fx+e))+9A-13B)}{3a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3*c^2/a*(sin(f*x+e)-1)*(sin(f*x+e)*(3*A-7*B)-B*cos(f*x+e)^2+9*A-13*B)/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(114) = 228$.
time = 0.51, size = 318, normalized size = 2.69

$$\frac{2\left(\frac{3\left(3c^{\frac{3}{2}}+\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)A}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}-\frac{2\left(7c^{\frac{3}{2}}+\frac{7c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{12c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{7c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)B}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*A/((a + a*sin(f*x
+ e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) -
 2*(7*c^(3/2) + 7*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 12*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 7*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*B/((a + a*sin(f*x + e
)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A]
time = 0.34, size = 71, normalized size = 0.60

$$\frac{2\left(Bc\cos\left(fx+e\right)^2-(3A-7B)c\sin\left(fx+e\right)-(9A-13B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{Ac\sqrt{-c\sin(e+fx)+c}}{\sin(e+fx)+1}\,dx+\int\left(-\frac{Ac\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin(e+fx)+1}\right)dx+\int\frac{Bc\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin(e+fx)+1}\,dx+\int\left(-\frac{Bc\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)}{\sin(e+fx)+1}\right)dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(-
A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integra
l(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integ
ral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x) + 1), x))/
a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(114) = 228.
time = 0.59, size = 373, normalized size = 3.16

$$\frac{4\sqrt{2}\sqrt{c}\left(\frac{3\left(Ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{a\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)}-\frac{3\,Ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\,Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{6\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{18\,Bc\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{3\,Bc\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}}{a\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^3}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 4/3*sqrt(2)*sqrt(c)*(3*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (3*A*c*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) - 7*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 18*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 1)^3))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{a+a\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),
 x)
```

3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=73

$$-\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

```
[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/c/f-(A-3*B)*c*cos(f*x+e)/a/f/(c-
c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3046, 2934, 2725}

$$-\frac{c(A-3B)\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 3*B)*c*Cos[e + f*x])/(a*f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*Sec
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a*c*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf} - \frac{(A-3B)\int$$
$$= -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec(e+fx)(c-c\sin(}{acf}$$

Mathematica [A]
time = 0.16, size = 44, normalized size = 0.60

$$\frac{2\sec(e+fx)(-A+2B+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x]),x]
```

```
[Out] (2*Sec[e + f*x]*(-A + 2*B + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Maple [A]
time = 3.78, size = 53, normalized size = 0.73

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(-B\sin(fx+e)+A-2B)}{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2*c/a*(sin(f*x+e)-1)*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(73) = 146$.

time = 0.57, size = 188, normalized size = 2.58

$$-\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}-\frac{A\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqr
t(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqrt(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Fricas [**A**]
time = 0.35, size = 47, normalized size = 0.64

$$\frac{2\left(B\sin\left(fx+e\right)-A+2B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2*(B*sin(f*x + e) - A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int\frac{B\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(73) = 146.
time = 0.50, size = 185, normalized size = 2.53

$$-\frac{2\sqrt{2}\left(A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{A\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\sqrt{c}}{af\left(\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -2*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - A*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - B*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1))*sqrt(c)/(a*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1))
```

Mupad [B]
time = 13.15, size = 128, normalized size = 1.75

$$\frac{2\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\,B\,\sin\left(2\,e+2\,f\,x\right)-2\,A\,\sin\left(2\,e+2\,f\,x\right)-4\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+7\,B\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+B\,\left(2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-1\right)\right)}{a\,f\,\left(4\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] (2*(-c*(sin(e + f*x) - 1))^(1/2)*(2*B*sin(2*e + 2*f*x) - 2*A*sin(2*e + 2*f*
x) - 4*A*(2*sin(e/2 + (f*x)/2)^2 - 1) + 7*B*(2*sin(e/2 + (f*x)/2)^2 - 1) +
B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a*f*(sin(e + f*x) + sin(3*e + 3*f*x
) + 4*sin(e + f*x)^2 - 4))
```

3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] 1/2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/
f*2^(1/2)/c^(1/2)-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

Rubi [A]
time = 0.18, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2728, 212}

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(Sqrt[2]*a*Sqrt[c]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
```

```
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} + \frac{(A+B)\int \ldots}{\ldots}$$

$$= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} - \frac{(A+B)\text{Subst}\ldots}{\ldots}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f} - \frac{(A-B)\sec\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 140, normalized size = 1.54

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+B-(1+i)\sqrt[4]{-1}(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af(1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e +
 f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 6.87, size = 130, normalized size = 1.43

method	result
default	$-\frac{(\sin(fx+e)-1)\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}\,A+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{}\right.\right.}{2a\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)*B)
/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(85) = 170$.
time = 0.35, size = 176, normalized size = 1.93

$$\frac{\sqrt{2}\,(A+B)\sqrt{c}\,\cos(fx+e)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\,\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx+e)+2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\sqrt{-c\sin(fx+e)+c}\,(A-B)}{4\,acf\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x +
 e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
 sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
 + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c)*
(A - B))/(a*c*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
+ c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + sqrt(-c*sin(e + f*x) + c)), x))/a
```

Giac [A]
time = 0.51, size = 154, normalized size = 1.69

$$\frac{\frac{\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\sqrt{2}\left(A\sqrt{c}-B\sqrt{c}\right)}{ac\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/4*(sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))) + 4*sqrt(2)*(A*sqrt(c) - B*sqrt(c))/(a*c*((cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
 x)
```

3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+1/8*(3*A-B)*arctanh(1/2*c
os(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-(A-B)
*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3046, 2934, 2729, 2728, 212}

$$\frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] ((3*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(4*Sqrt[2]*a*c^(3/2)*f) + ((3*A - B)*Cos[e + f*x])/(4*a*f*(c - c*Sin[e
+ f*x])^(3/2)) - ((A - B)*Sec[e + f*x])/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{(3A}{}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} - \frac{(3A}{}$$

$$= \frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f} + \frac{(3A-B}{4af(c-c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 284, normalized size = 2.09

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-(1+i)\sqrt[4]{-1}(3A-B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(A+B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{4af(1+\sin(e+fx))(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (1 + I)*
(-1)^(1/4)*(3*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]) + 2*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(4*
a*f*(1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A]
time = 5.90, size = 225, normalized size = 1.65

method	result
default	$-\frac{\sin(fx+e)\left(3A\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c-B\sqrt{c+c\sin(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*
arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-6*A*c^(3/2)+2*B*c^(3/
2))-3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)/c^(1/2))*c+B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x
+e))^(1/2)*2^(1/2)/c^(1/2))*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

Fricas [**A**]
time = 0.36, size = 250, normalized size = 1.84

$$-\frac{\sqrt{2}\left((3A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-(3A-B)\cos\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos(fx+e)+\sin(fx+e)+1\right)+3c\cos(fx+e)+\left(c\cos(fx+e)-2c\right)\sin(fx+e)+2c}{\cos(fx+e)^2+\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left((3A-B)\sin\left(fx+e\right)-A+3B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{16\left(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x + e
))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqr
t(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e)
 - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c*sin(
f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] (Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
 + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(125) = 250$.
time = 0.51, size = 426, normalized size = 3.13

$$\frac{\frac{2\sqrt{2}\left(3A\sqrt{c}-B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(A+B+\frac{14A\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{14B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3A\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{ac^{\frac{3}{2}}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/32*(2*sqrt(2)*(3*A*sqrt(c) - B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^2*sgn(sin(-1/4*pi + 1/
```

```
2*f*x + 1/2*e))) + sqrt(2)*(A + B + 14*A*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 14*B*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*A*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + B*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a*c^(3/2)*(
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) +
 (cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),
 x)
```

3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=180

$$\frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}-$$

```
[Out] 3/32*(5*A-3*B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A+B)*sec(f*x+e)
/a/c/f/(c-c*sin(f*x+e))^(3/2)+3/64*(5*A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)
*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(5/2)/f*2^(1/2)-1/8*(5*A-3*B)*sec(f*x+
e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2938, 2766, 2729, 2728, 212}

$$\frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}-\frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (3*(5*A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(32*Sqrt[2]*a*c^(5/2)*f) + (3*(5*A - 3*B)*Cos[e + f*x])/(32*a*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((A + B)*Sec[e + f*x])/(4*a*c*f*(c - c*Sin[e +
 f*x])^(3/2)) - ((5*A - 3*B)*Sec[e + f*x])/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{(5A-3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+}}}{8ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} +$$

$$= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{3(5A}{32acf(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.58, size = 404, normalized size = 2.24

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(7A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-(3+3i)\sqrt[4]{-1}(5A-3B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(7A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{32af(1+\sin(e+fx))(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*A -
 B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
+ Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]) + 8*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]) + 2*(7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(32*a*f*(1 + Sin[e + f*
x])*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(157) = 314.
time = 8.98, size = 350, normalized size = 1.94

method	result
default	$-\frac{\sin(fx+e)\left(-30A\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}+40Ac^{\frac{5}{2}}+18B\sqrt{c+c\sin\left(fx+e\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/64/c^(9/2)/a*(sin(f*x+e)*(-30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*A*c^(5/2)+18*B*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^
2-24*B*c^(5/2))+(-15*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+30*A*c^(5/2)+9*B*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*c^(5/2
))*cos(f*x+e)^2+30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-24*A*c^(5/2)-18*B*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*B*c^(5/2)
)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

Fricas [A]
time = 0.37, size = 304, normalized size = 1.69

$$-\frac{3\sqrt{2}\left((5A-3B)\cos\left(fx+e\right)^{3}+2\,(5A-3B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(5A-3B)\cos\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^{2}-2\sqrt{2}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3\,c\cos(fx+e)+(c\cos(fx+e)-2\,c)\sin(fx+e)+2\,c}{\cos(fx+e)^{2}+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(3\,(5A-3B)\cos\left(fx+e\right)^{2}+4\,(5A-3B)\sin\left(fx+e\right)-12A+20B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{128\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/128*(3*sqrt(2)*((5*A - 3*B)*cos(f*x + e)^3 + 2*(5*A - 3*B)*cos(f*x + e)*
sin(f*x + e) - 2*(5*A - 3*B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 -
```

```
 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) +
 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*
x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(5*A
 - 3*B)*cos(f*x + e)^2 + 4*(5*A - 3*B)*sin(f*x + e) - 12*A + 20*B)*sqrt(-c*
sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x
 + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(165) = 330.
time = 0.63, size = 528, normalized size = 2.93

$$\frac{\frac{12\sqrt{2}\left(5A\sqrt{c}-3B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}-\frac{16A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{90A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{54B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{ac^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{128\sqrt{2}\left(A\sqrt{c}-B\sqrt{c}\right)}{ac^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\frac{16\sqrt{2}Aac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\sqrt{2}Aac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{\sqrt{2}Bac^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}}{a^2c^6}}{512f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/512*(12*sqrt(2)*(5*A*sqrt(c) - 3*B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 16*A*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 90*A*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2 - 54*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(a*c^3*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 + 128*sqrt(2)*(A*sqrt(c) - B*sqrt(c))/(a*c^3*((cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) - (16*sqrt(2)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 sqrt(2)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - sqrt(2)*B*a*c
^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^6))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),
 x)
```

3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=242

$$\frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f}-\frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f}-64$$

```
[Out] -512/105*(7*A-13*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-64/105*(7*A
-13*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-16/105*(7*A-13*B)*c*sec(
f*x+e)*(c-c*sin(f*x+e))^(7/2)/a^2/f-1/21*(7*A-13*B)*sec(f*x+e)*(c-c*sin(f*x
+e))^(9/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(13/2)/a^2/c^2/f+2
048/105*(7*A-13*B)*c^4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]

time = 0.44, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$\frac{2048c^4(7A-13B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f}-\frac{512c^3(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}-\frac{64c^2(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f}-\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f}-\frac{16c(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (2048*(7*A - 13*B)*c^4*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*a^2*f) -
 (512*(7*A - 13*B)*c^3*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(105*a^2*f)
 - (64*(7*A - 13*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(105*a^2*f
) - (16*(7*A - 13*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*a^2*f)
 - ((7*A - 13*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(21*a^2*f) - ((A
- B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
```

```
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/}}{a^2c^2} \\
&= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f} - \frac{(7A-}{} \\
&= -\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-}{} \\
&= -\frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7}{} \\
&= -\frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{1}{} \\
&= -\frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \\
&= \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f} - \frac{5}{}
\end{aligned}$$

Mathematica [A]
time = 4.68, size = 176, normalized size = 0.73

$$\frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(95550A-179340B-72(203A-402B)\cos(2(e+fx))+6(7A-38B)\cos(4(e+fx))+119952A\sin(e+fx)-219618B\sin(e+fx)+784A\sin(3(e+fx))-2131B\sin(3(e+fx))+15B\sin(5(e+fx)))}{840a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(95550*
A - 179340*B - 72*(203*A - 402*B)*Cos[2*(e + f*x)] + 6*(7*A - 38*B)*Cos[4*(
e + f*x)] + 119952*A*Sin[e + f*x] - 219618*B*Sin[e + f*x] + 784*A*Sin[3*(e
+ f*x)] - 2131*B*Sin[3*(e + f*x)] + 15*B*Sin[5*(e + f*x)]))/(840*a^2*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 5.53, size = 143, normalized size = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)(15B\sin(fx+e)(\cos^4(fx+e))+(196A-544B)(\cos^2(fx+e))\sin(fx+e)+(7448A-13592B)\sin(fx+e)+(21A-114}{105a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/105*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(15*B*sin(f*x+e)*cos(f*x+e)^4+
(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(7448*A-13592*B)*sin(f*x+e)+(21*A-114
*B)*cos(f*x+e)^4+(-1848*A+3732*B)*cos(f*x+e)^2+6888*A-13032*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(230) = 460$.
time = 0.56, size = 826, normalized size = 3.41

$$-\frac{2\left(\frac{7\left(723c^{\frac{9}{2}}+\frac{2184c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{5370c^{\frac{9}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10696c^{\frac{9}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15021c^{\frac{9}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{21168c^{\frac{9}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{20748c^{\frac{9}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{21168c^{\frac{9}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{15021c^{\frac{9}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{10696c^{\frac{9}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{5370c^{\frac{9}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{2184c^{\frac{9}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{723c^{\frac{9}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)A}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{9}{2}}}-\frac{2\left(4707c^{\frac{9}{2}}+\frac{14121c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{35250c^{\frac{9}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{68549c^{\frac{9}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{99549c^{\frac{9}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{134802c^{\frac{9}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{138012c^{\frac{9}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{134802c^{\frac{9}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{99549c^{\frac{9}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{68549c^{\frac{9}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{35250c^{\frac{9}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{14121c^{\frac{9}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{4707c^{\frac{9}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)B}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{9}{2}}}\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 537
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(
9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^2 + 3*a^2*sin(f*x + e)/(c
```

```
os(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2
)) - 2*(4707*c^(9/2) + 14121*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3525
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 68549*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 99549*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 134802*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 138012*c^(9/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 134802*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 99549*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 68549*c^(9/2
)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 35250*c^(9/2)*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 14121*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 47
07*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*B/((a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)^(9/2)))/f
```

Fricas [A]
time = 0.36, size = 162, normalized size = 0.67

$$\frac{2\left(3\left(7A-38B\right)c^{4}\cos\left(fx+e\right)^{4}-12\left(154A-311B\right)c^{4}\cos\left(fx+e\right)^{2}+24\left(287A-543B\right)c^{4}+\left(15Bc^{4}\cos\left(fx+e\right)^{4}+4\left(49A-136B\right)c^{4}\cos\left(fx+e\right)^{2}+8\left(931A-1699B\right)c^{4}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{105\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x +
 e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136*B
)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8856 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(230) = 460.
time = 0.82, size = 1034, normalized size = 4.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -16/105*sqrt(2)*sqrt(c)*(35*(11*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 17*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*c^4*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 36*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*c^4*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 1)^3) - (511*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 1069*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3262*A*c^4*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) + 6958*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 842
1*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 18459*B*c^4*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^2 - 10780*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^3 + 24220*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 7105*A*c^4*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 13195*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4 - 2310*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 + 3990*B*c^4
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 + 315*A*c^4*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^6 - 525*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^6*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^6)/(a^2*((
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
1)^7))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{9/2}}{(a+a\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
2, x)
```

3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f}-\frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}-\frac{4(5A}{}$$

```
[Out] -32/15*(5*A-11*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-4/15*(5*A-11*
B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/15*(5*A-11*B)*sec(f*x+e)*(c-
c*sin(f*x+e))^(7/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^
2/c^2/f+128/15*(5*A-11*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]

time = 0.37, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$\frac{128c^3(5A-11B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}-\frac{32c^2(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}-\frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f}-\frac{4c(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (128*(5*A - 11*B)*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a^2*f) - (
32*(5*A - 11*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (
4*(5*A - 11*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*a^2*f) - ((5*
A - 11*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*a^2*f) - ((A - B)*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/}}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{(5A-}{}$$

$$= -\frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-}{}$$

$$= -\frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A}{}$$

$$= -\frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4}{}$$

$$= \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(}{}$$

Mathematica [A]

time = 1.89, size = 159, normalized size = 0.79

$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(-2100A+4725B+12(25A-62B)\cos(2(e+fx))+3B\cos(4(e+fx))-2730A\sin(e+fx)+5838B\sin(e+fx)-10A\sin(3(e+fx))+46B\sin(3(e+fx)))}{60a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
-2100*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)]
 - 2730*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 46*B
*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
 + f*x])^2)
```

Maple [A]
time = 7.25, size = 121, normalized size = 0.60

method	result
default	$\frac{2c^4(\sin(fx+e)-1)((-5A+23B)\sin(fx+e)(\cos^2(fx+e))+(-340A+724B)\sin(fx+e)+3B(\cos^4(fx+e))+(75A-189B)(\cos^2(fx+e))}{15a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*((-5*A+23*B)*sin(f*x+e)*cos(f*x+
e)^2+(-340*A+724*B)*sin(f*x+e)+3*B*cos(f*x+e)^4+(75*A-189*B)*cos(f*x+e)^2-3
00*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(191) = 382.
time = 0.61, size = 726, normalized size = 3.61

$$-\frac{2\left(\frac{5\left(45c^{\frac{7}{2}}+\frac{138c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{285c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{544c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{630c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{812c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{630c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{544c^{\frac{7}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{285c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{138c^{\frac{7}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{45c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)A}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}-\frac{2\left(249c^{\frac{7}{2}}+\frac{747c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{1611c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2896c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3612c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4298c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{3612c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{2896c^{\frac{7}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{1611c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{747c^{\frac{7}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{249c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)B}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^
(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^
(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2896*c
^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 361
2*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*sin(f*x + e)^7
```

```
/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/2)*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Fricas [**A**]
time = 0.37, size = 141, normalized size = 0.70

$$-\frac{2\,\left(3\,Bc^{3}\cos\left(fx+e\right)^{4}+3\,(25\,A-63\,B)c^{3}\cos\left(fx+e\right)^{2}-12\,(25\,A-57\,B)c^{3}-\left((5\,A-23\,B)c^{3}\cos\left(fx+e\right)^{2}+4\,(85\,A-181\,B)c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\,\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(25
*A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a
^2*f*cos(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(191) = 382.
time = 0.75, size = 818, normalized size = 4.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -16/15*sqrt(2)*sqrt(c)*(5*(4*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*
B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*A*c^3*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1
```

```
/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 6*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 1)^3) - (20*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 53*B*
c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 85*A*c^3*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 235*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 125*A*c^3*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 365*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2 - 75*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 165*B*c^3*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^4 - 30*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^2*((cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5))
/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\sin(e + fx))\,(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
2, x)
```

3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{(A-3B)\,\mathrm{s}}{}$$

```
[Out] -8/3*(A-3*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-3*B)*sec(f*x+
e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/
2)/a^2/c^2/f+32/3*(A-3*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]

time = 0.32, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}+\frac{32c^2(A-3B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f}-\frac{8c(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (32*(A - 3*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - (8*(A
- 3*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - 3*B)*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(9/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
```

```
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,d}{a^2c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} - \frac{(A-3B)}{}$$
$$= -\frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)}{}$$
$$= -\frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3}{}$$
$$= \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3}{}$$

Mathematica [A]
time = 0.80, size = 130, normalized size = 0.84

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(-50A+160B+6(A-4B)\cos(2(e+fx))+(-72A+201B)\sin(e+fx)+B\sin(3(e+fx)))}{6a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -1/6*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-
50*A + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x]
+ B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Si
n[e + f*x])^2)
```

Maple [A]
time = 5.92, size = 105, normalized size = 0.68

method	result	size
default	$-\frac{2c^{3}(\sin(fx+e)-1)\left(-B(\cos^{2}(fx+e))\sin(fx+e)+(18A-50B)\sin(fx+e)+(-3A+12B)(\cos^{2}(fx+e))+14A-46B\right)}{3a^{2}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\,f}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(-B*cos(f*x+e)^2*sin(f*x+e)+(18*
A-50*B)*sin(f*x+e)+(-3*A+12*B)*cos(f*x+e)^2+14*A-46*B)/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(146) = 292$.
time = 0.54, size = 625, normalized size = 4.06

$$-\frac{2\left(\frac{\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{90\,c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{11\,c^{\frac{5}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}\right)A}{\left(a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}-\frac{2\left(17\,c^{\frac{5}{2}}+\frac{51\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{92\,c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{149\,c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{150\,c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{149\,c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{92\,c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{51\,c^{\frac{5}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{17\,c^{\frac{5}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}\right)B}{\left(a^{2}+\frac{3\,a^{2}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^{2}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(17
*c^(5/2) + 51*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 92*c^(5/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 149*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 150*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 149*c^(5/2)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 92*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 + 51*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(5/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Fricas [A]
time = 0.36, size = 117, normalized size = 0.76

$$-\frac{2\left(3\,(A-4\,B)c^{2}\cos\left(fx+e\right)^{2}-2\,(7\,A-23\,B)c^{2}+\left(Bc^{2}\cos\left(fx+e\right)^{2}-2\,(9\,A-25\,B)c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x
+ e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f
*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(146) = 292.
time = 0.62, size = 309, normalized size = 2.01

$$\frac{32\,\sqrt{2}\left(Ac^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,Bc^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{3\,Ac^{2}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}+\frac{9\,Bc^{2}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}+\frac{2\,Ac^{2}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{2\,Bc^{2}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}\right)\sqrt{c}}{3\,a^{2}f\left(\frac{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 32/3*sqrt(2)*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 3*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
 + 9*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 2*A*c^2*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^3 + 2*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*s
qrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^2 - 1)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
2, x)
```

3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=115

$$\frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{(A-B)\sec^3(e+$$

```
[Out] -1/3*(A-7*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3
*(c-c*sin(f*x+e))^(7/2)/a^2/c^2/f+4/3*(A-7*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))
^(1/2)/a^2/f
```

Rubi [A]
time = 0.27, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{4c(A-7B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (4*(A - 7*B)*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - 7*B
)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x
]^3*(c - c*Sin[e + f*x])^(7/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
```

```
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f} - \frac{(A-7B}{}$$

$$= -\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B}{}$$

$$= \frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-7B}{}$$

Mathematica [A]
time = 0.48, size = 113, normalized size = 0.98

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(2A-23B+3B\cos(2(e+fx))+6(A-5B)\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)]
 + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 6.46, size = 81, normalized size = 0.70

method	result	size
default	$-\frac{2c^2(\sin(fx+e)-1)(\sin(fx+e)(3A-15B)+3B(\cos^2(fx+e))+A-13B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(sin(f*x+e)*(3*A-15*B)+3*B*cos(f
*x+e)^2+A-13*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(109) = 218$.
time = 0.54, size = 522, normalized size = 4.54

$$-\frac{2\left(\frac{\left(c^{\frac{3}{2}}+\frac{6c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{12c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)A}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}-\frac{2\left(5c^{\frac{3}{2}}+\frac{15c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{21c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{21c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{15c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{5c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)B}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*A
/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*sin(f*x + e)/(
cos(f*x + e) + 1) + 21*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*c^(
3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 21*c^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^(3/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A]
time = 0.35, size = 86, normalized size = 0.75

$$\frac{2\left(3\,Bc\cos\left(fx+e\right)^2+3\,(A-5\,B)c\sin\left(fx+e\right)+(A-13\,B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{Ac\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{Ac\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\right)dx+\int \frac{Bc\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{Bc\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\left(e+fx\right)}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\right)dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) +
 1), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x
)**2 + 2*sin(e + f*x) + 1), x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin
(e + f*x)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(-B*c*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
 x))/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(109) = 218.
time = 0.62, size = 299, normalized size = 2.60

$$-\frac{4\,\sqrt{2}\,\sqrt{c}\left(\frac{3\,B c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)}+\frac{A c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,B c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{3\,Ac\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-\frac{9\,Bc\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-\frac{3\,Bc\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}}{a^2\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -4/3*sqrt(2)*sqrt(c)*(3*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^2*((cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1))
+ (A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + 3*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 9*B*c*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) - 3*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2
*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
 + 1)^3))/f
```

Mupad [B]
time = 17.18, size = 492, normalized size = 4.28

$$-\frac{\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{2\,B\,c}{a^2\,f}-\frac{B\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}}{a^2\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{2\,B\,c}{3\,a^2\,f}-\frac{c\,\left(2\,A-3\,B\right)}{3\,a^2\,f}-\frac{2\,c\,\left(3\,A-2\,B\right)}{3\,a^2\,f}+\frac{c\,\left(A\,2\mathrm{i}-B\,3\mathrm{i}\right)\,1\mathrm{i}}{3\,a^2\,f}+\frac{c\,\left(A\,3\mathrm{i}-B\,2\mathrm{i}\right)\,2\mathrm{i}}{3\,a^2\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^3}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{c\,\left(A-B\right)\,4\mathrm{i}}{a^2\,f}+\frac{c\,\left(A\,1\mathrm{i}-B\,2\mathrm{i}\right)}{a^2\,f}+\frac{c\,\left(A\,1\mathrm{i}+B\,2\mathrm{i}\right)}{3\,a^2\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{4\,B\,c}{a^2\,f}+\frac{c\,\left(A\,1\mathrm{i}-B\,2\mathrm{i}\right)\,4\mathrm{i}}{a^2\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((2*B*c)/(3*a^2*f) - (c*(2*A - 3*B))/(3*a^2*f) - (2*c*(3*A
 - 2*B))/(3*a^2*f) + (c*(A*2i - B*3i)*1i)/(3*a^2*f) + (c*(A*3i - B*2i)*2i)/
(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - ((c -
 c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((2*B*c
)/(a^2*f) - (B*c*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i) - 1i)
 - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*((c*(A - B)*4i)/(a^2*f) + (c*(A*1i - B*2i))/(a^2*f) + (
c*(A*1i + B*2i))/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
 + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2)*((4*B*c)/(a^2*f) + (c*(A*1i - B*2i)*4i)/(a^2*f
)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=78

$$-\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^2/c^2/f-1/3*(A+5*B)*sec(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]

time = 0.20, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3046, 2934, 2752}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}-\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] -1/3*((A + 5*B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a^2*f) - ((A - B)*S
ec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*c^2*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f} + \frac{(A+5B)}{}$$

$$= -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec}{}$$

Mathematica [A]
time = 0.22, size = 87, normalized size = 1.12

$$-\frac{2(A+2B+3B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^2,x]
```

```
[Out] (-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A]
time = 5.18, size = 63, normalized size = 0.81

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(3B\sin(fx+e)+A+2B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(74) = 148$.

time = 0.53, size = 371, normalized size = 4.76

$$\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{A\left(\sqrt{c}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] 2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sq
rt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Fricas [A]

time = 0.34, size = 65, normalized size = 0.83

$$-\frac{2\,(3\,B\sin(fx+e)+A+2\,B)\sqrt{-c\sin(fx+e)+c}}{3\,(a^2f\cos(fx+e)\sin(fx+e)+a^2f\cos(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x
+ e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sqrt{-c\sin(e+fx)+c}}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx+\int\frac{B\sqrt{-c\sin(e+fx)+c}\,\sin(e+fx)}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**2
+ 2*sin(e + f*x) + 1), x))/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(74) = 148.
time = 0.61, size = 236, normalized size = 3.03

$$\frac{\sqrt{2}\left(A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{12B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3A\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{3B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\sqrt{c}}{3a^2f\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/3*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 12*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2 + 3*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)
*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1) + 1)^3)
```

Mupad [B]
time = 17.51, size = 137, normalized size = 1.76

$$\frac{4\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(B\,3\mathrm{i}+2\,A\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+4\,B\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-B\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}\right)}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^3\,\left(1+{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*(B*3i + 2*A*exp(e*1i + f*x*1i) + 4*B*exp(e*1i + f*x*1i)
- B*exp(e*2i + f*x*2i)*3i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*1i
+ f*x*1i)*1i + 1))
```

3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}-\frac{(A-B)\sec^3(e+$$

```
[Out] -1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f+1/4*(A+B)*arctanh(
1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/f*2^(1/2)/c^(1/2
)-1/2*(A+B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f
```

Rubi [A]

time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2934, 2754, 2728, 212}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(2*Sqrt[2]*a^2*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(2*a^2*c*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2
*c^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{(A+B)\int}{}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3}{}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3}{}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A+B)\mathrm{se}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 176, normalized size = 1.30

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(-A+B)-3(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(3+3i)\sqrt[4]{-1}(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)}{6a^2f(1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 + 3*
I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sq
rt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 7.08, size = 168, normalized size = 1.24

method	result
default	$-\frac{(\sin(fx+e)-1)\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\,(1+\sin\,(fx+e))}\ \sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))^{\frac{3}{2}}cA-6A\,c^{\frac{5}{2}}\sin(fx+e)+3\sqrt{2}\ \operatorname{arct}}{12a^2c^{\frac{5}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/12*(sin(f*x+e)-1)*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A-6*A*c^(5/2)*sin(f*x+e)+3*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2
)*c*B-6*B*c^(5/2)*sin(f*x+e)-10*A*c^(5/2)-2*B*c^(5/2))/a^2/c^(5/2)/(1+sin(f
*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
 + c)), x)
```

Fricas [A]
time = 0.38, size = 236, normalized size = 1.75

$$\frac{3\sqrt{2}\left((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(3(A+B)\sin(fx+e)+5A+B\right)\sqrt{-c\sin(fx+e)+c}}{24\left(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx+\int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e +
f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e
+ f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x))/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(122) = 244.
time = 0.53, size = 271, normalized size = 2.01

$$\frac{\frac{3\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{8\sqrt{2}\left(2A\sqrt{c}+B\sqrt{c}+\frac{3A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))) + 8*sqrt(2)*(2*A*sqrt(c) + B*sqrt(c) + 3*A*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*B*sqrt(
c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
) + 3*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^2)/(a^2*c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
), x)
```

3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}-\frac{(A \cdots}{}$$

```
[Out] 1/8*(5*A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)+1/16*(5*A+B)*arctanh(1/
2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(3/2)/f*2^(1/2)-
1/6*(5*A+B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^
3*(c-c*sin(f*x+e))^(1/2)/a^2/c^2/f
```

Rubi [A]
time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3046, 2934, 2766, 2729, 2728, 212}

$$\frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((5*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(8*Sqrt[2]*a^2*c^(3/2)*f) + ((5*A + B)*Cos[e + f*x])/(8*a^2*f*(c - c*Si
n[e + f*x])^(3/2)) - ((5*A + B)*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e +
 f*x]]) - ((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{(5A+B)\,\int}{}$$

$$= -\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-}}{3a^2c^2f}$$

$$= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} -$$

$$= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} -$$

$$= \frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f} + \frac{(5A+}{8a^2f(c-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.58, size = 300, normalized size = 1.71

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-12A\cos^2(e+fx)+4(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-(3+3i)\sqrt[4]{-1}(5A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+6(A+B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)}{24a^2f(1+\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^3))/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(3/2))
```

Maple [A]
time = 6.74, size = 258, normalized size = 1.47

method	result

default	$-\frac{\sin(fx+e)\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)(c+c\sin(fx+e))^{\frac{3}{2}}cA+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/48/c^(7/2)/a^2*(sin(f*x+e)*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A+3*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*B-20*A*c^(5/2)-4*
B*c^(5/2))+(30*A*c^(5/2)+6*B*c^(5/2))*cos(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A-3*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)
*c*B-4*A*c^(5/2)-20*B*c^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.37, size = 222, normalized size = 1.27

$$\frac{3\sqrt{2}\,(5A+B)\sqrt{c}\,\cos(fx+e)^3\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\,\sqrt{-c\sin(fx+e)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(3(5A+B)\cos(fx+e)^2-2(5A+B)\sin(fx+e)-2A-10B\right)\sqrt{-c\sin(fx+e)+c}}{96a^2c^2f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2
*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1
) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x
+ e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A +
 B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(160) = 320.
time = 0.58, size = 523, normalized size = 2.99

$$\frac{\frac{6\sqrt{2}\left(5A\sqrt{c}+B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}-\frac{10A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{2B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2c^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{16\sqrt{2}\left(7A\sqrt{c}-B\sqrt{c}+\frac{12A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{3B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2c^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{192f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
[Out] 1/192*(6*sqrt(2)*(5*A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^2*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 10*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
2*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*c^2*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 16*sqrt(2)*(7*A*
sqrt(c) - B*sqrt(c) + 12*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c
^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
),x)
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
), x)
```

3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}+\frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}+\frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 5/64*(7*A-B)*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+1/24*(7*A-B)*sec(f*x
+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+5/128*(7*A-B)*arctanh(1/2*cos(f*x+e)*c^(
1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-5/48*(7*A-B)*sec
(f*x+e)/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^3/a^2/c^2/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3046, 2934, 2760, 2766, 2729, 2728, 212}

$$\frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}}+\frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (5*(7*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(64*Sqrt[2]*a^2*c^(5/2)*f) + (5*(7*A - B)*Cos[e + f*x])/(64*a^2*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((7*A - B)*Sec[e + f*x])/(24*a^2*c*f*(c - c*Si
n[e + f*x])^(3/2)) - (5*(7*A - B)*Sec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Si
n[e + f*x]]) - ((A - B)*Sec[e + f*x]^3)/(3*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(7A-B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))}}{6a^2c}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+f}}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx}{48a^2c^2f\sqrt{c-c\sin(e+}}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx)}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx)}$$

$$= \frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f} + \frac{5(7}{64a^2c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 430, normalized size = 1.91

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(11A+3B)\cos^3(e+fx)+16(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+24(-3A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+12(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3-(15+15i)\sqrt[4]{-1}(7A-B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+24(A+B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+6(11A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)}{192a^2f(1+\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(
7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(A
 + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(11*A +
3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos[(e + f*x
```

```
)/2] + Sin[(e + f*x)/2])^3))/(192*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(198) = 396$.
time = 8.99, size = 426, normalized size = 1.89

method	result
default	$-\frac{30B(c(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c^2+105A(c(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/384/c^(11/2)/a^2*(30*B*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2+105*A*(c*(1+sin(f*x+e)
))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(
f*x+e)^2*c^2-15*B*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-210*A*(c*(1+sin(f*x+e)))^(3/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)
*c^2-210*A*c^(7/2)*sin(f*x+e)^3+30*B*c^(7/2)*sin(f*x+e)^3+70*A*c^(7/2)*sin(
f*x+e)^2-10*B*c^(7/2)*sin(f*x+e)^2+322*A*c^(7/2)*sin(f*x+e)-46*B*c^(7/2)*si
n(f*x+e)+105*A*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e
)))^(1/2)*2^(1/2)/c^(1/2))*c^2-15*B*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctan
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-86*A*c^(7/2)+122*B*c^(7
/2))/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.37, size = 300, normalized size = 1.33

$$-\frac{15\sqrt{2}\left((7A-B)\cos(fx+e)^3\sin(fx+e)-(7A-B)\cos(fx+e)^3\right)\sqrt{c}\,\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,\sqrt{c}\,(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(5(7A-B)\cos(fx+e)^2-\left(15(7A-B)\cos(fx+e)^2+56A-8B\right)\sin(fx+e)+8A-56B\right)\sqrt{-c\sin(fx+e)+c}}{768\left(a^2c^3f\cos(fx+e)^3\sin(fx+e)-a^2c^3f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/768*(15*sqrt(2)*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(f
*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e)
+ c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(
f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*(7*A
- B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*sin(f*
x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. 2(208) = 416.
time = 0.67, size = 787, normalized size = 3.50

$$\frac{\frac{60\sqrt{2}\left(7A\sqrt{c}-B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^3\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}-\frac{24A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{8B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{210A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{30B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}{a^2c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{256\sqrt{2}\left(5A\sqrt{c}-2B\sqrt{c}+\frac{9A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{3B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{6A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{3B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2c^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\left(\frac{24\sqrt{2}Aa^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{8\sqrt{2}Ba^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{\sqrt{2}Aa^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-\frac{\sqrt{2}Ba^2c^{\frac{7}{2}}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^4c^6}}{3072f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/3072*(60*sqrt(2)*(7*A*sqrt(c) - B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^3*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 24*A*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 8*B
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1) + 210*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2 - 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2/(a^2*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(5*A*sqrt(c) - 2*B*sqrt(c) + 9*A*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 3*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 6*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
```

```
- 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^3*((cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - 3*(24*sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 8*sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
 - sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - sqrt(2)*B
*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^4*c^6))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
), x)
```

3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=242

$$-\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}+\frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-$$

```
[Out] -2048/15*(A-3*B)*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+512/5*(A-3*B
)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-64/5*(A-3*B)*c*sec(f*x+e)^3
*(c-c*sin(f*x+e))^(7/2)/a^3/f-16/15*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(
9/2)/a^3/f-1/5*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^3/c/f-1/5*(A-
B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(15/2)/a^3/c^3/f
```

Rubi [A]
time = 0.43, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}-\frac{2048c^3(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}+\frac{512c^2(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf}-\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f}-\frac{64c(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-2048*(A - 3*B)*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f)
+ (512*(A - 3*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) -
 (64*(A - 3*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (16
*(A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*f) - ((A - 3*
B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c*f) - ((A - B)*Sec[e
 + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
```

```
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}\cdots}{a^3c^3} \\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(A-3B\cdots}{\cdots} \\
&= -\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B\cdots}{\cdots} \\
&= -\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-\cdots}{\cdots} \\
&= -\frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A\cdots}{\cdots} \\
&= \frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{64(A\cdots}{\cdots} \\
&= -\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{51\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 2.88, size = 176, normalized size = 0.73

$$-\frac{c^4(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}\,(11298A-33516B-40(137A-402B)\cos(2(e+fx))-10(A-6B)\cos(4(e+fx))+15600A\sin(e+fx)-47430B\sin(e+fx)-400A\sin(3(e+fx))+1335B\sin(3(e+fx))-3B\sin(5(e+fx)))}{120a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/120*(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 33516
*B - 40*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)] +
15600*A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1335
*B*Sin[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 5.46, size = 143, normalized size = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)(3B\sin(fx+e)(\cos^4(fx+e))+(100A-336B)(\cos^2(fx+e))\sin(fx+e)+(-1000A+3048B)\sin(fx+e)+(5A-30}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(3*B*sin(f*x+e)*cos(f*x+e)^4+
(100*A-336*B)*cos(f*x+e)^2*sin(f*x+e)+(-1000*A+3048*B)*sin(f*x+e)+(5*A-30*B
)*cos(f*x+e)^4+(680*A-1980*B)*cos(f*x+e)^2-1048*A+3096*B)/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(230) = 460.
time = 0.55, size = 1025, normalized size = 4.24

$$\frac{2\left(\frac{\left(363c^{\frac{9}{2}}+\frac{1800c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{5301c^{\frac{9}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{11600c^{\frac{9}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{21343c^{\frac{9}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{30200c^{\frac{9}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{40065c^{\frac{9}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{40800c^{\frac{9}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{40065c^{\frac{9}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{30200c^{\frac{9}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{21343c^{\frac{9}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{11600c^{\frac{9}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{5301c^{\frac{9}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}+\frac{1800c^{\frac{9}{2}}\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}}+\frac{363c^{\frac{9}{2}}\sin(fx+e)^{14}}{(\cos(fx+e)+1)^{14}}\right)A}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{9}{2}}}-\frac{6\left(181c^{\frac{9}{2}}+\frac{905c^{\frac{9}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2627c^{\frac{9}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5870c^{\frac{9}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{10521c^{\frac{9}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{15351c^{\frac{9}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{19695c^{\frac{9}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{20772c^{\frac{9}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{19695c^{\frac{9}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{15351c^{\frac{9}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{10521c^{\frac{9}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{5870c^{\frac{9}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{2627c^{\frac{9}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}+\frac{905c^{\frac{9}{2}}\sin(fx+e)^{13}}{(\cos(fx+e)+1)^{13}}+\frac{181c^{\frac{9}{2}}\sin(fx+e)^{14}}{(\cos(fx+e)+1)^{14}}\right)B}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{9}{2}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 30
200*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9
/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(c
```

```
os(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*A/
((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2)) - 6*(181*c^(9/2) + 905*c^(9/
2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2627*c^(9/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 5870*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10521*c^(9/
2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15351*c^(9/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 19695*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 20772
*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 19695*c^(9/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15351*c^(9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
 10521*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 5870*c^(9/2)*sin(f*x
 + e)^11/(cos(f*x + e) + 1)^11 + 2627*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e)
 + 1)^12 + 905*c^(9/2)*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 181*c^(9/2)*
sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(9/2)))/f
```

Fricas [A]
time = 0.38, size = 176, normalized size = 0.73

$$-\frac{2\left(5\,(A-6\,B)c^4\cos\left(fx+e\right)^4+20\,(34\,A-99\,B)c^4\cos\left(fx+e\right)^2-8\,(131\,A-387\,B)c^4+\left(3\,Bc^4\cos\left(fx+e\right)^4+4\,(25\,A-84\,B)c^4\cos\left(fx+e\right)^2-8\,(125\,A-381\,B)c^4\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^2
 - 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*co
s(f*x + e)^2 - 8*(125*A - 381*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(
f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8856 deep
```

Giac [A]
time = 0.71, size = 417, normalized size = 1.72

$$-\frac{1024\,\sqrt{2}\left(Ac^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,Bc^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{5\,Ac^4\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}+\frac{15\,Bc^4\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}+\frac{10\,Ac^4\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^4}-\frac{30\,Bc^4\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^4}-\frac{6\,Ac^4\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}-\frac{6\,Bc^4\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}\right)\sqrt{c}}{15\,a^3f\left(\frac{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1024/15*sqrt(2)*(A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^4*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^2 + 15*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*A*c^4*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^4 - 30*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^4 - 6*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 6*B*c^4*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)^5)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3, x)
```

3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3} dx$

Optimal. Leaf size=209

$$-\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}+\frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-$$

```
[Out] -128/15*(3*A-13*B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+32/5*(3*A-
13*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-4/5*(3*A-13*B)*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-1/15*(3*A-13*B)*sec(f*x+e)^3*(c-c*sin(f*x+e
))^(9/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(13/2)/a^3/c^3/f
```

Rubi [A]
time = 0.38, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}-\frac{128c^2(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf}-\frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f}+\frac{32c(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-128*(3*A - 13*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f
) + (32*(3*A - 13*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f)
 - (4*(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (
(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*c*f) - ((A
- B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/}}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f} - \frac{(3A-}{}$$

$$= -\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-}{}$$

$$= -\frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A}{}$$

$$= \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3}{}$$

$$= -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} +$$

Mathematica [A]
time = 1.80, size = 158, normalized size = 0.76

$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(1092A-4557B+(-540A+2200B)\cos(2(e+fx))+5B\cos(4(e+fx))+1410A\sin(e+fx)-6390B\sin(e+fx)-30A\sin(3(e+fx))+170B\sin(3(e+fx)))}{60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
1092*A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)]
 + 1410*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 170*
B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[
e + f*x])^3)
```

Maple [A]
time = 5.49, size = 121, normalized size = 0.58

method	result
default	$\frac{2c^4(\sin(fx+e)-1)((-15A+85B)\sin(fx+e)(\cos^2(fx+e))+(180A-820B)\sin(fx+e)+5B(\cos^4(fx+e))+(-135A+545B)(\cos^2(fx+e}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*((-15*A+85*B)*sin(f*x+e)*cos(f
*x+e)^2+(180*A-820*B)*sin(f*x+e)+5*B*cos(f*x+e)^4+(-135*A+545*B)*cos(f*x+e)
^2+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 926 vs. 2(199) = 398.
time = 0.56, size = 926, normalized size = 4.43

$$\frac{2\left(\frac{3\left(23c^{\frac{7}{2}}+\frac{110c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{318c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{590c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{1065c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{1220c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{1540c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1220c^{\frac{7}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{1065c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{590c^{\frac{7}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{318c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{110c^{\frac{7}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{23c^{\frac{7}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)A}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}-\frac{2\left(147c^{\frac{7}{2}}+\frac{735c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{1992c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4015c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6605c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{8370c^{\frac{7}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{9520c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{8370c^{\frac{7}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{6605c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{4015c^{\frac{7}{2}}\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{1992c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{735c^{\frac{7}{2}}\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}+\frac{147c^{\frac{7}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)B}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{7}{2}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(3*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(
7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 106
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
110*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
 - 2*(147*c^(7/2) + 735*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1992*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4015*c^(7/2)*sin(f*x + e)^3/(cos(
```

```
f*x + e) + 1)^3 + 6605*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8370*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9520*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 8370*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 660
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4015*c^(7/2)*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 + 1992*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
+ 735*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 147*c^(7/2)*sin(f*x +
 e)^12/(cos(f*x + e) + 1)^12)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/
2)))/f
```

Fricas [**A**]
time = 0.37, size = 157, normalized size = 0.75

$$\frac{2\left(5\,Bc^{3}\cos\left(fx+e\right)^{4}-5\left(27\,A-109\,B\right)c^{3}\cos\left(fx+e\right)^{2}+4\left(51\,A-211\,B\right)c^{3}-5\left(\left(3\,A-17\,B\right)c^{3}\cos\left(fx+e\right)^{2}-4\left(9\,A-41\,B\right)c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51*
A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(199) = 398.
time = 0.81, size = 818, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -4/15*sqrt(2)*sqrt(c)*(5*(3*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 19*
B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c^3*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 42*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^2)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) - 1)^3) - (33*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 113*
B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 150*A*c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 490*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 240*A*c^3*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 740*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^2 + 90*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 390*B*c^3*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*c^3*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^4 - 75*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*((cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^
5))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\ (c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
3, x)
```

3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=160

$$-\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}+\frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-\frac{(A-}{}$$

```
[Out] -32/15*(A-11*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+8/5*(A-11*B)*se
c(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-1/5*(A-11*B)*sec(f*x+e)^3*(c-c*sin(
f*x+e))^(7/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c^
3/f
```

Rubi [A]

time = 0.33, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}-\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}+\frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-\frac{32c(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-32*(A - 11*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) + (
8*(A - 11*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) - ((A - 1
1*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}}{a^3c^3} \\
&= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f} - \frac{(A-11}{} \\
&= -\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} - \frac{(A-B}{} \\
&= \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-11}{} \\
&= -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{8(A}{}
\end{aligned}$$

Mathematica [A]
time = 0.83, size = 132, normalized size = 0.82

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,(58A-488B-30(A-8B)\cos(2(e+fx))+5(8A-133B)\sin(e+fx)+15B\sin(3(e+fx)))}{30a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^3,x]

[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
58*A - 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*x]
```

```
 + 15*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
+ Sin[e + f*x])^3)
```

Maple [A]
time = 5.53, size = 105, normalized size = 0.66

method	result	size
default	$\frac{2c^{3}(\sin(fx+e)-1)(15B(\cos^{2}(fx+e))\sin(fx+e)+(10A-170B)\sin(fx+e)+(-15A+120B)(\cos^{2}(fx+e))+22A-182B)}{15a^{3}(1+\sin(fx+e))^{2}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)+
(10*A-170*B)*sin(f*x+e)+(-15*A+120*B)*cos(f*x+e)^2+22*A-182*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(152) = 304$.
time = 0.54, size = 825, normalized size = 5.16

$$\frac{2\left(\frac{\left(7c^{\frac{5}{2}}+\frac{20c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{95c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{80c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{250c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{120c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{250c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{80c^{\frac{5}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{95c^{\frac{5}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{20c^{\frac{5}{2}}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{7c^{\frac{5}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)A}{\left(a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}-\frac{2\left(31c^{\frac{5}{2}}+\frac{155c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{395c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{680c^{\frac{5}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{1030c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{1050c^{\frac{5}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{1030c^{\frac{5}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{680c^{\frac{5}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{395c^{\frac{5}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}+\frac{155c^{\frac{5}{2}}\sin(fx+e)^{9}}{(\cos(fx+e)+1)^{9}}+\frac{31c^{\frac{5}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)B}{\left(a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{5}{2}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(31*c^(5/2) + 155*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 395*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 680*
c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1030*c^(5/2)*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 1050*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10
30*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 680*c^(5/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 395*c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1
55*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 31*c^(5/2)*sin(f*x + e)^10
```

```
/(cos(f*x + e) + 1)^10)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Fricas [A]
time = 0.36, size = 133, normalized size = 0.83

$$-\frac{2\left(15\,(A-8\,B)c^2\cos\left(fx+e\right)^2-2\,(11\,A-91\,B)c^2-5\left(3\,Bc^2\cos\left(fx+e\right)^2+2\,(A-17\,B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*c
os(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(
a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x
+ e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(152) = 304.
time = 0.66, size = 474, normalized size = 2.96

$$\frac{4\sqrt{2}\sqrt{c}\left(\frac{15\,Bc^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)}+\frac{4\,Ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-29\,Bc^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{20\,Ac^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-\frac{130\,Bc^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+\frac{40\,Ac^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}-\frac{200\,Bc^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^2}-\frac{90\,Bc^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}-\frac{15\,Bc^2\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^4}}{a^3\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)^5}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 4/15*sqrt(2)*sqrt(c)*(15*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^3*((c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1
)) + (4*A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 29*B*c^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 20*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 130*B
*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
```

```
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 40*A*c^2*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2 - 200*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*B*c^
2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 15*B*c^2*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^4)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) + 1)^5))/f
```

Mupad [B]
time = 22.79, size = 904, normalized size = 5.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*B*c^2)/(a^3*f) - (B*c^2*exp(e*1i + f*x*1i)*2i)/(a^3*f)))/(exp(e*1i + f*x*
1i) - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*7i)*1i)/(3*a^3*f) - (2*c^2*(7*
A - 12*B))/(3*a^3*f) + (c^2*(A*23i - B*28i)*2i)/(3*a^3*f) - (c^2*(42*A - 67
*B))/(15*a^3*f) + (2*B*c^2)/(3*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1
i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i
)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i)*4i)/(a^3*f) + (
4*B*c^2)/(a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)) -
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((8*c^2*(A*1i - B*1i))/(a^3*f) + (c^2*(A*1i - B*3i))/(2*a^
3*f) + (c^2*(A*11i - B*1i))/(10*a^3*f) + (c^2*(12*A - 17*B)*1i)/(4*a^3*f) +
 (c^2*(52*A - 47*B)*1i)/(4*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i +
f*x*1i) + 1i)^4) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i))/(a^3*f) + (c^2*(A*
5i - B*4i))/(3*a^3*f) + (c^2*(A - 2*B)*8i)/(a^3*f)))/((exp(e*1i + f*x*1i) -
 1i)*(exp(e*1i + f*x*1i) + 1i)^2) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*5i)*1i
)/(5*a^3*f) - (c^2*(4*A - 3*B))/(a^3*f) - (c^2*(2*A - 5*B))/(5*a^3*f) + (c^
2*(A*4i - B*3i)*1i)/(a^3*f) - (c^2*(10*A - 11*B))/(5*a^3*f) + (c^2*(A*10i -
 B*11i)*1i)/(5*a^3*f) + (2*B*c^2)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)^5)
```

3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=121

$$\frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}-\frac{(A-B)\sec^5(e$$

```
[Out] 4/15*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f-1/5*(A+9*B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))
^(9/2)/a^3/c^3/f
```

Rubi [A]
time = 0.27, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3046, 2934, 2753, 2752}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}+\frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (4*(A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) - ((A +
9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
```

```
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f} + \frac{(A+9B}{}$$

$$= -\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B}{}$$

$$= \frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(A+9}{}$$

Mathematica [A]
time = 0.50, size = 113, normalized size = 0.93

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2A+27B-15B\cos(2(e+fx))+10(A+3B)\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*x
)] + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 5.59, size = 83, normalized size = 0.69

method	result	size
default	$-\frac{2c^{2}(\sin(fx+e)-1)(\sin(fx+e)(5A+15B)-15B(\cos^{2}(fx+e))-A+21B)}{15a^{3}(1+\sin(fx+e))^{2}\cos(fx+e)\sqrt{c-c\sin\left(fx+e\right)}\, f}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(sin(f*x+e)*(5*A+15*B)-15*B*c
os(f*x+e)^2-A+21*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(115) = 230$.
time = 0.54, size = 719, normalized size = 5.94

$$\frac{2\left(\frac{\left(c^{\frac{3}{2}}-\frac{10c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{4c^{\frac{3}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{30c^{\frac{3}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{30c^{\frac{3}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{4c^{\frac{3}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-\frac{10c^{\frac{3}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{c^{\frac{3}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}\right)A}{\left(a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{3}{2}}}-\frac{6\left(c^{\frac{3}{2}}+\frac{5c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{14c^{\frac{3}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{15c^{\frac{3}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{26c^{\frac{3}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{15c^{\frac{3}{2}}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}+\frac{14c^{\frac{3}{2}}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}+\frac{5c^{\frac{3}{2}}\sin(fx+e)^{7}}{(\cos(fx+e)+1)^{7}}+\frac{c^{\frac{3}{2}}\sin(fx+e)^{8}}{(\cos(fx+e)+1)^{8}}\right)B}{\left(a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{3}{2}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 6
*(c^(3/2) + 5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 14*c^(3/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 15*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 26*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 14*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 5*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A]
time = 0.35, size = 102, normalized size = 0.84

$$\frac{2\left(15\,Bc\cos\left(fx+e\right)^{2}-5\left(A+3\,B\right)c\sin\left(fx+e\right)+\left(A-21\,B\right)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*sq
rt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*
x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(115) = 230.
time = 0.59, size = 395, normalized size = 3.26

$$-\frac{2\sqrt{2}\left(Ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+9\,Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{5\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{45\,Bc\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{5\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{75\,Bc\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{15\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{15\,Bc\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)\sqrt{c}}{15\,a^3f\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -2/15*sqrt(2)*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 45*B*c
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 + 75*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*A*c*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^
3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
3)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 1)^5)
```

Mupad [**B**]
time = 19.16, size = 683, normalized size = 5.64

$$\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{4\,B\,c}{5\,a^3\,f}-\frac{2\,c\,(2\,A-3\,B)}{5\,a^3\,f}-\frac{4\,c\,(3\,A-2\,B)}{5\,a^3\,f}+\frac{c\,(A\,2\mathrm{i}-B\,3\mathrm{i})\,2\mathrm{i}}{5\,a^3\,f}+\frac{c\,(A\,3\mathrm{i}-B\,2\mathrm{i})\,4\mathrm{i}}{5\,a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^5}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{2\,B\,c}{3\,a^3\,f}+\frac{c\,(A\,2\mathrm{i}-B\,5\mathrm{i})\,2\mathrm{i}}{3\,a^3\,f}-\frac{2\,c\,(10\,A-13\,B)}{3\,a^3\,f}+\frac{c\,(A\,8\mathrm{i}-B\,13\mathrm{i})\,2\mathrm{i}}{15\,a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^3}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(-\frac{B\,c\,1\mathrm{i}}{a^3\,f}+\frac{c\,(2\,A-3\,B)\,1\mathrm{i}}{3\,a^3\,f}+\frac{2\,c\,(A\,1\mathrm{i}-B\,3\mathrm{i})}{a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{c\,(A-B)\,4\mathrm{i}}{a^3\,f}-\frac{B\,c\,1\mathrm{i}}{2\,a^3\,f}+\frac{c\,(8\,A-3\,B)\,1\mathrm{i}}{10\,a^3\,f}+\frac{c\,(A\,1\mathrm{i}-B\,2\mathrm{i})}{a^3\,f}+\frac{c\,(A\,7\mathrm{i}-B\,6\mathrm{i})}{a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^4}+\frac{4\,B\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{a^3\,f\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((4*B*c)/(5*a^3*f) - (2*c*(2*A - 3*B))/(5*a^3*f) - (4*c*(3
*A - 2*B))/(5*a^3*f) + (c*(A*2i - B*3i)*2i)/(5*a^3*f) + (c*(A*3i - B*2i)*4i
)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5) - (ex
p(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*
1i)/2))^(1/2)*((2*B*c)/(3*a^3*f) + (c*(A*2i - B*5i)*2i)/(3*a^3*f) - (2*c*(1
0*A - 13*B))/(3*a^3*f) + (c*(A*8i - B*13i)*2i)/(15*a^3*f)))/((exp(e*1i + f*
x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp
(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(2*A - 3*B)
*1i)/(3*a^3*f) - (B*c*1i)/(a^3*f) + (2*c*(A*1i - B*3i))/(a^3*f)))/((exp(e*1
i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c
*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(A -
B)*4i)/(a^3*f) - (B*c*1i)/(2*a^3*f) + (c*(8*A - 3*B)*1i)/(10*a^3*f) + (c*(A
*1i - B*2i))/(a^3*f) + (c*(A*7i - B*6i))/(a^3*f)))/((exp(e*1i + f*x*1i) - 1
i)*(exp(e*1i + f*x*1i) + 1i)^4) + (4*B*c*exp(e*1i + f*x*1i)*(c - c*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i
+ f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=85

$$-\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}$$

```
[Out] -1/15*(3*A+7*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c/f-1/5*(A-B)*sec(f
*x+e)^5*(c-c*sin(f*x+e))^(7/2)/a^3/c^3/f
```

Rubi [A]
time = 0.20, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3046, 2934, 2752}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}-\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] -1/15*((3*A + 7*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(a^3*c*f) - (
(A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c^3*f)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}$$
$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f} + \frac{(3A+7B)}{}$$
$$= -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf} - \frac{(A-B)}{}$$

Mathematica [A]
time = 0.23, size = 89, normalized size = 1.05

$$-\frac{2(3A+2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^3,x]
```

```
[Out] (-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 5.92, size = 65, normalized size = 0.76

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(5B\sin(fx+e)+3A+2B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/15*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(81) = 162$.

time = 0.58, size = 547, normalized size = 6.44

$$\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{5\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5\sqrt{c}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{3A\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)))/f
```

Fricas [A]

time = 0.36, size = 83, normalized size = 0.98

$$\frac{2\,(5\,B\sin(fx+e)+3\,A+2\,B)\sqrt{-c\sin(fx+e)+c}}{15\,\left(a^3f\cos(fx+e)^3-2\,a^3f\cos(fx+e)\sin(fx+e)-2\,a^3f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*
x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(81) = 162.
time = 0.54, size = 390, normalized size = 4.59

$$\frac{\sqrt{2}\left(3\,A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+7\,B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{20\,B\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+\frac{30\,A\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}+\frac{10\,B\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{2}}+\frac{60\,B\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{15\,A\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{4}}+\frac{15\,B\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{4}}\right)\sqrt{c}}{30\,a^{3}f\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 20*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*A*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1)^2 + 60*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^4 + 15*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)*sqrt(c
)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1) + 1)^5)
```

Mupad [**B**]
time = 17.54, size = 479, normalized size = 5.64

$$\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(\frac{8\,B}{5\,a^3\,f}-\frac{16\,A-8\,B}{10\,a^3\,f}+\frac{\left(A\,16\mathrm{i}-B\,8\mathrm{i}\right)\,1\mathrm{i}}{10\,a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^5}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(\frac{4\,B}{3\,a^3\,f}-\frac{16\,A-16\,B}{30\,a^3\,f}+\frac{\left(A\,80\mathrm{i}-B\,120\mathrm{i}\right)\,1\mathrm{i}}{30\,a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^3}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(-\frac{B\,1\mathrm{i}}{a^3\,f}+\frac{A\,16\mathrm{i}-B\,16\mathrm{i}}{40\,a^3\,f}+\frac{A\,80\mathrm{i}-B\,80\mathrm{i}}{40\,a^3\,f}+\frac{\left(160\,A-120\,B\right)\,1\mathrm{i}}{40\,a^3\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^4}-\frac{B\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,8\mathrm{i}}{3\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((8*B)/(5*a^3*f) - (16*A - 8*B)/(10*a^3*f) + ((A*16i - B*8
i)*1i)/(10*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5)
 - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*((4*B)/(3*a^3*f) - (16*A - 16*B)/(30*a^3*f) + ((A*80i -
 B*120i)*1i)/(30*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) +
1i)^3) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2)*((A*16i - B*16i)/(40*a^3*f) - (B*1i)/(a^3*f) + (A
*80i - B*80i)/(40*a^3*f) + ((160*A - 120*B)*1i)/(40*a^3*f)))/((exp(e*1i + f
*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (B*exp(e*1i + f*x*1i)*(c - c*((
exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*8i)/(3*a^3*f
*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2)
```

3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=174

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}-\frac{(A+B)\sec^3(e+$$

```
[Out] -1/6*(A+B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*(A-B)*sec(f*x+
e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^3/f+1/8*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1
/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/f*2^(1/2)/c^(1/2)-1/4*(A+B)*sec(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a^3/c/f
```

Rubi [A]
time = 0.29, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3046, 2934, 2754, 2728, 212}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}+\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(4*Sqrt[2]*a^3*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(4*a^3*c*f) - ((A + B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3
*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{(A+B)\int$$

$$= -\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\mathrm{se}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f} - \frac{(A+B)\mathrm{se}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 204, normalized size = 1.17

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(-A+B)-10(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-15(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-(15+15i)\sqrt[4]{-1}(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)}{60a^3f(1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(A
 + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(A +
 B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^5))/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e +
f*x]])
```

Maple [A]
time = 7.18, size = 200, normalized size = 1.15

method	result
default	$-\frac{(\sin(fx+e)-1)\left(-30A\,c^{\frac{9}{2}}\left(\sin^2(fx+e)\right)-30B\,c^{\frac{9}{2}}\left(\sin^2(fx+e)\right)-80A\,c^{\frac{9}{2}}\sin(fx+e)-80B\,c^{\frac{9}{2}}\sin(fx+e)+15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c}}{}\right.\right.}{120a^3c^{\frac{9}{2}}(1+\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/120*(sin(f*x+e)-1)*(-30*A*c^(9/2)*sin(f*x+e)^2-30*B*c^(9/2)*sin(f*x+e)^2
-80*A*c^(9/2)*sin(f*x+e)-80*B*c^(9/2)*sin(f*x+e)+15*2^(1/2)*arctanh(1/2*(c*
(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*A-74*c^
(9/2)*A+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c
*(1+sin(f*x+e)))^(5/2)*c^2*B-26*c^(9/2)*B)/a^3/c^(9/2)/(1+sin(f*x+e))^2/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
 + c)), x)
```

Fricas [A]
time = 0.38, size = 284, normalized size = 1.63

$$\frac{15\sqrt{2}\left((A+B)\cos(fx+e)^3-2(A+B)\cos(fx+e)\sin(fx+e)-2(A+B)\cos(fx+e)\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(15(A+B)\cos(fx+e)^2-40(A+B)\sin(fx+e)-52A-28B\right)\sqrt{-c\sin(fx+e)+c}}{240\left(a^3cf\cos(fx+e)^3-2a^3cf\cos(fx+e)\sin(fx+e)-2a^3cf\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x
+ e) - 2*(A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*co
s(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (
cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(A + B)*cos(f*x
 + e)^2 - 40*(A + B)*sin(f*x + e) - 52*A - 28*B)*sqrt(-c*sin(f*x + e) + c))
/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*
cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(159) = 318.
time = 0.57, size = 472, normalized size = 2.71

$$\frac{\frac{15\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\sqrt{2}\left(23A\sqrt{c}+17B\sqrt{c}+\frac{70A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{70B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{140A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{80B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{90A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{90B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{45A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}+\frac{15B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)}{a^3c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/240*(15*sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) + 4*sqrt(2)*(23*A*sqrt(c) + 17*B*sqrt(c) + 70*A*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 70
```

```
*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + 140*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^2 + 80*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 90*A*sqrt(c)*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 90*B*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^3 + 45*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^4 + 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*c*((cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\sin\left(e + f\,x\right)}{\left(a + a\,\sin\left(e + f\,x\right)\right)^3\,\sqrt{c - c\,\sin\left(e + f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
), x)
```

3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}+\frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}}-\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] 1/16*(7*A+3*B)*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*(A-B)*sec(f*x+e)
^5*(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f+1/32*(7*A+3*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-1/12*(7*A+3*B)
*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-1/30*(7*A+3*B)*sec(f*x+e)^3*(c-c
*sin(f*x+e))^(1/2)/a^3/c^2/f
```

Rubi [A]
time = 0.32, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3046, 2934, 2754, 2766, 2729, 2728, 212}

$$\frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}}-\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((7*A + 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + ((7*A + 3*B)*Cos[e + f*x])/(16*a^3*f*(c
- c*Sin[e + f*x])^(3/2)) - ((7*A + 3*B)*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c -
c*Sin[e + f*x]]) - ((7*A + 3*B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3
0*a^3*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c
^3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,d}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{(7A+3B}{}$$

$$= -\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)}{}$$

$$= -\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{}}{30a^3c^2}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} + \frac{(7A}{16a^3f(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.95, size = 357, normalized size = 1.59

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-40*A*Cos[e + f*x]^2 + 24*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2 - 30*(3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^4 + 15*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (15 + 15*I)*(-1)^(1/4)*(7*A + 3*B)
*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(A + B)*Si
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(240*a^3*f*(1 + Si
n[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A]
time = 6.86, size = 308, normalized size = 1.38

method	result
default	$-\frac{45B(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-210A\,c^{\frac{7}{2}}\left(\sin^3(fx+e)\right)-90B\,c^{\frac{7}{2}}\left(\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/480/c^(9/2)/a^3*(45*B*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1
+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-210*A*c^(7/2)*sin(f*x+e)^
3-90*B*c^(7/2)*sin(f*x+e)^3-350*A*c^(7/2)*sin(f*x+e)^2-150*B*c^(7/2)*sin(f*
x+e)^2+42*A*c^(7/2)*sin(f*x+e)+18*B*c^(7/2)*sin(f*x+e)+105*A*(c*(1+sin(f*x+
e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*si
n(f*x+e)*c-105*A*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x
+e)))^(1/2)*2^(1/2)/c^(1/2))*c-45*B*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctan
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+278*A*c^(7/2)-18*B*c^(7/2
))/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.38, size = 298, normalized size = 1.33

$$\frac{15\sqrt{2}\left((7A+3B)\cos(fx+e)^3\sin(fx+e)+(7A+3B)\cos(fx+e)^3\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(25(7A+3B)\cos(fx+e)^2+3\left(5(7A+3B)\cos(fx+e)^2-28A-12B\right)\sin(fx+e)-36A-84B\right)\sqrt{-c\sin(fx+e)+c}}{960\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/960*(15*sqrt(2)*((7*A + 3*B)*cos(f*x + e)^3*sin(f*x + e) + (7*A + 3*B)*co
s(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*c
os(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)
```

```
*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(25*(7*A + 3*B)*cos(f*x + e)^2 + 3*(
5*(7*A + 3*B)*cos(f*x + e)^2 - 28*A - 12*B)*sin(f*x + e) - 36*A - 84*B)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*
cos(f*x + e)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(207) = 414.
time = 0.63, size = 682, normalized size = 3.04

$$\frac{\frac{30\sqrt{2}\left(7A\sqrt{c}+3B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{15\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{15\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}-\frac{14A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3c^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{32\sqrt{2}\left(29A\sqrt{c}+6B\sqrt{c}+\frac{100A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{30B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{170A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{30B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\frac{120A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{30B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{45A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^4}\right)}{a^3c^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{1920f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/1920*(30*sqrt(2)*(7*A*sqrt(c) + 3*B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) + 15*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 14*A*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1) - 6*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*c^2*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 32*sqrt(2)
*(29*A*sqrt(c) + 6*B*sqrt(c) + 100*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*B*sqrt(c)*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 170*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
2 + 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^2 + 120*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*A*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)
/(a^3*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
), x)
```

3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=258

$$\frac{7(9A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}+\frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}}+\frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3}}$$

```
[Out] 7/128*(9*A+B)*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/240*(9*A+B)*sec(f
*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/256*(9*A+B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(5/2)/f*2^(1/2)-7/96*(9*A+B)*s
ec(f*x+e)/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/30*(9*A+B)*sec(f*x+e)^3/a^3/c^
2/f/(c-c*sin(f*x+e))^(1/2)-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/a^
3/c^3/f
```

Rubi [A]
time = 0.38, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3046, 2934, 2766, 2760, 2729, 2728, 212}

$$\frac{7(9A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}}+\frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (7*(9*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(128*Sqrt[2]*a^3*c^(5/2)*f) + (7*(9*A + B)*Cos[e + f*x])/(128*a^3*c*f
*(c - c*Sin[e + f*x])^(3/2)) + (7*(9*A + B)*Sec[e + f*x])/(240*a^3*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (7*(9*A + B)*Sec[e + f*x])/(96*a^3*c^2*f*Sqrt[c -
 c*Sin[e + f*x]]) - ((9*A + B)*Sec[e + f*x]^3)/(30*a^3*c^2*f*Sqrt[c - c*Sin
[e + f*x]]) - ((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*c^3*
f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{(9A+B)\,\int}{}$$

$$= -\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{}}{5a^3c^3f}$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+f}}$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx}{96a^3c^2f\sqrt{c-c\sin(e+f}}$$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx}{240a^3cf(c-c\sin(e+fx}$$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx}{240a^3cf(c-c\sin(e+fx}$$

$$= \frac{7(9A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f} + \frac{7(9A}{128a^3c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.51, size = 479, normalized size = 1.86

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (105 + 10
5*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
```

```
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(1920*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A]
time = 8.25, size = 410, normalized size = 1.59

method	result
default	$-\frac{\left(1260c^{\frac{9}{2}}A+140c^{\frac{9}{2}}B\right)\sin(fx+e)(\cos^2(fx+e))+\left(-1890\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)(c+c\sin(fx+e))^{\frac{5}{2}}c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3840/c^(13/2)/a^3*((1260*c^(9/2)*A+140*c^(9/2)*B)*sin(f*x+e)*cos(f*x+e)^
2+(-1890*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*s
in(f*x+e))^(5/2)*c^2*A+864*c^(9/2)*A-210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e
))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B+96*c^(9/2)*B)*sin(f*
x+e)+(-1890*c^(9/2)*A-210*c^(9/2)*B)*cos(f*x+e)^4+(-945*2^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+252*c
^(9/2)*A-105*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c
+c*sin(f*x+e))^(5/2)*c^2*B+28*c^(9/2)*B)*cos(f*x+e)^2+1890*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+96
*c^(9/2)*A+210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
(c+c*sin(f*x+e))^(5/2)*c^2*B+864*c^(9/2)*B)/(1+sin(f*x+e))^2/(sin(f*x+e)-1)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.40, size = 256, normalized size = 0.99

$$\frac{105\sqrt{2}(9A+B)\sqrt{c}\cos(fx+e)^5\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e)-2c)\sin(fx+e)+2c}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(105(9A+B)\cos(fx+e)^4-14(9A+B)\cos(fx+e)^2-2\left(35(9A+B)\cos(fx+e)^2+216A+24B\right)\sin(fx+e)-48A-432B\right)\sqrt{-c\sin(fx+e)+c}}{7680a^3c^3f\cos(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2
 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e)
 + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(
f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(105*
(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A + B)*co
s(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin(f*x +
 e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 946 vs. 2(239) = 478.
time = 0.69, size = 946, normalized size = 3.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/30720*(420*sqrt(2)*(9*A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^3*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 32*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
16*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 378*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 42*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2/(a^3*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(54*A*sqrt(c) - 4*B*sqrt(c) + 195*A*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) - 5*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) + 315*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(
```

```
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 25*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 225*A*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3
- 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^3 + 75*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*c^3*((cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*(32*sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1) + 16*sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) - sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
 sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^6*c^6))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\sin\left(e + f\,x\right)}{\left(a + a\,\sin\left(e + f\,x\right)\right)^3\,\left(c - c\,\sin\left(e + f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
), x)
```

3.131 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=94

$$-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/4*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)+1/5
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] -1/4*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin[
e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,(c-$$
$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e}{4f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.71, size = 118, normalized size = 1.26

$$-\frac{c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(4(-60A+23B)\sin(e+fx)+4\cos(2(e+fx))(-35A+25B+4(5A-6B)\sin(e+fx))+\cos(4(e+fx))(5A-15B+4B\sin(e+fx)))}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] -1/160*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]
]*(4*(-60*A + 23*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-35*A + 25*B + 4*(5*
A - 6*B)*Sin[e + f*x]) + Cos[4*(e + f*x)]*(5*A - 15*B + 4*B*Sin[e + f*x])))
/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.
time = 22.54, size = 174, normalized size = 1.85

method	result
default	$\frac{\left(-4B\left(\cos^4(fx+e)\right)+5A\left(\cos^2(fx+e)\right)\sin(fx+e)-15B\left(\cos^2(fx+e)\right)\sin(fx+e)-20A\left(\cos^2(fx+e)\right)+28B\left(\cos^2(fx+e)\right)-35A\sin(f}{20f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-3\left(\cos^2(fx+e)\right)-4\sin(f\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*sin
(f*x+e)-20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)+25*B*sin(f*x+e)
+40*A-24*B)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(
cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(7/2), x)
```

Fricas [A]
time = 0.38, size = 148, normalized size = 1.57

$$-\frac{\left(5\,(A-3\,B)c^3\cos\left(fx+e\right)^4-40\,(A-B)c^3\cos\left(fx+e\right)^2+5\,(7\,A-5\,B)c^3+4\,\left(Bc^3\cos\left(fx+e\right)^4+(5\,A-7\,B)c^3\cos\left(fx+e\right)^2-2\,(5\,A-3\,B)c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{20\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/20*(5*(A - 3*B)*c^3*cos(f*x + e)^4 - 40*(A - B)*c^3*cos(f*x + e)^2 + 5*(
7*A - 5*B)*c^3 + 4*(B*c^3*cos(f*x + e)^4 + (5*A - 7*B)*c^3*cos(f*x + e)^2 -
 2*(5*A - 3*B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 159, normalized size = 1.69

$$-\frac{4\left(8\,Bc^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-5\,Ac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-5\,Bc^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\right)\sqrt{a}\,\sqrt{c}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/5*(8*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 5*A*c^3*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^8 - 5*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 16.17, size = 173, normalized size = 1.84

$$-\frac{c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(100\,B\,\cos\left(e+f\,x\right)-140\,A\,\cos\left(e+f\,x\right)-135\,A\,\cos\left(3\,e+3\,f\,x\right)+5\,A\,\cos\left(5\,e+5\,f\,x\right)+85\,B\,\cos\left(3\,e+3\,f\,x\right)-15\,B\,\cos\left(5\,e+5\,f\,x\right)-240\,A\,\sin\left(2\,e+2\,f\,x\right)+40\,A\,\sin\left(4\,e+4\,f\,x\right)+90\,B\,\sin\left(2\,e+2\,f\,x\right)-48\,B\,\sin\left(4\,e+4\,f\,x\right)+2\,B\,\sin\left(6\,e+6\,f\,x\right)\right)}{160\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] -(c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(100*B*cos
(e + f*x) - 140*A*cos(e + f*x) - 135*A*cos(3*e + 3*f*x) + 5*A*cos(5*e + 5*f
*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x
) + 40*A*sin(4*e + 4*f*x) + 90*B*sin(2*e + 2*f*x) - 48*B*sin(4*e + 4*f*x) +
 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

3.132 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=94

$$-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/3*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+1/4
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/3*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin[
e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,(c-$$

$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e}{3f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.57, size = 102, normalized size = 1.09

$$\frac{c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(3B\cos(4(e+fx))+16(7A-2B)\sin(e+fx)-4\cos(2(e+fx))(-12A+9B+4(A-2B)\sin(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A - 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(-12*A
+ 9*B + 4*(A - 2*B)*Sin[e + f*x])))/(96*f)
```

Maple [A]
time = 0.38, size = 129, normalized size = 1.37

method	result
default	$\frac{(3B(\cos^2(fx+e))\sin(fx+e)+4A(\cos^2(fx+e))-8B(\cos^2(fx+e))+12A\sin(fx+e)-9B\sin(fx+e)-16A+8B)(-c(\sin(fx+e)-1))^{\frac{5}{2}}}{12f(\cos^2(fx+e)+2\sin(fx+e)-2)\cos(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2-8*B*cos(f*x+e)^2+12*A*
sin(f*x+e)-9*B*sin(f*x+e)-16*A+8*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a
*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(5/2), x)
```

Fricas [A]
time = 0.36, size = 128, normalized size = 1.36

$$\frac{\left(3 Bc^{2} \cos \left(fx+e\right)^{4}+12 \left(A-B\right)c^{2} \cos \left(fx+e\right)^{2}-3 \left(4 A-3 B\right)c^{2}-4 \left(\left(A-2 B\right)c^{2} \cos \left(fx+e\right)^{2}-2 \left(2 A-B\right)c^{2}\right) \sin \left(fx+e\right)\right) \sqrt{a \sin \left(fx+e\right)+a} \sqrt{-c \sin \left(fx+e\right)+c}}{12 f \cos \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*c^2*cos(f*x + e)^4 + 12*(A - B)*c^2*cos(f*x + e)^2 - 3*(4*A - 3*B
)*c^2 - 4*((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(2*A - B)*c^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.57, size = 159, normalized size = 1.69

$$-\frac{4 \left(3 Bc^{2} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{8}-2 Ac^{2} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{6}-2 Bc^{2} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{6}\right) \sqrt{a} \sqrt{c}}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/3*(3*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*c^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^6 - 2*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 14.96, size = 149, normalized size = 1.59

$$\frac{c^{2} \sqrt{a \left(\sin \left(e+f x\right)+1\right)} \sqrt{-c \left(\sin \left(e+f x\right)-1\right)} \left(48 A \cos \left(e+f x\right)-36 B \cos \left(e+f x\right)+48 A \cos \left(3 e+3 f x\right)-33 B \cos \left(3 e+3 f x\right)+3 B \cos \left(5 e+5 f x\right)+112 A \sin \left(2 e+2 f x\right)-8 A \sin \left(4 e+4 f x\right)-32 B \sin \left(2 e+2 f x\right)+16 B \sin \left(4 e+4 f x\right)\right)}{96 f \left(\cos \left(2 e+2 f x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(e
 + f*x) - 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) - 33*B*cos(3*e + 3*f*x)
 + 3*B*cos(5*e + 5*f*x) + 112*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x) - 3
2*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1)
)
```

3.133 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=94

$$-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+1/3
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/2*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[
e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,(c-$$

$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e}{2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.39, size = 84, normalized size = 0.89

$$\frac{c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(2(6A-B)\sin(e+fx)+\cos(2(e+fx))(3A-3B+2B\sin(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2*(6*A
 - B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*A - 3*B + 2*B*Sin[e + f*x])))/(12*
f)
```

Maple [A]
time = 0.38, size = 91, normalized size = 0.97

method	result	si
default	$\frac{(-2B(\cos^2(fx+e))+3A\sin(fx+e)-3B\sin(fx+e)-6A+2B)(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)\sqrt{a(1+\sin(fx+e))}}{6f(\sin(fx+e)-1)\cos(fx+e)}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(-2*B*cos(f*x+e)^2+3*A*sin(f*x+e)-3*B*sin(f*x+e)-6*A+2*B)*(-c*(sin(f*
x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2), x)
```

Fricas [A]
time = 0.37, size = 98, normalized size = 1.04

$$\frac{\left(3\left(A-B\right)c\cos\left(fx+e\right)^2-3\left(A-B\right)c+2\left(Bc\cos\left(fx+e\right)^2+\left(3A-B\right)c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(3*(A - B)*c*cos(f*x + e)^2 - 3*(A - B)*c + 2*(B*c*cos(f*x + e)^2 + (3*
A - B)*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/
(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c(\sin\left(e+fx\right)-1)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*(A + B*s
in(e + f*x)), x)
```

Giac [A]
time = 0.55, size = 153, normalized size = 1.63

$$-\frac{2\left(4\,Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6-3\,Ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-3\,Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -2/3*(4*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 3*A*c*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^4 - 3*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 1.79, size = 122, normalized size = 1.30

$$\frac{c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\,A\,\cos\left(e+f\,x\right)-3\,B\,\cos\left(e+f\,x\right)+3\,A\,\cos\left(3\,e+3\,f\,x\right)-3\,B\,\cos\left(3\,e+3\,f\,x\right)+12\,A\,\sin\left(2\,e+2\,f\,x\right)-2\,B\,\sin\left(2\,e+2\,f\,x\right)+B\,\sin\left(4\,e+4\,f\,x\right)\right)}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
f*x) - 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) - 3*B*cos(3*e + 3*f*x) + 12*
A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(cos
(2*e + 2*f*x) + 1))
```

3.134 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\sqrt{c-c\sin($

Optimal. Leaf size=92

$$-\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 1/2*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)-a*(A+B
)*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Si
n[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.12, size = 63, normalized size = 0.68

$$\frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,(-B\cos(2(e+fx))+4A\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(-(B*Cos[2*(e + f*x)]) + 4*A*Sin[e
 + f*x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

Maple [A]
time = 0.37, size = 57, normalized size = 0.62

method	result	size
default	$\frac{(B\sin(fx+e)+2A)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}{2f\cos(fx+e)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x
+e)))^(1/2)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c), x)
```

Fricas [A]
time = 0.36, size = 66, normalized size = 0.72

$$-\frac{\left(B\cos\left(fx+e\right)^{2}-2\,A\sin\left(fx+e\right)-B\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(
e + f*x)), x)
```

Giac [A]
time = 0.51, size = 148, normalized size = 1.61

$$-\frac{2\left(B\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+A\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -2*(B*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + A*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 0.94, size = 75, normalized size = 0.82

$$-\frac{\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(B\,\cos\left(e+f\,x\right)+B\,\cos\left(3\,e+3\,f\,x\right)-4\,A\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(B*cos(e + f*x
) + B*cos(3*e + 3*f*x) - 4*A*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1)
)
```

3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=100

$$-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -a*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)+a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3050, 2817, 2816, 2746, 31}

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx - \frac{B\int \sqrt{a+a\sin(e}{}$$

$$= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} + \frac{(a(A+B)c\cos(e}{\sqrt{a+a\sin(e+f}}$$

$$= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} - \frac{(a(A+B)\cos(e+}{f\sqrt{a+a}}$$

$$= -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{aB\cos(e}{cf}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.81, size = 125, normalized size = 1.25

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-\left((A+B)\left(\log\left(e^{i(e+fx)}\right)-2\log\left(-i+e^{i(e+fx)}\right)\right)\right)+B\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-((A +
 B)*(Log[E^(I*(e + f*x))] - 2*Log[-I + E^(I*(e + f*x))])) + B*Sin[e + f*x])
)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(92) = 184$.
time = 0.34, size = 394, normalized size = 3.94

method	result
default	$-\frac{\left(A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-2A\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-2A\sin(fx+e)\ln\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/f*(A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-2
*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+B
*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)-2*B
*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f*x+e)*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln(2/(1+cos(f*x+e)))+2*A*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(1+cos(f*x+e)))+B*sin(f*x+e)+2*B*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B)*(a*(1+sin(f*x+e)))^(1/2)/(-1+co
s(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(1/2)
```

Maxima [A]
time = 0.51, size = 187, normalized size = 1.87

$$\frac{B\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}+\frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(c+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+A\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
+ A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

Giac [A]
time = 0.49, size = 148, normalized size = 1.48

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\,B\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(A\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*B*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))/(sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*
(A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sqrt(c)*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2)/(c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+a*B*cos(
f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3050, 2816, 2746, 31, 2817}

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + (a*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e}{\sqrt{c-c\sin(e}}}{c}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(aB\,\mathrm{co}}{\sqrt{a+a\,\mathrm{si}}}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{(aB\cos(e}{cf\sqrt{a}}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos}{cf\sqrt{a+a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.67, size = 118, normalized size = 1.19

$$\frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\left(A+B-B\log\left(e^{i(e+fx)}\right)+2B\log\left(-i+e^{i(e+fx)}\right)+B\left(\log\left(e^{i(e+fx)}\right)-2\log\left(-i+e^{i(e+fx)}\right)\right)\sin(e+fx)\right)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(A + B - B*Log[E^(I*(e + f*x))] +
2*B*Log[-I + E^(I*(e + f*x))] + B*(Log[E^(I*(e + f*x))] - 2*Log[-I + E^(I*(
e + f*x))])*Sin[e + f*x]))/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(91) = 182$.
time = 0.35, size = 403, normalized size = 4.07

method	result
default	$\left(-2B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))+2B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)\cos(fx+e)+B\ln\left(\frac{2}{1+\cos(fx+e)}\right)(\mathrm{co}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*B*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+B*ln(2/(1+cos(f
*x+e)))*cos(f*x+e)^2-B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x
+e)^2-A*sin(f*x+e)*cos(f*x+e)-2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))-4*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*co
s(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+2*B*ln
(2/(1+cos(f*x+e)))*sin(f*x+e)+A*sin(f*x+e)+4*B*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+B*sin(f*x+e)-2*B*ln(2/(1+cos(f*x+e)))-A-B)*(a*(1+sin(f*x+e))
)^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(3/2), x)
```

Giac [A]
time = 0.48, size = 118, normalized size = 1.19

$$-\frac{\left(4\,B\sqrt{c}\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{A\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)\sqrt{a}}{2\,c^{2}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(4*B*sqrt(c)*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + (A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sq
rt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
)*sqrt(a)/(c^2*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.137 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-a*B*
cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3050, 2817}

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e}{(c-c\sin(e+f}}{c}$$

$$= \frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{}{cf\sqrt{a+}}$$

Mathematica [A]
time = 0.37, size = 101, normalized size = 1.10

$$\frac{\sqrt{a(1+\sin(e+fx))}\,(A-B+2B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))
```

Maple [A]
time = 0.31, size = 137, normalized size = 1.49

method	result
default	$-\frac{\left(A\left(\cos^2(fx+e)\right)-A\sin(fx+e)\cos(fx+e)-B\left(\cos^2(fx+e)\right)+B\sin(fx+e)\cos(fx+e)+2A\cos(fx+e)+3A\sin(fx+e)-B\sin(fx+e\right.}{2f(-1+\cos(fx+e)-\sin(fx+e))(-c(\sin(fx+e)-1))^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(A*cos(f*x+e)^2-A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2+B*sin(f*x+e)*
cos(f*x+e)+2*A*cos(f*x+e)+3*A*sin(f*x+e)-B*sin(f*x+e)-3*A+B)*sin(f*x+e)*(a*
(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(5/2), x)
```

Fricas [A]
time = 0.39, size = 94, normalized size = 1.02

$$-\frac{(2\,B\sin\left(fx+e\right)+A-B)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(c^3f\cos\left(fx+e\right)^3+2\,c^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f
*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(5/2), x)
```

Giac [A]
time = 0.48, size = 118, normalized size = 1.28

$$\frac{\left(4\,B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-A\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{8\,c^3f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/8*(4*B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^2 - A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(c)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^3*f*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)-1/2*
a*B*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e}{(c-c\sin(e+f}}{c}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{}{2cf\sqrt{a-}}$$

Mathematica [**A**]
time = 0.43, size = 103, normalized size = 1.10

$$\frac{\sqrt{a(1+\sin(e+fx))}\,(2A-B+3B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
 f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(82) = 164.
time = 0.32, size = 205, normalized size = 2.18

method	result
default	$\frac{(2A(\cos^3(fx+e))+2A(\cos^2(fx+e))\sin(fx+e)-B(\cos^3(fx+e))-B(\cos^2(fx+e))\sin(fx+e)-8A(\cos^2(fx+e))+6A\sin(fx+e)\cos(}{6f(-1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(2*A*cos(f*x+e)^3+2*A*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e)^3-B*cos(f*
x+e)^2*sin(f*x+e)-8*A*cos(f*x+e)^2+6*A*sin(f*x+e)*cos(f*x+e)+4*B*cos(f*x+e)
^2-3*B*sin(f*x+e)*cos(f*x+e)-8*A*cos(f*x+e)-14*A*sin(f*x+e)+B*cos(f*x+e)+4*
B*sin(f*x+e)+14*A-4*B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-s
in(f*x+e))/(-c*(sin(f*x+e)-1))^(7/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(7/2), x)
```

Fricas [A]
time = 0.37, size = 114, normalized size = 1.21

$$-\frac{(3\,B\sin\left(fx+e\right)+2\,A-B)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x +
 e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.53, size = 118, normalized size = 1.26

$$\frac{\left(3\,B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-A\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-B\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{24\,c^4f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 1/24*(3*B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(c)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^4*f*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

Mupad [B]
time = 17.63, size = 153, normalized size = 1.63

$$-\frac{2\,A\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}-B\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}+3\,B\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\frac{9\,c^4\,f\,\cos(3\,e+3\,f\,x)}{2}+\frac{21\,c^4\,f\,\sin(2\,e+2\,f\,x)}{2}-\frac{3\,c^4\,f\,\sin(4\,e+4\,f\,x)}{4}-\frac{21\,c^4\,f\,\cos(e+f\,x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] -(2*A*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) - B*(a + a*sin(
e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*B*sin(e + f*x)*(a + a*sin(e
+ f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e + 3*f*x))/2 + (
21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4 - (21*c^4*f*cos
(e + f*x))/2)
```

3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a+a\sin(e+fx)}}-\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+f}{15f}$$

```
[Out] -1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/30*a^2*
(3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-1/15*a*(
3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(3A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/30*(a^2*(3*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a
*Sin[e + f*x]]) - (a*(3*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(7/2))/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(
c - c*Sin[e + f*x])^(7/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{6f}$$

$$= -\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin}}{15}$$

$$= -\frac{a^2(3A-B)\cos(e+fx)(c-c\sin}{30f\sqrt{a+a\sin(e+fx}}$$

Mathematica [A]
time = 1.02, size = 205, normalized size = 1.40

$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(15(16A-11B)\cos(2(e+fx))+30(2A-B)\cos(4(e+fx))+5B\cos(6(e+fx))+840A\sin(e+fx)-240B\sin(e+fx)+60A\sin(3(e+fx))+40B\sin(3(e+fx))-12A\sin(5(e+fx))+24B\sin(5(e+fx)))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*S
in[e + f*x]]*(15*(16*A - 11*B)*Cos[2*(e + f*x)] + 30*(2*A - B)*Cos[4*(e + f
*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] - 240*B*Sin[e + f*x] + 60*
A*Sin[3*(e + f*x)] + 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x)] + 24*B*S
in[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)
```

Maple [A]
time = 0.41, size = 185, normalized size = 1.27

method	result
default	$\frac{(5B\sin(fx+e)(\cos^4(fx+e))+6A(\cos^4(fx+e))-12B(\cos^4(fx+e))+15A(\cos^2(fx+e))\sin(fx+e)-10B(\cos^2(fx+e))\sin(fx+e)-1}{30f(\cos^2(fx+e)+2\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4-12*B*cos(f*x+e)^4+15*A
*cos(f*x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2+4*B
*cos(f*x+e)^2+15*A*sin(f*x+e)-10*B*sin(f*x+e)-24*A+8*B)*(-c*(sin(f*x+e)-1))
^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/co
s(f*x+e)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [**A**]
time = 0.39, size = 156, normalized size = 1.07

$$\frac{\left(5\,Bac^{3}\cos\left(fx+e\right)^{6}+15\,(A-B)ac^{3}\cos\left(fx+e\right)^{4}-5\,(3\,A-2\,B)ac^{3}-2\,\left(3\,(A-2\,B)ac^{3}\cos\left(fx+e\right)^{4}-2\,(3\,A-B)ac^{3}\cos\left(fx+e\right)^{2}-4\,(3\,A-B)ac^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a*c^3*cos(f*x + e)^6 + 15*(A - B)*a*c^3*cos(f*x + e)^4 - 5*(3*A -
 2*B)*a*c^3 - 2*(3*(A - 2*B)*a*c^3*cos(f*x + e)^4 - 2*(3*A - B)*a*c^3*cos(f
*x + e)^2 - 4*(3*A - B)*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 262, normalized size = 1.79

$$\frac{8 \left(20 Bac^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{12} - 12 Aac^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{10} - 36 Bac^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{10} + 15 Aac^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{8} + 15 Bac^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{8}\right) \sqrt{a} \sqrt{c}}{15 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 8/15*(20*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 12*A*a*c^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^10 - 36*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 15*A*a*c^3*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^8 + 15*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sq
rt(a)*sqrt(c)/f
```

Mupad [B]
time = 17.53, size = 323, normalized size = 2.21

$$\frac{\mathrm{e}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{96\,f}-\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin(e+f\,x)\,(A\,7\mathrm{i}-B\,2\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{4\,f}+\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,(2\,A-B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{16\,f}+\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,(16\,A-11\,B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}-\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,(A\,3\mathrm{i}+B\,2\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{24\,f}+\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin(5\,e+5\,f\,x)\,(A\,1\mathrm{i}-B\,2\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{40\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((B*a*c^3*exp(e*6i + f*x*6
i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) - (a*c^3*exp(e*6i +
f*x*6i)*sin(e + f*x)*(A*7i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f) + (
a*c^3*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A - B)*(a + a*sin(e + f*x))^(1
/2))/(16*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A - 11*B)*(a +
 a*sin(e + f*x))^(1/2))/(32*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(A*3i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) + (a*c^3*exp(e*6i + f*
x*6i)*sin(5*e + 5*f*x)*(A*1i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f))
)/(2*cos(e + f*x))
```

3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a+a\sin(e+fx)}}-\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+f}{20f}$$

```
[Out] -1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/30*a^2*
(5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/20*a*(
5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(5A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{20f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/30*(a^2*(5*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a
*Sin[e + f*x]]) - (a*(5*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(5/2))/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(
c - c*Sin[e + f*x])^(5/2))/(5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{5f}$$

$$= -\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin}}{20}$$

$$= -\frac{a^2(5A-B)\cos(e+fx)(c-c\sin}{30f\sqrt{a+a\sin(e+fx}}$$

Mathematica [A]
time = 1.10, size = 172, normalized size = 1.18

$$\frac{c^2(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(4(100A-11B)\sin(e+fx)+3\cos(4(e+fx))(5A-5B+4B\sin(e+fx))+4\cos(2(e+fx))(15(A-B)+4(5A+2B)\sin(e+fx)))}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(4*(100*A - 11*B)*Sin[e + f*x] + 3*Cos[4*(e + f*x)]*(5*A - 5*B + 4*B*
Sin[e + f*x]) + 4*Cos[2*(e + f*x)]*(15*(A - B) + 4*(5*A + 2*B)*Sin[e + f*x]
)))/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^3)
```

Maple [A]
time = 0.38, size = 147, normalized size = 1.01

method	result
default	$\frac{(-12B(\cos^4(fx+e))+15A(\cos^2(fx+e))\sin(fx+e)-15B(\cos^2(fx+e))\sin(fx+e)-20A(\cos^2(fx+e))+4B(\cos^2(fx+e))+15A\sin(}{60f(\sin(fx+e)-1)\cos(fx+e)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/60/f*(-12*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*s
in(f*x+e)-20*A*cos(f*x+e)^2+4*B*cos(f*x+e)^2+15*A*sin(f*x+e)-15*B*sin(f*x+e
)-40*A+8*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(
sin(f*x+e)-1)/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [A]
time = 0.40, size = 133, normalized size = 0.91

$$\frac{\left(15\,(A-B)ac^2\cos\left(fx+e\right)^4-15\,(A-B)ac^2+4\left(3\,Bac^2\cos\left(fx+e\right)^4+(5\,A-B)ac^2\cos\left(fx+e\right)^2+2\,(5\,A-B)ac^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{60\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/60*(15*(A - B)*a*c^2*cos(f*x + e)^4 - 15*(A - B)*a*c^2 + 4*(3*B*a*c^2*cos
(f*x + e)^4 + (5*A - B)*a*c^2*cos(f*x + e)^2 + 2*(5*A - B)*a*c^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(137) = 274.
time = 0.58, size = 362, normalized size = 2.48

$$-\frac{4\left(24 Bac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + 15 Aac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - 75 Bac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - 40 Aac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + 80 Bac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + 30 Aac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - 30 Bac^2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{15 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] -4/15*(24*B*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*A*a*c^2*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - 75*B*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 40*A*a*c^2*co
s(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 80*B*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 30
*A*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 30*B*a*c^2*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 16.57, size = 174, normalized size = 1.19

$$\frac{a c^2 \sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,\left(60\,A\cos(e+f\,x)-60\,B\cos(e+f\,x)+75\,A\cos(3\,e+3\,f\,x)+15\,A\cos(5\,e+5\,f\,x)-75\,B\cos(3\,e+3\,f\,x)-15\,B\cos(5\,e+5\,f\,x)+400\,A\sin(2\,e+2\,f\,x)+40\,A\sin(4\,e+4\,f\,x)-50\,B\sin(2\,e+2\,f\,x)+16\,B\sin(4\,e+4\,f\,x)+6\,B\sin(6\,e+6\,f\,x)\right)}{480\,f\,(\cos(2\,e+2\,f\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*cos
(e + f*x) - 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*f*
x) - 75*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) + 400*A*sin(2*e + 2*f*x)
 + 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}}-\frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}{3f}-\frac{B\cos(e$$

```
[Out] -1/4*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/f-1/3*a^2*A
*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*A*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{aA\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -1/3*(a^2*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[e +
f*x]]) - (a*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2))/(3*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(3/2))/(4*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{4f}$$
$$= -\frac{aA\cos(e+fx)\sqrt{a+a\sin(e+f}}{3f}$$
$$= -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.51, size = 96, normalized size = 0.72

$$-\frac{c\sec^3(e+fx)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(-12B\cos(2(e+fx))-3B\cos(4(e+fx))+8A(9\sin(e+fx)+\sin(3(e+fx))))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/96*(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sq
rt[c - c*Sin[e + f*x]]*(-12*B*Cos[2*(e + f*x)] - 3*B*Cos[4*(e + f*x)] + 8*A
*(9*Sin[e + f*x] + Sin[3*(e + f*x)])))/f
```

Maple [A]
time = 0.35, size = 86, normalized size = 0.64

method	result	size
default	$\frac{\left(3B\left(\cos^2(fx+e)\right)\sin(fx+e)+4A\left(\cos^2(fx+e)\right)+3B\sin(fx+e)+8A\right)\left(-c(\sin(fx+e)-1)\right)^{\frac{3}{2}}\sin(fx+e)\left(a(1+\sin(fx+e))\right)^{\frac{3}{2}}}{12f\cos(fx+e)^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+3*B*sin(f*x+e)+8*A)*(-
c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [A]
time = 0.38, size = 89, normalized size = 0.66

$$-\frac{\left(3\,Bac\cos\left(fx+e\right)^{4}-3\,Bac-4\,\left(Aac\cos\left(fx+e\right)^{2}+2\,Aac\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/12*(3*B*a*c*cos(f*x + e)^4 - 3*B*a*c - 4*(A*a*c*cos(f*x + e)^2 + 2*A*a*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(125) = 250.
time = 0.53, size = 252, normalized size = 1.88

$$\frac{4\,\left(3\,Bac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,Aac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-6\,Bac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,Aac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Bac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 4/3*(3*B*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*c*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - 6*B*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*A*a*c*cos(-1/4*pi + 1/2
*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) + 3*B*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 1.84, size = 103, normalized size = 0.77

$$-\frac{a\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,B\,\cos\left(e+f\,x\right)+15\,B\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(5\,e+5\,f\,x\right)-80\,A\,\sin\left(2\,e+2\,f\,x\right)-8\,A\,\sin\left(4\,e+4\,f\,x\right)\right)}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*B*cos(
e + f*x) + 15*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) - 80*A*sin(2*e + 2*
f*x) - 8*A*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=96

$$\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)+1/3*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin}}{a}$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+}{2f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 0.40, size = 81, normalized size = 0.84

$$-\frac{a\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(-2(6A+B)\sin(e+fx)+\cos(2(e+fx))(3(A+B)+2B\sin(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -1/12*(a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-2*(6*A + B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*(A + B) + 2*B*Sin[e + f*x])
))/f
```

Maple [A]
time = 0.38, size = 91, normalized size = 0.95

method	result	s
default	$\frac{(-2B(\cos^2(fx+e))+3A\sin(fx+e)+3B\sin(fx+e)+6A+2B)\sqrt{-c(\sin(fx+e)-1)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}}{6f(1+\sin(fx+e))\cos(fx+e)}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(-2*B*cos(f*x+e)^2+3*A*sin(f*x+e)+3*B*sin(f*x+e)+6*A+2*B)*(-c*(sin(f*
x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(1+sin(f*x+e))/cos(f*x+e
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c), x)

Fricas [A]
time = 0.37, size = 93, normalized size = 0.97

$$-\frac{\left(3\,(A+B)a\cos\left(fx+e\right)^{2}-3\,(A+B)a+2\,\left(Ba\cos\left(fx+e\right)^{2}-(3\,A+B)a\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] -1/6*(3*(A + B)*a*cos(f*x + e)^2 - 3*(A + B)*a + 2*(B*a*cos(f*x + e)^2 - (3*A + B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x)), x)

Giac [A]
time = 0.51, size = 153, normalized size = 1.59

$$-\frac{2\left(4\,Ba\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Aa\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,Ba\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] -2/3*(4*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*A*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f

Mupad [B]
time = 14.29, size = 122, normalized size = 1.27

$$-\frac{a\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\,A\,\cos\left(e+f\,x\right)+3\,B\,\cos\left(e+f\,x\right)+3\,A\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(3\,e+3\,f\,x\right)-12\,A\,\sin\left(2\,e+2\,f\,x\right)-2\,B\,\sin\left(2\,e+2\,f\,x\right)+B\,\sin\left(4\,e+4\,f\,x\right)\right)}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
 f*x) + 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12
*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(co
s(2*e + 2*f*x) + 1))
```

3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(}{2f\sqrt{c-}}$$

```
[Out] -1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-2*a^2*(A+
B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)-a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2819, 2816, 2746, 31}

$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a^2*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+(A+B)\int \cdots \\
&= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e\cdots}{2\cdots} \\
&= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e\cdots}{2\cdots} \\
&= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e\cdots}{2\cdots} \\
&= -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.48, size = 136, normalized size = 0.94

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\left(-B\cos(2(e+fx))+16(A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4(A+2B)\sin(e+fx)\right)}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-
(B*Cos[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 4*(A + 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqr
t[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(131) = 262$.
time = 0.38, size = 494, normalized size = 3.41

method	result
default	$-\frac{\left(B\left(\cos^{3}(fx+e)\right)+B\left(\cos^{2}(fx+e)\right)\sin(fx+e)+2A\left(\cos^{2}(fx+e)\right)-2A\sin(fx+e)\cos(fx+e)+4A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-8A\,\mathrm{cc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2-2*A*sin(f
*x+e)*cos(f*x+e)+4*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-8*A*cos(f*x+e)*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-8*A
*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)^2-4*B
*sin(f*x+e)*cos(f*x+e)+4*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-8*B*cos(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*ln(2/(1+cos(f*x+e)))*sin(f*x+
e)-8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*sin(f*x+e)
-4*A*ln(2/(1+cos(f*x+e)))+8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*
cos(f*x+e)+3*B*sin(f*x+e)-4*B*ln(2/(1+cos(f*x+e)))+8*B*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))-2*A-3*B)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)*sin(f*
x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(-c*(sin(e
+ f*x) - 1)), x)
```

Giac [A]
time = 0.56, size = 256, normalized size = 1.77

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\left(Aa\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}Bac^{\frac{3}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Aac^{\frac{3}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Bac^{\frac{3}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] sqrt(2)*sqrt(a)*(sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-cos(-1/4*pi + 1/2*f*
x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (sqrt(2)*B*a*c^
(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*A*a*c^(3/2)*cos(-1/4*pi + 1/2
*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) + sqrt(2)*B*a*c^(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^2)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{a(A+3B)\cos(}{2cf\sqrt{}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*(A
+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+1/2*a*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$\frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (a^2*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 3*B)*Cos[e + f*x]*Sqrt[
a + a*Sin[e + f*x]])/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B}{cf\sqrt{a+a}}$$

Mathematica [**A**]

time = 0.60, size = 210, normalized size = 1.33

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(4A+3B+B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+12B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-2\left(-B+2(A+3B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/2*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
*A + 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(142) = 284$.
time = 0.35, size = 748, normalized size = 4.73

method	result
default	$\frac{\left(-2A-4B+6B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)\cos(fx+e)-3B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)\cos(fx+e)+A\left(\cos^2(fx+e)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(-B*cos(f*x+e)^2*sin(f*x+e)-2*A-4*B+6*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-3*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f
*x+e)+4*B*sin(f*x+e)+2*A*sin(f*x+e)-B*cos(f*x+e)^3+B*cos(f*x+e)+2*A*cos(f*x
+e)^2+4*B*cos(f*x+e)^2+A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+A*cos(f*x+e)*ln(
2/(1+cos(f*x+e)))-6*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)
^2+6*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-3*B*sin(f*x+e)*cos(f*x+e)-6*B*cos(f*
x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*sin(f*x+e)*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-2*A*cos
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*sin(f*x+e)*ln(-(-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+2*A*c
os(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e
)*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*A*ln(2/(1+cos(f*x+e)))+4*A*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))-6*B*ln(2/(1+cos(f*x+e)))+12*B*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))+3*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-2*A*sin(f*x+e)*cos(f*x+
e))*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+
e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(152) = 304$.
time = 0.53, size = 394, normalized size = 2.49

$$-\frac{B\left(\frac{6\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{2\left(\frac{3\,a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}-\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\,a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c^{\frac{3}{2}}-\frac{2\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)+A\left(\frac{2\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{4\,a^{\frac{3}{2}}\sqrt{c}\,\sin(fx+e)}{\left(c^2-\frac{2\,c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] -(B*(6*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 3*a^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 2*(3*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) - 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c^(3/2) - 2*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) + A*(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/c^(3/2) - a^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4
*a^(3/2)*sqrt(c)*sin(f*x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(
f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f*
x) - 1))**(3/2), x)
```

Giac [A]
time = 0.52, size = 236, normalized size = 1.49

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}Ba\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{3}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ba\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-8\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+8\right)}{c^{2}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(Aa\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ba\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)c^{2}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))/(c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(
2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-8*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 8)/
(c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=149

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}}-\frac{a^2B\cos(e+fx)\log(1}{c^2f\sqrt{a+a\sin(e+fx)}\ \sqrt{}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a*B*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^2*B*cos(f*x+e)
*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2818, 2816, 2746, 31}

$$-\frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\ \sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f
*x])^(3/2)) - (a^2*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\int \frac{(a+a\sin\cdots}{(c-c\sin\cdots}}{\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e\cdots}{cf(\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e\cdots}{cf(\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e\cdots}{cf(\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e\cdots}{cf(\cdots}$$

Mathematica [A]
time = 0.66, size = 198, normalized size = 1.33

$$\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(B\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-B\left(2+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\left(A+3B+4B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos[
2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(135) = 270$.
time = 0.35, size = 594, normalized size = 3.99

method	result
default	$\frac{\left(-A+3B-4B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)\cos(fx+e)+2B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)\cos(fx+e)-3B\sin(fx+e)+A\sin}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))*cos(f*x+e)^3+2*B*cos(f*x+e)^2*sin(f*x+e)-A+3*B-4*B*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+2*B*ln(2/(1+cos(f*x+e
)))*sin(f*x+e)*cos(f*x+e)-3*B*sin(f*x+e)+A*sin(f*x+e)+2*B*cos(f*x+e)^3-2*B*
cos(f*x+e)+B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)-2*B*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+A*cos(f*x+e)^2-3*B*co
s(f*x+e)^2+6*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-4*B*
ln(2/(1+cos(f*x+e)))*sin(f*x+e)+B*sin(f*x+e)*cos(f*x+e)+4*B*cos(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+4*B*ln(2/(1+cos(f
*x+e)))-8*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*ln(2/(1+cos(f*x+
e)))*cos(f*x+e)^2-A*sin(f*x+e)*cos(f*x+e))*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*
x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))
^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 -
(c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f*
x) - 1))**(5/2), x)
```

Giac [A]
time = 0.54, size = 215, normalized size = 1.44

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{4\sqrt{2}\,Ba\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\left(Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*sqrt(a)*(4*sqrt(2)*B*a*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
+ 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))) + sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*
B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(A*a*sqrt(c)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\ (a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}+\frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*(
A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A]
time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3051, 2821}

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(5/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\int}{}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\,\mathrm{c}}{24}$$

Mathematica [A]
time = 0.69, size = 125, normalized size = 1.30

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(A+4B-3B\cos(2(e+fx))+3(A-B)\sin(e+fx))}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] -1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A
 + 4*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(c^3*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(84) = 168$.
time = 0.35, size = 223, normalized size = 2.32

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(A(\cos^3(fx+e))+A(\cos^2(fx+e))\sin(fx+e)+B(\cos^3(fx+e))+B(\cos^2(fx+e))\sin(fx+e)-4A(\cos^2(f\right.}{6f(-c(\sin(fx+e)-1))^{\frac{7}{2}}(\cos(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(A*cos(f*x+e)^3+A*cos(f*x+e)^2*si
n(f*x+e)+B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)-4*A*cos(f*x+e)^2+3*A*sin(
f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)-7*A*cos(f*x+e)
-10*A*sin(f*x+e)-B*cos(f*x+e)+2*B*sin(f*x+e)+10*A-2*B)/(-c*(sin(f*x+e)-1))^
(7/2)/(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A]
time = 0.39, size = 134, normalized size = 1.40

$$\frac{\left(6\,Ba\cos\left(fx+e\right)^{2}-3\,(A-B)a\sin\left(fx+e\right)-(A+7\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\,\left(3\,c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4
*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(90) = 180.
time = 0.50, size = 193, normalized size = 2.01

$$-\frac{\left(12\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-3\,Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-9\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{24\,c^{4}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -1/24*(12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^4 - 3*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 - 9*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
```

```
))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 2*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(
c^4*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e-}{96c^2f(c}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(9/2)+1/24*(
A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/96*(A
-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A]
time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(7/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)
)/(96*c^2*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\,c}{24}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\,c}{24}$$

Mathematica [A]
time = 0.89, size = 123, normalized size = 0.84

$$\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(2A+3B-3B\cos(2(e+fx))+4A\sin(e+fx))}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A +
3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.32, size = 217, normalized size = 1.49

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(A(\cos^4(fx+e))-A(\cos^3(fx+e))\sin(fx+e)+4A(\cos^3(fx+e))+5A(\cos^2(fx+e))\sin(fx+e)-12A(\cos\right.}{6f(-c(\sin(fx+e)-1))^{\frac{9}{2}}(\cos(fx+e)\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*si
n(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2+7*A
*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)-10*A*cos(
f*x+e)-17*A*sin(f*x+e)+3*B*sin(f*x+e)+17*A-3*B)/(-c*(sin(f*x+e)-1))^(9/2)/(
cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [**A**]
time = 0.41, size = 144, normalized size = 0.99

$$-\frac{\left(3\,Ba\cos\left(fx+e\right)^{2}-2\,Aa\sin\left(fx+e\right)-\left(A+3\,B\right)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\,\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\,\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(
f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f
*x + e))*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 193, normalized size = 1.32

$$-\frac{\left(12\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-4\,Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-12\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{96\,c^{5}f\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] -1/96*(12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^4 - 4*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 - 12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 3*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/
(c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8
)
```

Mupad [B]
time = 18.99, size = 245, normalized size = 1.68

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}+\frac{32\,A\,a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}-\frac{8\,B\,a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^5\,f}\right)}{84\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}-54\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)+16\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((8*a*exp(e*5i + f*x*5i)*(2*A + 3*B)*(a + a*sin
(e + f*x))^(1/2))/(3*c^5*f) + (32*A*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a +
a*sin(e + f*x))^(1/2))/(3*c^5*f) - (8*B*a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*
x)*(a + a*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i
) - 54*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e +
 5*f*x) - 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*si
n(4*e + 4*f*x))
```

3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{40cf(c-c\sin(e+fx))^{9/2}}-\frac{a^2(\ldots}{120c^2f\sqrt{a+a\ldots}}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(11/2)-1/12
0*a^2*(3*A-7*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1
/2)+1/40*a*(3*A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))
^(9/2)
```

Rubi [A]
time = 0.25, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2818, 2817}

$$-\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + (a*(3*A - 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(40*c*f*(
c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 7*B)*Cos[e + f*x])/(120*c^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-7B)}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B}{40}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B}{40}$$

Mathematica [A]
time = 1.27, size = 126, normalized size = 0.82

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(9(A+B)-10B\cos(2(e+fx))+5(3A+B)\sin(e+fx))}{60c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] -1/60*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
9*(A + B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(c^5*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f
*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(136) = 272.
time = 0.36, size = 339, normalized size = 2.20

method	result

default	$-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}(9A(\cos^5(fx+e))+9A(\cos^4(fx+e))\sin(fx+e)-B(\cos^5(fx+e))-B\sin(fx+e)(\cos^4(fx+e))-54A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/60/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(9*A*cos(f*x+e)^5+9*A*cos(f*x+e
)^4*sin(f*x+e)-B*cos(f*x+e)^5-B*sin(f*x+e)*cos(f*x+e)^4-54*A*cos(f*x+e)^4+4
5*A*cos(f*x+e)^3*sin(f*x+e)+6*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-10
8*A*cos(f*x+e)^3-153*A*cos(f*x+e)^2*sin(f*x+e)+12*B*cos(f*x+e)^3+17*B*cos(f
*x+e)^2*sin(f*x+e)+288*A*cos(f*x+e)^2-135*A*sin(f*x+e)*cos(f*x+e)-52*B*cos(
f*x+e)^2+35*B*sin(f*x+e)*cos(f*x+e)+159*A*cos(f*x+e)+294*A*sin(f*x+e)-11*B*
cos(f*x+e)-46*B*sin(f*x+e)-294*A+46*B)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+
e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Fricas [A]
time = 0.40, size = 166, normalized size = 1.08

$$-\frac{(20\,Ba\cos(fx+e)^2-5\,(3\,A+B)a\sin(fx+e)-(9\,A+19\,B)a)\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{60\,(5\,c^6f\cos(fx+e)^5-20\,c^6f\cos(fx+e)^3+16\,c^6f\cos(fx+e)-(c^6f\cos(fx+e)^5-12\,c^6f\cos(fx+e)^3+16\,c^6f\cos(fx+e))\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a)
*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5
 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5
- 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 193, normalized size = 1.25

$$-\frac{\left(40\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-45\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+12\,Aa\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+12\,Ba\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{960\,c^{6}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] -1/960*(40*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4 - 15*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 - 45*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 12*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt
(a)/(c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^10)
```

Mupad [B]
time = 20.23, size = 279, normalized size = 1.81

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(A+B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,48{}\mathrm{i}}{5\,c^6\,f}-\frac{B\,a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{3\,c^6\,f}+\frac{16\,a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\left(A\,3{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^6\,f}\right)}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(A + B)*(a + a*sin(e + f
*x))^(1/2)*48i)/(5*c^6*f) - (B*a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2)*32i)/(3*c^6*f) + (16*a*exp(e*6i + f*x*6i)*sin(e + f*x)
*(A*3i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f)))/(cos(e + f*x)*exp(e*
6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f
*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + ex
p(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)
*2i)
```

3.149 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=198

$$-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a+a\sin(e+fx)}}-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e}{105f}$$

```
[Out] -1/42*a*(7*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-
1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/105*a^3*
(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a^
2*(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.31, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{105f}-\frac{a(7A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{42f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/105*(a^3*(7*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a^2*(7*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(
c - c*Sin[e + f*x])^(7/2))/(105*f) - (a*(7*A - B)*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(42*f) - (B*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(7*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
```

```
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/}}{7f} \\ &= -\frac{a(7A-B)\cos(e+fx)(a+a\sin(e}{42f} \\ &= -\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{105} \\ &= -\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e}{105f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [**A**]
time = 1.77, size = 223, normalized size = 1.13

$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\,(525(A-B)\cos(2(e+fx))+210(A-B)\cos(4(e+fx))+35A\cos(6(e+fx))-35B\cos(6(e+fx))+4200A\sin(e+fx)-525B\sin(e+fx)+700A\sin(3(e+fx))+35B\sin(3(e+fx))+84A\sin(5(e+fx))+63B\sin(5(e+fx))+15B\sin(7(e+fx)))}{6720f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*
Sin[e + f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)]
+ 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] - 525
*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A*Sin
[5*(e + f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [**A**]
time = 0.45, size = 203, normalized size = 1.03

method	result
default	$\frac{\left(-30B\left(\cos^{6}(fx+e)\right)+35A\left(\cos^{4}(fx+e)\right)\sin(fx+e)-35B\sin(fx+e)\left(\cos^{4}(fx+e)\right)-42A\left(\cos^{4}(fx+e)\right)+6B\left(\cos^{4}(fx+e)\right)+35A\left(\cos^{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/210/f*(-30*B*cos(f*x+e)^6+35*A*cos(f*x+e)^4*sin(f*x+e)-35*B*sin(f*x+e)*co
s(f*x+e)^4-42*A*cos(f*x+e)^4+6*B*cos(f*x+e)^4+35*A*cos(f*x+e)^2*sin(f*x+e)-
35*B*cos(f*x+e)^2*sin(f*x+e)-56*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2+35*A*sin(f*
x+e)-35*B*sin(f*x+e)-112*A+16*B)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1
+sin(f*x+e)))^(5/2)/(sin(f*x+e)-1)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A]
time = 0.44, size = 168, normalized size = 0.85

$$\frac{\left(35\,(A-B)a^{2}c^{3}\cos\left(fx+e\right)^{6}-35\,(A-B)a^{2}c^{3}+2\left(15\,Ba^{2}c^{3}\cos\left(fx+e\right)^{6}+3\,(7\,A-B)a^{2}c^{3}\cos\left(fx+e\right)^{4}+4\,(7\,A-B)a^{2}c^{3}\cos\left(fx+e\right)^{2}+8\,(7\,A-B)a^{2}c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{210\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^2
*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^2*
c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(186) = 372.
time = 0.64, size = 480, normalized size = 2.42

$$\frac{16\left(120\,Ba^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+70\,Aa^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-490\,Ba^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-252\,Aa^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+756\,Ba^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+315\,Aa^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-525\,Ba^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-140\,Aa^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+140\,Ba^2c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 16/105*(120*B*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 70*A*a^2*c^3*cos(-1/
4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 490*B*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 25
2*A*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 756*B*a^2*c^3*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 315*A*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 525*B*a^2*c^
3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 140*A*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) + 140*B*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 18.17, size = 383, normalized size = 1.93

$$\frac{{\mathrm{e}}^{-e\,7{}\mathrm{i}-f\,x\,7{}\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,1{}\mathrm{i}-B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,5{}\mathrm{i}}{32\,f}-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\left(A\,1{}\mathrm{i}-B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1{}\mathrm{i}}{16\,f}-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(6\,e+6\,f\,x\right)\,\left(A\,1{}\mathrm{i}-B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1{}\mathrm{i}}{96\,f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,\left(4\,A+3\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{5\,a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\left(8\,A-B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\left(20\,A+B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{96\,f}+\frac{B\,a^2\,c^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(7\,e+7\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^2*c^3*exp(e*7i + f*x*7
i)*sin(5*e + 5*f*x)*(4*A + 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^2*
c^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^
(1/2)*1i)/(16*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i - B*1
```

```
i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(
2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (5*a^2*c
^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A - B)*(a + a*sin(e + f*x))^(1/2))/(3
2*f) + (a^2*c^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A + B)*(a + a*sin(e
 + f*x))^(1/2))/(96*f) + (B*a^2*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a
+ a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=180

$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}}-\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{aA\,\mathrm{co}}{}$$

```
[Out] -1/5*a*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/6*B*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f-2/15*a^3*A*cos(f*
x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*A*cos(f*x+e)*(
c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.31, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2A\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{aA\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (-2*a^3*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a^2*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2))/(5*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2))/(5*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(5/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(I
```

```
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{6f}$$

$$= -\frac{aA\cos(e+fx)(a+a\sin(e+fx)}{5f}$$

$$= -\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+f}}{5f}$$

$$= -\frac{2a^3A\cos(e+fx)(c-c\sin(e+f}{15f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.52, size = 113, normalized size = 0.63

$$\frac{a^2c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-75B\cos(2(e+fx))-30B\cos(4(e+fx))-5B\cos(6(e+fx))+600A\sin(e+fx)+100A\sin(3(e+fx))+12A\sin(5(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-75*B*Cos[2*(e + f*x)] - 30*B*Cos[4*(e + f*x)] - 5*B*Cos[6*(e + f*x)] + 600
*A*Sin[e + f*x] + 100*A*Sin[3*(e + f*x)] + 12*A*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A]
time = 0.38, size = 114, normalized size = 0.63

method	result

default	$\frac{(5B\sin(fx+e)(\cos^4(fx+e))+6A(\cos^4(fx+e))+5B(\cos^2(fx+e))\sin(fx+e)+8A(\cos^2(fx+e))+5B\sin(fx+e)+16A)(-c(\sin(fx+e)}{30f\cos(fx+e)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4+5*B*cos(f*x+e)^2*sin(f
*x+e)+8*A*cos(f*x+e)^2+5*B*sin(f*x+e)+16*A)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f
*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [A]
time = 0.40, size = 124, normalized size = 0.69

$$-\frac{\left(5\,Ba^2c^2\cos\left(fx+e\right)^6-5\,Ba^2c^2-2\left(3\,Aa^2c^2\cos\left(fx+e\right)^4+4\,Aa^2c^2\cos\left(fx+e\right)^2+8\,Aa^2c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/30*(5*B*a^2*c^2*cos(f*x + e)^6 - 5*B*a^2*c^2 - 2*(3*A*a^2*c^2*cos(f*x +
e)^4 + 4*A*a^2*c^2*cos(f*x + e)^2 + 8*A*a^2*c^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(168) = 336.
time = 0.57, size = 376, normalized size = 2.09

$$-\frac{16 \left(10 \, B a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{12} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) + 6 \, A a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{10} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) - 30 \, B a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{10} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) - 15 \, A a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{8} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) + 30 \, B a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{8} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) + 10 \, A a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{6} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) - 10 \, B a^{2} c^{2} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{6} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right) \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right)\right) \sqrt{a} \sqrt{c}}{15 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] -16/15*(10*B*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a^2*c^2*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 30*B*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*
a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a^2*c^2*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) + 10*A*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*B*a^2*c^2*cos(-1/
4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 16.00, size = 131, normalized size = 0.73

$$-\frac{a^2\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,B\,\cos\left(e+f\,x\right)+105\,B\,\cos\left(3\,e+3\,f\,x\right)+35\,B\,\cos\left(5\,e+5\,f\,x\right)+5\,B\,\cos\left(7\,e+7\,f\,x\right)-700\,A\,\sin\left(2\,e+2\,f\,x\right)-112\,A\,\sin\left(4\,e+4\,f\,x\right)-12\,A\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] -(a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*B*
cos(e + f*x) + 105*B*cos(3*e + 3*f*x) + 35*B*cos(5*e + 5*f*x) + 5*B*cos(7*e
 + 7*f*x) - 700*A*sin(2*e + 2*f*x) - 112*A*sin(4*e + 4*f*x) - 12*A*sin(6*e
+ 6*f*x)))/(960*f*(cos(2*e + 2*f*x) + 1))
```

3.151 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=142

$$\frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{20f}$$

```
[Out] -1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(5*A
+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/20*(5*
A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.24, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$\frac{c^2(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{c(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{20f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] ((5*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((5*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c
- c*Sin[e + f*x]])/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{5f}$$

$$= \frac{(5A+B)c\cos(e+fx)(a+a\sin(e}{20f}$$

$$= \frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e}{30f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 1.19, size = 165, normalized size = 1.16

$$-\frac{c(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\,(4(100A+11B)\sin(e+fx)+4\cos(2(e+fx))(-15(A+B)+4(5A-2B)\sin(e+fx))-3\cos(4(e+fx))(5(A+B)+4B\sin(e+fx)))}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/480*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]*(4*(100*A + 11*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-15*(A + B) +
 4*(5*A - 2*B)*Sin[e + f*x]) - 3*Cos[4*(e + f*x)]*(5*(A + B) + 4*B*Sin[e +
f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^5)
```

Maple [A]
time = 0.38, size = 147, normalized size = 1.04

method	result
default	$\frac{(-12B(\cos^4(fx+e))+15A(\cos^2(fx+e))\sin(fx+e)+15B(\cos^2(fx+e))\sin(fx+e)+20A(\cos^2(fx+e))+4B(\cos^2(fx+e))+15A\sin(}{60f(1+\sin(fx+e))\cos(fx+e)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/60/f*(-12*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*s
in(f*x+e)+20*A*cos(f*x+e)^2+4*B*cos(f*x+e)^2+15*A*sin(f*x+e)+15*B*sin(f*x+e
)+40*A+8*B)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(
1+sin(f*x+e))/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [A]
time = 0.42, size = 126, normalized size = 0.89

$$-\frac{\left(15\,(A+B)a^2c\cos\left(fx+e\right)^4-15\,(A+B)a^2c+4\left(3\,Ba^2c\cos\left(fx+e\right)^4-(5\,A+B)a^2c\cos\left(fx+e\right)^2-2\,(5\,A+B)a^2c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{60\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/60*(15*(A + B)*a^2*c*cos(f*x + e)^4 - 15*(A + B)*a^2*c + 4*(3*B*a^2*c*co
s(f*x + e)^4 - (5*A + B)*a^2*c*cos(f*x + e)^2 - 2*(5*A + B)*a^2*c)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 262, normalized size = 1.85

$$\frac{4 \left(24 B a^{2} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{10} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 15 A a^{2} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{8} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 45 B a^{2} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{8} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 20 A a^{2} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{6} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 20 B a^{2} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{6} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)\right) \sqrt{a} \sqrt{c}}{15 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 4/15*(24*B*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*A*a^2*c*cos(-1/4*pi +
1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) - 45*B*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*A*a^2*c*cos
(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt
(a)*sqrt(c)/f
```

Mupad [B]
time = 16.32, size = 174, normalized size = 1.23

$$-\frac{a^{2} c \sqrt{a \left(\sin\left(e + f x\right) + 1\right)} \sqrt{-c \left(\sin\left(e + f x\right) - 1\right)} \left(60 A \cos\left(e + f x\right) + 60 B \cos\left(e + f x\right) + 75 A \cos\left(3 e + 3 f x\right) + 15 A \cos\left(5 e + 5 f x\right) + 75 B \cos\left(3 e + 3 f x\right) + 15 B \cos\left(5 e + 5 f x\right) - 400 A \sin\left(2 e + 2 f x\right) - 40 A \sin\left(4 e + 4 f x\right) - 50 B \sin\left(2 e + 2 f x\right) + 16 B \sin\left(4 e + 4 f x\right) + 6 B \sin\left(6 e + 6 f x\right)\right)}{480 f \left(\cos\left(2 e + 2 f x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*co
s(e + f*x) + 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*f
*x) + 75*B*cos(3*e + 3*f*x) + 15*B*cos(5*e + 5*f*x) - 400*A*sin(2*e + 2*f*x
) - 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=96

$$\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin}}{a}$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+}{3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 0.57, size = 102, normalized size = 1.06

$$\frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(3B\cos(4(e+fx))+16(7A+2B)\sin(e+fx)-4\cos(2(e+fx))(12A+9B+4(A+2B)\sin(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A + 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(12*A +
 9*B + 4*(A + 2*B)*Sin[e + f*x])))/(96*f)
```

Maple [A]
time = 0.39, size = 131, normalized size = 1.36

method	result
default	$\frac{(-3B(\cos^2(fx+e))\sin(fx+e)-4A(\cos^2(fx+e))-8B(\cos^2(fx+e))+12A\sin(fx+e)+9B\sin(fx+e)+16A+8B)\sqrt{-c\,(\sin\,(fx}}{12f(2\sin(fx+e)-(\cos^2(fx+e))+2)\cos(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(-3*B*cos(f*x+e)^2*sin(f*x+e)-4*A*cos(f*x+e)^2-8*B*cos(f*x+e)^2+12*A
*sin(f*x+e)+9*B*sin(f*x+e)+16*A+8*B)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(
a*(1+sin(f*x+e)))^(5/2)/(2*sin(f*x+e)-cos(f*x+e)^2+2)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [**A**]
time = 0.41, size = 124, normalized size = 1.29

$$\frac{\left(3\,Ba^{2}\cos\left(fx+e\right)^{4}-12\,\left(A+B\right)a^{2}\cos\left(fx+e\right)^{2}+3\,\left(4\,A+3\,B\right)a^{2}-4\,\left(\left(A+2\,B\right)a^{2}\cos\left(fx+e\right)^{2}-2\,\left(2\,A+B\right)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*a^2*cos(f*x + e)^4 - 12*(A + B)*a^2*cos(f*x + e)^2 + 3*(4*A + 3*B
)*a^2 - 4*((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(2*A + B)*a^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [**A**]
time = 0.51, size = 159, normalized size = 1.66

$$-\frac{4\,\left(3\,Ba^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,Aa^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,Ba^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/3*(3*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - 2*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [**B**]
time = 2.70, size = 149, normalized size = 1.55

$$-\frac{a^{2}\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)+36\,B\,\cos\left(e+f\,x\right)+48\,A\,\cos\left(3\,e+3\,f\,x\right)+33\,B\,\cos\left(3\,e+3\,f\,x\right)-3\,B\,\cos\left(5\,e+5\,f\,x\right)-112\,A\,\sin\left(2\,e+2\,f\,x\right)+8\,A\,\sin\left(4\,e+4\,f\,x\right)-32\,B\,\sin\left(2\,e+2\,f\,x\right)+16\,B\,\sin\left(4\,e+4\,f\,x\right)\right)}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(
e + f*x) + 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) + 33*B*cos(3*e + 3*f*x
) - 3*B*cos(5*e + 5*f*x) - 112*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x) -
32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1
))
```

3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=193

$$-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos}{2f}$$

```
[Out] -1/2*a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-4*a^3*(A+B)*c
os(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-
2*a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2819, 2816, 2746, 31}

$$-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-4*a^3*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*(a + a*S
in[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a}{\sqrt{}}$$

$$= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(}{3}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+}{}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+}{}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+}{}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A}{}$$

Mathematica [A]
time = 1.01, size = 177, normalized size = 0.92

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-3(A+3B)\cos(2(e+fx))+96A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+96B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+(36A+51B)\sin(e+fx)-B\sin(3(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(
-3*(A + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] + 96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e +
f*x] - B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt
[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(173) = 346$.
time = 0.38, size = 590, normalized size = 3.06

method	result
default	$\frac{\left(-15A-17B-3A\cos(fx+e)+17B\sin(fx+e)+15A\sin(fx+e)-9B\cos(fx+e)+7B\left(\cos^2(fx+e)\right)\sin(fx+e)+3A\left(\cos^2(fx+e)\right)\sin(fx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(3*A*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^2*sin(f*x+e)-15*A-17*B-3*
A*cos(f*x+e)+17*B*sin(f*x+e)+15*A*sin(f*x+e)+3*A*cos(f*x+e)^3+9*B*cos(f*x+e
)^3-9*B*cos(f*x+e)+2*B*cos(f*x+e)^3*sin(f*x+e)-2*B*cos(f*x+e)^4+15*A*cos(f*
x+e)^2+19*B*cos(f*x+e)^2+24*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+24*B*ln(2/(1+
cos(f*x+e)))*sin(f*x+e)-26*B*sin(f*x+e)*cos(f*x+e)-48*B*cos(f*x+e)*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+24*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-48*A*cos(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))+24*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-24*A*ln(2/(1+c
os(f*x+e)))+48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*B*ln(2/(1+co
s(f*x+e)))+48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-18*A*sin(f*x+e)*
cos(f*x+e))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-
3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(si
n(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.59, size = 318, normalized size = 1.65

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(4Ba^2c^{\frac{5}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Aa^2c^{\frac{5}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ba^2c^{\frac{5}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6Aa^2c^{\frac{5}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6Ba^2c^{\frac{5}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(a)*(6*(sqrt(2)*A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + sqrt(2)*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-co
s(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 + sqrt(2)*(4*B*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + 3*A*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x +
```

```
 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a^2*c^(5/2)*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2*c^(5/2)
*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(c^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}+\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(A+2B)\,c}{cf}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a*
(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+4*a^3*
(A+2*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+2*a^2*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x
+e))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a(A+2B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^3*(A + 2*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(A + 2*B)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 2*B)*Co
s[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+2B}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3(A+2}{cf\sqrt{a+}}$$

Mathematica [A]
time = 1.15, size = 231, normalized size = 1.10

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(28A+16B+2(2A+7B)\cos(2(e+fx))+64A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+128B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(8A+31B-64(A+2B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+B\sin(3(e+fx))\right)}{8cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/8*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(28*A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8*A
+ 31*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x
] + B*Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(190) = 380$.
time = 0.31, size = 845, normalized size = 4.02

method	result

default	$\frac{\left(12A+22B-32B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)\cos(fx+e)+16B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)\cos(fx+e)-8A(\cos^2(fx+e)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(2*A*cos(f*x+e)^2*sin(f*x+e)+6*B*cos(f*x+e)^2*sin(f*x+e)+12*A+22*B-32
*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+16*B*ln
(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)-2*A*cos(f*x+e)-22*B*sin(f*x+e)-12*
A*sin(f*x+e)+2*A*cos(f*x+e)^3+7*B*cos(f*x+e)^3-7*B*cos(f*x+e)+B*cos(f*x+e)^
3*sin(f*x+e)-B*cos(f*x+e)^4-12*A*cos(f*x+e)^2-21*B*cos(f*x+e)^2-8*A*cos(f*x
+e)^2*ln(2/(1+cos(f*x+e)))-8*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+32*B*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-32*B*ln(2/(1+cos(f*x+e)))*
sin(f*x+e)+15*B*sin(f*x+e)*cos(f*x+e)+32*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+64*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-16*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+16*A*cos(f*x+e)*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+32*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-16*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-16*A*cos(f*x+e)*sin(f*x+e)*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(
1+cos(f*x+e)))+16*A*ln(2/(1+cos(f*x+e)))-32*A*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))+32*B*ln(2/(1+cos(f*x+e)))-64*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))+16*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1
6*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+10*A*sin(f*x+e)*cos(f*x+e))*(a*(1+sin
(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*cos(
f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}}-\frac{a^3(A+5B)\cos(e}{c^2f\sqrt{a+a\sin(e}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-1/4*a*
(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*(A
+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)-1/2*a^2*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (a^3*(A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]]
)/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + 5*B
)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)}{4}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)}{4}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)}{4}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)}{4}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)}{4}$$

Mathematica [A]
time = 0.77, size = 207, normalized size = 0.98

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(2(A+B)-4(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2(A+5B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A +
 B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. 2(190) = 380.
time = 0.31, size = 1092, normalized size = 5.15

method	result	size
default	Expression too large to display	1092

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/f*(5*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-10*B*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))*cos(f*x+e)^3+2*A*cos(f*x+e)^2*sin(f*x+e)+9*B*cos(f*x+e)^2*
sin(f*x+e)+2*A+14*B-20*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x
+e)*cos(f*x+e)+10*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)-2*A*cos(f*x+
e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)^3*ln(2/(1+cos(
f*x+e)))-2*A*cos(f*x+e)-14*B*sin(f*x+e)-2*A*sin(f*x+e)+2*A*cos(f*x+e)^3+8*B
*cos(f*x+e)^3-8*B*cos(f*x+e)-B*cos(f*x+e)^3*sin(f*x+e)+5*B*ln(2/(1+cos(f*x+
e)))*cos(f*x+e)^2*sin(f*x+e)-10*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^4-2*A*cos(f*x+e)^2-15*B*cos(f*x+e)^2
-3*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-2*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+
30*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-20*B*ln(2/(1+c
os(f*x+e)))*sin(f*x+e)+6*B*sin(f*x+e)*cos(f*x+e)+20*B*cos(f*x+e)*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+40*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-2*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))+A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-4*A*ln(2/(1+cos
(f*x+e)))*sin(f*x+e)+4*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-10*B*cos(f*x+
e)*ln(2/(1+cos(f*x+e)))-4*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+4*A*ln(2/(
1+cos(f*x+e)))-8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+20*B*ln(2/(1+
cos(f*x+e)))-40*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*A*cos(f*x+e)
^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-15*B*ln(2/(1+cos(f*x+e)))*cos
(f*x+e)^2)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3
*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin
(f*x+e)-1))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(203) = 406$.
time = 0.70, size = 546, normalized size = 2.58

$$-\frac{\left(\frac{8 a^{\frac{5}{2}} \sqrt{c} \sin(fx+e)^2}{\left(c^3-\frac{4 c^3 \sin(fx+e)}{\cos(fx+e)+1}+\frac{6 c^3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4 c^3 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^3 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2}-\frac{2 a^{\frac{5}{2}} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}}+\frac{a^{\frac{5}{2}} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}}\right)A-B\left(\frac{10 a^{\frac{5}{2}} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}}-\frac{5 a^{\frac{5}{2}} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}}+\frac{2\left(\frac{5 a^{\frac{5}{2}} \sin(fx+e)}{\cos(fx+e)+1}-\frac{16 a^{\frac{5}{2}} \sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{14 a^{\frac{5}{2}} \sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{16 a^{\frac{5}{2}} \sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5 a^{\frac{5}{2}} \sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{c^{\frac{5}{2}}-\frac{4 c^{\frac{5}{2}} \sin(fx+e)}{\cos(fx+e)+1}+\frac{7 c^{\frac{5}{2}} \sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{8 c^{\frac{5}{2}} \sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{7 c^{\frac{5}{2}} \sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4 c^{\frac{5}{2}} \sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{5}{2}} \sin(fx+e)^6}{(\cos(fx+e)+1)^6}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] -((8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e
) + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^
(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))*A - B*(10*a^(5/
2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 5*a^(5/2)*log(sin(f*x
```

```
 + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + 2*(5*a^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) - 16*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*a^(5/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 16*a^(5/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 5*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(5/2) - 4*c^
(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 8*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*c^(5/2)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin
(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.56, size = 311, normalized size = 1.47

$$\frac{\sqrt{2}\left(\frac{4\sqrt{2}Ba^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\sqrt{2}\left(Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-32\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+32\right)}{c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(3Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\left(Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{a}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(4*sqrt(2)*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*sq
rt(2)*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sqrt(c)*
```

```
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-32*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 32)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^2*sqrt(
c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a^2*sqrt(c)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) - 4*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
2*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}}{\left(c-c\,\sin(e+f\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(5/2),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=196

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2B\cos(e+fx)\sqrt{a+a}}{c^2f(c-c\sin(e+f}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^2*B*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^3*B*cos(f*x+e)*l
n(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2818, 2816, 2746, 31}

$$\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{a^2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e
 + f*x])^(5/2)) + (a^2*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c -
 c*Sin[e + f*x])^(3/2)) + (a^3*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\int \frac{(a+a\,\mathrm{si}}{(c-c\,\mathrm{si}}}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e}{2cf}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e}{2cf}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e}{2cf}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e}{2cf}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e}{2cf}$$

Mathematica [A]
time = 0.79, size = 204, normalized size = 1.04

$$\frac{\left(4(A+B)-6(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3(A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+6B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] ((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(176) = 352$.
time = 0.34, size = 832, normalized size = 4.24

method	result
default	$\frac{\left(3B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^4(fx+e)\right)-6B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^4(fx+e)\right)-3B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^3(fx+e)\right)\sin(fx+e)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(9*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-18*B*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))*cos(f*x+e)^3+A*cos(f*x+e)^2*sin(f*x+e)+13*B*cos(f*x+e)^2*
sin(f*x+e)+4*A+16*B-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x
+e)*cos(f*x+e)+12*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)-3*B*ln(2/(1+
cos(f*x+e)))*cos(f*x+e)^3*sin(f*x+e)-16*B*sin(f*x+e)-4*A*sin(f*x+e)+6*B*cos
(f*x+e)^3-6*B*cos(f*x+e)-7*B*cos(f*x+e)^3*sin(f*x+e)+12*B*ln(2/(1+cos(f*x+e
)))*cos(f*x+e)^2*sin(f*x+e)-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^4-5*A*cos(f*x+e)^2-23*B*cos(f*x+e)^
2-A*cos(f*x+e)^3*sin(f*x+e)+6*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*
cos(f*x+e)^3*sin(f*x+e)+48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos
(f*x+e)^2+3*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^4-6*B*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*cos(f*x+e)^4-24*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+10*B*
sin(f*x+e)*cos(f*x+e)+24*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*cos(f
*x+e)*ln(2/(1+cos(f*x+e)))+A*cos(f*x+e)^4+24*B*ln(2/(1+cos(f*x+e)))-48*B*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*B*ln(2/(1+cos(f*x+e)))*cos(f*x+
e)^2+4*A*sin(f*x+e)*cos(f*x+e))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(
```

```
f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*s
in(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos
(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.55, size = 289, normalized size = 1.47

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{6\sqrt{2}Ba^2\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{7}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3\left(Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+10Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\left(Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*sqrt(a)*(6*sqrt(2)*B*a^2*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) - sqrt(2)*(3*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 5*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1
/2*f*x + 1/2*e)^4 + A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*
B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*(A*a^2*sqrt(c)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)^3*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{(A-7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*(
A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A]
time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3051, 2821}

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c -
c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\int}{}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\,\mathrm{c}}{48}$$

Mathematica [A]
time = 1.91, size = 145, normalized size = 1.51

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(5A-5B-3(A-B)\cos(2(e+fx))+(4A+17B)\sin(e+fx)-3B\sin(3(e+fx)))}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A
- 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[3*
(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(84) = 168$.
time = 0.32, size = 309, normalized size = 3.22

method	result
default	$-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(A\left(\cos^4(fx+e)\right)-A\left(\cos^3(fx+e)\right)\sin(fx+e)-B\left(\cos^4(fx+e)\right)+B\left(\cos^3(fx+e)\right)\sin(fx+e)+4A\left(\cos^3\right.\right.}{6f(-c(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*s
in(f*x+e)-B*cos(f*x+e)^4+B*cos(f*x+e)^3*sin(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos
(f*x+e)^2*sin(f*x+e)+2*B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)-9*A*cos(f*x
+e)^2+4*A*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^2-4*B*sin(f*x+e)*cos(f*x+e)-
10*A*cos(f*x+e)-14*A*sin(f*x+e)-2*B*cos(f*x+e)+2*B*sin(f*x+e)+14*A-2*B)/(-c
*(sin(f*x+e)-1))^(9/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2
-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [A]
time = 0.38, size = 176, normalized size = 1.83

$$-\frac{\left(3\,(A-B)a^2\cos\left(fx+e\right)^2-4\,(A-B)a^2+2\left(3\,Ba^2\cos\left(fx+e\right)^2-(A+5\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x + e
)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x
 + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(90) = 180.
time = 0.55, size = 282, normalized size = 2.94

$$-\frac{\left(24\,Ba^2\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\,Aa^2\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-42\,Ba^2\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\,Aa^2\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+28\,Ba^2\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\,Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{48\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^4c^5f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] -1/48*(24*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 6*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - 42*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*A*a^2*sqrt(c)*cos(-1/4*pi +
1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 28*B*a^2*sqrt(c)*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*a^2
*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*sqrt(c)*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^4*
c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(9/2), x)
```

3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A-4B)\cos(e-}{240c^2f(c}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/40
*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/240
*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A]
time = 0.26, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(9/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2))/(240*c^2*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\,\mathrm{c}}{40}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\,\mathrm{c}}{40}$$

Mathematica [A]
time = 2.66, size = 146, normalized size = 1.00

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(-36A-6B+10(2A+B)\cos(2(e+fx))-5(8A+13B)\sin(e+fx)+15B\sin(3(e+fx)))}{120c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36*
A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 15*
B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(128) = 256$.
time = 0.39, size = 368, normalized size = 2.52

method	result

default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}(4A(\cos^5(fx+e))+4A(\cos^4(fx+e))\sin(fx+e)-B(\cos^5(fx+e))-B\sin(fx+e)(\cos^4(fx+e))-24A(c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/30/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(4*A*cos(f*x+e)^5+4*A*cos(f*x+e)
^4*sin(f*x+e)-B*cos(f*x+e)^5-B*sin(f*x+e)*cos(f*x+e)^4-24*A*cos(f*x+e)^4+20
*A*cos(f*x+e)^3*sin(f*x+e)+6*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-48*
A*cos(f*x+e)^3-68*A*cos(f*x+e)^2*sin(f*x+e)-3*B*cos(f*x+e)^3+2*B*cos(f*x+e)
^2*sin(f*x+e)+118*A*cos(f*x+e)^2-50*A*sin(f*x+e)*cos(f*x+e)-22*B*cos(f*x+e)
^2+20*B*sin(f*x+e)*cos(f*x+e)+74*A*cos(f*x+e)+124*A*sin(f*x+e)+4*B*cos(f*x+
e)-16*B*sin(f*x+e)-124*A+16*B)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^3-cos
(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)+4*
sin(f*x+e)+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Fricas [A]
time = 0.38, size = 194, normalized size = 1.33

$$-\frac{\left(5\,(2\,A+B)a^2\cos\left(fx+e\right)^2-2\,(7\,A+2\,B)a^2+5\,\left(3\,Ba^2\cos\left(fx+e\right)^2-2\,(A+2\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos(
f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*
c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c
^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(137) = 274.
time = 0.57, size = 282, normalized size = 1.93

$$\frac{\left(30\,Ba^{2}\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+10\,Aa^{2}\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-40\,Ba^{2}\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-5\,Aa^{2}\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+20\,Ba^{2}\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Aa^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,Ba^{2}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{240\,\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{5}c^{6}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] 1/240*(30*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 10*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) - 40*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1
/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*a^2*sqrt(c)*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*sqrt(c)*
cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*a^
2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^2*sqrt(c)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^5
*c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [**B**]
time = 20.76, size = 341, normalized size = 2.34

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{16\,a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(A\,6{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^6\,f}-\frac{16\,a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,2{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^6\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\left(8\,A+13\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,8{}\mathrm{i}}{3\,c^6\,f}-\frac{B\,a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,8{}\mathrm{i}}{c^6\,f}\right)}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((16*a^2*exp(e*6i + f*x*6i)*(A*6i + B*1i)*(a +
a*sin(e + f*x))^(1/2))/(5*c^6*f) - (16*a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f
*x)*(A*2i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f) + (a^2*exp(e*6i + f
*x*6i)*sin(e + f*x)*(8*A + 13*B)*(a + a*sin(e + f*x))^(1/2)*8i)/(3*c^6*f) -
 (B*a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*8i)/
(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*
e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*
6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e
*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=196

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e-}{160c^2f(c}$$

```
[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(13/2)+1/40
*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/16
0*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)+1/
960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^3/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A]
time = 0.33, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5
/2))/(160*c^2*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
a*Sin[e + f*x])^(5/2))/(960*c^3*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\,c}{40c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\,c}{40c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\,c}{40c}$$

Mathematica [**A**]
time = 3.59, size = 144, normalized size = 0.73

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(29A+13B-15(A+B)\cos(2(e+fx))+6(6A+7B)\sin(e+fx)-10B\sin(3(e+fx)))}{120c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^6\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*A
 + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B*S
in[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(172) = 344.
time = 0.43, size = 423, normalized size = 2.16

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(-444A+52B+242A\cos(fx+e)-52B\sin(fx+e)+444A\sin(fx+e)-6B\cos(fx+e)+29B\left(\cos^2(fx+e\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/60/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-343*A*cos(f*x+e)^2*sin(f*x+e)+
29*B*cos(f*x+e)^2*sin(f*x+e)-444*A+52*B+242*A*cos(f*x+e)-52*B*sin(f*x+e)+44
4*A*sin(f*x+e)-7*A*cos(f*x+e)^5*sin(f*x+e)+B*cos(f*x+e)^5*sin(f*x+e)-224*A*
cos(f*x+e)^3+12*B*cos(f*x+e)^3-6*B*cos(f*x+e)-17*B*cos(f*x+e)^3*sin(f*x+e)+
24*B*cos(f*x+e)^4+545*A*cos(f*x+e)^2-75*B*cos(f*x+e)^2+119*A*cos(f*x+e)^3*s
in(f*x+e)+49*A*cos(f*x+e)^4*sin(f*x+e)+42*A*cos(f*x+e)^5-6*B*cos(f*x+e)^5-B
*cos(f*x+e)^6+46*B*sin(f*x+e)*cos(f*x+e)-168*A*cos(f*x+e)^4+7*A*cos(f*x+e)^
6-202*A*sin(f*x+e)*cos(f*x+e)-7*B*sin(f*x+e)*cos(f*x+e)^4)/(-c*(sin(f*x+e)-
1))^(13/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*cos(f*x+e
)*sin(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(13/2), x)
```

Fricas [A]
time = 0.41, size = 209, normalized size = 1.07

$$\frac{\left(15\,(A+B)a^2\cos\left(fx+e\right)^2-2\,(11\,A+7\,B)a^2+2\,\left(10\,Ba^2\cos\left(fx+e\right)^2-(9\,A+13\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{60\,\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\,\left(3\,c^7f\cos\left(fx+e\right)^5-16\,c^7f\cos\left(fx+e\right)^3+16\,c^7f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] 1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos(
f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c
```

```
^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 1
6*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 282, normalized size = 1.44

$$-\frac{\left(40\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+15\,Aa^2\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-45\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-6\,Aa^2\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+18\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Aa^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,Ba^2\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{960\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^6c^7f\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] -1/960*(40*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 15*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 45*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x +
1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*a^2*sqrt(c)*cos(-1/4*pi
+ 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a^2*sqrt(c)
*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*a
^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*sqrt(c)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^
6*c^7*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [**B**]
time = 20.71, size = 357, normalized size = 1.82

$$\frac{\sqrt{c-c\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\left(A\,29{}\mathrm{i}+B\,13{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,16{}\mathrm{i}}{15\,c^7\,f}-\frac{a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,1{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,16{}\mathrm{i}}{c^7\,f}-\frac{32\,a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\left(6\,A+7\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}+\frac{32\,B\,a^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^7\,f}\right)}{-858\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+858\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(7\,e+7\,f\,x\right)+1144\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)-416\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)+24\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(13/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*7i + f*x*7i)*(A*29i + B*13i)*(a + a
*sin(e + f*x))^(1/2)*16i)/(15*c^7*f) - (a^2*exp(e*7i + f*x*7i)*cos(2*e + 2*
f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^7*f) - (32*a^2*exp(e*
7i + f*x*7i)*sin(e + f*x)*(6*A + 7*B)*(a + a*sin(e + f*x))^(1/2))/(5*c^7*f)
```

```
 + (32*B*a^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)
)/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858*cos(e + f*x)*e
xp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x) + 2*exp(e*7i +
f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e + 2*f*x) - 416*e
xp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*sin(6*e + 6*f*x)
)
```

3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=250

$$-\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a+a\sin(e+fx)}}-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+$$

```
[Out] -1/84*a^2*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/
f-1/72*a*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/f
-1/9*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/f-1/315*a^4
*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1/2)-1/126*a
^3*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.39, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{126f}-\frac{a^2(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{84f}-\frac{a(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9/2}}{72f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] -1/315*(a^4*(9*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(f*Sqrt[a +
a*Sin[e + f*x]]) - (a^3*(9*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(9/2))/(126*f) - (a^2*(9*A - B)*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(84*f) - (a*(9*A - B)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(9/2))/(72*f) - (B*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(9*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
```

```
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{9f}$$

$$= -\frac{a(9A-B)\cos(e+fx)(a+a\sin(}{72}$$

$$= -\frac{a^2(9A-B)\cos(e+fx)(a+a\sin}{84}$$

$$= -\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin}}{126}$$

$$= -\frac{a^4(9A-B)\cos(e+fx)(c-c\sin}{315f\sqrt{a+a\sin(e+fx}}$$

Mathematica [**A**]
time = 4.56, size = 269, normalized size = 1.08

$$\frac{a^3c^4(-1+\sin(e+fx))^4(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(17640(A-B)\cos(2(e+fx))+8820(A-B)\cos(4(e+fx))+2520A\cos(6(e+fx))-2520B\cos(6(e+fx))+315A\cos(8(e+fx))-315B\cos(8(e+fx))+176400A\sin(e+fx)-17640B\sin(e+fx)+35280A\sin(3(e+fx))+7056A\sin(5(e+fx))+2016B\sin(5(e+fx))+720A\sin(7(e+fx))+900B\sin(7(e+fx))+140B\sin(9(e+fx)))}{322560f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(9/2),x]
```

```
[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(17640*(A - B)*Cos[2*(e + f*x)] + 8820*(A - B)
*Cos[4*(e + f*x)] + 2520*A*Cos[6*(e + f*x)] - 2520*B*Cos[6*(e + f*x)] + 315
*A*Cos[8*(e + f*x)] - 315*B*Cos[8*(e + f*x)] + 176400*A*Sin[e + f*x] - 1764
0*B*Sin[e + f*x] + 35280*A*Sin[3*(e + f*x)] + 7056*A*Sin[5*(e + f*x)] + 201
```

```
6*B*Sin[5*(e + f*x)] + 720*A*Sin[7*(e + f*x)] + 900*B*Sin[7*(e + f*x)] + 14
0*B*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A]
time = 0.43, size = 259, normalized size = 1.04

method	result
default	$\frac{\left(-280B\left(\cos^{8}\left(fx+e\right)\right)+315A\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)-315B\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)-360A\left(\cos^{6}\left(fx+e\right)\right)+40B\left(\cos^{6}\left(fx+e\right)\right)+315A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2520/f*(-280*B*cos(f*x+e)^8+315*A*cos(f*x+e)^6*sin(f*x+e)-315*B*cos(f*x+e
)^6*sin(f*x+e)-360*A*cos(f*x+e)^6+40*B*cos(f*x+e)^6+315*A*cos(f*x+e)^4*sin(
f*x+e)-315*B*sin(f*x+e)*cos(f*x+e)^4-432*A*cos(f*x+e)^4+48*B*cos(f*x+e)^4+3
15*A*cos(f*x+e)^2*sin(f*x+e)-315*B*cos(f*x+e)^2*sin(f*x+e)-576*A*cos(f*x+e)
^2+64*B*cos(f*x+e)^2+315*A*sin(f*x+e)-315*B*sin(f*x+e)-1152*A+128*B)*(-c*(s
in(f*x+e)-1))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)-1)/cos(
f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [A]
time = 0.43, size = 192, normalized size = 0.77

$$\frac{\left(315\,(A-B)a^{3}c^{4}\cos\left(fx+e\right)^{8}-315\,(A-B)a^{3}c^{4}+8\,\left(35\,Ba^{3}c^{4}\cos\left(fx+e\right)^{8}+5\,(9\,A-B)a^{3}c^{4}\cos\left(fx+e\right)^{6}+6\,(9\,A-B)a^{3}c^{4}\cos\left(fx+e\right)^{4}+8\,(9\,A-B)a^{3}c^{4}\cos\left(fx+e\right)^{2}+16\,(9\,A-B)a^{3}c^{4}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2520\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] 1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B*
a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)*a
```

```
^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B)*a
^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(235) = 470.
time = 0.66, size = 584, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] -32/315*(560*B*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^18*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 315*A*a^3*c^4*cos(-
1/4*pi + 1/2*f*x + 1/2*e)^16*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)) - 2835*B*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^1
6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 1440*A*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5760*B*a^3*c^4*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 2520*A*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5880*
B*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2016*A*a^3*c^4*cos(-1/4*pi + 1/2
*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3024*B*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 630*A*a^3*c
^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 630*B*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 20.03, size = 482, normalized size = 1.93

$$\frac{\mathrm{e}^{-e\,9\mathrm{i}-f\,x\,9\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(-\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,7\mathrm{i}}{64\,f}-\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,7\mathrm{i}}{128\,f}-\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{64\,f}-\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\cos(8\,e+8\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{512\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\sin(5\,e+5\,f\,x)\,(7\,A+2\,B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\sin(7\,e+7\,f\,x)\,(4\,A+5\,B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{896\,f}+\frac{7\,A\,a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\sin(e+f\,x)\,(10\,A-B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{B\,a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\,\sin(9\,e+9\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{1152\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9
/2),x)
```

```
[Out] (exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^4*exp(e*9i + f*x*9
i)*sin(5*e + 5*f*x)*(7*A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^3*
c^4*exp(e*9i + f*x*9i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^
(1/2)*7i)/(128*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos(6*e + 6*f*x)*(A*1i - B*
1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(64*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos
(8*e + 8*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(512*f) - (a^3*c
^4*exp(e*9i + f*x*9i)*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(
1/2)*7i)/(64*f) + (a^3*c^4*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(4*A + 5*B)*
(a + a*sin(e + f*x))^(1/2))/(896*f) + (7*A*a^3*c^4*exp(e*9i + f*x*9i)*sin(3
*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*9i + f*x*
9i)*sin(e + f*x)*(10*A - B)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (B*a^3*c^4
*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/2))/(1152*f)))
/(2*cos(e + f*x))
```

3.161 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=226

$$-\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}}-\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2A}{}$$

```
[Out] -1/7*a^2*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a
*A*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/8*B*cos(f*x
+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2)/f-2/35*a^4*A*cos(f*x+e)*(
c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^3*A*cos(f*x+e)*(c-c*s
in(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.38, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$-\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3A\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2A\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{7f}-\frac{aA\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (-2*a^4*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^3*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(7/2))/(35*f) - (a^2*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(7/2))/(7*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
*(c - c*Sin[e + f*x])^(7/2))/(8*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
```

```
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/}}{8f}$$

$$= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5}}{7f}$$

$$= -\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))}{7f}$$

$$= -\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+f}}{35f}$$

$$= -\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx)}{35f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 1.06, size = 135, normalized size = 0.60

$$\frac{a^3c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-1960B\cos(2(e+fx))-980B\cos(4(e+fx))-280B\cos(6(e+fx))-35B\cos(8(e+fx))+19600A\sin(e+fx)+3920A\sin(3(e+fx))+784A\sin(5(e+fx))+80A\sin(7(e+fx)))}{35840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-1960*B*Cos[2*(e + f*x)] - 980*B*Cos[4*(e + f*x)] - 280*B*Cos[6*(e + f*x)]
- 35*B*Cos[8*(e + f*x)] + 19600*A*Sin[e + f*x] + 3920*A*Sin[3*(e + f*x)] +
784*A*Sin[5*(e + f*x)] + 80*A*Sin[7*(e + f*x)]))/(35840*f)
```

Maple [A]
time = 0.43, size = 142, normalized size = 0.63

method	result
default	$\frac{(35B(\cos^6(fx+e))\sin(fx+e)+40A(\cos^6(fx+e))+35B\sin(fx+e)(\cos^4(fx+e))+48A(\cos^4(fx+e))+35B(\cos^2(fx+e))\sin(fx+e)-}{280f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/280/f*(35*B*cos(f*x+e)^6*sin(f*x+e)+40*A*cos(f*x+e)^6+35*B*sin(f*x+e)*cos
(f*x+e)^4+48*A*cos(f*x+e)^4+35*B*cos(f*x+e)^2*sin(f*x+e)+64*A*cos(f*x+e)^2+
35*B*sin(f*x+e)+128*A)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(7/2)/cos(f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A]
time = 0.42, size = 142, normalized size = 0.63

$$-\frac{\left(35\,Ba^3c^3\cos\left(fx+e\right)^8-35\,Ba^3c^3-8\left(5\,Aa^3c^3\cos\left(fx+e\right)^6+6\,Aa^3c^3\cos\left(fx+e\right)^4+8\,Aa^3c^3\cos\left(fx+e\right)^2+16\,Aa^3c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*x
 + e)^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*a^
3*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(211) = 422.
time = 0.61, size = 480, normalized size = 2.12

$$\frac{32 \left(35 B a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{16} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 20 A a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{14} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 140 B a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{14} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 70 A a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{12} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 210 B a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{12} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 84 A a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{10} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 140 B a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{10} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 35 A a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{8} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 35 B a^{3} c^{3} \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{8} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)\right) \sqrt{a} \sqrt{c}}{35 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] 32/35*(35*B*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^16*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*A*a^3*c^3*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 140*B*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 70*A
*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 210*B*a^3*c^3*cos(-1/4*pi + 1/2*f
*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + 84*A*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 140*B*a^3*c^3*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 35*A*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
+ 35*B*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 17.65, size = 384, normalized size = 1.70

$$\frac{\mathrm{e}^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,A\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}-\frac{7\,B\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}-\frac{7\,B\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{128\,f}-\frac{B\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}-\frac{B\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(8\,e+8\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{512\,f}+\frac{7\,A\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,A\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(5\,e+5\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{A\,a^3\,c^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*A*a^3*c^3*exp(e*8i +
f*x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*B*a^3*c^3*exp(
e*8i + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*B*a
^3*c^3*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128
```

```
*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(
1/2))/(64*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(512*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(
a + a*sin(e + f*x))^(1/2))/(32*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(5*e
 + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (A*a^3*c^3*exp(e*8i + f*x*8
i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.162 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=192

$$\frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e-}}{105f}$$

```
[Out] 1/42*(7*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f-1
/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2)/f+1/105*(7*A+
B)*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/105*(7*
A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.32, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$\frac{c^3(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2c^2(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}+\frac{c(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{42f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] ((7*A + B)*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(105*f*Sqrt[c - c*S
in[e + f*x]]) + (2*(7*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sq
rt[c - c*Sin[e + f*x]])/(105*f) + ((7*A + B)*c*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
```

```
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{7f} \\ &= \frac{(7A+B)c\cos(e+fx)(a+a\sin(e}{42f} \\ &= \frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(}{105} \\ &= \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e}{105f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 1.39, size = 232, normalized size = 1.21

$$\frac{a^3c^2(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(-525(A+B)\cos(2(e+fx))-210(A+B)\cos(4(e+fx))-35A\cos(6(e+fx))-35B\cos(6(e+fx))+4200A\sin(e+fx)+525B\sin(e+fx)+700A\sin(3(e+fx))-35B\sin(3(e+fx))+84A\sin(5(e+fx))-63B\sin(5(e+fx))-15B\sin(7(e+fx)))}{6720f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)*C
os[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Si
n[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(e +
f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e + f*x
)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^7)
```

Maple [A]
time = 0.49, size = 203, normalized size = 1.06

method	result
default	$\frac{(-30B(\cos^6(fx+e))+35A(\cos^4(fx+e))\sin(fx+e)+35B\sin(fx+e)(\cos^4(fx+e))+42A(\cos^4(fx+e))+6B(\cos^4(fx+e))+35A(\cos^2(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/210/f*(-30*B*cos(f*x+e)^6+35*A*cos(f*x+e)^4*sin(f*x+e)+35*B*sin(f*x+e)*co
s(f*x+e)^4+42*A*cos(f*x+e)^4+6*B*cos(f*x+e)^4+35*A*cos(f*x+e)^2*sin(f*x+e)+
35*B*cos(f*x+e)^2*sin(f*x+e)+56*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2+35*A*sin(f*
x+e)+35*B*sin(f*x+e)+112*A+16*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1
+sin(f*x+e)))^(7/2)/(1+sin(f*x+e))/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [A]
time = 0.44, size = 158, normalized size = 0.82

$$-\frac{\left(35\,(A+B)a^3c^2\cos\left(fx+e\right)^6-35\,(A+B)a^3c^2+2\left(15\,Ba^3c^2\cos\left(fx+e\right)^6-3\,(7\,A+B)a^3c^2\cos\left(fx+e\right)^4-4\,(7\,A+B)a^3c^2\cos\left(fx+e\right)^2-8\,(7\,A+B)a^3c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{210\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a^
3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a^3
*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(180) = 360.
time = 0.58, size = 376, normalized size = 1.96

$$-\frac{16\left(120\,Ba^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+70\,Aa^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-350\,Ba^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-168\,Aa^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+336\,Ba^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+105\,Aa^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-105\,Ba^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] -16/105*(120*B*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 70*A*a^3*c^2*cos(-1
/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) - 350*B*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 1
68*A*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 336*B*a^3*c^2*cos(-1/4*pi + 1
/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 105*A*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 105*B*a^3*c
^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 18.43, size = 383, normalized size = 1.99

$$\frac{e^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,5\mathrm{i}}{32\,f}+\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,(A\,1\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{16\,f}+\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,(A\,1\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,1\mathrm{i}}{96\,f}+\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(5\,e+5\,f\,x)\,(4\,A-3\,B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,(20\,A-B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{96\,f}+\frac{5\,a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(e+f\,x)\,(8\,A+B)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}-\frac{B\,a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^2*exp(e*7i + f*x*7
i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (
a^3*c^2*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*
x))^(1/2)*1i)/(16*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i +
 B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) + (a^3*c^2*exp(e*7i + f*x*7i)*
sin(5*e + 5*f*x)*(4*A - 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^2
*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A - B)*(a + a*sin(e + f*x))^(1/2))
/(96*f) + (5*a^3*c^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A + B)*(a + a*sin(e
 + f*x))^(1/2))/(32*f) - (B*a^3*c^2*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a
+ a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.163 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=142

$$\frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}$$

```
[Out] -1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(3*A
+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/15*(3*
A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.24, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3052, 2819, 2817}

$$\frac{c^2(3A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{c(3A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] ((3*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((3*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c -
 c*Sin[e + f*x])^(3/2))/(6*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))}{6f}$$

$$= \frac{(3A+B)c\cos(e+fx)(a+a\sin(e}{15f}$$

$$= \frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e}{30f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 1.26, size = 212, normalized size = 1.49

$$-\frac{a^3c(-1+\sin(e+fx))(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-15(16A+11B)\cos(2(e+fx))-30(2A+B)\cos(4(e+fx))+5B\cos(6(e+fx))+840A\sin(e+fx)+240B\sin(e+fx)+60A\sin(3(e+fx))-40B\sin(3(e+fx))-12A\sin(5(e+fx))-24B\sin(5(e+fx)))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/960*(a^3*c*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-15*(16*A + 11*B)*Cos[2*(e + f*x)] - 30*(2
*A + B)*Cos[4*(e + f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] + 240*
B*Sin[e + f*x] + 60*A*Sin[3*(e + f*x)] - 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5
*(e + f*x)] - 24*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A]
time = 0.44, size = 187, normalized size = 1.32

method	result
default	$\frac{(-5B\sin(fx+e)(\cos^4(fx+e))-6A(\cos^4(fx+e))-12B(\cos^4(fx+e))+15A(\cos^2(fx+e))\sin(fx+e)+10B(\cos^2(fx+e))\sin(fx+e)+}{30f(2\sin(fx+e)-(\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(-5*B*sin(f*x+e)*cos(f*x+e)^4-6*A*cos(f*x+e)^4-12*B*cos(f*x+e)^4+15*
A*cos(f*x+e)^2*sin(f*x+e)+10*B*cos(f*x+e)^2*sin(f*x+e)+12*A*cos(f*x+e)^2+4*
B*cos(f*x+e)^2+15*A*sin(f*x+e)+10*B*sin(f*x+e)+24*A+8*B)*(-c*(sin(f*x+e)-1)
)^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(2*sin(f*x+e)-cos(f*x+e)^2+2)/c
os(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [A]
time = 0.40, size = 150, normalized size = 1.06

$$\frac{\left(5\,Ba^3c\cos\left(fx+e\right)^6-15\left(A+B\right)a^3c\cos\left(fx+e\right)^4+5\left(3\,A+2\,B\right)a^3c-2\left(3\left(A+2\,B\right)a^3c\cos\left(fx+e\right)^4-2\left(3\,A+B\right)a^3c\cos\left(fx+e\right)^2-4\left(3\,A+B\right)a^3c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a^3*c*cos(f*x + e)^6 - 15*(A + B)*a^3*c*cos(f*x + e)^4 + 5*(3*A +
 2*B)*a^3*c - 2*(3*(A + 2*B)*a^3*c*cos(f*x + e)^4 - 2*(3*A + B)*a^3*c*cos(f
*x + e)^2 - 4*(3*A + B)*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 262, normalized size = 1.85

$$\frac{8\left(20\,Ba^{3}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+12\,Aa^{3}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-36\,Ba^{3}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-15\,Aa^{3}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+15\,Ba^{3}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 8/15*(20*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c*cos(-1/4*pi +
1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - 36*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^3*c*c
os(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sq
rt(a)*sqrt(c)/f
```

Mupad [B]
time = 18.17, size = 321, normalized size = 2.26

$$-\frac{e^{-e\,6i-f\,x\,6i}\,\sqrt{c-c\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos\left(4\,e+4\,f\,x\right)\,\left(2\,A+B\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}}{16\,f}-\frac{B\,a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos\left(6\,e+6\,f\,x\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}}{96\,f}+\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\sin\left(e+f\,x\right)\,\left(A\,7i+B\,2i\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}\,1i}{4\,f}+\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos\left(2\,e+2\,f\,x\right)\,\left(16\,A+11\,B\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}}{32\,f}+\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\sin\left(3\,e+3\,f\,x\right)\,\left(A\,3i-B\,2i\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}\,1i}{24\,f}-\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\sin\left(5\,e+5\,f\,x\right)\,\left(A\,1i+B\,2i\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}\,1i}{40\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c*exp(e*6i + f*x*6i
)*cos(4*e + 4*f*x)*(2*A + B)*(a + a*sin(e + f*x))^(1/2))/(16*f) - (B*a^3*c*
exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (a
^3*c*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i + B*2i)*(a + a*sin(e + f*x))^(1/
2)*1i)/(4*f) + (a^3*c*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A + 11*B)*(a
+ a*sin(e + f*x))^(1/2))/(32*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x
)*(A*3i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) - (a^3*c*exp(e*6i + f
*x*6i)*sin(5*e + 5*f*x)*(A*1i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f)
))/(2*cos(e + f*x))
```

3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=96

$$\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin}}{a}$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+}{4f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 0.68, size = 121, normalized size = 1.26

$$\frac{a^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(4(60A+23B)\sin(e+fx)+\cos(4(e+fx))(5A+15B+4B\sin(e+fx))-4\cos(2(e+fx))(5(7A+5B)+4(5A+6B)\sin(e+fx)))}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(6
0*A + 23*B)*Sin[e + f*x] + Cos[4*(e + f*x)]*(5*A + 15*B + 4*B*Sin[e + f*x])
 - 4*Cos[2*(e + f*x)]*(5*(7*A + 5*B) + 4*(5*A + 6*B)*Sin[e + f*x])))/(160*f
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(84) = 168$.
time = 0.38, size = 174, normalized size = 1.81

method	result
default	$\frac{(-4B(\cos^4(fx+e))+5A(\cos^2(fx+e))\sin(fx+e)+15B(\cos^2(fx+e))\sin(fx+e)+20A(\cos^2(fx+e))+28B(\cos^2(fx+e))-35A\sin(f}{20f((\cos^2(fx+e))\sin(fx+e)+3(\cos^2(fx+e))-4\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*sin
(f*x+e)+20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)-25*B*sin(f*x+e)
-40*A-24*B)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(
cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [A]
time = 0.38, size = 147, normalized size = 1.53

$$\frac{\left(5\left(A+3B\right)a^{3}\cos\left(fx+e\right)^{4}-40\left(A+B\right)a^{3}\cos\left(fx+e\right)^{2}+5\left(7A+5B\right)a^{3}+4\left(Ba^{3}\cos\left(fx+e\right)^{4}-\left(5A+7B\right)a^{3}\cos\left(fx+e\right)^{2}+2\left(5A+3B\right)a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{20f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(f*x + e)^4 - 40*(A + B)*a^3*cos(f*x + e)^2 + 5*(7
*A + 5*B)*a^3 + 4*(B*a^3*cos(f*x + e)^4 - (5*A + 7*B)*a^3*cos(f*x + e)^2 +
2*(5*A + 3*B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 159, normalized size = 1.66

$$-\frac{4\left(8Ba^{3}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5Aa^{3}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5Ba^{3}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -4/5*(8*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a^3*cos(-1/4*pi + 1/2*f*
x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 5*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 16.68, size = 173, normalized size = 1.80

$$-\frac{a^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(140\,A\,\cos\left(e+f\,x\right)+100\,B\,\cos\left(e+f\,x\right)+135\,A\,\cos\left(3\,e+3\,f\,x\right)-5\,A\,\cos\left(5\,e+5\,f\,x\right)+85\,B\,\cos\left(3\,e+3\,f\,x\right)-15\,B\,\cos\left(5\,e+5\,f\,x\right)-240\,A\,\sin\left(2\,e+2\,f\,x\right)+40\,A\,\sin\left(4\,e+4\,f\,x\right)-90\,B\,\sin\left(2\,e+2\,f\,x\right)+48\,B\,\sin\left(4\,e+4\,f\,x\right)-2\,B\,\sin\left(6\,e+6\,f\,x\right)\right)}{160\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(140*A*cos
(e + f*x) + 100*B*cos(e + f*x) + 135*A*cos(3*e + 3*f*x) - 5*A*cos(5*e + 5*f
*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x
) + 40*A*sin(4*e + 4*f*x) - 90*B*sin(2*e + 2*f*x) + 48*B*sin(4*e + 4*f*x) -
 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=239

$$-\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+B)\cos}{f}$$

```
[Out] -a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3*a
*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-1/4*B*cos
(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)-8*a^4*(A+B)*cos(f*x
+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^3*
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2819, 2816, 2746, 31}

$$-\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-8*a^4*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (4*a^3*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + B)*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x
]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f
*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+\\
&= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e\\
&= -\frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+\\
&= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+\\
&= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+\\
&= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+\\
&= -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3(A
\end{aligned}
$$

Mathematica [A]
time = 1.85, size = 183, normalized size = 0.77

$$
-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\left(-12(8A+15B)\cos(2(e+fx))+3B\cos(4(e+fx))+1536(A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+24(29A+36B)\sin(e+fx)-8(A+4B)\sin(3(e+fx))\right)}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[
a*(1 + Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f
*x)] + 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A + 3
6*B)*Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(215) = 430.
time = 0.32, size = 671, normalized size = 2.81

method	result
default	$-\frac{\left(64A+67B+24A\cos(fx+e)-67B\sin(fx+e)-64A\sin(fx+e)+45B\cos(fx+e)-32B\left(\cos^2(fx+e)\right)\sin(fx+e)+4A\left(\cos^4(fx+e)\right)-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/12/f*(-20*A*cos(f*x+e)^2*sin(f*x+e)-32*B*cos(f*x+e)^2*sin(f*x+e)+64*A+67
*B+24*A*cos(f*x+e)-67*B*sin(f*x+e)-64*A*sin(f*x+e)-24*A*cos(f*x+e)^3-48*B*c
os(f*x+e)^3+45*B*cos(f*x+e)-16*B*cos(f*x+e)^3*sin(f*x+e)+13*B*cos(f*x+e)^4-
68*A*cos(f*x+e)^2-80*B*cos(f*x+e)^2-4*A*cos(f*x+e)^3*sin(f*x+e)-96*A*cos(f*
x+e)*ln(2/(1+cos(f*x+e)))+3*B*cos(f*x+e)^5-96*B*ln(2/(1+cos(f*x+e)))*sin(f*
x+e)+112*B*sin(f*x+e)*cos(f*x+e)+192*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+192*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-96*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+192*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))+192*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))-96*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+4*A*cos(f*x+e)^4+96*A*ln(2/(
1+cos(f*x+e)))-192*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+96*B*ln(2/(
1+cos(f*x+e)))-192*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+88*A*sin(f*
x+e)*cos(f*x+e)+3*B*sin(f*x+e)*cos(f*x+e)^4)*(a*(1+sin(f*x+e)))^(7/2)/(cos(
f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8
*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin
(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(231) = 462.
time = 0.65, size = 494, normalized size = 2.07

$$\frac{2\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}Ba^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\sqrt{2}Aa^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\sqrt{2}Ba^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\sqrt{2}Aa^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\sqrt{2}Ba^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\sqrt{2}Aa^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\sqrt{2}Ba^3c^{\frac{7}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^4}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*sqrt(a)*(6*(sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-co
s(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
 + (3*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*A*a^3
*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 3*sqrt(2)*A*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
 + 3*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*A*a^3*
c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)))/c^4)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}+\frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^3(3A+5B)}{cf}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a^
2*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+1/
6*a*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)+
4*a^4*(3*A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)+2*a^3*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c
-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$\frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{a(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^4*(3*A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^3*(3*A + 5*B)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a^2*(3*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*
x]]) + (a*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)}{\ldots} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(3A+5B\ldots}{\ldots} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+5\ldots}{\ldots} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5\ldots}{\ldots} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5\ldots}{\ldots} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5\ldots}{\ldots} \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^4(3A+5\ldots}{cf\sqrt{a+\ldots}}
\end{aligned}
$$

Mathematica [**A**]
time = 2.32, size = 292, normalized size = 1.08

$$
\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-132A-45B-2(27A+59B)\cos(2(e+fx))+B\cos(4(e+fx))-576A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-960B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-117A\sin(e+fx)-279B\sin(e+fx)+576A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+960B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-3A\sin(3(e+fx))-13B\sin(3(e+fx))\right)}{24cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-132
*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 960*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] - 13*B*Sin[3*(e + f*x)
]))/(24*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[
c - c*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 926 vs. 2(245) = 490.
time = 0.31, size = 927, normalized size = 3.42

method	result	size
default	Expression too large to display	927

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(-24*A*cos(f*x+e)^2*sin(f*x+e)-48*B*cos(f*x+e)^2*sin(f*x+e)-102*A-16
6*B+240*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-
120*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)+27*A*cos(f*x+e)+166*B*sin(
f*x+e)+102*A*sin(f*x+e)-27*A*cos(f*x+e)^3-61*B*cos(f*x+e)^3+59*B*cos(f*x+e)
-13*B*cos(f*x+e)^3*sin(f*x+e)+11*B*cos(f*x+e)^4+99*A*cos(f*x+e)^2+155*B*cos
(f*x+e)^2-3*A*cos(f*x+e)^3*sin(f*x+e)+72*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e))
)+72*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-240*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))*cos(f*x+e)^2+2*B*cos(f*x+e)^5+240*B*ln(2/(1+cos(f*x+e)))*sin(f
*x+e)-107*B*sin(f*x+e)*cos(f*x+e)-240*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-480*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+144*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-144*A*cos(f*x+e)*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-288*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))+120*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+3*A*cos(f*x+e)^4+144*A*co
s(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-72*A*cos(f*x
+e)*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-144*A*ln(2/(1+cos(f*x+e)))+288*A*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-240*B*ln(2/(1+cos(f*x+e)))+480*B*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))+120*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-75*A*sin(f
*x+e)*cos(f*x+e)+2*B*sin(f*x+e)*cos(f*x+e)^4)*(a*(1+sin(f*x+e)))^(7/2)/(cos
(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-
8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(si
n(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*si
n(f*x + e) - 2*c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 411, normalized size = 1.52

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{6\left(3\sqrt{2}Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\sqrt{2}Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c^2\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{6\left(\sqrt{2}Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)c^2\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(4Ba^3c^{\frac{9}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Aa^3c^{\frac{9}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+9Ba^3c^{\frac{9}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+12Aa^3c^{\frac{9}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+24Ba^3c^{\frac{9}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{c^6\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(a)*(6*(3*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 5*sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*lo
g(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) - 6*(sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sq
rt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/((cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(4
*B*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 3*A*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 9*B*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*B*a^3*c^(9/2)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(c^6*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}} \, dx$

Optimal. Leaf size=263

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}}-\frac{6a^4(A+3B)\cos}{c^2f\sqrt{a+a\sin(}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(5/2)-1/2*a*
(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)-3/4*a^
2*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)-6*
a^4*(A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)-3*a^3*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c
*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$-\frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^3(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (6*a^4*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x
]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*(A +
 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]
]) - (3*a^2*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c^2*f*Sqr
t[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)}{2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)}{2}$$

Mathematica [A]
time = 1.69, size = 251, normalized size = 0.95

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}\left(16(A+B)-16(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+B\cos(2(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-48(A+3B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-4(A+6B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin(e+fx)\right)}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A
+ B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(e
+ f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(A
 + 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(237) = 474$.
time = 0.31, size = 1189, normalized size = 4.52

method	result	size
default	Expression too large to display	1189

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(36*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-72*B*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*cos(f*x+e)^3+22*A*cos(f*x+e)^2*sin(f*x+e)+63*B*cos(f*x+
e)^2*sin(f*x+e)+32*A+100*B-144*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
*sin(f*x+e)*cos(f*x+e)+72*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)-24*A
*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*A*cos(f*x+e)^3*
ln(2/(1+cos(f*x+e)))-20*A*cos(f*x+e)-100*B*sin(f*x+e)-32*A*sin(f*x+e)+20*A*
cos(f*x+e)^3+53*B*cos(f*x+e)^3-54*B*cos(f*x+e)-10*B*cos(f*x+e)^3*sin(f*x+e)
+36*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)-72*B*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+9*B*cos(f*x+e)^4-34*A*cos(f
*x+e)^2-109*B*cos(f*x+e)^2-2*A*cos(f*x+e)^3*sin(f*x+e)-36*A*cos(f*x+e)^2*ln
(2/(1+cos(f*x+e)))-24*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+216*B*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+B*cos(f*x+e)^5-144*B*ln(2/(1+cos
(f*x+e)))*sin(f*x+e)+46*B*sin(f*x+e)*cos(f*x+e)+144*B*cos(f*x+e)*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+288*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))-24*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-48*A*ln(2/
(1+cos(f*x+e)))*sin(f*x+e)+48*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+96*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-72*B*
cos(f*x+e)*ln(2/(1+cos(f*x+e)))+2*A*cos(f*x+e)^4-48*A*cos(f*x+e)*sin(f*x+e)
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*sin(f*x+e)*ln(2
/(1+cos(f*x+e)))+48*A*ln(2/(1+cos(f*x+e)))-96*A*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))+144*B*ln(2/(1+cos(f*x+e)))-288*B*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+72*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-108*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+12*A*sin(f*x+e)*cos(f*x+e)+B*sin
(f*x+e)*cos(f*x+e)^4)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*s
in(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x
+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3
- (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 433, normalized size = 1.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(6*sqrt(2)*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-cos(-1/4
*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 2
*(sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*A*a^3*c^(7/
2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)))/c^6 + (5*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
```

```
/2*e)) + 9*sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(3
*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*B*a^
3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=264

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2(A+7B)\cos(e}{4c^2f(c}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(7/2)-1/12*a
*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)+1/4*a
^2*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a
^4*(A+7*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*si
n(f*x+e))^(1/2)+1/2*a^3*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-
c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a^3(A+7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^3f\sqrt{c-c\sin(e+fx)}}+\frac{a^2(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*c*f*(c
- c*Sin[e + f*x])^(5/2)) + (a^2*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(4*c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^4*(A + 7*B)*Cos[e + f*x]*
Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (a^3*(A + 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^3*f*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{(A+7B)\int}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)}{12}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)}{12}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)}{12}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)}{12}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)}{12}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)}{12}$$

Mathematica [**A**]
time = 1.98, size = 244, normalized size = 0.92

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}\left(8(A+B)-6(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+18(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+6(A+7B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6+3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6\sin(e+fx)\right)}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A +
 B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1454 vs. 2(236) = 472.
time = 0.31, size = 1455, normalized size = 5.51

method	result	size

default	Expression too large to display	1455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(63*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-126*B*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*cos(f*x+e)^3+6*A*cos(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-3*A*cos(f*x+e)^3*sin(f*x+e)*ln(2/(1+cos(f*x+e)
))+14*A*cos(f*x+e)^2*sin(f*x+e)+98*B*cos(f*x+e)^2*sin(f*x+e)+20*A+116*B-168
*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+84*B*ln
(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)-18*A*cos(f*x+e)^3*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+9*A*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-6*A*cos(f*
x+e)-21*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3*sin(f*x+e)-116*B*sin(f*x+e)-20*
A*sin(f*x+e)+6*A*cos(f*x+e)^3+57*B*cos(f*x+e)^3-54*B*cos(f*x+e)-41*B*cos(f*
x+e)^3*sin(f*x+e)+84*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)-168*B*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+44*B*cos(
f*x+e)^4-28*A*cos(f*x+e)^2-160*B*cos(f*x+e)^2-8*A*cos(f*x+e)^3*sin(f*x+e)-2
4*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-12*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+
42*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3*sin(f*x+e)+336
*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-3*B*cos(f*x+e)^5
+3*A*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))-6*A*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))+21*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^4-42*B*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^4-168*B*ln(2/(1+cos(f*x+e)))
*sin(f*x+e)+62*B*sin(f*x+e)*cos(f*x+e)+168*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))+336*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))-24*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-24*A*ln(2/(1+cos(f*
x+e)))*sin(f*x+e)+24*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-84*B*cos(f*x+e
)*ln(2/(1+cos(f*x+e)))+8*A*cos(f*x+e)^4-24*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f
*x+e)))+24*A*ln(2/(1+cos(f*x+e)))-48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))+168*B*ln(2/(1+cos(f*x+e)))-336*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))+48*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-168*B*
ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+14*A*sin(f*x+e)*cos(f*x+e)-3*B*sin(f*x+e)
*cos(f*x+e)^4)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+
e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin
(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(252) = 504$.

time = 0.55, size = 809, normalized size = 3.06

$$-\frac{B\left(\frac{42\,a^{\frac{7}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{7}{2}}}-\frac{21\,a^{\frac{7}{2}}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{c^{\frac{7}{2}}}+\frac{2\left(\frac{21\,a^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{102\,a^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{227\,a^{\frac{7}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{228\,a^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{227\,a^{\frac{7}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-\frac{102\,a^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{21\,a^{\frac{7}{2}}\sin\left(fx+e\right)^7}{\left(\cos\left(fx+e\right)+1\right)^7}\right)}{c^{\frac{7}{2}}-\frac{6\,c^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{16\,c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{26\,c^{\frac{7}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{30\,c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{26\,c^{\frac{7}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{16\,c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{6\,c^{\frac{7}{2}}\sin\left(fx+e\right)^7}{\left(\cos\left(fx+e\right)+1\right)^7}+\frac{c^{\frac{7}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}}\right)+A\left(\frac{6\,a^{\frac{7}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{7}{2}}}-\frac{3\,a^{\frac{7}{2}}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{c^{\frac{7}{2}}}+\frac{4\left(\frac{3\,a^{\frac{7}{2}}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{6\,a^{\frac{7}{2}}\sqrt{c}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{22\,a^{\frac{7}{2}}\sqrt{c}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{6\,a^{\frac{7}{2}}\sqrt{c}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{3\,a^{\frac{7}{2}}\sqrt{c}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{c^4-\frac{6\,c^4\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{15\,c^4\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{20\,c^4\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{15\,c^4\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{6\,c^4\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{c^4\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] -1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/2)
*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2)*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(7/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7/2)*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x + e)/(cos(f*x
 + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 22*a^(
7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2)*sqrt(c)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^4*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
5*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos
(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.58, size = 376, normalized size = 1.42

$$-\frac{\sqrt{2}\left(\frac{12\sqrt{2}Ba^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{7}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{6\sqrt{2}\left(Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-192\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+192\right)}{c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(11Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+41Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+18\left(Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-3\left(9Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+31Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{a}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*(12*sqrt(2)*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 6
*sqrt(2)*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a^3*sqrt(
c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-192*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^2 + 192)/(c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(11*A*a^3
*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 41*B*a^3*sqrt(c)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 18*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e)^4 - 3*(9*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 31*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*c^4*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=247

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}}+\frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)-1/3*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*B*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)-a^3*B*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)-a^4*B*cos(f*x+e)*ln(
1-sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2818, 2816, 2746, 31}

$$-\frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^3B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e
 + f*x])^(7/2)) + (a^2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*
(c - c*Sin[e + f*x])^(5/2)) - (a^3*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(c^3*f*(c - c*Sin[e + f*x])^(3/2)) - (a^4*B*Cos[e + f*x]*Log[1 - Sin[e + f
*x]])/(c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\int \frac{(a+a\sin(}{(c-c\sin(}}{a}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+}{3cf(c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+}{3cf(c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+}{3cf(c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+}{3cf(c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+}{3cf(c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+}{3cf(c}$$

Mathematica [A]
time = 1.79, size = 238, normalized size = 0.96

$$\frac{\left(6(A+B)-4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+9(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-3(A+7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6-6B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] ((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A
+ 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. 2(221) = 442.
time = 0.38, size = 1019, normalized size = 4.13

method	result	size

default	Expression too large to display	1019

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(-24*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3+48*B*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*cos(f*x+e)^3+3*A*cos(f*x+e)^2*sin(f*x+e)-39*B*cos(f*x+e
)^2*sin(f*x+e)+6*A-34*B+48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin
(f*x+e)*cos(f*x+e)-24*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)+12*B*ln(
2/(1+cos(f*x+e)))*cos(f*x+e)^3*sin(f*x+e)+34*B*sin(f*x+e)-6*A*sin(f*x+e)-28
*B*cos(f*x+e)^3+20*B*cos(f*x+e)+11*B*cos(f*x+e)^3*sin(f*x+e)-36*B*ln(2/(1+c
os(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)+72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-19*B*cos(f*x+e)^4-9*A*cos(f*x+e)^2+53*B*co
s(f*x+e)^2-3*A*cos(f*x+e)^3*sin(f*x+e)-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))*cos(f*x+e)^3*sin(f*x+e)-120*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))*cos(f*x+e)^2+8*B*cos(f*x+e)^5-15*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e
)^4+30*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^4+48*B*ln(2/
(1+cos(f*x+e)))*sin(f*x+e)-14*B*sin(f*x+e)*cos(f*x+e)-48*B*cos(f*x+e)*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+24*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+3*A*cos(f*x+e)^4
-6*B*cos(f*x+e)^5*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)
^5*ln(2/(1+cos(f*x+e)))-6*B*cos(f*x+e)^4*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)^4*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-48*B*l
n(2/(1+cos(f*x+e)))+96*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+60*B*ln
(2/(1+cos(f*x+e)))*cos(f*x+e)^2+6*A*sin(f*x+e)*cos(f*x+e)+8*B*sin(f*x+e)*co
s(f*x+e)^4)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+
3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*
x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*
cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*
c^5)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 354, normalized size = 1.43

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{24\sqrt{2}Ba^3\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{9}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(12\left(Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6-3Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-47Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-18\left(Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+11Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+4\left(3Aa^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+41Ba^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^4c^5\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*sqrt(a)*(24*sqrt(2)*B*a^3*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(9/2)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) - sqrt(2)*(12*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 7*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e)^6 - 3*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
47*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*(A*a^3*sqrt(c)*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 4*(3*A*a^3*sqrt(c)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 41*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2 - 1)^4*c^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(9/2), x)
```

3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{(A-9B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/80
*(A-9*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A]

time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3051, 2821}

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c
- c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)}{8}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 434 vs. 2(96) = 192.
time = 6.61, size = 434, normalized size = 4.52

$$\frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}+\frac{(-3A-5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}+\frac{2(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2
)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + S
in[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*
(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c
- c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(
1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(11/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(84) = 168.
time = 0.37, size = 389, normalized size = 4.05

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(A(\cos^5(fx+e))+A(\cos^4(fx+e))\sin(fx+e)+B(\cos^5(fx+e))+B\sin(fx+e)(\cos^4(fx+e))-6A(\cos^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/10/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(A*cos(f*x+e)^5+A*cos(f*x+e)^4*s
in(f*x+e)+B*cos(f*x+e)^5+B*sin(f*x+e)*cos(f*x+e)^4-6*A*cos(f*x+e)^4+5*A*cos
(f*x+e)^3*sin(f*x+e)+4*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-17*A*cos(
f*x+e)^3-22*A*cos(f*x+e)^2*sin(f*x+e)-7*B*cos(f*x+e)^3-2*B*cos(f*x+e)^2*sin
(f*x+e)+32*A*cos(f*x+e)^2-10*A*sin(f*x+e)*cos(f*x+e)-8*B*cos(f*x+e)^2+10*B*
sin(f*x+e)*cos(f*x+e)+26*A*cos(f*x+e)+36*A*sin(f*x+e)+6*B*cos(f*x+e)-4*B*si
n(f*x+e)-36*A+4*B)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^4+cos(f*x+e)^3*si
n(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+
e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(90) = 180.
time = 0.41, size = 212, normalized size = 2.21

$$\frac{\left(10\,Ba^3\cos\left(fx+e\right)^4-5\,(A+7\,B)a^3\cos\left(fx+e\right)^2+2\,(3\,A+13\,B)a^3-5\,\left((A-B)a^3\cos\left(fx+e\right)^2-2\,(A-B)a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{10\,\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] 1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A + 1
3*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 2
0*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12
*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(90) = 180.
time = 0.57, size = 359, normalized size = 3.74

$$\frac{\left(40\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+10\,Aa^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-90\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-10\,Aa^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+90\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\,Aa^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-45\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Aa^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+9\,Ba^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{80\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{5}c^{6}f\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] 1/80*(40*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 10*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - 90*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^3*sqrt(c)*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*B*a^3*sqrt(c)*
cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*
a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - 45*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*
B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi +
1/2*f*x + 1/2*e)^2 - 1)^5*c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{7/2}}{\left(c-c\,\sin(e+f\,x)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(11/2), x)
```

3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60
*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/48
0*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A]
time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(480*c^2*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)}{6}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)}{6}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 442 vs. 2(146) = 292.
time = 6.67, size = 442, normalized size = 3.03

$$\frac{4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} - \frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} + \frac{3(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} + \frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} + \frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2
)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(13/2)) + (3*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(13/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(13/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(13/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(128) = 256$.
time = 0.41, size = 393, normalized size = 2.69

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(3A\left(\cos^{6}(fx+e)\right)-3A\left(\cos^{5}(fx+e)\right)\sin(fx+e)+18A\left(\cos^{5}(fx+e)\right)+21A\left(\cos^{4}(fx+e)\right)\sin(fx+e)-72A\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/30/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(3*A*cos(f*x+e)^6-3*A*cos(f*x+e)
^5*sin(f*x+e)+18*A*cos(f*x+e)^5+21*A*cos(f*x+e)^4*sin(f*x+e)-72*A*cos(f*x+e
)^4+51*A*cos(f*x+e)^3*sin(f*x+e)+15*B*cos(f*x+e)^4-15*B*cos(f*x+e)^3*sin(f*
x+e)-106*A*cos(f*x+e)^3-157*A*cos(f*x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^3+5*B
*cos(f*x+e)^2*sin(f*x+e)+235*A*cos(f*x+e)^2-78*A*sin(f*x+e)*cos(f*x+e)-35*B
*cos(f*x+e)^2+30*B*sin(f*x+e)*cos(f*x+e)+118*A*cos(f*x+e)+196*A*sin(f*x+e)+
10*B*cos(f*x+e)-20*B*sin(f*x+e)-196*A+20*B)/(-c*(sin(f*x+e)-1))^(13/2)/(cos
(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-
8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(13/2), x)
```

Fricas [A]
time = 0.43, size = 228, normalized size = 1.56

$$-\frac{\left(15\,Ba^{3}\cos\left(fx+e\right)^{4}-15\,(A+3\,B)a^{3}\cos\left(fx+e\right)^{2}+6\,(3\,A+5\,B)a^{3}-2\,\left(5\,(A+B)a^{3}\cos\left(fx+e\right)^{2}-(11\,A+5\,B)a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,\left(c^{7}f\cos\left(fx+e\right)^{7}-18\,c^{7}f\cos\left(fx+e\right)^{5}+48\,c^{7}f\cos\left(fx+e\right)^{3}-32\,c^{7}f\cos\left(fx+e\right)+2\,\left(3\,c^{7}f\cos\left(fx+e\right)^{5}-16\,c^{7}f\cos\left(fx+e\right)^{3}+16\,c^{7}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(15*B*a^3*cos(f*x + e)^4 - 15*(A + 3*B)*a^3*cos(f*x + e)^2 + 6*(3*A +
 5*B)*a^3 - 2*(5*(A + B)*a^3*cos(f*x + e)^2 - (11*A + 5*B)*a^3)*sin(f*x + e
```

```
))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7
 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e
) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x
+ e))*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(137) = 274.
time = 0.55, size = 359, normalized size = 2.46

$$-\frac{\left(60\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+20\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-100\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-15\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+75\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-30\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Aa^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\,Ba^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{480\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^6c^7f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] -1/480*(60*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 20*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 100*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^3*sqrt(c)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 75*B*a^3*sqrt(
c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6
*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) - 30*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 5*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2 - 1)^6*c^7*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [**B**]
time = 22.85, size = 406, normalized size = 2.78

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{56\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\left(4\,A+5\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}+\frac{a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\left(A\,1{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{3\,c^7\,f}-\frac{32\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A+2\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^7\,f}+\frac{8\,B\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^7\,f}-\frac{a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\left(A\,13{}\mathrm{i}+B\,5{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{5\,c^7\,f}\right)}{-858\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+858\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(7\,e+7\,f\,x\right)+1144\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)-416\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)+24\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((56*a^3*exp(e*7i + f*x*7i)*(4*A + 5*B)*(a + a
*sin(e + f*x))^(1/2))/(5*c^7*f) + (a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*
(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*c^7*f) - (32*a^3*exp(e*7i
+ f*x*7i)*cos(2*e + 2*f*x)*(A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(c^7*f) +
(8*B*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(c
^7*f) - (a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(A*13i + B*5i)*(a + a*sin(e +
f*x))^(1/2)*32i)/(5*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858
*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x)
+ 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e +
 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*s
in(6*e + 6*f*x))
```

3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}}+\frac{(3A-11B)\,\mathrm{co}}{840c^2}$$

```
[Out] 1/14*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/16
8*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2)+
1/840*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(
11/2)+1/6720*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*
x+e))^(9/2)
```

Rubi [A]

time = 0.33, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}+\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}}+\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x]
)^(15/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(168*c*f
*(c - c*Sin[e + f*x])^(13/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(7/2))/(840*c^2*f*(c - c*Sin[e + f*x])^(11/2)) + ((3*A - 11*B)*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2))/(6720*c^3*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cdots}{\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cdots}{16\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cdots}{16\cdots}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cdots}{16\cdots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 442 vs. 2(202) = 404.
time = 6.74, size = 442, normalized size = 2.19

$$\frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} - \frac{2(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} + \frac{6(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} + \frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}} + \frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(15/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2
)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
```

```
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(15/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(178) = 356$.
time = 0.47, size = 505, normalized size = 2.50

method	result
default	$-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(5016A-472B-2748A\cos(fx+e)+472B\sin(fx+e)-5016A\sin(fx+e)-4B\cos(fx+e)+39A(\cos^7(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/420/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(5136*A*cos(f*x+e)^2*sin(f*x+e
)-352*B*cos(f*x+e)^2*sin(f*x+e)+5016*A-472*B+39*A*cos(f*x+e)^6*sin(f*x+e)-2
748*A*cos(f*x+e)+472*B*sin(f*x+e)-5016*A*sin(f*x+e)+273*A*cos(f*x+e)^5*sin(
f*x+e)-21*B*cos(f*x+e)^5*sin(f*x+e)+3225*A*cos(f*x+e)^3-65*B*cos(f*x+e)^3-4
*B*cos(f*x+e)+287*B*cos(f*x+e)^3*sin(f*x+e)-380*B*cos(f*x+e)^4-7404*A*cos(f
*x+e)^2+828*B*cos(f*x+e)^2-1911*A*cos(f*x+e)^3*sin(f*x+e)-3*B*cos(f*x+e)^6*
sin(f*x+e)-1209*A*cos(f*x+e)^4*sin(f*x+e)-936*A*cos(f*x+e)^5+72*B*cos(f*x+e
)^5+24*B*cos(f*x+e)^6-476*B*sin(f*x+e)*cos(f*x+e)+3120*A*cos(f*x+e)^4-3*B*c
os(f*x+e)^7+39*A*cos(f*x+e)^7-312*A*cos(f*x+e)^6+2268*A*sin(f*x+e)*cos(f*x+
e)+93*B*sin(f*x+e)*cos(f*x+e)^4)/(-c*(sin(f*x+e)-1))^(15/2)/(cos(f*x+e)^4+c
os(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e
)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(15/2), x)
```

Fricas [A]
time = 0.45, size = 249, normalized size = 1.23

$$-\frac{\left(140\,Ba^{3}\cos\left(fx+e\right)^{4}-7\,(27\,A+61\,B)a^{3}\cos\left(fx+e\right)^{2}+4\,(57\,A+71\,B)a^{3}-7\left(5\,(3\,A+5\,B)a^{3}\cos\left(fx+e\right)^{2}-4\,(9\,A+7\,B)a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{420\left(7\,c^{8}f\cos\left(fx+e\right)^{7}-56\,c^{8}f\cos\left(fx+e\right)^{5}+112\,c^{8}f\cos\left(fx+e\right)^{3}-64\,c^{8}f\cos\left(fx+e\right)-\left(c^{8}f\cos\left(fx+e\right)^{7}-24\,c^{8}f\cos\left(fx+e\right)^{5}+80\,c^{8}f\cos\left(fx+e\right)^{3}-64\,c^{8}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="fricas")
```

```
[Out] -1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*(
57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^3)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*
cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^
8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8
*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2)
,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 359, normalized size = 1.78

$$\frac{\left(280\,B a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+105\,A a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-385\,B a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-63\,A a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+231\,B a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+21\,A a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-77\,B a^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,A a^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+11\,B a^3\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{6720\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^7c^8f\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="giac")
```

```
[Out] 1/6720*(280*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 105*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 385*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 63*A*a^3*sqrt(c)*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 231*B*a^3*sq
rt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 21*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) - 77*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - 3*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 11*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(
-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^7*c^8*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))
```

Mupad [**B**]
time = 25.26, size = 827, normalized size = 4.09

$$-\frac{\sqrt{c-c\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(\frac{B\,a^3\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,16{}\mathrm{i}}{3\,c^8\,f}+\frac{B\,a^3\,{\mathrm{e}}^{e\,12{}\mathrm{i}+f\,x\,12{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,16{}\mathrm{i}}{3\,c^8\,f}-\frac{a^3\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(A\,3{}\mathrm{i}+B\,5{}\mathrm{i}\right)\,8{}\mathrm{i}}{3\,c^8\,f}+\frac{a^3\,{\mathrm{e}}^{e\,11{}\mathrm{i}+f\,x\,11{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(A\,3{}\mathrm{i}+B\,5{}\mathrm{i}\right)\,8{}\mathrm{i}}{3\,c^8\,f}-\frac{a^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(27\,A+41\,B\right)\,16{}\mathrm{i}}{15\,c^8\,f}-\frac{a^3\,{\mathrm{e}}^{e\,10{}\mathrm{i}+f\,x\,10{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(27\,A+41\,B\right)\,16{}\mathrm{i}}{15\,c^8\,f}+\frac{a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(A\,43{}\mathrm{i}+B\,29{}\mathrm{i}\right)\,8{}\mathrm{i}}{5\,c^8\,f}-\frac{a^3\,{\mathrm{e}}^{e\,9{}\mathrm{i}+f\,x\,9{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(A\,43{}\mathrm{i}+B\,29{}\mathrm{i}\right)\,8{}\mathrm{i}}{5\,c^8\,f}+\frac{a^3\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\sqrt{a+a\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(89\,A+82\,B\right)\,32{}\mathrm{i}}{35\,c^8\,f}\right)}{1+910\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}-2002\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}+2002\,{\mathrm{e}}^{e\,10{}\mathrm{i}+f\,x\,10{}\mathrm{i}}-910\,{\mathrm{e}}^{e\,12{}\mathrm{i}+f\,x\,12{}\mathrm{i}}+90\,{\mathrm{e}}^{e\,14{}\mathrm{i}+f\,x\,14{}\mathrm{i}}-{\mathrm{e}}^{e\,16{}\mathrm{i}+f\,x\,16{}\mathrm{i}}-90\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,14{}\mathrm{i}-{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,350{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,1638{}\mathrm{i}-{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,1430{}\mathrm{i}-{\mathrm{e}}^{e\,9{}\mathrm{i}+f\,x\,9{}\mathrm{i}}\,1430{}\mathrm{i}+{\mathrm{e}}^{e\,11{}\mathrm{i}+f\,x\,11{}\mathrm{i}}\,1638{}\mathrm{i}-{\mathrm{e}}^{e\,13{}\mathrm{i}+f\,x\,13{}\mathrm{i}}\,350{}\mathrm{i}+{\mathrm{e}}^{e\,15{}\mathrm{i}+f\,x\,15{}\mathrm{i}}\,14{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(15/2),x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(B*a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
 f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^8*f) + (B*a^3*exp(e*12i + f*x*12i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^
8*f) - (a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) + (a^3*exp(e*11i +
f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^
(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)
/(15*c^8*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)/(15*c^8*f) + (a^3*e
xp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) - (a^3*exp(e*9i + f*x*9i)*(a +
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*43i
+ B*29i)*8i)/(5*c^8*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*
1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(89*A + 82*B)*32i)/(35*c^8*f)
))/(exp(e*1i + f*x*1i)*14i - 90*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*350
i + 910*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*1638i - 2002*exp(e*6i + f*x
*6i) - exp(e*7i + f*x*7i)*1430i - exp(e*9i + f*x*9i)*1430i + 2002*exp(e*10i
 + f*x*10i) + exp(e*11i + f*x*11i)*1638i - 910*exp(e*12i + f*x*12i) - exp(e
*13i + f*x*13i)*350i + 90*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*14i -
 exp(e*16i + f*x*16i) + 1)
```

3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}+\frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}}+\frac{(A-3B)\cos(e+}{224c^2f(c-}$$

```
[Out] 1/16*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(17/2)+1/56
*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(15/2)+1/22
4*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(13/2)+1
/1120*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(11/
2)+1/8960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^4/f/(c-c*sin(f*x+e))^
(9/2)
```

Rubi [A]
time = 0.40, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(17/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x]
)^(17/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c
- c*Sin[e + f*x])^(15/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(224*c^2*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^(7/2))/(1120*c^3*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*
B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8960*c^4*f*(c - c*Sin[e + f*x]
)^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
```

```
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)}{56}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)}{56}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)}{56}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)}{56}$$

Mathematica [A]
time = 6.71, size = 436, normalized size = 1.77

$$\frac{(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}+\frac{(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(17/2),x]
```

```
[Out] ((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)
)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
 (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x
]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x]
)^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[
e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 +
 Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*
Sin[e + f*x])^(17/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(216) = 432$.
time = 0.54, size = 560, normalized size = 2.28

method	result
default	$-\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(3076A-268B-1608A\cos(fx+e)+268B\sin(fx+e)-3076A\sin(fx+e)+64B\cos(fx+e)+96A(\cos^7($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/140/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(3880*A*cos(f*x+e)^2*sin(f*x+e
)-300*B*cos(f*x+e)^2*sin(f*x+e)+3076*A-268*B+108*A*cos(f*x+e)^6*sin(f*x+e)-
1608*A*cos(f*x+e)+268*B*sin(f*x+e)-3076*A*sin(f*x+e)+372*A*cos(f*x+e)^5*sin
(f*x+e)-31*B*cos(f*x+e)^5*sin(f*x+e)+2332*A*cos(f*x+e)^3-136*B*cos(f*x+e)^3
+64*B*cos(f*x+e)+164*B*cos(f*x+e)^3*sin(f*x+e)-275*B*cos(f*x+e)^4-5348*A*co
s(f*x+e)^2+504*B*cos(f*x+e)^2+12*A*cos(f*x+e)^8-1548*A*cos(f*x+e)^3*sin(f*x
+e)-9*B*cos(f*x+e)^6*sin(f*x+e)-1332*A*cos(f*x+e)^4*sin(f*x+e)-960*A*cos(f*
x+e)^5+80*B*cos(f*x+e)^5-12*A*cos(f*x+e)^7*sin(f*x+e)+B*cos(f*x+e)^7*sin(f*
x+e)+40*B*cos(f*x+e)^6-204*B*sin(f*x+e)*cos(f*x+e)+2880*A*cos(f*x+e)^4-8*B*
cos(f*x+e)^7+96*A*cos(f*x+e)^7-480*A*cos(f*x+e)^6+1468*A*sin(f*x+e)*cos(f*x
+e)-B*cos(f*x+e)^8+111*B*sin(f*x+e)*cos(f*x+e)^4)/(-c*(sin(f*x+e)-1))^(17/2
)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f
*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="maxima")
```

[Out] Timed out

Fricas [A]
time = 0.47, size = 259, normalized size = 1.05

$$\frac{\left(35\,Ba^{3}\cos\left(fx+e\right)^{4}-56\,(A+2\,B)a^{3}\cos\left(fx+e\right)^{2}+4\,(17\,A+19\,B)a^{3}-4\,\left(7\,(A+2\,B)a^{3}\cos\left(fx+e\right)^{2}-2\,(9\,A+8\,B)a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{140\,\left(c^{9}f\cos\left(fx+e\right)^{9}-32\,c^{9}f\cos\left(fx+e\right)^{7}+160\,c^{9}f\cos\left(fx+e\right)^{5}-256\,c^{9}f\cos\left(fx+e\right)^{3}+128\,c^{9}f\cos\left(fx+e\right)+8\,\left(c^{9}f\cos\left(fx+e\right)^{7}-10\,c^{9}f\cos\left(fx+e\right)^{5}+24\,c^{9}f\cos\left(fx+e\right)^{3}-16\,c^{9}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="fricas")
```

```
[Out] 1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A
+ 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*x +
 e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*cos(
f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f*co
s(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2)
,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 359, normalized size = 1.46

$$-\frac{\left(140\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+56\,Aa^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-168\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-28\,Aa^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+84\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Aa^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-24\,Ba^{3}\sqrt{c}\,\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Aa^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{8960\,\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{8}c^{9}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="giac")
```

```
[Out] -1/8960*(140*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 56*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 168*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 28*A*a^3*sqrt(c)*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 84*B*a^3*sqr
t(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 8*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x
```

```
 + 1/2*e)) - 24*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*
pi + 1/2*f*x + 1/2*e)^2 - 1)^8*c^9*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 28.34, size = 841, normalized size = 3.42

$$\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{8\,B\,a^3\,e^{e\,5i+f\,x\,5i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{c^9\,f}+\frac{8\,B\,a^3\,e^{e\,13i+f\,x\,13i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{c^9\,f}-\frac{64\,a^3\,e^{e\,6i+f\,x\,6i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(A\,1i+B\,2i\right)}{5\,c^9\,f}-\frac{32\,a^3\,e^{e\,7i+f\,x\,7i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(8\,A+11\,B\right)}{5\,c^9\,f}+\frac{64\,a^3\,e^{e\,12i+f\,x\,12i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(A\,1i+B\,2i\right)}{5\,c^9\,f}-\frac{32\,a^3\,e^{e\,11i+f\,x\,11i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(8\,A+11\,B\right)}{5\,c^9\,f}+\frac{64\,a^3\,e^{e\,8i+f\,x\,8i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(A\,13i+B\,10i\right)}{7\,c^9\,f}-\frac{64\,a^3\,e^{e\,10i+f\,x\,10i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(A\,13i+B\,10i\right)}{7\,c^9\,f}+\frac{16\,a^3\,e^{e\,9i+f\,x\,9i}\sqrt{a+a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\left(64\,A+53\,B\right)}{7\,c^9\,f}\right)}{1+1700\,e^{e\,4i+f\,x\,4i}-6188\,e^{e\,6i+f\,x\,6i}+4862\,e^{e\,8i+f\,x\,8i}+4862\,e^{e\,10i+f\,x\,10i}-6188\,e^{e\,12i+f\,x\,12i}+1700\,e^{e\,14i+f\,x\,14i}-119\,e^{e\,16i+f\,x\,16i}+e^{e\,18i+f\,x\,18i}-119\,e^{e\,2i+f\,x\,2i}+e^{e\,1i+f\,x\,1i}\,16i-e^{e\,3i+f\,x\,3i}\,544i+e^{e\,5i+f\,x\,5i}\,3808i-e^{e\,7i+f\,x\,7i}\,7072i+e^{e\,11i+f\,x\,11i}\,7072i-e^{e\,13i+f\,x\,13i}\,3808i+e^{e\,15i+f\,x\,15i}\,544i-e^{e\,17i+f\,x\,17i}\,16i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(17/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
8*B*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
+ f*x*1i)*1i)/2))^(1/2))/(c^9*f) + (8*B*a^3*exp(e*13i + f*x*13i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^9*f) - (6
4*a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f) - (32*a^3*exp(e*7i + f*x*7i)
*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(8
*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f)
- (32*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*8i + f*
x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/
2)*(A*13i + B*10i))/(7*c^9*f) - (64*a^3*exp(e*10i + f*x*10i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*13i + B*10i))/
(7*c^9*f) + (16*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(64*A + 53*B))/(7*c^9*f)))/(exp(e*1i +
f*x*1i)*16i - 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*544i + 1700*exp(e
*4i + f*x*4i) + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i + f*x*6i) - exp(e*
7i + f*x*7i)*7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(e*10i + f*x*10i) +
exp(e*11i + f*x*11i)*7072i - 6188*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13
i)*3808i + 1700*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*544i - 119*exp(
e*16i + f*x*16i) - exp(e*17i + f*x*17i)*16i + exp(e*18i + f*x*18i) + 1)
```

3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=197

$$\frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{(A-B)c\cos(}{2f\sqrt{}}$$

```
[Out] 1/2*(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*
B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+4*(A-B)*c^3*co
s(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2
*(A-B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2819, 2816, 2746, 31}

$$\frac{4c^3(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c^2(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{c(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (4*(A - B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*(A - B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[
e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*(c - c*Si
n[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-}{\sqrt{a+}}$$

$$= \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+}{3f\sqrt{}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c}{}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c}{}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c}{}$$

$$= \frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2(A-B)}{}$$

Mathematica [A]
time = 0.88, size = 185, normalized size = 0.94

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\left(3(A-3B)\cos(2(e+fx))-96A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+96B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(36A-51B)\sin(e+fx)+B\sin(3(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -1/12*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt
[c - c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (
36*A - 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(177) = 354$.
time = 0.38, size = 601, normalized size = 3.05

method	result
default	$-\frac{\left(48A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-48B\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-48B\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-48*B*sin
(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-48*B*ln(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))+48*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-3*A
*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^2*sin(f*x+e)-15*A+17*B-3*A*cos(f*x+
e)+17*B*sin(f*x+e)-15*A*sin(f*x+e)+3*A*cos(f*x+e)^3-9*B*cos(f*x+e)^3+9*B*co
s(f*x+e)+2*B*cos(f*x+e)^3*sin(f*x+e)+2*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2-19*
B*cos(f*x+e)^2-48*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4
8*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*l
n(2/(1+cos(f*x+e)))+24*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-26*B*sin(f*x+e)*co
s(f*x+e)-24*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-24*B*cos(f*x+e)*ln(2/(1+cos(f
*x+e)))-24*A*ln(2/(1+cos(f*x+e)))+24*B*ln(2/(1+cos(f*x+e)))+18*A*sin(f*x+e)
*cos(f*x+e))*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e
)-3*cos(f*x+e)^2+2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1
+sin(f*x+e)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*
x + e) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.59, size = 317, normalized size = 1.61

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{6\sqrt{2}\left(A\sqrt{a}\,c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}\,c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(4Ba^{\frac{5}{2}}c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6-3Aa^{\frac{5}{2}}c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+3Ba^{\frac{5}{2}}c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-6Aa^{\frac{5}{2}}c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+6Ba^{\frac{5}{2}}c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(c)*(6*sqrt(2)*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-2*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^2 + 2)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sq
rt(2)*(4*B*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^6 - 3*A*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^4 + 3*B*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
```

```
1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 6*A*a^(5/2)*c^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 6*B*a^(5/2)*c^2*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\,(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=146

$$\frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}-\frac{B\cos(e+fx)(c-}{2f\sqrt{a+a\sin}}$$

```
[Out] -1/2*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+2*(A-B)*c
^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2819, 2816, 2746, 31}

$$\frac{2c^2(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*(A - B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e +
f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-}{\sqrt{a}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+}{2f\sqrt{}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+}{2f\sqrt{}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+}{2f\sqrt{}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)}{}$$

Mathematica [A]
time = 0.47, size = 146, normalized size = 1.00

$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}\left(B\cos(2(e+fx))-4\left(4(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(A-2B)\sin(e+fx)\right)\right)}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -1/4*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(132) = 264$.
time = 0.33, size = 507, normalized size = 3.47

method	result
default	$\frac{\left(-B\left(\cos^3(fx+e)\right)+B\left(\cos^2(fx+e)\right)\sin(fx+e)+2A\left(\cos^2(fx+e)\right)+2A\sin(fx+e)\cos(fx+e)+4A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-8A\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2+2*A*sin(f
*x+e)*cos(f*x+e)+4*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-8*A*cos(f*x+e)*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+8*A
*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^2-4*B
*sin(f*x+e)*cos(f*x+e)-4*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+8*B*cos(f*x+e)*l
n(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*B*ln(2/(1+cos(f*x+e)))*sin(f*x+
e)-8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)
-4*A*ln(2/(1+cos(f*x+e)))+8*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+B*
cos(f*x+e)+3*B*sin(f*x+e)+4*B*ln(2/(1+cos(f*x+e)))-8*B*ln(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))-2*A+3*B)*(-c*(sin(f*x+e)-1))^(3/2)/(cos(f*x+e)^2-cos
(f*x+e)*sin(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c*
sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(a*(sin(e
+ f*x) + 1)), x)
```

Giac [A]
time = 0.55, size = 260, normalized size = 1.78

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{\sqrt{2}\left(A\sqrt{a}\,c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}\,c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}Ba^{\frac{3}{2}}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-\sqrt{2}Aa^{\frac{3}{2}}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{2}Ba^{\frac{3}{2}}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -sqrt(2)*sqrt(c)*(sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-sin(-1/4*pi + 1/2*f
*x + 1/2*e)^2 + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (sqrt(2)*B*a^(
3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - sqrt(2)*A*a^(3/2)*c*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^2 + sqrt(2)*B*a^(3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/a^2)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] (A-B)*c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)-B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3050, 2817, 2816, 2746, 31}

$$\frac{c(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{B\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A+$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(a(-A+B)c\cos(e}{\sqrt{a+a\sin(e+f}$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{((-A+B)c\cos(e}{f\sqrt{a+as}$$

$$= \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx}{f\sqrt{a-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.79, size = 126, normalized size = 1.31

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-\left((A-B)\left(\log\left(e^{i(e+fx)}\right)-2\log\left(i+e^{i(e+fx)}\right)\right)\right)+B\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e
+ f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-((A - B)*(Log[E^(I*(e + f*x))] - 2
*Log[I + E^(I*(e + f*x))])) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(88) = 176$.
time = 0.31, size = 403, normalized size = 4.20

method	result
default	$\frac{\left(A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2A\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-A\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+2A\sin(fx+e)\ln\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))-A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+2*A*sin(f*x+e)*ln(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+
e)-B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))+B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-2*B*sin(f*x+e)*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-A*ln(2/(1+cos(f*x+e)))+2*A*ln(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)+B*ln(2/(1+cos(f*x+e)))-2*B*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+B)*(-c*(sin(f*x+e)-1))^(1/2)/(-1+co
s(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [A]
time = 0.52, size = 188, normalized size = 1.96

$$\frac{B\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}}-\frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(a+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)-A\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
- A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e)
 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f
*x) + 1)), x)
```

Giac [A]
time = 0.50, size = 150, normalized size = 1.56

$$\frac{\sqrt{2}\left(\frac{2\sqrt{2}\,B\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+4\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{c}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*B*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^2/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*
(A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)))*log(-4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 4)/(a*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=113

$$-\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)\log(1+\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)+1/2*(A-B)*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/
(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3048, 2816, 2746, 31}

$$\frac{(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] -1/2*((A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*Cos[e + f*x]*Log[1 + Sin[e + f*x]])
/(2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3048

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist
[(A*b + a*B)/(2*a*b), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] + Dist[(B*c + A*d)/(2*c*d), Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a +
b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a} + \frac{(A-B)\int \frac{\sqrt{c-c\mathrm{s}}}{\sqrt{a+a\mathrm{s}}}}{2c}$$

$$= \frac{(a(A-B)\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)c\,\mathrm{c}}{2\sqrt{a+a\sin}}$$

$$= \frac{((A-B)\cos(e+fx))\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} -$$

$$= -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)}{2f\sqrt{a+}}$$

Mathematica [A]
time = 0.23, size = 97, normalized size = 0.86

$$-\frac{\cos(e+fx)\left((A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*((A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (-A + B
)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*
Sqrt[c - c*Sin[e + f*x]]))
```

Maple [A]
time = 0.28, size = 167, normalized size = 1.48

method	result
default	$\frac{\left(A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-A\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-B\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right.\right.}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-A*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-B*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln(2/(1+cos(f*x+e))))*cos(f*x+e)/(a*(1
+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(a*c*cos(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e +
f*x) - 1))), x)
```

Giac [A]
time = 0.48, size = 150, normalized size = 1.33

$$\frac{\frac{2\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{\sqrt{a}\,c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{a\,c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/2*(2*(A*sqrt(c) + B*sqrt(c))*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(sq
rt(a)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))) - (A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(-sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^2 + 1)/(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*(A
-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2820, 3855}

$$\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\,\ldots}{\ldots}$$

$$= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{((A-\ldots}{2c\sqrt{a+\ldots}}$$

$$= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{(A-B\ldots}{2cf\sqrt{a+\ldots}}$$

Mathematica [A]
time = 0.38, size = 191, normalized size = 1.85

$$\frac{\left(A+B+(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+(A-B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f\sqrt{a(1+\sin(e+fx))}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)),x]
```

```
[Out] ((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*x])]
*(c - c*Sin[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(91) = 182.
time = 0.32, size = 308, normalized size = 2.99

method	result
default	$-\frac{\left(A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-B\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(f\ldots}{\sin(fx+e)}\ldots\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e
)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))-A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+A*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-A*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))-B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e))*cos(f*x+e)/
(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2)), x)
```

Fricas [A]
time = 0.44, size = 363, normalized size = 3.52

$$\left[-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx+e))\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)+2\,\sqrt{ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{\cos(fx+e)^3}\right)+2\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,(A+B)}{4\,(ac^2f\cos(fx+e)\sin(fx+e)-ac^2f\cos(fx+e))},\ -\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,(A+B)}{2\,(ac^2f\cos(fx+e)\sin(fx+e)-ac^2f\cos(fx+e))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c)
*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*sin(f*
x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a(\sin(e+fx)+1)}\,(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
 - 1))**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(98) = 196.
time = 0.53, size = 227, normalized size = 2.20

$$-\frac{\frac{\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}}{ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/4*((A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(-sin(-1/4*pi + 1/2*f*x +
1/2*e)^2 + 1)/(a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) - 2*(A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(abs(sin(
-1/4*pi + 1/2*f*x + 1/2*e)))/(a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))
/(a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\text{ta}}{4c^2f\sqrt{a+a}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A
-B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A-B)*
arctanh(sin(f*x+e))*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)) + ((A - B)*Cos[e + f*x])/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\ \ldots}{\ldots}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{4cf\sqrt{a+\ldots}}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{4cf\sqrt{a+\ldots}}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{4cf\sqrt{a+\ldots}}$$

Mathematica [A]
time = 0.42, size = 222, normalized size = 1.45

$$\frac{\left(A+B+(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+(A-B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{4f\sqrt{a(1+\sin(e+fx))}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(5/2)),x]
```

```
[Out] ((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[Co
s[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
```

```
Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/
2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(135) = 270$.
time = 0.34, size = 471, normalized size = 3.08

method	result
default	$\frac{\left(A(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))*cos(f*x+e)^2+B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
*cos(f*x+e)^2+2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A
*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)^2-2*B
*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)*ln(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+2*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+3*A*sin(f*x+e)+2*B
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))+B*sin(f*x+e)-2*A)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(
sin(f*x+e)-1))^(5/2)
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2)), x)
```

Fricas [A]
time = 0.44, size = 456, normalized size = 2.98

$$\left[-\frac{((A-B)\cos(fx+e)^3+2(A-B)\cos(fx+e)\sin(fx+e)-2(A-B)\cos(fx+e))\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)+2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)-2((A-B)\sin(fx+e)-2A)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{8(ac^3f\cos(fx+e)^3+2ac^3f\cos(fx+e)\sin(fx+e)-2ac^3f\cos(fx+e))}, -\frac{((A-B)\cos(fx+e)^3+2(A-B)\cos(fx+e)\sin(fx+e)-2(A-B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)-((A-B)\sin(fx+e)-2A)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{4(ac^3f\cos(fx+e)^3+2ac^3f\cos(fx+e)\sin(fx+e)-2ac^3f\cos(fx+e))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A
 - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
 + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
 e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
+ e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x + e)^3
+ 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt(-a*c)*
arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*c
os(f*x + e)*sin(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin{(e + fx)}}{\sqrt{a(\sin{(e + fx)} + 1)}\,(-c(\sin{(e + fx)} - 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
 - 1))**(5/2)), x)
```

Giac [A]
time = 0.55, size = 263, normalized size = 1.72

$$-\frac{\frac{2\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}}{ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/16*(2*(A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(-sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 + 1)/(a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) - 4*(A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(abs(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*(A*sqrt(a)*sqrt(c) - B*sqrt(a)*sq
rt(c))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqr
t(c))/(a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}-\frac{(3A-5\ldots$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*(
3*A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-1
/6*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)
-4*(3*A-5*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)-2*(3*A-5*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$-\frac{4c^4(3A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c^3(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c^2(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{c(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(3*A - 5*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(3*A - 5*B)*c^3*Cos[e + f*x]*Sqrt
[c - c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((3*A - 5*B)*c^2*Cos
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((
3*A - 5*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[a + a*Sin
[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(2*f*(a + a
*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(3A-5B)}{}$$

$$= -\frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B}{}$$

$$= -\frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-}{}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-}{}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-}{}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-}{}$$

$$= -\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(3A}{}$$

Mathematica [A]
time = 2.32, size = 271, normalized size = 1.00

$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(132A-45B+2(27A-59B)\cos(2(e+fx))+B\cos(4(e+fx))+576A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-960B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-117A\sin(e+fx)+279B\sin(e+fx)+576A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-960B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-3A\sin(3(e+fx))+13B\sin(3(e+fx))\right)}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/24*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
132*A - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 576*
A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*(e + f
*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(245) = 490$.
time = 0.32, size = 948, normalized size = 3.50

method	result	size
default	Expression too large to display	948

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(288*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-480*B*si
n(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-480*B*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))+288*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+
24*A*cos(f*x+e)^2*sin(f*x+e)-48*B*cos(f*x+e)^2*sin(f*x+e)-102*A+166*B-120*B
*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)+27*A*cos(f*x+e)+166*B*sin(f*x+e
)-102*A*sin(f*x+e)-27*A*cos(f*x+e)^3+61*B*cos(f*x+e)^3-59*B*cos(f*x+e)-13*B
*cos(f*x+e)^3*sin(f*x+e)-11*B*cos(f*x+e)^4+99*A*cos(f*x+e)^2-155*B*cos(f*x+
e)^2-144*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+240*B*cos(
f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)^3*sin(f*x+
e)+72*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+72*A*cos(f*x+e)*ln(2/(1+cos(f*x+e
)))-2*B*cos(f*x+e)^5+240*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-107*B*sin(f*x+e)
*cos(f*x+e)-144*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-120*B*cos(f*x+e)*ln(2/(1+
cos(f*x+e)))+3*A*cos(f*x+e)^4+72*A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f*x+e)
))-144*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2
40*B*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-144*A
*ln(2/(1+cos(f*x+e)))+240*B*ln(2/(1+cos(f*x+e)))-120*B*ln(2/(1+cos(f*x+e)))
*cos(f*x+e)^2+75*A*sin(f*x+e)*cos(f*x+e)-144*A*ln(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))*cos(f*x+e)^2+240*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)*cos(f*x+e)^2+2*B*sin(f*x+e)*cos(f*x+e)^4)*(-c*(sin(f*x+e)-1))^(7/2)/(cos(
f*x+e)^4-cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3+4*cos(f*x+e)^2*sin(f*x+e)-8
*cos(f*x+e)^2+4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)-8*sin(f*x+e)+8)/(a*(1+si
n(f*x+e)))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.60, size = 413, normalized size = 1.52

$$\frac{\sqrt{2}\,\sqrt{c}\left(\frac{6\left(3\,\sqrt{2}\,A\sqrt{a}\,c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-5\,\sqrt{2}\,B\sqrt{a}\,c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{6\left(\sqrt{2}\,A\sqrt{a}\,c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\sqrt{2}\,B\sqrt{a}\,c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\sqrt{2}\left(4\,Ba^{\frac{9}{2}}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-3\,Aa^{\frac{9}{2}}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+9\,Ba^{\frac{9}{2}}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-12\,Aa^{\frac{9}{2}}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ba^{\frac{9}{2}}c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{a^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(c)*(6*(3*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - 5*sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log
(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))) - 6*(sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - sqr
t(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/((sin(-1/4*pi + 1/2
*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(4*
B*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^6 - 3*A*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^4 + 9*B*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 12*A*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 24*B*a^(9/2)*c^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^6*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}-\frac{(A-2B)c}{2}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*(
A-2*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-4*(A-
2*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)-2*(A-2*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*
x+e))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$-\frac{4c^3(A-2B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2c^2(A-2B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c(A-2B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(A - 2*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(A - 2*B)*c^2*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - 2*B)*c*Cos[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*C
os[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-2B)}{}$$

$$= -\frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)}{}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2}{}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2}{}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2}{}$$

$$= -\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(A-}{}$$

Mathematica [A]
time = 1.07, size = 212, normalized size = 1.01

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(28A-16B+2(2A-7B)\cos(2(e+fx))+64A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-128B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(-8A+31B+64(A-2B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+B\sin(3(e+fx))\right)}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/8*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2
8*A - 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*A +
 31*B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]
 + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Si
n[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(190) = 380$.
time = 0.32, size = 865, normalized size = 4.12

method	result

default	$\frac{\left(32A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-64B\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-64B\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(32*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-64*B*sin(
f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-64*B*ln(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))+32*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*A*
cos(f*x+e)^2*sin(f*x+e)-6*B*cos(f*x+e)^2*sin(f*x+e)-12*A+22*B-16*B*ln(2/(1+
cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)+22*B*sin(f*x+e)-12*A*sin(
f*x+e)-2*A*cos(f*x+e)^3+7*B*cos(f*x+e)^3-7*B*cos(f*x+e)-B*cos(f*x+e)^3*sin(
f*x+e)-B*cos(f*x+e)^4+12*A*cos(f*x+e)^2-21*B*cos(f*x+e)^2-16*A*cos(f*x+e)*l
n(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+32*B*cos(f*x+e)*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))+8*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+8*A*cos(f*x
+e)*ln(2/(1+cos(f*x+e)))+32*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-15*B*sin(f*x+
e)*cos(f*x+e)-16*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-16*B*cos(f*x+e)*ln(2/(1+
cos(f*x+e)))+8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-16*A*cos(f*x+e)
*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+32*B*cos(f*x+e)*sin(
f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-16*A*ln(2/(1+cos(f*x+e)))
+32*B*ln(2/(1+cos(f*x+e)))-16*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+10*A*sin(
f*x+e)*cos(f*x+e)-16*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e
)^2+32*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2)*(-c*(sin(
f*x+e)-1))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*cos
(f*x+e)*sin(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.50, size = 266, normalized size = 1.27

$$-\frac{2\left(B\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+A\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5B\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\left(A\sqrt{a}c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2B\sqrt{a}c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)-\frac{A\sqrt{a}c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)\sqrt{c}}{a^2f\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -2*(B*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + A*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 5*B*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - 2*B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(ab
s(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) - B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/cos(-1/4*
pi + 1/2*f*x + 1/2*e)^2)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}}-\frac{(A-B)\cos(}{2f(}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(3/2)-(A-3*
B)*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+
e))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2819, 2816, 2746, 31}

$$-\frac{c^2(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -(((A - 3*B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])) - ((A - 3*B)*c*Cos[e + f*x]*Sqrt[c - c*S
in[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c -
 c*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-3B)}{}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)}{}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)}{}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)}{}$$

$$= -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{(A-3}{}$$

Mathematica [A]

time = 0.61, size = 190, normalized size = 1.19

$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(4A-3B-B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-12B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2\left(B+2(A-3B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*A
 - 3*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
- 12*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(143) = 286$.
time = 0.32, size = 774, normalized size = 4.87

method	result
default	$-\frac{\left(-4A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+12B\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+12B\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/f*(-4*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+12*B*sin(f
*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+12*B*ln(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))-4*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+B*cos(
f*x+e)^2*sin(f*x+e)+2*A-4*B+3*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)-
4*B*sin(f*x+e)+2*A*sin(f*x+e)-B*cos(f*x+e)^3+B*cos(f*x+e)-2*A*cos(f*x+e)^2+
4*B*cos(f*x+e)^2+2*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-
6*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)^2*ln
(2/(1+cos(f*x+e)))-A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-6*B*ln(2/(1+cos(f*x+e)
))*sin(f*x+e)+3*B*sin(f*x+e)*cos(f*x+e)+2*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)
+3*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f*
x+e)))+2*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
-6*B*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*A*l
n(2/(1+cos(f*x+e)))-6*B*ln(2/(1+cos(f*x+e)))+3*B*ln(2/(1+cos(f*x+e)))*cos(f
*x+e)^2-2*A*sin(f*x+e)*cos(f*x+e)+2*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))*cos(f*x+e)^2-6*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e
)^2)*(-c*(sin(f*x+e)-1))^(3/2)/(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*
x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(153) = 306$.
time = 0.53, size = 395, normalized size = 2.48

$$-\frac{B\left(\frac{6\,c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{3\,c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{2\left(\frac{3\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^{\frac{3}{2}}+\frac{2\,a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)-A\left(\frac{2\,c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{4\sqrt{a}\,c^{\frac{3}{2}}\sin(fx+e)}{\left(a^2+\frac{2\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] -(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) - A*(2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^(3/2) - c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4
*sqrt(a)*c^(3/2)*sin(f*x + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(a\,(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f*
x) + 1))**(3/2), x)
```

Giac [A]
time = 0.53, size = 237, normalized size = 1.49

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}\,Bc\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}\,c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\sqrt{a}\,c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-8\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+8\right)}{a^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(A\sqrt{a}\,c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}\,c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)a^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{c}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^2/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(
2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-8*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 8)/
(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+B*c*cos
(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3050, 2816, 2746, 31, 2817}

$$\frac{Bc\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])) + (B*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin}}{(a+a\sin(e}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc}{\sqrt{a+a}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(}{af\sqrt{}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\,c}{af\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.81, size = 157, normalized size = 1.57

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\,\left(A-B+B\log\left(e^{i(e+fx)}\right)-2B\log\left(i+e^{i(e+fx)}\right)+B\left(\log\left(e^{i(e+fx)}\right)-2\log\left(i+e^{i(e+fx)}\right)\right)\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(A - B +
B*Log[E^(I*(e + f*x))] - 2*B*Log[I + E^(I*(e + f*x))] + B*(Log[E^(I*(e + f*
x))] - 2*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(92) = 184$.
time = 0.30, size = 413, normalized size = 4.13

method	result
default	$-\frac{\left(2B\cos(fx+e)\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+2B\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)-B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\text{si}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/f*(2*B*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+
2*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*ln(2/(1+cos(f
*x+e)))*sin(f*x+e)*cos(f*x+e)-B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+A*sin(f*x
+e)*cos(f*x+e)+A*cos(f*x+e)^2-4*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))+2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-B*si
n(f*x+e)*cos(f*x+e)+2*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-B*cos(f*x+e)^2-B*co
s(f*x+e)*ln(2/(1+cos(f*x+e)))-A*sin(f*x+e)-4*B*ln(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))+B*sin(f*x+e)+2*B*ln(2/(1+cos(f*x+e)))-A+B)*(-c*(sin(f*x+e)-1
))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(3/2), x)
```

Giac [A]
time = 0.52, size = 120, normalized size = 1.20

$$-\frac{\left(4\,B\sqrt{a}\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{A\sqrt{a}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-B\sqrt{a}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)\sqrt{c}}{2\,a^{2}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(4*B*sqrt(a)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - (A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sq
rt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=103

$$-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*(
A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2820, 3855}

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
+ f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{(A+}{\quad}\\ &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{((A}{2a\sqrt{c}}\\ &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{(A-}{2af\sqrt{}}\end{aligned}$$

Mathematica [**A**]
time = 0.39, size = 186, normalized size = 1.81

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+B-(A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+(A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)}{2f(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/
2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(91) = 182.
time = 0.30, size = 307, normalized size = 2.98

method	result
default	$\frac{\left(A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-A\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+B\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(f}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-A*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))+A*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))-B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e))*cos(f*x+e)/(
a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Fricas [A]
time = 0.44, size = 355, normalized size = 3.45

$$\left[\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)-2\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}(A-B)}{4(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e))}, -\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}(A-B)}{2(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)*
log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*sin(f*x
 + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}\sqrt{-c(\sin(e + fx) - 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(98) = 196.
time = 0.50, size = 222, normalized size = 2.16

$$-\frac{\frac{2\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^{\frac{3}{2}}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\left(A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^2c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}}{a^2c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/4*(2*(A*sqrt(c) + B*sqrt(c))*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a
^(3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))) - (A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))*log(-cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 + 1)/(a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))/(a^2*c*cos(
-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{A\tan}{2acf\sqrt{a+}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*A
*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*A*arctanh
(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2acf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(3/2)) + (A*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\int}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af}$$

Mathematica [A]
time = 0.48, size = 178, normalized size = 1.19

$$-\frac{\cos(e+fx)\left(2B-A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+A\cos(2(e+fx))\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+2A\sin(e+fx)\right)}{4cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] -1/4*(Cos[e + f*x]*(2*B - A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + A*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2
*A*Sin[e + f*x]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqr
t[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.30, size = 131, normalized size = 0.87

method	result
default	$\frac{\left(A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-A(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-B(\cos^2(fx+e))+A\sin(fx+e)+B}{2f(a(1+\sin(fx+e)))^{\frac{3}{2}}(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-A*cos(f*x+
e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2+A*sin(f*x+e)
+B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Fricas [A]
time = 0.47, size = 292, normalized size = 1.95

$$\left[\frac{\sqrt{ac}\,A\cos(fx+e)^3\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\sqrt{ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{\cos(fx+e)^3}\right)+2\,(A\sin(fx+e)+B)\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{4\,a^2c^2f\cos(fx+e)^3},\ -\frac{\sqrt{-ac}\,A\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3-(A\sin(fx+e)+B)\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{2\,a^2c^2f\cos(fx+e)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x +
 e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*
x + e))/cos(f*x + e)^3) + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a)*s
qrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*A*ar
ctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}\,(-c(\sin(e + fx) - 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f
*x) - 1))**(3/2)), x)
```

Giac [A]
time = 0.52, size = 239, normalized size = 1.59

$$\frac{\frac{2\,A\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{\frac{3}{2}}c^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} - \frac{4\,A\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{\frac{3}{2}}c^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} + \frac{2\,A\sqrt{a}\,\sqrt{c}\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-A\sqrt{a}\,\sqrt{c}+B\sqrt{a}\,\sqrt{c}}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)a^{2}c^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/8*(2*A*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^(3/2)*c^(3/2)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*A*lo
g(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(3/2)*c^(3/2)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*A*sqrt(a)*sqrt(c
)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))
/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 - cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)*a^2
*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\sin(e + f\,x)}{(a + a\,\sin(e + f\,x))^{3/2}\,(c - c\,\sin(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{8acf\sqrt{a+}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+1/8*(
3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A-B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$\frac{(3A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] -1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A - B)*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A\ldots}{\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{8af\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{8af\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{8af\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{\ldots}{8af\ldots}$$

Mathematica [A]
time = 0.65, size = 306, normalized size = 1.41

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(2A\cos^2(e+fx)+(-A+B)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4+(A+B)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2+(-3A+B)\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2+(3A-B)\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2\right)}{8f(a(1+\sin(e+fx)))^{3/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 +
(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/
2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(193) = 386$.

time = 0.29, size = 435, normalized size = 2.00

method	result
default	$\frac{\left(3A(\cos^2(fx+e))\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-3A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))\sin(fx+e)-B\ln\left(-\frac{-1}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(3*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-3*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-B
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+B*ln(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e
)^2*sin(f*x+e)-3*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
3*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*B*cos(f*x+e)^
2*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*ln
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+A*cos(f*x+e)^2-3*B*co
s(f*x+e)^2+3*A*sin(f*x+e)-B*sin(f*x+e)-A+3*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))
^(3/2)/(-c*(sin(f*x+e)-1))^(5/2)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Fricas [**A**]

time = 0.45, size = 467, normalized size = 2.15

$$\left[-\frac{\left((3A-B)\cos(fx+e)^3\sin(fx+e)-(3A-B)\cos(fx+e)^3\right)\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)+2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)+2\left((3A-B)\cos(fx+e)^2+(3A-B)\sin(fx+e)-A+3B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16\left(a^2c^3f\cos(fx+e)^3\sin(fx+e)-a^2c^3f\cos(fx+e)^3\right)}, -\frac{\left((3A-B)\cos(fx+e)^3\sin(fx+e)-(3A-B)\cos(fx+e)^3\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\left((3A-B)\cos(fx+e)^2+(3A-B)\sin(fx+e)-A+3B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{8\left(a^2c^3f\cos(fx+e)^3\sin(fx+e)-a^2c^3f\cos(fx+e)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*
sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
 + 2*((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f
*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8*(((3*A - B)*cos(f*x + e)^3*sin(f*
x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) +
((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*sin(f*
x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x +
e) - a^2*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.52, size = 323, normalized size = 1.49

$$\frac{\frac{2\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{a^{2}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^{2}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-3\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2A\sqrt{a}\sqrt{c}-2B\sqrt{a}\sqrt{c}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{2}a^{2}c^{3}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/32*(2*(3*A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(-cos(-1/4*pi + 1/2*f*
x + 1/2*e)^2 + 1)/(a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - 4*(3*A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))*log(
abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*(3*A*sqrt(a)*sqrt(c) - B*s
qrt(a)*sqrt(c))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 3*(3*A*sqrt(a)*sqrt(c) -
 B*sqrt(a)*sqrt(c))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2*A*sqrt(a)*sqrt(c)
- 2*B*sqrt(a)*sqrt(c))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^2*c^3*co
```

```
s(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=323

$$\frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}+\frac{(3A-7B}{}$$

```
[Out] 1/4*(3*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2
)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(5/2)+(3*A
-7*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+1/
3*(3*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1
/2)+8*(3*A-7*B)*c^5*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2
)/(c-c*sin(f*x+e))^(1/2)+4*(3*A-7*B)*c^4*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/
a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\frac{8c^5(3A-7B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{4c^4(3A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^2(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (8*(3*A - 7*B)*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*(3*A - 7*B)*c^4*Cos[e + f*x]*Sqr
t[c - c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^3*
Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 ((3*A - 7*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a
+ a*Sin[e + f*x]]) + ((3*A - 7*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2)
)/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e +
 f*x])^(9/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A-7I}{}$$

$$= \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B}{}$$

$$= \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-}{}$$

$$= \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-}{}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-}{}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-}{}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-}{}$$

$$= \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4(3A}{}$$

Mathematica [A]
time = 4.10, size = 286, normalized size = 0.89

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{9/2}\left(-96(A-B)+192(2A-3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-3(A-7B)\cos(2(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+192(3A-7B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-3(28A-97B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin(e+fx)-B\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin(3(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2)*(-96*(A -
 B) + 192*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 3*(A - 7*B)
*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 192*(3*A - 7*B)
*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^4 - 3*(28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*
x] - B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[3*(e + f*x)]))/(12*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(293) = 586$.

time = 0.32, size = 1301, normalized size = 4.03

method	result	size
default	Expression too large to display	1301

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(-1152*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2688*
B*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2688*B*ln(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))-1152*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))-336*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-255*A*cos(f*x+e)^2*sin(f*x+e)
+581*B*cos(f*x+e)^2*sin(f*x+e)+396*A-932*B+672*B*ln(2/(1+cos(f*x+e)))*sin(f
*x+e)*cos(f*x+e)+144*A*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-222*A*cos(f*x+e)-9
32*B*sin(f*x+e)+396*A*sin(f*x+e)+2*B*cos(f*x+e)^5*sin(f*x+e)+219*A*cos(f*x+
e)^3-473*B*cos(f*x+e)^3+490*B*cos(f*x+e)-108*B*cos(f*x+e)^3*sin(f*x+e)+336*
B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)-93*B*cos(f*x+e)^4-429*A*cos(
f*x+e)^2+1023*B*cos(f*x+e)^2+576*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))-1344*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+
36*A*cos(f*x+e)^3*sin(f*x+e)-432*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-288*A*
cos(f*x+e)*ln(2/(1+cos(f*x+e)))-3*A*cos(f*x+e)^4*sin(f*x+e)+3*A*cos(f*x+e)^
5-17*B*cos(f*x+e)^5+2*B*cos(f*x+e)^6-1344*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)
+442*B*sin(f*x+e)*cos(f*x+e)-144*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(1+cos(f*x+
e)))+576*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+672*B*cos(f*x+e)*ln(2/(1+cos(f*x
+e)))+33*A*cos(f*x+e)^4-288*A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+28
8*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-672*
B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+576*A*
cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-1344*B*cos
(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-288*A*cos(f*x
+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+672*B*cos(f*x+e)^3*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+576*A*ln(2/(1+cos(f*x+e)))-1344*B*ln(2/
(1+cos(f*x+e)))+1008*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-174*A*sin(f*x+e)*c
os(f*x+e)+864*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-201
6*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+15*B*sin(f*x+e)
*cos(f*x+e)^4)*(-c*(sin(f*x+e)-1))^(9/2)/(cos(f*x+e)^4*sin(f*x+e)+cos(f*x+e
)^5+4*cos(f*x+e)^3*sin(f*x+e)-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)-8*c
os(f*x+e)^3-8*cos(f*x+e)*sin(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8*cos(f*x
+e)-16)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 4*B)*c^4*cos(f*x + e)^4 - 4*(2*A - 3*B)*c^4*cos(f*x + e)^2
+ 8*(A - B)*c^4 + (B*c^4*cos(f*x + e)^4 + 4*(A - 2*B)*c^4*cos(f*x + e)^2 -
8*(A - B)*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x +
 e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 475, normalized size = 1.47

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{12\left(3\sqrt{2}A\sqrt{a}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\sqrt{2}B\sqrt{a}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(7A\sqrt{a}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-11B\sqrt{a}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\left(2A\sqrt{a}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\sqrt{a}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(4Ba^{\frac{13}{2}}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6-3Aa^{\frac{13}{2}}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+15Ba^{\frac{13}{2}}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-18Aa^{\frac{13}{2}}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+54Ba^{\frac{13}{2}}c^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^9\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(c)*(12*(3*sqrt(2)*A*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) - 7*sqrt(2)*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*l
og(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))) + 3*sqrt(2)*(7*A*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
```

```
11*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*(2*A*sqrt(a)*c^4*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x +
 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(4*B*a^
(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^6 - 3*A*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^4 + 15*B*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 18*A*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 54*B*a^(13/2)*c^4*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^9*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}+\frac{3(A-3B)c}{4}$$

```
[Out] 1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-
1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(5/2)+3/4*(A
-3*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+6*
(A-3*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+3*(A-3*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a
*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\frac{6c^4(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3c^3(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (6*(A - 3*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (3*(A - 3*B)*c^3*Cos[e + f*x]*Sqrt[c
- c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*(A - 3*B)*c^2*Cos[
e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (
(A - 3*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*a*f*(a + a*Sin[e +
f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*(a +
a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-3B}{}$$

$$= \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)}{}$$

$$= \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3}{}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3}{}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3}{}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3}{}$$

$$= \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3(A-}{}$$

Mathematica [**A**]
time = 1.70, size = 243, normalized size = 0.92

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{7/2}\left(-16(A-B)+16(3A-5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+B\cos(2(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+48(A-3B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-4(A-6B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin(e+fx)\right)}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A -
 B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e +
 f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4*(A
- 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. $2(237) = 474$.
time = 0.31, size = 1219, normalized size = 4.63

method	result	size
default	Expression too large to display	1219

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-96*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+288*B*si
n(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+288*B*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))-96*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-3
6*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3-22*A*cos(f*x+e)^2*sin(f*x+e)+63*B*cos
(f*x+e)^2*sin(f*x+e)+32*A-100*B+72*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*
x+e)+12*A*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-20*A*cos(f*x+e)-100*B*sin(f*x+e
)+32*A*sin(f*x+e)+20*A*cos(f*x+e)^3-53*B*cos(f*x+e)^3+54*B*cos(f*x+e)-10*B*
cos(f*x+e)^3*sin(f*x+e)+36*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)-9
*B*cos(f*x+e)^4-34*A*cos(f*x+e)^2+109*B*cos(f*x+e)^2+48*A*cos(f*x+e)*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-144*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)^3*sin(f*x+e)-36*A*cos(f*x+e)^2*ln(2/(1
+cos(f*x+e)))-24*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-B*cos(f*x+e)^5-144*B*ln(
2/(1+cos(f*x+e)))*sin(f*x+e)+46*B*sin(f*x+e)*cos(f*x+e)-12*A*cos(f*x+e)^2*s
in(f*x+e)*ln(2/(1+cos(f*x+e)))+48*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+72*B*co
s(f*x+e)*ln(2/(1+cos(f*x+e)))+2*A*cos(f*x+e)^4-24*A*cos(f*x+e)*sin(f*x+e)*l
n(2/(1+cos(f*x+e)))+24*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x
+e)^2*sin(f*x+e)-72*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)
^2*sin(f*x+e)+48*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))-144*B*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))-24*A*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+72*B*cos(
f*x+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+48*A*ln(2/(1+cos(f*x+e)
))-144*B*ln(2/(1+cos(f*x+e)))+108*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-12*A*
sin(f*x+e)*cos(f*x+e)+72*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f
*x+e)^2-216*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+B*sin
(f*x+e)*cos(f*x+e)^4)*(-c*(sin(f*x+e)-1))^(7/2)/(cos(f*x+e)^4-cos(f*x+e)^3*
sin(f*x+e)+3*cos(f*x+e)^3+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+4*cos(f*
x+e)*sin(f*x+e)-4*cos(f*x+e)-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(5/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2), x)

[Out] Timed out

Giac [A]
time = 0.61, size = 434, normalized size = 1.65

$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{6\sqrt{2}\left(A\sqrt{a}c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\sqrt{a}c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(\sqrt{2}Ba^{\frac{7}{2}}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-\sqrt{2}Aa^{\frac{7}{2}}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+5\sqrt{2}Ba^{\frac{7}{2}}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)}{a^{6}}+\frac{5\sqrt{2}A\sqrt{a}c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-9\sqrt{2}B\sqrt{a}c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\left(3\sqrt{2}A\sqrt{a}c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5\sqrt{2}B\sqrt{a}c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{2}a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*sqrt(c)*(6*sqrt(2)*(A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(sqrt(2)*B*a^(7/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - sqrt(2)*A*a^(7/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 5*sqrt(2)*B*a^(7/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/a^6 + (5*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +

```
1/2*e)) - 9*sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(
3*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*sqrt(2)*B*s
qrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\ (c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=211

$$\frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}}+\frac{(A-5B)c\cos}{4af(}$$

```
[Out] 1/4*(A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-
1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(5/2)+(A-5*B
)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+1/2*(A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin
(f*x+e))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3051, 2818, 2819, 2816, 2746, 31}

$$\frac{c^3(A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2(A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c(A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A - 5*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - 5*B)*c^2*Cos[e + f*x]*Sqrt[c - c*
Sin[e + f*x]])/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((A - 5*B)*c*Cos[e + f*
x]*(c - c*Sin[e + f*x])^(3/2))/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B
)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-5B}{}$$

$$= \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)}{}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B}{}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B}{}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B}{}$$

$$= \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-5B}{}$$

Mathematica [A]
time = 0.77, size = 199, normalized size = 0.94

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{5/2}\left(-2A+2B+4(A-2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+2(A-5B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+B\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A + 2
*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1119 vs. $2(189) = 378$.
time = 0.30, size = 1120, normalized size = 5.31

method	result	size
default	Expression too large to display	1120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/f*(-8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+40*B*sin(f*
x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+40*B*ln(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))-8*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-5*B*ln(
2/(1+cos(f*x+e)))*cos(f*x+e)^3-2*A*cos(f*x+e)^2*sin(f*x+e)+9*B*cos(f*x+e)^2
*sin(f*x+e)+2*A-14*B+10*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)+A*cos(
f*x+e)^3*ln(2/(1+cos(f*x+e)))-2*A*cos(f*x+e)-14*B*sin(f*x+e)+2*A*sin(f*x+e)
+2*A*cos(f*x+e)^3-8*B*cos(f*x+e)^3+8*B*cos(f*x+e)-B*cos(f*x+e)^3*sin(f*x+e)
+5*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e)^4-2*A*cos(f*
x+e)^2+15*B*cos(f*x+e)^2+4*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))-20*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-3*A*cos(
f*x+e)^2*ln(2/(1+cos(f*x+e)))-2*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-20*B*ln(2
/(1+cos(f*x+e)))*sin(f*x+e)+6*B*sin(f*x+e)*cos(f*x+e)-A*cos(f*x+e)^2*sin(f*
x+e)*ln(2/(1+cos(f*x+e)))+4*A*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+10*B*cos(f*x+
e)*ln(2/(1+cos(f*x+e)))-2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+2*A*
ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-10*B*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x
+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-20*B*cos(f*x+e)*s
in(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^3*ln(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+10*B*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))+4*A*ln(2/(1+cos(f*x+e)))-20*B*ln(2/(1+cos(f*x+e)))+
15*B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+6*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))*cos(f*x+e)^2-30*B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos
(f*x+e)^2)*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-
3*cos(f*x+e)^2+2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+s
in(f*x+e)))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(202) = 404$.
time = 0.53, size = 544, normalized size = 2.58

$$\frac{\left(\frac{8\sqrt{a}\,c^{\frac{5}{2}}\sin(fx+e)^2}{\left(a^3+\frac{4a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2}-\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{5}{2}}}+\frac{c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{5}{2}}}\right)A+B\left(\frac{10c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{5}{2}}}-\frac{5c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{5}{2}}}-\frac{2\left(\frac{5c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{16c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{14c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{16c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^{\frac{5}{2}}+\frac{4a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{7a^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{8a^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{7a^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4a^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{a^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] ((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c^(5/2
)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(sin(f*x
```

```
+ e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)/(cos(f
*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*c^(5/2)*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/2) + 4*a^(
5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^(5/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin
(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.53, size = 312, normalized size = 1.48

$$\frac{\sqrt{2}\left(\frac{4\sqrt{2}\,Bc^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\sqrt{2}\left(A\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-5B\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-32\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+32\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3A\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7B\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\left(A\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2B\sqrt{a}\,c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\sqrt{c}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(4*sqrt(2)*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sq
rt(2)*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*sqrt(a)*c^2*
```

```
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-32*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 32)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*sqrt(a)*c
^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*sqrt(a)*c^2*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) - 4*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
2*B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))))*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)-B*c^2
*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)-B*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.26, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2818, 2816, 2746, 31}

$$-\frac{Bc^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -((B*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) - (B*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{B\int \frac{(c-c\sin}{(a+a\sin}}{}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+}{4f(a+}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+}{4f(a+}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+}{4f(a+}$$

$$= -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(}{af}$$

Mathematica [A]

time = 0.66, size = 179, normalized size = 1.20

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(B\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-B\left(2+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\left(A-3B-4B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2*
(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(135) = 270$.
time = 0.31, size = 611, normalized size = 4.10

method	result
default	$-\frac{\left(8B\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+8B\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-A-3B+2B\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/f*(8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+8*B*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-B*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^3+2*B
*cos(f*x+e)^2*sin(f*x+e)-A-3*B+2*B*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+
e)-3*B*sin(f*x+e)-A*sin(f*x+e)-2*B*cos(f*x+e)^3+2*B*cos(f*x+e)+B*ln(2/(1+co
s(f*x+e)))*cos(f*x+e)^2*sin(f*x+e)+A*cos(f*x+e)^2+3*B*cos(f*x+e)^2-4*B*cos(
f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*B*ln(2/(1+cos(f*x+e)))*
sin(f*x+e)+B*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*B*
ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-4*B*cos(
f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*B*cos(f*x+e)
^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*B*ln(2/(1+cos(f*x+e)))+3*B*
ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+A*sin(f*x+e)*cos(f*x+e)-6*B*ln(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2)*(-c*(sin(f*x+e)-1))^(3/2)/(cos(
f*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e))
)^(5/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(a\,(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f*
x) + 1))**(5/2), x)
```

Giac [A]
time = 0.52, size = 181, normalized size = 1.21

$$\frac{\left(8\,B\sqrt{a}\,c\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{A\sqrt{a}\,c\mathrm{sgn}(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))-B\sqrt{a}\,c\mathrm{sgn}(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))-2\left(A\sqrt{a}\,c\mathrm{sgn}(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))-3\,B\sqrt{a}\,c\mathrm{sgn}(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))\right)\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)^2}{\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)^4}\right)\sqrt{c}}{4\,a^3f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/4*(8*B*sqrt(a)*c*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + (A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B
*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(A*sqrt(a)*c*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/cos(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt
(c)/(a^3*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-B*c
*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3050, 2817}

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/2*((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*
Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(}{(a+a\sin(e}$$

$$= -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{af(a+}$$

Mathematica [A]
time = 0.36, size = 99, normalized size = 1.05

$$-\frac{\sqrt{a(1+\sin(e+fx))}\,(A+B+2B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[
e + f*x]])/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 0.29, size = 135, normalized size = 1.44

method	result
default	$-\frac{\sin(fx+e)\sqrt{-c(\sin(fx+e)-1)}\,(A\sin(fx+e)\cos(fx+e)+A(\cos^2(fx+e))+B\sin(fx+e)\cos(fx+e)+B(\cos^2(fx+e))}{2f(a(1+\sin(fx+e)))^{\frac{5}{2}}(-1+\cos(fx+e)+\sin(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(1/2)*(A*sin(f*x+e)*cos(f*x+e)+A*cos(
f*x+e)^2+B*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2-3*A*sin(f*x+e)+2*A*cos(f*x+
e)-B*sin(f*x+e)-3*A-B)/(a*(1+sin(f*x+e)))^(5/2)/(-1+cos(f*x+e)+sin(f*x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(5/2), x)
```

Fricas [A]
time = 0.38, size = 92, normalized size = 0.98

$$\frac{(2 B \sin(fx+e) + A + B)\sqrt{a \sin(fx+e) + a}\,\sqrt{-c \sin(fx+e) + c}}{2 \left(a^3 f \cos(fx+e)^3 - 2 a^3 f \cos(fx+e)\sin(fx+e) - 2 a^3 f \cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*
cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sin(e+fx)-1)}\,(A + B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(5/2), x)
```

Giac [A]
time = 0.48, size = 117, normalized size = 1.24

$$\frac{\left(4 B\sqrt{a}\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + A\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - B\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right)\sqrt{c}}{8 a^3 f \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] 1/8*(4*B*sqrt(a)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) + A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/(a^3*f*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 14.86, size = 156, normalized size = 1.66

$$-\frac{2\sqrt{-c\,(\sin(e+f\,x)-1)}\,\left(A\sin(2\,e+2\,f\,x)+3\,B\,\sin(2\,e+2\,f\,x)-2\,A\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)-3\,B\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+B\left(2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-1\right)\right)}{a^2\,f\,\sqrt{a\,(\sin(e+f\,x)+1)}\,\left(-8\sin(e+f\,x)^2+4\,\sin(e+f\,x)+2\sin(2\,e+2\,f\,x)^2+4\,\sin(3\,e+3\,f\,x)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(A*sin(2*e + 2*f*x) + 3*B*sin(2*e + 2*f*x
) - 2*A*(2*sin(e/2 + (f*x)/2)^2 - 1) - 3*B*(2*sin(e/2 + (f*x)/2)^2 - 1) + B
*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(
4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)
^2 + 8))
```

3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=151

$$-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\,\mathrm{t}}{4a^2f\sqrt{a+}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*(
A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*(A+B)
*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
+ f*x]]) - ((A + B)*Cos[e + f*x])/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c
- c*Sin[e + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+ \cdots}{\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{\cdots}{4af(a \cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{\cdots}{4af(a \cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{\cdots}{4af(a \cdots}$$

Mathematica [A]
time = 0.45, size = 214, normalized size = 1.42

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(-A+B-(A+B)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2-(A+B)\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4+(A+B)\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4\right)}{4f(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin
```

```
[(e + f*x)/2])^4))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(133) = 266$.
time = 0.32, size = 471, normalized size = 3.12

method	result
default	$-\frac{\left(-A\left(\cos^{2}(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^{2}(fx+e)\right)-B\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(-A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*cos(f*x+e)^2+B*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))*cos(f*x+e)^2+2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2
*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)^2+2
*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f*x+e)*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))-2*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-3*A*sin(f*x+e)+2
*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))+B*sin(f*x+e)-2*A)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c
*(sin(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Fricas [A]
time = 0.44, size = 440, normalized size = 2.91

$$\left[\frac{\left((A+B)\cos(fx+e)^{3}-2\,(A+B)\cos(fx+e)\sin(fx+e)-2\,(A+B)\cos(fx+e)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos(fx+e)^{3}-2\,ac\cos(fx+e)-2\,\sqrt{ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{\cos(fx+e)^{3}}\right)+2\,\left((A+B)\sin(fx+e)+2\,A\right)\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{8\,\left(a^{3}cf\cos(fx+e)^{3}-2\,a^{3}cf\cos(fx+e)\sin(fx+e)-2\,a^{3}cf\cos(fx+e)\right)},\ -\frac{\left((A+B)\cos(fx+e)^{3}-2\,(A+B)\cos(fx+e)\sin(fx+e)-2\,(A+B)\cos(fx+e)\right)\sqrt{-ac}\,\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)-\left((A+B)\sin(fx+e)+2\,A\right)\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{4\,\left(a^{3}cf\cos(fx+e)^{3}-2\,a^{3}cf\cos(fx+e)\sin(fx+e)-2\,a^{3}cf\cos(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A
+ B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
- 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
e))/cos(f*x + e)^3) + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x +
 e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x + e)^3 -
 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(-a*c)*a
rctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*co
s(f*x + e)*sin(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f
*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin{\left(e + fx \right)}}{\left(a\left(\sin{\left(e + fx \right)} + 1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sin{\left(e + fx \right)} - 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

Giac [A]
time = 0.51, size = 260, normalized size = 1.72

$$-\frac{\frac{2\left(A\sqrt{c}+B\sqrt{c}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^{\frac{5}{2}}c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^3c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/16*(2*(A*sqrt(c) + B*sqrt(c))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)
/(a^(5/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))) - 4*(A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))*log(abs(sin(-1/4*pi +
 1/2*f*x + 1/2*e)))/(a^3*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) + (2*(A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 - 3*A*sqrt(a)*sqrt(c) - B*sqrt(a)*sqrt(c))/((si
n(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\quad}{8a^2f\sqrt{c}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-1/8*(
3*A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/8*(3*
A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(3A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{(3A+B)\cos(e+fx)}{8af(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*
x])^(3/2)) - ((3*A + B)*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(3/2)) + ((3*A + B)*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin
[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A + B)*ArcTanh[Sin[e + f*x]]*C
os[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{(3A \cdots}{\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{\cdots}{8af(\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{\cdots}{8af(\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{\cdots}{8af(\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{\cdots}{8af(\cdots}$$

Mathematica [A]
time = 0.69, size = 305, normalized size = 1.47

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2A\cos^2(e+fx)+(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-(3A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+(3A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)}{8f(a(1+\sin(e+fx)))^{5/2}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 +
 (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/
2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(184) = 368$.
time = 0.30, size = 435, normalized size = 2.09

method	result
default	$-\frac{\left(3A\left(\cos^2(fx+e)\right)\sin(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-3A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+B\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(3*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-3*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+
B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-B*ln(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-2*A*cos(f*x+
e)^2*sin(f*x+e)+3*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-3*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*B*cos(f*x+e)
^2*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*l
n(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+A*cos(f*x+e)^2+3*B*c
os(f*x+e)^2-3*A*sin(f*x+e)-B*sin(f*x+e)-A-3*B)*cos(f*x+e)/(a*(1+sin(f*x+e))
)^(5/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Fricas [**A**]
time = 0.47, size = 449, normalized size = 2.16

$$\left[\frac{\left((3A+B)\cos(fx+e)^3\sin(fx+e)+(3A+B)\cos(fx+e)^3\right)\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)-2\left((3A+B)\cos(fx+e)^2-(3A+B)\sin(fx+e)-A-3B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)},\ -\frac{\left((3A+B)\cos(fx+e)^3\sin(fx+e)+(3A+B)\cos(fx+e)^3\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\left((3A+B)\cos(fx+e)^2-(3A+B)\sin(fx+e)-A-3B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{8\left(a^3c^2f\cos(fx+e)^3\sin(fx+e)+a^3c^2f\cos(fx+e)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*s
qrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
- 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*
x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*(((3*A + B)*cos(f*x + e)^3*sin(f*x
 + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + (
(3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e
) + a^3*c^2*f*cos(f*x + e)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.54, size = 318, normalized size = 1.53

$$\frac{\frac{2\left(3A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(3A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\log\left(\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^3c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(3A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-\left(3A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}\right)\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/32*(2*(3*A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))*log(-cos(-1/4*pi + 1/2*f*
x + 1/2*e)^2 + 1)/(a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - 4*(3*A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))*log(
abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*(3*A*sqrt(a)*sqrt(c) + B*s
qrt(a)*sqrt(c))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 - (3*A*sqrt(a)*sqrt(c) + B
*sqrt(a)*sqrt(c))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - A*sqrt(a)*sqrt(c) + B*
sqrt(a)*sqrt(c))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^3*c^2*cos(-1/4*p
```

```
i + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=245

$$-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}+\frac{}{8a^2f\sqrt{a}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*A
*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*A*cos(f*x
+e)/a^2/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*cos(f*x+e)/a^
2/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*arctanh(sin(f*x+e
))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3051, 2822, 2820, 3855}

$$\frac{3A\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3A\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{A\cos(e+fx)}{2af(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] -1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*
x])^(5/2)) - (A*Cos[e + f*x])/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[
e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (3*A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x
])/(8*a^2*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{A\int}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af}$$

Mathematica [A]
time = 0.64, size = 246, normalized size = 1.00

$$\frac{\sec^3(e+fx)\left(16B-9A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12A\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-3A\cos(4(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+9A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+22A\sin(e+fx)+6A\sin(3(e+fx))\right)}{64a^2c^2f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(16*B - 9*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*A
*Cos[2*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]) - 3*A*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*A*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + 22*A*Sin[e + f*x] + 6*A*Sin[3*(e + f*x
)]))/(64*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.34, size = 152, normalized size = 0.62

method	result
default	$\frac{\left(3A\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^4(fx+e)\right)-3A\left(\cos^4(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-2B\left(\cos^4(fx+e)\right)+3A\left(\cos^2(fx+\right.\right.}{8f\left(a(1+\sin(fx+e))\right)^{\frac{5}{2}}\left(-c(\sin(fx+e)-1)\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(3*A*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^4-3*A*cos(
f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^4+3*A*co
s(f*x+e)^2*sin(f*x+e)+2*A*sin(f*x+e)+2*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/
2)/(-c*(sin(f*x+e)-1))^(5/2)
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Fricas [A]
time = 0.47, size = 328, normalized size = 1.34

$$\left[\frac{3\sqrt{ac}A\cos(fx+e)^5\log\left(-\frac{ac\cos(fx+e)^3-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{\cos(fx+e)^3}\right)+2\left((3A\cos(fx+e)^2+2A)\sin(fx+e)+2B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16a^3c^3f\cos(fx+e)^5}, -\frac{3\sqrt{-ac}A\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^5-\left((3A\cos(fx+e)^2+2A)\sin(fx+e)+2B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{8a^3c^3f\cos(fx+e)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*
x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin
(f*x + e))/cos(f*x + e)^3) + 2*((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2
*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x
+ e)^5), -1/8*(3*sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (
(3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2*B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.56, size = 286, normalized size = 1.17

$$-\frac{\frac{12 A \log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{a^{\frac{5}{2}}c^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{24 A \log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^{\frac{5}{2}}c^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{12 A\sqrt{a}\sqrt{c}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-18 A\sqrt{a}\sqrt{c}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+4 A\sqrt{a}\sqrt{c}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+A\sqrt{a}\sqrt{c}+B\sqrt{a}\sqrt{c}}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)^{2}a^{3}c^{3}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] -1/64*(12*A*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^(5/2)*c^(5/2)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 24*
A*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(5/2)*c^(5/2)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (12*A*sqrt(a)*s
qrt(c)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 18*A*sqrt(a)*sqrt(c)*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^4 + 4*A*sqrt(a)*sqrt(c)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
 + A*sqrt(a)*sqrt(c) + B*sqrt(a)*sqrt(c))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^
4 - sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)^2*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=174

$$\frac{2^{\frac{1}{2}+n}c(B(m-n)+A(1+m+n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)\,(}{f(1+2m)(1+m+n)}$$

```
[Out] 2^(1/2+n)*c*(B*(m-n)+A*(1+m+n))*cos(f*x+e)*hypergeom([1/2+m, 1/2-n],[3/2+m]
,1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x
+e))^(-1+n)/f/(1+2*m)/(1+m+n)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+
e))^n/f/(1+m+n)
```

Rubi [A]

time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3052, 2824, 2768, 72, 71}

$$\frac{c2^{n+\frac{1}{2}}(A(m+n+1)+B(m-n))\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(1-2n);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)(m+n+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*(B*(m - n) + A*(1 + m + n))*Cos[e + f*x]*Hypergeometric2F1[(
1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*
x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 +
 2*m)*(1 + m + n)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n)/(f*(1 + m + n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(1+m+n)}$$

$$= \frac{2^{\frac{1}{2}+n}c\left(A+\frac{B(m-n)}{1+m+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 12.41, size = 1931, normalized size = 11.10

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (-4*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1
+ 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*(a + a*Sin[e + f*x])^m*(A + B*Sin[
e + f*x])*(c - c*Sin[e + f*x])^n*Tan[(-e + Pi/2 - f*x)/4])/(f*((8*B*AppellF
1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2
 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*Appell
F1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/4]^2 + 4*n*(8*B*AppellF1[1/2
 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/
2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2])*Cot[(-e + Pi/2 - f*x)/2]*Tan[(-e + Pi/2 - f*x)/4] + 4*(
m + n)*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m,
 1 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)
/4]^2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2 + (2
*(1/2 + n)*((A + B)*(2*m*AppellF1[3/2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + (1 + 2*(m + n))
*AppellF1[3/2 + n, -2*m, 2*(1 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2
, -Tan[(-e + Pi/2 - f*x)/4]^2]) + 8*B*(2*m*AppellF1[3/2 + n, 1 - 2*m, 3 + 2
*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
 + (3 + 2*(m + n))*AppellF1[3/2 + n, -2*m, 2*(2 + m + n), 5/2 + n, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]) - 16*B*(m*AppellF1[3/2 +
n, 1 - 2*m, 2*(1 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + n, -2*m, 3 + 2*m + 2*n, 5/2
+ n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Sec[(-e + P
i/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^2)/(3/2 + n) - (4*m*(8*B*AppellF1[
1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 +
 n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1
```

```
[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^
2)/(-1 + Tan[(-e + Pi/2 - f*x)/4]^2) - 4*m*(8*B*AppellF1[1/2 + n, -2*m, 2*(
1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m, 3
+ 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2])*Tan[(-e + Pi/2 - f*x)/4]*Tan[(-e + Pi/2 - f*x)/2]))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(A+B\sin \left(fx+e\right)\right) \left(c-c\sin \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin \left(e+fx\right)+1\right)\right)^{m} \left(-c \left(\sin \left(e+fx\right)-1\right)\right)^{n} \left(A+B\sin \left(e+fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*(A + B*sin(e
+ f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

3.196 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=145

$$\frac{2^{\frac{1}{2}+m}a^4c^3(B(3-m)-A(4+m))\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{\frac{1}{2}-m}(a}{7f(4+m)}$$

```
[Out] 1/7*2^(1/2+m)*a^4*c^3*(B*(3-m)-A*(4+m))*cos(f*x+e)^7*hypergeom([7/2, 1/2-m]
,[9/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
/(4+m)-a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)
```

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2768, 72, 71}

$$\frac{a^4c^32^{m+\frac{1}{2}}(B(3-m)-A(m+4))\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f(m+4)}-\frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*a^4*c^3*(B*(3 - m) - A*(4 + m))*Cos[e + f*x]^7*Hypergeometric2
F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-4 + m))/(7*f*(4 + m)) - (a^3*B*c^3*Cos[e + f*x]^7*(a +
 a*Sin[e + f*x])^(-3 + m))/(f*(4 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+f$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx}{f(4+m)}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx}{f(4+m)}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx}{f(4+m)}$$

$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\left(A-\frac{B(3-m)}{4+m}\right)\cos^7(e+fx}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(A+B\sin \left(fx+e\right)\right) \left(c-c\sin \left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*(a*sin
(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int \left(-A(a\sin \left(e+fx\right)+a)^{m}\right)\, dx+\int 3A(a\sin \left(e+fx\right)+a)^{m}\sin \left(e+fx\right)dx+\int \left(-3A(a\sin \left(e+fx\right)+a)^{m}\sin^{2}\left(e+fx\right)\right)\, dx+\int A(a\sin \left(e+fx\right)+a)^{m}\sin^{3}\left(e+fx\right)dx+\int \left(-B(a\sin \left(e+fx\right)+a)^{m}\sin \left(e+fx\right)\right)\, dx+\int 3B(a\sin \left(e+fx\right)+a)^{m}\sin^{2}\left(e+fx\right)dx+\int \left(-3B(a\sin \left(e+fx\right)+a)^{m}\sin^{3}\left(e+fx\right)\right)\, dx+\int B(a\sin \left(e+fx\right)+a)^{m}\sin^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(3*A*(a*sin(e + f*
x) + a)**m*sin(e + f*x), x) + Integral(-3*A*(a*sin(e + f*x) + a)**m*sin(e +
 f*x)**2, x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3, x) + Int
egral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(3*B*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)**2, x) + Integral(-3*B*(a*sin(e + f*x) + a)**m*s
in(e + f*x)**3, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**4, x)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

3.197 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=145

$$\frac{2^{\frac{1}{2}+m}a^3c^2(B(2-m)-A(3+m))\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{\frac{1}{2}-m}(a}{5f(3+m)}$$

```
[Out] 1/5*2^(1/2+m)*a^3*c^2*(B*(2-m)-A*(3+m))*cos(f*x+e)^5*hypergeom([5/2, 1/2-m]
,[7/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
/(3+m)-a^2*B*c^2*cos(f*x+e)^5*(a+a*sin(f*x+e))^(-2+m)/f/(3+m)
```

Rubi [A]

time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2768, 72, 71}

$$\frac{a^3c^22^{m+\frac{1}{2}}(B(2-m)-A(m+3))\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f(m+3)}-\frac{a^2Bc^2\cos^5(e+fx)(a\sin(e+fx)+a)^{m-2}}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a^3*c^2*(B*(2 - m) - A*(3 + m))*Cos[e + f*x]^5*Hypergeometric2
F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-3 + m))/(5*f*(3 + m)) - (a^2*B*c^2*Cos[e + f*x]^5*(a +
 a*Sin[e + f*x])^(-2 + m))/(f*(3 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+f$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx}{f(3+m)}$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx}{f(3+m)}$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx}{f(3+m)}$$

$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\left(A-\frac{B(2-m)}{3+m}\right)\cos^5(e+fx}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] $Aborted
```

Maple [F]
time = 1.24, size = 0, normalized size = 0.00

$$\int (a+a\sin (fx+e))^{m} (A+B\sin (fx+e)) (c-c\sin (fx+e))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2} \left(\int A(a\sin (e+fx)+a)^{m}\, dx + \int \left(-2A(a\sin (e+fx)+a)^{m} \sin (e+fx)\right) dx + \int A(a\sin (e+fx)+a)^{m} \sin^{2}(e+fx)\, dx + \int B(a\sin (e+fx)+a)^{m} \sin (e+fx)\, dx + \int \left(-2B(a\sin (e+fx)+a)^{m} \sin^{2}(e+fx)\right) dx + \int B(a\sin (e+fx)+a)^{m} \sin^{3}(e+fx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(A*(a*sin(e + f*x) + a)**m, x) + Integral(-2*A*(a*sin(e + f*x
) + a)**m*sin(e + f*x), x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x
)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-
2*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e + f*x
) + a)**m*sin(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=139

$$\frac{2^{\frac{1}{2}+m}a^2c(B(1-m)-A(2+m))\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{\frac{1}{2}-m}(a-}{3f(2+m)}$$

```
[Out] 1/3*2^(1/2+m)*a^2*c*(B*(1-m)-A*(2+m))*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],[
5/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(
2+m)-a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)
```

Rubi [A]

time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3046, 2939, 2768, 72, 71}

$$\frac{a^2c2^{m+\frac{1}{2}}(B(1-m)-A(m+2))\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f(m+2)}-\frac{aBc\cos^3(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a^2*c*(B*(1 - m) - A*(2 + m))*Cos[e + f*x]^3*Hypergeometric2F1
[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a +
 a*Sin[e + f*x])^(-2 + m))/(3*f*(2 + m)) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin
[e + f*x])^(-1 + m))/(f*(2 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))}{f(2+m)}$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))}{f(2+m)}$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))}{f(2+m)}$$

$$= -\frac{2^{\frac{1}{2}+m}a^2c\left(A-\frac{B(1-m)}{2+m}\right)\cos^3(e+fx)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.66, size = 462, normalized size = 3.32

$$\frac{i4^{-1-m}ce^{ifmx}\left(1+ie^{-i(e+fx)}\right)^{-2m}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\right)^{2m}\left(-\frac{iBe^{-i(2e+f(2+m)x)}\,{}_2F_1\left(-2-m,-2m;-1-m;-ie^{-i(e+fx)}\right)}{2+m}+\frac{2(-iA+B)e^{-i(e+f(1+m)x)}\,{}_2F_1\left(-1-m,-2m;-m;-ie^{-i(e+fx)}\right)}{1+m}+\frac{2iAe^{i(e-f(-1+m)x)}\,{}_2F_1\left(1-m,-2m;2-m;-ie^{-i(e+fx)}\right)}{-1+m}+\frac{2Be^{i(e-f(-1+m)x)}\,{}_2F_1\left(1-m,-2m;2-m;-ie^{-i(e+fx)}\right)}{-1+m}+\frac{iBe^{2ie-if(-2+m)x}\,{}_2F_1\left(2-m,-2m;3-m;-ie^{-i(e+fx)}\right)}{-2+m}+\frac{4Ae^{-ifmx}\,{}_2F_1\left(-2m,-m;1-m;-ie^{-i(e+fx)}\right)}{m}\right)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^m\sin^{-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] (I*4^(-1 - m)*c*E^(I*f*m*x)*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)*
(e + f*x))))^(2*m)*(((-I)*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, (-I)/E^
(I*(e + f*x))])/(E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*((-I)*A + B)*Hyper
geometric2F1[-1 - m, -2*m, -m, (-I)/E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*
x))*(1 + m)) + ((2*I)*A*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -
2*m, 2 - m, (-I)/E^(I*(e + f*x))])/(-1 + m) + (2*B*E^(I*(e - f*(-1 + m)*x))
*Hypergeometric2F1[1 - m, -2*m, 2 - m, (-I)/E^(I*(e + f*x))])/(-1 + m) + (I
*B*E^((2*I)*e - I*f*(-2 + m)*x)*Hypergeometric2F1[2 - m, -2*m, 3 - m, (-I)/
E^(I*(e + f*x))])/(-2 + m) + (4*A*Hypergeometric2F1[-2*m, -m, 1 - m, (-I)/E
^(I*(e + f*x))])/(E^(I*f*m*x)*m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x])
)^m)/((1 + I/E^(I*(e + f*x)))^(2*m)*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2*Sin[(2*e + Pi + 2*f*x)/4]^(2*m))
```

Maple [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int \left(a +a \sin \left(f x +e \right)\right)^{m} \left(A +B \sin \left(f x +e \right)\right) \left(c -c \sin \left(f x +e \right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(f
*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-A(a\sin(e+fx)+a)^m\right)dx+\int A(a\sin(e+fx)+a)^m\sin(e+fx)\,dx+\int\left(-B(a\sin(e+fx)+a)^m\sin(e+fx)\right)dx+\int B(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) +
a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x),
x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)\right.}{f(1+m)}$$

```
[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2830, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{(A+Am+Bm)}{} \\
&= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{((A+Am+Bm)}{} \\
&= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.28, size = 275, normalized size = 2.35

$$
-\frac{(a(1+\sin(e+fx)))^m\left(\frac{\sqrt[4]{-1}\,2^{-1-2m}Be^{-\frac{3}{2}i(e+fx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\right)^{1+2m}\left(e^{2i(e+fx)}(-1+m)\,{}_2F_1\left(1,m;-m;-ie^{-i(e+fx)}\right)-(1+m)\,{}_2F_1\left(1,2+m;2-m;-ie^{-i(e+fx)}\right)\right)}{-1+m^2}+\frac{2\sqrt{2}A\cos^{1+2m}\left(\frac{1}{4}(2e-\pi+2fx)\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{(1+2m)\sqrt{1-\sin(e+fx)}}\right)\sin^{-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*A*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (A+B\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=123

$$\frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\sin(e}{cfm}$$

```
[Out] 2^(1/2+m)*(A*m+B*m+B)*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec
(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/c/f/m-B*sec(f*x+e)*(a+a*s
in(f*x+e))^(1+m)/a/c/f/m
```

Rubi [A]

time = 0.21, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2768, 72, 71}

$$\frac{2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cfm}-\frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{acfm}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin
[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x]
)^m)/(c*f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}(A+B\sin(e+fx))}{ac}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{(B+Am+Bm}{}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(a(B+Am+\right.}{}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(2^{-\frac{1}{2}+m}a(B+\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx\right.}{cf}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 25.73, size = 6999, normalized size = 56.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{c-c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin(e+fx)-1}\,dx + \int \frac{B(a\sin(e+fx)+a)^m\sin(e+fx)}{\sin(e+fx)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x) + Integral(B*(a
*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{c-c\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{2^{\frac{1}{2}+m}(A(1-m)-B(2+m))\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a}{3ac^2f(1-m)}$$

```
[Out] 1/3*2^(1/2+m)*(A*(1-m)-B*(2+m))*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin(
f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/
(1-m)+B*sec(f*x+e)^3*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)
```

Rubi [A]
time = 0.22, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2768, 72, 71}

$$\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2c^2f(1-m)}+\frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2))\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*(A*(1 - m) - B*(2 + m))*Hypergeometric2F1[-3/2, 1/2 - m, -1/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(1 + m))/(3*a*c^2*f*(1 - m)) + (B*Sec[e + f*x]^3*(a + a*Sin[e
+ f*x])^(2 + m))/(a^2*c^2*f*(1 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}(A+B\sin(e+fx))}{a^2c^2}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(A-\frac{B(2+m)}{1-m}\right)\int}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(\left(A-\frac{B(2+m)}{1-m}\right)\mathrm{s}\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(}{1}\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx\right.}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.59, size = 9240, normalized size = 62.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a \sin \left(f x+e\right)\right)^{m} \left(A+B \sin \left(f x+e\right)\right)}{\left(c-c \sin \left(f x+e\right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 +
 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\left(a \sin \left(e+f x\right)+a\right)^{m}}{\sin ^{2}\left(e+f x\right)-2 \sin \left(e+f x\right)+1}\, d x+\int \frac{B\left(a \sin \left(e+f x\right)+a\right)^{m} \sin \left(e+f x\right)}{\sin ^{2}\left(e+f x\right)-2 \sin \left(e+f x\right)+1}\, d x}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),
 x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x)**2 - 2*
sin(e + f*x) + 1), x))/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x
)
```

3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{2^{\frac{1}{2}+m}(A(2-m)-B(3+m))\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a}{5a^2c^3f(2-m)}$$

```
[Out] 1/5*2^(1/2+m)*(A*(2-m)-B*(3+m))*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin(
f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/
f/(2-m)+B*sec(f*x+e)^5*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)
```

Rubi [A]
time = 0.22, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3046, 2939, 2768, 72, 71}

$$\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{a^3c^3f(2-m)}+\frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3))\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*(A*(2 - m) - B*(3 + m))*Hypergeometric2F1[-5/2, 1/2 - m, -3/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(2 + m))/(5*a^2*c^3*f*(2 - m)) + (B*Sec[e + f*x]^5*(a + a*Sin[
e + f*x])^(3 + m))/(a^3*c^3*f*(2 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}(A+B\sin(e+fx))}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(A-\frac{B(3+m)}{2-m}\right)\int}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(\left(A-\frac{B(3+m)}{2-m}\right)\mathrm{s}\right.}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(}{2}\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx\right.}{5a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.82, size = 12302, normalized size = 83.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 2.00, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx+e))^{m}(A + B\sin(fx+e))}{(c - c\sin(fx+e))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2
 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3, x
)
```

3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+$$

```
[Out] -2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3052, 2824, 2746, 70}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+\ldots}{\sqrt{c-\ldots}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+\ldots}{\sqrt{a+\ldots}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e\ldots}{f\sqrt{a\ldots}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+\ldots}{\ldots}$$

Mathematica [A]
time = 5.59, size = 200, normalized size = 1.69

$$\frac{2^{-\frac{3}{2}-2m}\left(-2^{3+2m}B+2^{1+2m}(A+B)\,{}_2F_1\left(1,1+2m;2(1+m);\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+(A+B)\,{}_2F_1\left(1+2m,1+2m;2(1+m);\frac{1}{2}\left(1-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sec^2\left(\frac{1}{8}(2e-\pi+2fx)\right)^{1+2m}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)}{(f+2fm)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
```

```
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Si
n[e + f*x]])
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(c+$$

[Out] -2*B*cos(f*x+e)*(c+c*sin(f*x+e))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)+(A+B)*cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)

Rubi [A]
time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3052, 2824, 2746, 70}

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],x]

[Out] (-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*x]])

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - (-A-B)\int \frac{(c+}{\sqrt{a}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)\cos(e}{\sqrt{a-}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)c\cos(e}{f\sqrt{a}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+f}{}$$

Mathematica [A]
time = 1.67, size = 200, normalized size = 1.69

$$\frac{2^{-\frac{3}{2}-2m}\left(-2^{3+2m}B+2^{1+2m}(A+B)\,{}_2F_1\left(1,1+2m;2(1+m);\sin\left(\tfrac{1}{4}(2e+\pi+2fx)\right)\right)+(A+B)\,{}_2F_1\left(1+2m,1+2m;2(1+m);\tfrac{1}{2}\left(1-\tan^2\left(\tfrac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sec^2\left(\tfrac{1}{8}(2e-\pi+2fx)\right)^{1+2m}\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^m\sin\left(\tfrac{1}{4}(2e+\pi+2fx)\right)}{(f+2fm)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
```

```
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[a - a*Si
n[e + f*x]])
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(fx+e))\,(c+c\sin(fx+e))^{m}}{\sqrt{a-a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
+ a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)^m/(a*sin(f*x + e) - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e+fx)+1))^{m}\,(A+B\sin(e+fx))}{\sqrt{-a\,(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f*
x) - 1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+c\,\sin\left(e+f\,x\right)\right)^m}{\sqrt{a-a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2
), x)
```

3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=275

$$-\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(5+2m)(7+2m)\left(3+8m+4m^2\right)\sqrt{c-c\sin(e+fx)}}-\frac{16c^2(B(5-2m)-A(7+2m))\cos(e+}{f(7+2m)\left(\right.}$$

```
[Out] -2*c*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
3/2)/f/(4*m^2+24*m+35)-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
5/2)/f/(7+2*m)-64*c^3*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f
/(5+2*m)/(7+2*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)-16*c^2*(B*(5-2*m)-A*(
7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(7+2*m)/(4*m
^2+16*m+15)
```

Rubi [A]
time = 0.34, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3052, 2819, 2817}

$$-\frac{64c^3(B(5-2m)-A(2m+7))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)(2m+7)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}-\frac{16c^2(B(5-2m)-A(2m+7))\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f(2m+7)\left(4m^2+16m+15\right)}-\frac{2c(B(5-2m)-A(2m+7))\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)(2m+7)}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-64*c^3*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(
f*(5 + 2*m)*(7 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (16*c^2
*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c
*Sin[e + f*x]])/(f*(7 + 2*m)*(15 + 16*m + 4*m^2)) - (2*c*(B*(5 - 2*m) - A*(
7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(
f*(5 + 2*m)*(7 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Si
n[e + f*x])^(5/2))/(f*(7 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(7+2m)}$$

$$=-\frac{2c(B(5-2m)-A(7+2m))\cos(e}{f(5}$$

$$=-\frac{16c^2(B(5-2m)-A(7+2m))\cos(}{f(3+2}$$

$$=-\frac{64c^3(B(5-2m)-A(7+2m))\cos(}{f(1+2m)(3+2m)(5+2m)(7+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.59, size = 667, normalized size = 2.43

$$\frac{(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{5/2}\left(\frac{(2100A-1575B+1272Am-110Bm+304Am^2-68Bm^2+32Am^3-8Bm^3)\left(\left(\frac{1}{8}+\frac{i}{8}\right)\cos\left(\frac{1}{2}(e+fx)\right)+\left(\frac{1}{8}-\frac{i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(1+2m)(3+2m)(5+2m)(7+2m)}+\frac{(2100A-1575B+1272Am-110Bm+304Am^2-68Bm^2+32Am^3-8Bm^3)\left(\left(\frac{1}{8}-\frac{i}{8}\right)\cos\left(\frac{1}{2}(e+fx)\right)+\left(\frac{1}{8}+\frac{i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(1+2m)(3+2m)(5+2m)(7+2m)}+\frac{(350A-385B+184Am-104Bm+24Am^2-12Bm^2)\left(\left(\frac{1}{8}-\frac{i}{8}\right)\cos\left(\frac{3}{2}(e+fx)\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\sin\left(\frac{3}{2}(e+fx)\right)\right)}{(3+2m)(5+2m)(7+2m)}+\frac{(350A-385B+184Am-104Bm+24Am^2-12Bm^2)\left(\left(\frac{1}{8}+\frac{i}{8}\right)\cos\left(\frac{3}{2}(e+fx)\right)-\left(\frac{1}{8}-\frac{i}{8}\right)\sin\left(\frac{3}{2}(e+fx)\right)\right)}{(3+2m)(5+2m)(7+2m)}+\frac{(14A-35B+4Am-6Bm)\left(\left(-\frac{1}{8}+\frac{i}{8}\right)\cos\left(\frac{5}{2}(e+fx)\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\sin\left(\frac{5}{2}(e+fx)\right)\right)}{(5+2m)(7+2m)}+\frac{(14A-35B+4Am-6Bm)\left(\left(-\frac{1}{8}-\frac{i}{8}\right)\cos\left(\frac{5}{2}(e+fx)\right)-\left(\frac{1}{8}-\frac{i}{8}\right)\sin\left(\frac{5}{2}(e+fx)\right)\right)}{(5+2m)(7+2m)}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)B\cos\left(\frac{7}{2}(e+fx)\right)-\left(\frac{1}{8}+\frac{i}{8}\right)B\sin\left(\frac{7}{2}(e+fx)\right)}{7+2m}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)B\cos\left(\frac{7}{2}(e+fx)\right)-\left(\frac{1}{8}-\frac{i}{8}\right)B\sin\left(\frac{7}{2}(e+fx)\right)}{7+2m}\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2100*A - 1575*B + 1
272*A*m - 110*B*m + 304*A*m^2 - 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 + I/8)
*Cos[(e + f*x)/2] + (1/8 - I/8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5
+ 2*m)*(7 + 2*m)) + ((2100*A - 1575*B + 1272*A*m - 110*B*m + 304*A*m^2 - 68
```

```
*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 - I/8)*Cos[(e + f*x)/2] + (1/8 + I/8)*Si
n[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((350*A - 385*
B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 - I/8)*Cos[(3*(e + f*x))
/2] - (1/8 + I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)) +
((350*A - 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 + I/8)*Cos
[(3*(e + f*x))/2] - (1/8 - I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)
*(7 + 2*m)) + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 + I/8)*Cos[(5*(e + f*x)
)/2] - (1/8 + I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((14*A -
35*B + 4*A*m - 6*B*m)*((-1/8 - I/8)*Cos[(5*(e + f*x))/2] - (1/8 - I/8)*Sin[
(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((1/8 - I/8)*B*Cos[(7*(e + f*x))
/2] - (1/8 + I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m) + ((1/8 + I/8)*B*Cos[(7
*(e + f*x))/2] - (1/8 - I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m)))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(274) = 548$.
time = 0.57, size = 763, normalized size = 2.77

$$-\frac{2\left(\frac{\left((4m^2+24m+43)a^m c^{\frac{5}{2}}-\frac{(12m^2+40m-15)a^m c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2(4m^2+8m+35)a^m c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2(4m^2+8m+35)a^m c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{(12m^2+40m-15)a^m c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{(4m^2+24m+43)a^m c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)Ae^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{(8m^3+36m^2+46m+15)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}-\frac{2\left((4m^2+40m+115)a^m c^{\frac{5}{2}}-\frac{2(4m^3+40m^2+115m)a^m c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2(12m^3+76m^2+97m+175)a^m c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{(16m^3+76m^2+260m-175)a^m c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{(16m^3+76m^2+260m-175)a^m c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2(12m^3+76m^2+97m+175)a^m c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{2(4m^3+40m^2+115m)a^m c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{(4m^2+40m+115)a^m c^{\frac{5}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(16m^4+128m^3+344m^2+352m+\frac{(16m^4+128m^3+344m^2+352m+105)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+105\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)*a^m*c^(
5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e)^3/(co
```

```
s(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^m*c^(5/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 128*m^3
 + 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m + 105)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(274) = 548$.
time = 0.43, size = 574, normalized size = 2.09

$$\frac{2\left(\left(8\,Bc^2m^3+36\,Bc^2m^2+46\,Bc^2m+15\,Bc^2\right)\cos\left(fx+e\right)^4+64\,(A+B)c^2m-\left(8\,(A-2\,B)c^2m^3+4\,(11\,A-28\,B)c^2m^2+2\,(31\,A-86\,B)c^2m+3\,(7\,A-20\,B)c^2\right)\cos\left(fx+e\right)^3+32\,(7\,A-5\,B)c^2+\left(8\,(A-B)c^2m^3+4\,(19\,A-11\,B)c^2m^2+190\,(A-B)c^2m+(77\,A-85\,B)c^2\right)\cos\left(fx+e\right)^2+2\left(8\,(A-B)c^2m^3+60\,(A-B)c^2m^2+2\,(79\,A-63\,B)c^2m+(161\,A-145\,B)c^2\right)\cos\left(fx+e\right)+\left(64\,(A+B)c^2m-\left(8\,Bc^2m^3+36\,Bc^2m^2+46\,Bc^2m+15\,Bc^2\right)\cos\left(fx+e\right)^3+32\,(7\,A-5\,B)c^2-\left(8\,(A-B)c^2m^3+4\,(11\,A-19\,B)c^2m^2+2\,(31\,A-63\,B)c^2m+3\,(7\,A-15\,B)c^2\right)\cos\left(fx+e\right)^2-2\left(8\,(A-B)c^2m^3+60\,(A-B)c^2m^2+2\,(63\,A-79\,B)c^2m+(49\,A-65\,B)c^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}\,\left(a\sin\left(fx+e\right)+a\right)^m}{16\,fm^4+128\,fm^3+344\,fm^2+352\,fm+\left(16\,fm^4+128\,fm^3+344\,fm^2+352\,fm+105\,f\right)\cos\left(fx+e\right)-\left(16\,fm^4+128\,fm^3+344\,fm^2+352\,fm+105\,f\right)\sin\left(fx+e\right)+105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 64
*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A -
 86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 + (8
*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A - 85
*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 + 2*(79
*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^2*m -
(8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3 + 32*(7
*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(31*A - 63
*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*m^3 + 60*
(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(f*x + e))*
sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(16*f*m^4 +
128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f
*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 10
5*f)*sin(f*x + e) + 105*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e)
+ a)^m, x)
```

Mupad [B]
time = 21.06, size = 749, normalized size = 2.72

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,c^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}-\frac{c^2\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(2100\,A-1575\,B+1272\,A\,m-110\,B\,m+304\,A\,m^2+32\,A\,m^3-68\,B\,m^2-8\,B\,m^3\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}+\frac{c^2\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(A\,2100{}\mathrm{i}-B\,1575{}\mathrm{i}+A\,m\,1272{}\mathrm{i}-B\,m\,110{}\mathrm{i}+A\,m^2\,304{}\mathrm{i}+A\,m^3\,32{}\mathrm{i}-B\,m^2\,68{}\mathrm{i}-B\,m^3\,8{}\mathrm{i}\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}-\frac{c^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\left(2\,m+1\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(350\,A-385\,B+184\,A\,m-104\,B\,m+24\,A\,m^2-12\,B\,m^2\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}+\frac{c^2\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\left(2\,m+1\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(A\,350{}\mathrm{i}-B\,385{}\mathrm{i}+A\,m\,184{}\mathrm{i}-B\,m\,104{}\mathrm{i}+A\,m^2\,24{}\mathrm{i}-B\,m^2\,12{}\mathrm{i}\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}-\frac{B\,c^2\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(8\,m^3+36\,m^2+46\,m+15\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}+\frac{c^2\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,m^2+8\,m+3\right)\,\left(14\,A-35\,B+4\,A\,m-6\,B\,m\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}-\frac{c^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,m^2+8\,m+3\right)\,\left(A\,14{}\mathrm{i}-B\,35{}\mathrm{i}+A\,m\,4{}\mathrm{i}-B\,m\,6{}\mathrm{i}\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}\right)}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}-\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(m^4\,16{}\mathrm{i}+m^3\,128{}\mathrm{i}+m^2\,344{}\mathrm{i}+m\,352{}\mathrm{i}+105{}\mathrm{i}\right)}{16\,m^4+128\,m^3+344\,m^2+352\,m+105}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((B*c^2*(a + a*sin(e + f*x))^m*(m*46i + m^2*36
i + m^3*8i + 15i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*
exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(2100*A - 1575*B + 1272*A*m - 110
*B*m + 304*A*m^2 + 32*A*m^3 - 68*B*m^2 - 8*B*m^3))/(4*f*(352*m + 344*m^2 +
128*m^3 + 16*m^4 + 105)) + (c^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^m*(
A*2100i - B*1575i + A*m*1272i - B*m*110i + A*m^2*304i + A*m^3*32i - B*m^2*6
8i - B*m^3*8i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*exp
(e*5i + f*x*5i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(350*A - 385*B + 184*A*m -
 104*B*m + 24*A*m^2 - 12*B*m^2))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
 105)) + (c^2*exp(e*2i + f*x*2i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*350i -
 B*385i + A*m*184i - B*m*104i + A*m^2*24i - B*m^2*12i))/(4*f*(352*m + 344*m
^2 + 128*m^3 + 16*m^4 + 105)) - (B*c^2*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^m*(46*m + 36*m^2 + 8*m^3 + 15))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^
4 + 105)) + (c^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3
)*(14*A - 35*B + 4*A*m - 6*B*m))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
 105)) - (c^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3)*(
A*14i - B*35i + A*m*4i - B*m*6i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4
+ 105))))/(exp(e*4i + f*x*4i) - (exp(e*3i + f*x*3i)*(m*352i + m^2*344i + m^
3*128i + m^4*16i + 105i))/(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105))
```

3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=166

$$\frac{4(A-B)c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}-\frac{2(A-3B)c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}-\frac{2Bc^2\cos(e}{a^2f(5+}$$

```
[Out] 4*(A-B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)-
2*(A-3*B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e)
)^(1/2)-2*B*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5+2*m)/(c-c*sin(f*
x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3052, 2819, 2817}

$$-\frac{8c^2(B(3-2m)-A(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}-\frac{2c(B(3-2m)-A(2m+5))\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f(2m+3)(2m+5)}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-8*c^2*(B*(3 - 2*m) - A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f
*(5 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (2*c*(B*(3 - 2*m)
- A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]]
)/(f*(3 + 2*m)*(5 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(I
```

```
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))}{f(5+2m)}$$

$$= -\frac{2c(B(3-2m)-A(5+2m))\cos(e}{f(3}$$

$$= -\frac{8c^2(B(3-2m)-A(5+2m))\cos(}{f(1+2m)(3+2m)(5+2m}$$

Mathematica [A]
time = 1.18, size = 174, normalized size = 1.05

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}\,(50A-39B+40Am-16Bm+8Am^2-4Bm^2+B(3+8m+4m^2)\cos(2(e+fx))-2(1+2m)(5A-9B+2Am-2Bm)\sin(e+fx))}{f(1+2m)(3+2m)(5+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3 +
 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)*Si
n[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c-c\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(169) = 338.
time = 0.55, size = 528, normalized size = 3.18

$$-\frac{2\left(\frac{\left(a^{m}c^{\frac{3}{2}}(2m+5)-\frac{a^{m}c^{\frac{3}{2}}(2m-3)\sin(fx+e)}{\cos(fx+e)+1}-\frac{a^{m}c^{\frac{3}{2}}(2m-3)\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{m}c^{\frac{3}{2}}(2m+5)\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)Ae^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)\right)}}{(4m^{2}+8m+3)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{3}{2}}}-\frac{2\left(a^{m}c^{\frac{3}{2}}(2m+9)-\frac{2(2m^{2}+9m)a^{m}c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{(4m^{2}+15)a^{m}c^{\frac{3}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{(4m^{2}+15)a^{m}c^{\frac{3}{2}}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{2(2m^{2}+9m)a^{m}c^{\frac{3}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{m}c^{\frac{3}{2}}(2m+9)\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)\right)}}{\left(8m^{3}+36m^{2}+46m+\frac{(8m^{3}+36m^{2}+46m+15)\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+15\right)\left(\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)^{\frac{3}{2}}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -2*((a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x + e) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c^(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + 3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(a^m*c^(3/2)*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*c^(3/2)*(2*m + 9)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f

Fricas [**A**]
time = 0.40, size = 323, normalized size = 1.95

$$\frac{2\left((4Bcm^{2}+8Bcm+3Bc)\cos(fx+e)^{3}+8(A+B)cm+(4Acm^{2}+12(A-B)cm+(5A-6B)c)\cos(fx+e)^{2}+4(5A-3B)c+(4(A-B)cm^{2}+4(5A-3B)cm+(25A-21B)c)\cos(fx+e)+(8(A+B)cm+(4Bcm^{2}+8Bcm+3Bc)\cos(fx+e)^{2}+4(5A-3B)c-(4(A-B)cm^{2}+4(3A-5B)cm+(5A-9B)c)\cos(fx+e))\sin(fx+e)\right)\sqrt{-c\sin(fx+e)+c}\,(a\sin(fx+e)+a)^{m}}{8fm^{3}+36fm^{2}+46fm+(8fm^{3}+36fm^{2}+46fm+15f)\cos(fx+e)-(8fm^{3}+36fm^{2}+46fm+15f)\sin(fx+e)+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m^2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4*(A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A + B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c - (4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2

```
+ 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 3
6*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
+ a)^m, x)
```

Mupad [B]
time = 19.20, size = 480, normalized size = 2.89

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{c\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(45\,A-30\,B+28\,A\,m+4\,B\,m+4\,A\,m^2\right)}{f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}+\frac{c\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(A\,45{}\mathrm{i}-B\,30{}\mathrm{i}+A\,m\,28{}\mathrm{i}+B\,m\,4{}\mathrm{i}+A\,m^2\,4{}\mathrm{i}\right)}{f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}+\frac{B\,c\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^2\,4{}\mathrm{i}+m\,8{}\mathrm{i}+3{}\mathrm{i}\right)}{2\,f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}+\frac{B\,c\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,m^2+8\,m+3\right)}{2\,f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}+\frac{c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(2\,m+1\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(10\,A-15\,B+4\,A\,m-2\,B\,m\right)}{2\,f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}+\frac{c\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\left(2\,m+1\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(A\,10{}\mathrm{i}-B\,15{}\mathrm{i}+A\,m\,4{}\mathrm{i}-B\,m\,2{}\mathrm{i}\right)}{2\,f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}\right)}{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}+\frac{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\left(8\,m^3+36\,m^2+46\,m+15\right)}{m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),
x)
[Out] ((c - c*sin(e + f*x))^(1/2)*((c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(
45*A - 30*B + 28*A*m + 4*B*m + 4*A*m^2))/(f*(m*46i + m^2*36i + m^3*8i + 15i
)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(A*45i - B*30i + A*m*28i
+ B*m*4i + A*m^2*4i))/(f*(m*46i + m^2*36i + m^3*8i + 15i)) + (B*c*(a + a*si
n(e + f*x))^m*(m*8i + m^2*4i + 3i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))
+ (B*c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3))/(2*f*(m
*46i + m^2*36i + m^3*8i + 15i)) + (c*exp(e*1i + f*x*1i)*(2*m + 1)*(a + a*si
n(e + f*x))^m*(10*A - 15*B + 4*A*m - 2*B*m))/(2*f*(m*46i + m^2*36i + m^3*8i
 + 15i)) + (c*exp(e*4i + f*x*4i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*10i -
B*15i + A*m*4i - B*m*2i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))))/(exp(e*3
i + f*x*3i) + (exp(e*2i + f*x*2i)*(46*m + 36*m^2 + 8*m^3 + 15))/(m*46i + m^
2*36i + m^3*8i + 15i))
```

3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

Optimal. Leaf size=104

$$\frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+2*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3050, 2817}

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 +
 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 3050

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin}}{a}$$
$$= \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 0.32, size = 116, normalized size = 1.12

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}\,(-2B+A(3+2m)+B(1+2m)\sin(e+fx))}{f(1+2m)(3+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*
m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(106) = 212.
time = 0.54, size = 345, normalized size = 3.32

$$-\frac{2\left(\frac{2\left(\frac{2a^m\sqrt{c}\,m\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^m\sqrt{c}\,m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-a^m\sqrt{c}-\frac{a^m\sqrt{c}\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(4m^2+8m+\frac{(4m^2+8m+3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+3\right)\sqrt{\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1}}+\frac{\left(a^m\sqrt{c}+\frac{a^m\sqrt{c}\,\sin(fx+e)}{\cos(fx+e)+1}\right)Ae^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{(2m+1)\sqrt{\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -2*(2*(2*a^m*sqrt(c)*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c)*m*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2 + 8
*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e
) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)))/f
```

Fricas [A]
time = 0.37, size = 173, normalized size = 1.66

$$-\frac{2\left(\left(2\,Bm+B\right)\cos\left(fx+e\right)^{2}-2\left(A+B\right)m-\left(2\,Am+3\,A-2\,B\right)\cos\left(fx+e\right)-\left(2\left(A+B\right)m+\left(2\,Bm+B\right)\cos\left(fx+e\right)+3\,A-B\right)\sin\left(fx+e\right)-3\,A+B\right)\sqrt{-c\sin\left(fx+e\right)+c}\,\left(a\sin\left(fx+e\right)+a\right)^{m}}{4\,fm^{2}+8\,fm+\left(4\,fm^{2}+8\,fm+3\,f\right)\cos\left(fx+e\right)-\left(4\,fm^{2}+8\,fm+3\,f\right)\sin\left(fx+e\right)+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x
+ e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) - 3*
A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m +
(4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x + e)
 + 3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e
 + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)

Mupad [B]
time = 1.44, size = 105, normalized size = 1.01

$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,\left(6\,A\,\cos(e+f\,x)-4\,B\,\cos(e+f\,x)+B\,\sin(2\,e+2\,f\,x)+4\,A\,m\,\cos(e+f\,x)+2\,B\,m\,\sin(2\,e+2\,f\,x)\right)}{f\,(\sin(e+f\,x)-1)\,(4\,m^2+8\,m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2), x)

[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(6*A*cos(e + f*x) - 4*B*cos(e + f*x) + B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2*f*x)))/(f*(sin(e + f*x) - 1)*(8*m + 4*m^2 + 3))

3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3052, 2824, 2746, 70}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+}{\sqrt{c}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e}{\sqrt{a}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e}{f\sqrt{c}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+}{}$$

Mathematica [A]
time = 1.56, size = 200, normalized size = 1.69

$$\frac{2^{-\frac{3}{2}-2m}\left(-2^{3+2m}B+2^{1+2m}(A+B)\,{}_2F_1\left(1,1+2m;2(1+m);\sin\left(\tfrac{1}{4}(2e+\pi+2fx)\right)\right)+(A+B)\,{}_2F_1\left(1+2m,1+2m;2(1+m);\tfrac{1}{2}\left(1-\tan^2\left(\tfrac{1}{8}(2e-\pi+2fx)\right)\right)\right)\sec^2\left(\tfrac{1}{8}(2e-\pi+2fx)\right)^{1+2m}\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sin\left(\tfrac{1}{4}(2e+\pi+2fx)\right)}{(f+2fm)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
```

```
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Si
n[e + f*x]])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m (A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c-c\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
), x)
```

3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}}+\frac{(A(1-2m)-B(3+2m))\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1\right.}{4cf(1+2m)\sqrt{c-c\sin(e+fx}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A*(1-
2*m)-B*(3+2*m))*cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))
*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3051, 2824, 2746, 70}

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3/
2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c*f*(1 + 2*m)*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(Bc(-\frac{3}{2}-m}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{((Bc(-\frac{3}{2}-}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(a(Bc(-\frac{3}{2}-}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(A(1-2m)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.57, size = 3178, normalized size = 23.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (2^(-3/2 - 2*m)*B*(-(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Cos[(-e + Pi/2
- f*x)/2]]) + Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(-e + Pi/2 - f*
```

x)/4]^2)/2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a + a*Sin[e + f*x])^m)/(f*m*(c - c*Sin[e + f*x])^(3/2)) - ((A + B)*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a + a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/(8*Sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*Cos[(-e + Pi/2 - f*x)/4]*(Cos[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/Sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4])/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/2*(m*AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m)*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2 + (m*AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/

```
2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)
^(2*m) + (m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e
+ Pi/2 - f*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(
2*m)*Sec[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/
(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m
, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(
-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4
]^4)^(2*m))/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-1/2*((1 + 2*m)*AppellF1[2
+ 2*m, 2*m, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + P
i/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2 +
2*m) - (m*(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + P
i/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4
]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]
^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*Appe
llF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[
(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]
^3*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 +
 2*m))/(1 + 2*m)))/(8*Sqrt[2])))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin{(e+fx)}+1))^m (A+B\sin{(e+fx)})}{(-c(\sin{(e+fx)}-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) -
 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin{(e+f\,x)})\,(a+a\,\sin{(e+f\,x)})^m}{(c-c\,\sin{(e+f\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2
), x)
```

3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}}+\frac{(A(3-2m)-B(5+2m))\cos(e+fx)\,{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(\right.}{16c^2f(1+2m)\sqrt{c-c\sin(e+}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(5/2)+1/16*(A*(3
-2*m)-B*(5+2*m))*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e)
)*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3051, 2824, 2746, 70}

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5/
2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(16*c^2*f*(1 + 2*
m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(Bc(-\frac{5}{2}-m}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{((Bc(-\frac{5}{2}-n}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(a^2(Bc(-\frac{5}{2}-}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(A(3-2m)-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.70, size = 8147, normalized size = 60.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x +
 e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m (A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) -
 1))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2
), x)
```

3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal. Leaf size=267

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)}+\frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin}{cf(5+2m)(}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m)/f/(7+2*m)+(3*A-
2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/c/f/(4*m^2
+24*m+35)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(-2-m)/c^2/f/(8*m^3+60*m^2+142*m+105)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin
(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^3/f/(16*m^4+128*m^3+344*m^2+352*m+105)
```

Rubi [A]
time = 0.30, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^3f(2m+5)(2m+7)\left(4m^2+8m+3\right)}+\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{c^2f(2m+7)\left(4m^2+16m+15\right)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}+\frac{(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{cf(2m+5)(2m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m))
/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-3 - m))/(c*f*(5 + 2*m)*(7 + 2*m)) + (2*(3*A - 2*B*(2
+ m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(c
^2*f*(7 + 2*m)*(15 + 16*m + 4*m^2)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(
a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^3*f*(5 + 2*m)*(7 +
2*m)*(3 + 8*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
```

```
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(7+2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(7+2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(7+2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(7+2}$$

Mathematica [**A**]
time = 17.91, size = 353, normalized size = 1.32

$$-\frac{2^{-18-m}\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\csc^{21}\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\sec^{7}\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-4-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\left(-96A+58B-176Am+64Bm-96Am^2+16Bm^2-16Am^3+4(2+m)(-3A+2B(2+m))\cos\left(2\left(-e+\frac{\pi}{2}-fx\right)\right)+3A\cos\left(3\left(-e+\frac{\pi}{2}-fx\right)\right)-4B\cos\left(3\left(-e+\frac{\pi}{2}-fx\right)\right)-2Bm\cos\left(3\left(-e+\frac{\pi}{2}-fx\right)\right)+(29+32m+8m^2)(3A-2B(2+m))\sin(e+fx)\right)}{f(1+2m)(3+2m)(5+2m)(7+2m)\left(-1+\cot^2\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-4 - m),x]
```

```
[Out] -((2^(-18 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^21*Sec[(-e
 + Pi/2 - f*x)/8]^7*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m)*(-
96*A + 58*B - 176*A*m + 64*B*m - 96*A*m^2 + 16*B*m^2 - 16*A*m^3 + 4*(2 + m)
*(-3*A + 2*B*(2 + m))*Cos[2*(-e + Pi/2 - f*x)] + 3*A*Cos[3*(-e + Pi/2 - f*x
)] - 4*B*Cos[3*(-e + Pi/2 - f*x)] - 2*B*m*Cos[3*(-e + Pi/2 - f*x)] + (29 +
```

```
32*m + 8*m^2)*(3*A - 2*B*(2 + m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5
+ 2*m)*(7 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^7*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-4 - m))))
```

Maple [F]
time = 1.37, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(A+B\sin \left(fx+e\right)\right) \left(c-c\sin \left(fx+e\right)\right)^{-4-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 4), x)
```

Fricas [A]
time = 0.41, size = 212, normalized size = 0.79

$$\frac{\left(4\,{\left(2\,B m^{2} - {\left(3\,A - 8\,B\right)} m - 6\,A + 8\,B\right)} \cos\left(f x + e\right)^{3} + {\left(8\,A m^{3} + 12\,{\left(4\,A - B\right)} m^{2} + 2\,{\left(47\,A - 24\,B\right)} m + 60\,A - 45\,B\right)} \cos\left(f x + e\right) - {\left(2\,{\left(2\,B m - 3\,A + 4\,B\right)} \cos\left(f x + e\right)^{3} - {\left(8\,B m^{3} - 12\,{\left(A - 4\,B\right)} m^{2} - 2\,{\left(24\,A - 47\,B\right)} m - 45\,A + 60\,B\right)} \cos\left(f x + e\right)\right)} \sin\left(f x + e\right)\right)} {\left(a \sin\left(f x + e\right) + a\right)}^{m} {\left(-c \sin\left(f x + e\right) + c\right)}^{-m - 4}}{16\,f m^{4} + 128\,f m^{3} + 344\,f m^{2} + 352\,f m + 105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="fricas")
```

```
[Out] (4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4*
A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m - 3*
A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*m -
 45*A + 60*B)*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*
x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 4), x)
```

Mupad [B]
time = 22.23, size = 368, normalized size = 1.38

$$-\frac{\sin\left(4\,e+4\,f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,B-3\,A+2\,B\,m\right)\,1{}\mathrm{i}}{4\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m+4}\,\left(m^4\,16{}\mathrm{i}+m^3\,128{}\mathrm{i}+m^2\,344{}\mathrm{i}+m\,352{}\mathrm{i}+105{}\mathrm{i}\right)}+\frac{\cos\left(e+f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(A\,168{}\mathrm{i}-B\,84{}\mathrm{i}+A\,m\,340{}\mathrm{i}-B\,m\,96{}\mathrm{i}+A\,m^2\,192{}\mathrm{i}+A\,m^3\,32{}\mathrm{i}-B\,m^2\,24{}\mathrm{i}\right)}{4\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m+4}\,\left(m^4\,16{}\mathrm{i}+m^3\,128{}\mathrm{i}+m^2\,344{}\mathrm{i}+m\,352{}\mathrm{i}+105{}\mathrm{i}\right)}+\frac{\sin\left(2\,e+2\,f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(2\,m^2+8\,m+7\right)\,\left(4\,B-3\,A+2\,B\,m\right)\,1{}\mathrm{i}}{f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m+4}\,\left(m^4\,16{}\mathrm{i}+m^3\,128{}\mathrm{i}+m^2\,344{}\mathrm{i}+m\,352{}\mathrm{i}+105{}\mathrm{i}\right)}+\frac{\cos\left(3\,e+3\,f\,x\right)\,\left(m+2\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(-A\,3{}\mathrm{i}+B\,4{}\mathrm{i}+B\,m\,2{}\mathrm{i}\right)}{f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m+4}\,\left(m^4\,16{}\mathrm{i}+m^3\,128{}\mathrm{i}+m^2\,344{}\mathrm{i}+m\,352{}\mathrm{i}+105{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 4),x)
```

```
[Out] (cos(e + f*x)*(a + a*sin(e + f*x))^m*(A*168i - B*84i + A*m*340i - B*m*96i +
 A*m^2*192i + A*m^3*32i - B*m^2*24i))/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*
352i + m^2*344i + m^3*128i + m^4*16i + 105i)) - (sin(4*e + 4*f*x)*(a + a*si
n(e + f*x))^m*(4*B - 3*A + 2*B*m)*1i)/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*
352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (sin(2*e + 2*f*x)*(a + a*si
n(e + f*x))^m*(8*m + 2*m^2 + 7)*(4*B - 3*A + 2*B*m)*1i)/(f*(c - c*sin(e + f
*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (cos(3*e +
3*f*x)*(m + 2)*(a + a*sin(e + f*x))^m*(B*4i - A*3i + B*m*2i))/(f*(c - c*sin
(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i))
```

3.212 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=191

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)}+\frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin}{cf(3+2m)(}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+(2*A-
B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(4*m^2
+16*m+15)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-
1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)
```

Rubi [A]
time = 0.22, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {3051, 2822, 2821}

$$\frac{(2A-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)(4m^2+8m+3)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}+\frac{(2A-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{cf(2m+3)(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))
/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-2 - m))/(c*f*(3 + 2*m)*(5 + 2*m)) + ((2*A - B*(3 + 2*
m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2
*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(5+2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(5+2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(5+2}$$

Mathematica [A]
time = 14.86, size = 269, normalized size = 1.41

$$\frac{2^{-13-m}\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\csc^{15}\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\sec^5\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-3-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\left(16A-9B+24Am-6Bm+8Am^2+(2A-3B-2Bm)\cos\left(2\left(-e+\frac{\pi}{2}-fx\right)\right)+2(3+2m)(-2A+B(3+2m))\sin(e+fx)\right)}{f(1+2m)(3+2m)(5+2m)\left(-1+\cot^2\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-3 - m),x]
```

```
[Out] (2^(-13 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^15*Sec[(-e +
 Pi/2 - f*x)/8]^5*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(16*
A - 9*B + 24*A*m - 6*B*m + 8*A*m^2 + (2*A - 3*B - 2*B*m)*Cos[2*(-e + Pi/2 -
 f*x)] + 2*(3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 +
2*m)*(5 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^5*Sin[(-e + Pi/2 - f*x)/2]
^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 3), x)
```

Fricas [A]
time = 0.41, size = 143, normalized size = 0.75

$$\frac{\left((2\,Bm-2\,A+3\,B)\cos\left(fx+e\right)^3+\left(4\,Bm^2-4\,(A-3\,B)m-6\,A+9\,B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(4\,Am^2+4\,(3\,A-B)m+9\,A-6\,B\right)\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}}{8\,fm^3+36\,fm^2+46\,fm+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B)
*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*x
+ e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8*f*m^3 + 36*f
*m^2 + 46*f*m + 15*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 3), x)
```

Mupad [B]
time = 15.48, size = 239, normalized size = 1.25

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(30\,A\,\cos\left(e+f\,x\right)-15\,B\,\cos\left(e+f\,x\right)-2\,A\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(3\,e+3\,f\,x\right)-12\,A\,\sin\left(2\,e+2\,f\,x\right)+18\,B\,\sin\left(2\,e+2\,f\,x\right)+8\,B\,m^2\,\sin\left(2\,e+2\,f\,x\right)+48\,A\,m\,\cos\left(e+f\,x\right)-10\,B\,m\,\cos\left(e+f\,x\right)+16\,A\,m^2\,\cos\left(e+f\,x\right)+2\,B\,m\,\cos\left(3\,e+3\,f\,x\right)-8\,A\,m\,\sin\left(2\,e+2\,f\,x\right)+24\,B\,m\,\sin\left(2\,e+2\,f\,x\right)\right)}{c^3\,f\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,\left(8\,m^3+36\,m^2+46\,m+15\right)\,\left(15\,\sin\left(e+f\,x\right)+6\,\cos\left(2\,e+2\,f\,x\right)-\sin\left(3\,e+3\,f\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 3),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*cos
(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2*e
 + 2*f*x) + 8*B*m^2*sin(2*e + 2*f*x) + 48*A*m*cos(e + f*x) - 10*B*m*cos(e +
 f*x) + 16*A*m^2*cos(e + f*x) + 2*B*m*cos(3*e + 3*f*x) - 8*A*m*sin(2*e + 2*
f*x) + 24*B*m*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*m + 3
6*m^2 + 8*m^3 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x
) - 10))
```

3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=114

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}+\frac{(A-2B(1+m))\cos(e+fx)(a+a\sin}{cf(1+2m)(3}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+(A-2*
B*(1+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(4*m^2+8
*m+3)
```

Rubi [A]
time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3051, 2821}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf(2m+1)(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))
/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c
- c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(3+2}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(3+2}$$

Mathematica [A]
time = 11.73, size = 211, normalized size = 1.85

$$-\frac{2^{-7-m}\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\csc^9\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\sec^3\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-2-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}(B-2A(1+m)+(A-2B(1+m))\sin(e+fx))}{f\left(3+8m+4m^2\right)\left(-1+\cot^2\left(\frac{1}{8}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-2 - m),x]
```

```
[Out] -((2^(-7 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^9*Sec[(-e +
 Pi/2 - f*x)/8]^3*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m)*(B -
 2*A*(1 + m) + (A - 2*B*(1 + m))*Sin[e + f*x]))/(f*(3 + 8*m + 4*m^2)*(-1 +
Cot[(-e + Pi/2 - f*x)/8]^2)^3*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))
```

Maple [F]
time = 0.59, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

Fricas [A]
time = 0.41, size = 94, normalized size = 0.82

$$\frac{((2\,Bm - A + 2\,B)\cos{(fx + e)}\sin{(fx + e)} + (2\,Am + 2\,A - B)\cos{(fx + e)})(a\sin{(fx + e)} + a)^m(-c\sin{(fx + e)} + c)^{-m-2}}{4\,fm^2 + 8\,fm + 3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x +
e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m
+ 3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

Mupad [B]
time = 14.16, size = 134, normalized size = 1.18

$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,(4\,A\,\cos(e+f\,x)-2\,B\,\cos(e+f\,x)-A\,\sin(2\,e+2\,f\,x)+2\,B\,\sin(2\,e+2\,f\,x)+4\,A\,m\,\cos(e+f\,x)+2\,B\,m\,\sin(2\,e+2\,f\,x))}{c^2\,f\,(-c\,(\sin(e+f\,x)-1))^m\,(4\,m^2+8\,m+3)\,(4\,\sin(e+f\,x)+\cos(2\,e+2\,f\,x)-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*e
 + 2*f*x) + 2*B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2*f
*x)))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m + 4*m^2 + 3)*(4*sin(e + f*x) +
cos(2*e + 2*f*x) - 3))
```

3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=163

$$\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}-\frac{2^{\frac{1}{2}-m}B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+\right.}{}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)-2^(1/
2-m)*B*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1-s
in(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)
```

Rubi [A]
time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3051, 2824, 2768, 72, 71}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))
/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2,
 (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 +
m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3051

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(1+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(1+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(1+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(1+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+}{f(1+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 11.56, size = 675, normalized size = 4.14

$$-\frac{2^{-m}(-3+2m)\cos^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\cot\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-1-m)}(a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\left(8B(1+2m)F_1\left(\frac{1}{2}-m;-2m,1;\frac{3}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)-(A+B)\left((-1+2m)\,{}_2F_1\left(-\frac{1}{2}-m,-2m;\frac{1}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)+(1+2m)\,{}_2F_1\left(\frac{1}{2}-m,-2m;\frac{3}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\right)}{f\left(-1+4m^2\right)\left(-64BmF_1\left(\frac{3}{2}-m;1-2m,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\sin^4\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)-32BF_1\left(\frac{3}{2}-m;-2m,2;\frac{5}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\sin^4\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)+(-3+2m)\left(-4BF_1\left(\frac{1}{2}-m;-2m,1;\frac{3}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\sin^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)+(A+B)\left(1+\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\left(1-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)^{2m}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] -(((-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/4]^2*Cot[(-e + Pi/2 - f*x)/4]*(a + a*Si
n[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - m)*(8*B*(1 +
2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - (A + B)*((-1 + 2*m)*Hyp
ergeometric2F1[-1/2 - m, -2*m, 1/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (1 +
2*m)*Hypergeometric2F1[1/2 - m, -2*m, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2]*
Tan[(-e + Pi/2 - f*x)/4]^2)))/(2^m*f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^
(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-1 - m))*(-64*B*m*AppellF1[
3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
 f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^4 - 32*B*AppellF1[3/2 - m, -2*m, 2, 5/
2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + P
i/2 - f*x)/4]^4 + (-3 + 2*m)*(-4*B*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/
2]^2 + (A + B)*(1 + Cos[(-e + Pi/2 - f*x)/2])*(1 - Tan[(-e + Pi/2 - f*x)/4]
^2)^(2*m)))))
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-m),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m}{(c - c\,\sin(e + f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 1),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 1), x)
```

3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=158

$$\frac{2^{\frac{1}{2}-m}c(A+2Bm)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}}}{f(1+2m)}$$

```
[Out] 2^(1/2-m)*c*(2*B*m+A)*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*s
in(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m
)/f/(1+2*m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/((c-c*sin(f*x+e))^m)
```

Rubi [A]
time = 0.18, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3052, 2824, 2768, 72, 71}

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)) - (B*Cos[
e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f}$$

$$= \frac{2^{\frac{1}{2}-m}c(A+2Bm)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 18.87, size = 1767, normalized size = 11.18

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^m,x]
```

```
[Out] (2^(2 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/
2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[
1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2]))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a + a*Sin[e + f*x])
^m*(A + B*Sin[e + f*x])*Tan[(-e + Pi/2 - f*x)/4])/(f*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(c - c*Sin[e + f*x])^m*(-(((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-Appel
lF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]
^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Sec[(-e + Pi/2 - f*x)/4]^2) + 4*m*((A +
B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3,
 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cot[(-
e + Pi/2 - f*x)/2]*Tan[(-e + Pi/2 - f*x)/4] + (2*(-1 + 2*m)*(2*(A + B)*m*Ap
pellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2] - 16*B*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 16*B*m*AppellF1[3/2 - m,
 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2] + A*AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Ta
n[(-e + Pi/2 - f*x)/4]^2] + B*AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 16*B*AppellF1[3/2 - m, -2
*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] +
24*B*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]
^2)/(-3 + 2*m) + (4*m*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m,
-2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
+ AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2]))*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^2
)/(-1 + Tan[(-e + Pi/2 - f*x)/4]^2) + 4*m*((A + B)*AppellF1[1/2 - m, -2*m,
1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*
(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Tan[(-e + Pi/2 - f*x)/4]*Tan[(-e
+ Pi/2 - f*x)/2]))
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{-m} (A+B\sin(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^
m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m, x
)
```

3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=170

$$\frac{2^{\frac{1}{2}-m}c^2(2A-B(1-2m))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin}{f(1+2m)}$$

```
[Out] 2^(1/2-m)*c^2*(2*A-B*(1-2*m))*cos(f*x+e)*hypergeom([1/2+m, -1/2+m],[3/2+m],
1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+
e))^(-1-m)/f/(1+2*m)-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
1-m)/f
```

Rubi [A]
time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2824, 2768, 72, 71}

$$\frac{c^2 2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - m
),x]
```

```
[Out] (2^(1/2 - m)*c^2*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)
) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(2
*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{2f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{2f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{2f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{2f}$$

$$= \frac{2^{\frac{1}{2}-m}c^2(2A-B(1-2m))\cos(e+f}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 89.50, size = 3601, normalized size = 21.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(1 - m),x]
```

```
[Out] (2^(5 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(
2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*(a + a*Sin[e + f*x])^m*(c - c*Sin
[e + f*x])^(1 - m)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2*((A*Cos[(-e + Pi/2 - f
*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x])/(Cos[Pi/4 + (e
 - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m)) + (A*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e - Pi/2
 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*
x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e -
 Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + Cos[Pi/4 + (e - P
i/2 + f*x)/2]*((-2*A*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 +
f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^
(2*m) - (2*B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi
/2 + f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m)))*Tan[(-e + Pi/2 - f*x)/4])/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(1 - m))*(1 - Tan[(-e + P
i/2 - f*x)/4]^2)^(2*m)*(-((2^(5 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 2,
3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*
B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(
-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/
4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/
2 - f*x)/2]^(2*m))) - (2^(3 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 -
m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*App
ellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2)/((-1 + 2*m)*Sin[(-e + Pi/2
- f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(5 - m)*m*((A
+ B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
```

```
-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2
- m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4
]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Sin[(-e + Pi/
2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e +
 Pi/2 - f*x)/4]^2)^(2*m)) + (2^(5 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 2
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A +
9*B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m,
 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos
[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e
 + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2
^(5 - m)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*((A + B)*(
-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi
/2 - f*x)/4])/(3/2 - m)) - ((1/2 - m)*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*
x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (A + 9*B)*(-(((1/2 - m)*m*Ap
pellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/
2 - m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e
 + Pi/2 - f*x)/4])/(2*(3/2 - m))) + 8*B*(((1/2 - m)*m*AppellF1[3/2 - m, 1 -
 2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m) + (5*(1/2 -
m)*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(
2*(3/2 - m)) + 2*(-(((1/2 - m)*m*AppellF1[3/2 -...
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e))(c - c\sin(fx+e))^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m
),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m
), x)
```

3.217 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=173

$$\frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin}{3f(1+2m)}$$

```
[Out] 1/3*2^(5/2-m)*c^3*(3*A-2*B*(1-m))*cos(f*x+e)*hypergeom([-3/2+m, 1/2+m],[3/2
+m],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(
f*x+e))^(-1-m)/f/(1+2*m)-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e
))^(2-m)/f
```

Rubi [A]
time = 0.23, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3052, 2824, 2768, 72, 71}

$$\frac{c^3 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{3f(2m+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - m
),x]
```

```
[Out] (2^(5/2 - m)*c^3*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(3*f*(1 + 2*
m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
(3*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3052

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e
, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
-2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{3f}$$

$$= \frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+f}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 61.82, size = 5163, normalized size = 29.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(2 - m),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.39, size = 0, normalized size = 0.00

$$\int (a+a\sin (fx+e))^{m} (A+B\sin (fx+e)) (c-c\sin (fx+e))^{2-m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m
),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m
), x)
```

3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{-3+n}}{f}$$

```
[Out] a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(-3+n)/f
```

Rubi [A]
time = 0.18, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3046, 2933}

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c$$
$$= \frac{a^3Bc^3\cos^7(e+fx)(c-c}{f}$$

Mathematica [A]
time = 0.38, size = 63, normalized size = 1.85

$$\frac{a^3B(c-c\sin(e+fx))^n(14\cos(e+fx)-6\cos(3(e+fx))+14\sin(2(e+fx))-\sin(4(e+fx)))}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 +
 n)*Sin[e + f*x]),x]
```

```
[Out] (a^3*B*(c - c*Sin[e + f*x])^n*(14*Cos[e + f*x] - 6*Cos[3*(e + f*x)] + 14*Si
n[2*(e + f*x)] - Sin[4*(e + f*x)]))/(8*f)
```

Maple [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{3}\left(c-c\sin\left(fx+e\right)\right)^{n}\left(B(3-n)-B(4+n)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.
time = 0.44, size = 84, normalized size = 2.47

$$-\frac{\left(3\,Ba^{3}\cos\left(fx+e\right)^{3}-4\,Ba^{3}\cos\left(fx+e\right)+\left(Ba^{3}\cos\left(fx+e\right)^{3}-4\,Ba^{3}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(-c\sin\left(fx+e\right)+c\right)^{n}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] -(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) + (B*a^3*cos(f*x + e)^3 - 4
*B*a^3*cos(f*x + e))*sin(f*x + e))*(-c*sin(f*x + e) + c)^n/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(31) = 62$.
time = 80.43, size = 898, normalized size = 26.41

$$\begin{cases} -\frac{Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{6Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{14Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{14Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{14Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{14Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{6Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{Ba^3\left(c-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^n}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} & \text{for}\: f \neq 0 \\ x\left(B(3-n) - B(n+4)\sin(e)\right)\left(a\sin(e)+a\right)^3\left(-c\sin(e)+c\right)^n & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Piecewise((-B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*
tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c - 2*c*tan(
e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
 f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/
2 + f*x/2)**2 + f) - 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)*
*2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/
2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*a**3*
(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**5
/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(
e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*t
an(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
 + 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/
2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/
2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c - 2*c*tan(e/2 +
f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**
8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2
)**2 + f) + B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
+ 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(3 - n) - B*(n + 4)*sin(e)
)*(a*sin(e) + a)**3*(-c*sin(e) + c)**n, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] integrate(-(B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

Mupad [B]
time = 14.45, size = 64, normalized size = 1.88

$$\frac{B\,a^3\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(n - 3) + B*sin(e + f*x)*(n + 4))*(a + a*sin(e + f*x))^3*(c - c*sin
(e + f*x))^n,x)
```

```
[Out] (B*a^3*(-c*(sin(e + f*x) - 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 14
*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$-\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(e+fx))^{-3+n}}{f}$$

```
[Out] -a^3*B*c^3*cos(f*x+e)^7*(c+c*sin(f*x+e))^(-3+n)/f
```

Rubi [A]
time = 0.15, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {3046, 2933}

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a - a\sin(e+fx))^3 (c + c\sin(e+fx))^n (B(3-n) + B(4+n)\sin(e+fx))\, dx = (a^3c^3) \int \cos^6(e+fx)(c$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c+}{f}$$

Mathematica [A]
time = 0.78, size = 67, normalized size = 1.97

$$-\frac{a^3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 +
 n)*Sin[e + f*x]),x]
[Out] -((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)
```

Maple [F]
time = 1.07, size = 0, normalized size = 0.00

$$\int (a - a\sin(fx+e))^3 (c + c\sin(fx+e))^n (B(3-n) + B(4+n)\sin(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
[Out] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
[Out] -integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.
time = 0.39, size = 83, normalized size = 2.44

$$\frac{\left(3\,Ba^3\cos(fx+e)^3 - 4\,Ba^3\cos(fx+e) - \left(Ba^3\cos(fx+e)^3 - 4\,Ba^3\cos(fx+e)\right)\sin(fx+e)\right)(c\sin(fx+e)+c)^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] (3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4*
B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(32) = 64$.
time = 80.81, size = 898, normalized size = 26.41

$$\begin{cases} \frac{Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{6Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan^{7}\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{14Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{14Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{14Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{14Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{6Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{Ba^{3}\left(c+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^{n}}{f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+f} & \text{for}\: f \neq 0 \\ x\left(B(3-n)+B(n+4)\sin(e)\right)\left(-a\sin(e)+a\right)^{3}\left(c\sin(e)+c\right)^{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**3*(c+c*sin(f*x+e))**n*(B*(3-n)+B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Piecewise((B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*t
an(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*t
an(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c + 2*c*tan(e
/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2
 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**
2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2
)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*a**3*(
c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**5/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
+ 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e
/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*ta
n(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
- 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2
 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2
 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c + 2*c*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**8
 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)
**2 + f) - B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n/(
f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 +
 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(3 - n) + B*(n + 4)*sin(e))
*(-a*sin(e) + a)**3*(c*sin(e) + c)**n, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] integrate(-(B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

Mupad [B]
time = 14.46, size = 61, normalized size = 1.79

$$-\frac{B\,a^3\,{\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^n\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(n - 3) - B*sin(e + f*x)*(n + 4))*(a - a*sin(e + f*x))^3*(c + c*sin
(e + f*x))^n,x)
```

```
[Out] -(B*a^3*(c*(sin(e + f*x) + 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14
*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

3.220 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$

Optimal. Leaf size=33

$$\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f}$$

```
[Out] a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f
```

Rubi [A]
time = 0.16, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3046, 2933}

$$\frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = (a^3c^3)\int \cos^6(e+fx$$

$$= \frac{a^3Bc^3\cos^7(e+fx)(a}{}$$

Mathematica [A]
time = 1.15, size = 66, normalized size = 2.00

$$\frac{Bc^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4
+ m)*Sin[e + f*x]),x]
[Out] (B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/f
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^3\left(B(-3+m)-B(4+m)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(35) = 70.
time = 0.39, size = 84, normalized size = 2.55

$$-\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)-\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 - 4
*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(31) = 62$.
time = 80.04, size = 898, normalized size = 27.21

$$\begin{cases} -\frac{Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}+\frac{6Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}-\frac{14Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}+\frac{14Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}-\frac{14Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}+\frac{14Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}-\frac{6Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f}+\frac{Bc^3\left(a+\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} & \text{for}\: f \neq 0 \\ x\left(B(m-3)-B(m+4)\sin(e)\right)\left(a\sin(e)+a\right)^m\left(-c\sin(e)+c\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3*(B*(-3+m)-B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Piecewise((-B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*
tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a + 2*a*tan(
e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
 f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/
2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)*
*2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/
2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*c**3*
(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**5
/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(
e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*t
an(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
 + 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/
2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/
2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a + 2*a*tan(e/2 +
f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**
8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2
)**2 + f) + B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
+ 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(m - 3) - B*(m + 4)*sin(e)
)*(a*sin(e) + a)**m*(-c*sin(e) + c)**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

Mupad [B]
time = 14.38, size = 61, normalized size = 1.85

$$\frac{B\,c^3\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - 3) - B*sin(e + f*x)*(m + 4))*(a + a*sin(e + f*x))^m*(c - c*sin(
e + f*x))^3,x)
```

```
[Out] (B*c^3*(a*(sin(e + f*x) + 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14*
sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$

Optimal. Leaf size=35

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{-3+m}}{f}$$

```
[Out] -a^3*B*c^3*cos(f*x+e)^7*(a-a*sin(f*x+e))^(-3+m)/f
```

Rubi [A]
time = 0.15, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {3046, 2933}

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{}$$

Mathematica [A]
time = 0.40, size = 61, normalized size = 1.74

$$\frac{Bc^3(a-a\sin(e+fx))^m(-14\cos(e+fx)+6\cos(3(e+fx))-14\sin(2(e+fx))+\sin(4(e+fx)))}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4
+ m)*Sin[e + f*x]),x]
```

```
[Out] (B*c^3*(a - a*Sin[e + f*x])^m*(-14*Cos[e + f*x] + 6*Cos[3*(e + f*x)] - 14*S
in[2*(e + f*x)] + Sin[4*(e + f*x)]))/(8*f)
```

Maple [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int (a-a\sin(fx+e))^m (c+c\sin(fx+e))^3 (B(-3+m)+B(4+m)\sin(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.
time = 0.37, size = 83, normalized size = 2.37

$$\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)+\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(-a\sin\left(fx+e\right)+a\right)^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] (3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) + (B*c^3*cos(f*x + e)^3 - 4*
B*c^3*cos(f*x + e))*sin(f*x + e))*(-a*sin(f*x + e) + a)^m/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(32) = 64$.
time = 81.10, size = 898, normalized size = 25.66

$$\begin{cases} \frac{Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{6Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{14Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} + \frac{14Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{14Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{14Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{6Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} - \frac{Bc^3\left(a-\frac{2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1}\right)^m}{f\tan^8\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+6f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+f} & \text{for}\: f \neq 0 \\ x\left(B(m-3)+B(m+4)\sin(e)\right)\left(-a\sin(e)+a\right)^m\left(c\sin(e)+c\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**3*(B*(-3+m)+B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Piecewise((B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*t
an(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*t
an(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a - 2*a*tan(e
/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e/2 +
f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2
 + f*x/2)**2 + f) + 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**
2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2
)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*c**3*(
a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**5/
(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4
+ 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e
/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)**8 + 4*f*ta
n(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f)
- 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2
 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2
 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a - 2*a*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*tan(e/2 + f*x/2)**8
 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)
**2 + f) - B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m/(
f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 +
 4*f*tan(e/2 + f*x/2)**2 + f), Ne(f, 0)), (x*(B*(m - 3) + B*(m + 4)*sin(e))
*(-a*sin(e) + a)**m*(c*sin(e) + c)**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

Mupad [B]
time = 14.36, size = 64, normalized size = 1.83

$$-\frac{B\,c^3\,{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - 3) + B*sin(e + f*x)*(m + 4))*(a - a*sin(e + f*x))^m*(c + c*sin(
e + f*x))^3,x)
```

```
[Out] -(B*c^3*(-a*(sin(e + f*x) - 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 1
4*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$

Optimal. Leaf size=36

$$\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f}$$

```
[Out] B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f
```

Rubi [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {3049}

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

Rule 3049

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a$$

Mathematica [A]
time = 0.34, size = 36, normalized size = 1.00

$$\frac{B\cos(e+fx)(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 +
 m + n)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \left(c - c\sin\left(fx + e\right)\right)^{n} \left(B(m - n) - B(1 + m + n)\sin\left(fx + e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] -integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m*
(-c*sin(f*x + e) + c)^n, x)
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.08

$$\frac{(a\sin\left(fx + e\right) + a)^{m}(-c\sin\left(fx + e\right) + c)^{n}B\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-B\left(\int \left(-m(a\sin\left(e + fx\right) + a)^{m}\left(-c\sin\left(e + fx\right) + c\right)^{n}\right) dx + \int n(a\sin\left(e + fx\right) + a)^{m}\left(-c\sin\left(e + fx\right) + c\right)^{n}\, dx + \int (a\sin\left(e + fx\right) + a)^{m}\left(-c\sin\left(e + fx\right) + c\right)^{n}\sin\left(e + fx\right) dx + \int m(a\sin\left(e + fx\right) + a)^{m}\left(-c\sin\left(e + fx\right) + c\right)^{n}\sin\left(e + fx\right) dx + \int n(a\sin\left(e + fx\right) + a)^{m}\left(-c\sin\left(e + fx\right) + c\right)^{n}\sin\left(e + fx\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n*(B*(m-n)-B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] -B*(Integral(-m*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Inte
gral(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Integral((a*s
in(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral(m*
(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integra
l(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9672 vs. $2(39) = 78$.
time = 45.40, size = 9672, normalized size = 268.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] (B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*ta
n(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*
e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*
tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) + 5
/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*
e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m*
log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1
/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(
1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x
 + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*ta
n(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x +
 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*ta
n(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 +
 abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x
 + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1
/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2
+ tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x +
e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/
2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*
```

```
tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x
+ e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x
 + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*ta
n(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + ta
n(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(-1/4*pi*m*sgn(2*a*tan(1/2*
f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*
a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/
2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x
 + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^
3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor
(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn(
4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x +
 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*
e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*e^(-m*log(
2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e
)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*
f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x + 1
/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/
2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + abs
(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1
/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e
) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + ta
n(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/
2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2
 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(
f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)
^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)
^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1
/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*ta
n(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/
2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*
x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*sin(2*pi*m*floor(-1/8*sgn(4*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4
*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*...
```

Mupad [B]

time = 13.54, size = 36, normalized size = 1.00

$$\frac{B\,\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - n) - B*sin(e + f*x)*(m + n + 1))*(a + a*sin(e + f*x))^m*(c - c*
sin(e + f*x))^n,x)
```

```
[Out] (B*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/f
```

3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$

Optimal. Leaf size=37

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c+c\sin(e+fx))^n}{f}$$

```
[Out] -B*cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n/f
```

Rubi [A]
time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = $\{3049\}$

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

Rule 3049

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a}{}$$

Mathematica [A]
time = 0.32, size = 37, normalized size = 1.00

$$-\frac{B\cos(e+fx)(c(1+\sin(e+fx)))^n(a-a\sin(e+fx))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 +
 m + n)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^n*(a - a*Sin[e + f*x])^m)/f)
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \left(a-a\sin \left(fx+e\right)\right)^{m} \left(c+c\sin \left(fx+e\right)\right)^{n} \left(B(m-n)+B(1+m+n)\sin \left(fx+e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m*
(c*sin(f*x + e) + c)^n, x)
```

Fricas [A]
time = 0.40, size = 40, normalized size = 1.08

$$-\frac{(-a\sin \left(fx+e\right)+a)^{m}(c\sin \left(fx+e\right)+c)^{n}B\cos \left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] -(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$B\left(\int m(-a\sin \left(e+fx\right)+a)^{m} \left(c\sin \left(e+fx\right)+c\right)^{n}\, dx+\int \left(-n(-a\sin \left(e+fx\right)+a)^{m} \left(c\sin \left(e+fx\right)+c\right)^{n}\right)\, dx+\int (-a\sin \left(e+fx\right)+a)^{m} \left(c\sin \left(e+fx\right)+c\right)^{n}\sin \left(e+fx\right)dx+\int m(-a\sin \left(e+fx\right)+a)^{m} \left(c\sin \left(e+fx\right)+c\right)^{n}\sin \left(e+fx\right)dx+\int n(-a\sin \left(e+fx\right)+a)^{m} \left(c\sin \left(e+fx\right)+c\right)^{n}\sin \left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**n*(B*(m-n)+B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] B*(Integral(m*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integr
al(-n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integral((-a*s
in(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral(m*(
-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral
(n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9673 vs. $2(40) = 80$.
time = 45.27, size = 9673, normalized size = 261.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] -(B*cos(2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2
*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4
*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) +
5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*
tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi*n)*e^(-m
*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan
(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*t
an(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x
+ 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*t
an(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2
+ abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x +
1/2*e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2
 + tan(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x +
 e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1
/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2
```

```
*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x
 + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*
x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(1/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + t
an(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*tan(-1/4*pi*m*sgn(2*a*tan(1/2
*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2
*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1
/2*f*x + 1/2*e)) - 1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*
x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)
^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floo
r(f*x/pi + e/pi + 1/2) + 1/2*pi*n*floor(f*x/pi + e/pi + 1/2) - 1/4*pi*m*sgn
(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x
+ 1/2*e)) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2
*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*e^(-m*log
(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*
e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1
/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/
2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1
/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x + 1/2*e)^2 + ab
s(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x +
1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*
e) + 2))*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + t
an(1/2*f*x + 1/2*e)^2 + 1)) + n*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^
2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan
(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e
)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e
)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/2*f*x +
1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*t
an(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1
/2*f*x + 1/2*e) + 2))*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + tan(f
*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1)))*sin(2*pi*n*floor(-1/8*sgn(4*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) +
4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8...
```

Mupad [B]
time = 13.50, size = 37, normalized size = 1.00

$$-\frac{B\,\cos\left(e+f\,x\right)\,{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,{\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - n) + B*sin(e + f*x)*(m + n + 1))*(a - a*sin(e + f*x))^m*(c + c*
sin(e + f*x))^n,x)
```

```
[Out] -(B*cos(e + f*x)*(-a*(sin(e + f*x) - 1))^m*(c*(sin(e + f*x) + 1))^n)/f
```

3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos^7(c+dx)}{7d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^3A\cos(\ldots}{}$$

```
[Out] 1/8*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+3/5*a^3*A*cos(d*x+c)^5/d-1/7*a^3*A*cos
(d*x+c)^7/d-1/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/12*a^3*A*cos(d*x+c)*sin(d*x
+c)^3/d+1/3*a^3*A*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]

time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3045, 2713, 2715, 8}

$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5*
d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[c +
 d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin^3(c+dx)+2a^3A\sin^4(c+dx)-2a^3\right.$$
$$=\left(a^3A\right)\int \sin^3(c+dx)\,dx-\left(a^3A\right)\int \sin^7(c+dx$$
$$=-\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}+\frac{a^3A\cos(c+d}{3}$$
$$=-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3}{}$$
$$=\frac{3}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+d}{5d}$$
$$=\frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+d}{5d}$$

Mathematica [A]
time = 0.11, size = 87, normalized size = 0.62

$$\frac{a^3A(840c+840dx-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c+dx))-15\cos(7(c+dx))-210\sin(2(c+dx))-210\sin(4(c+dx))+70\sin(6(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Co
s[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c
+ d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A]
time = 0.24, size = 158, normalized size = 1.13

method	result
risch	$\frac{a^3xA}{8}-\frac{13a^3\cos(dx+c)A}{64d}-\frac{a^3\cos(7dx+7c)A}{448d}+\frac{\sin(6dx+6c)a^3A}{96d}+\frac{7a^3\cos(5dx+5c)A}{320d}-\frac{\sin(4dx+4c)a^3A}{32d}-$ 5

derivativedivides	$\frac{\frac{a^3 A\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}-2a^3A\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos}{6}\right.}{d}$
default	$\frac{\frac{a^3 A\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}-2a^3A\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos}{6}\right.}{d}$
norman	$-\frac{44a^3A}{105d}+\frac{a^3xA}{8}-\frac{4a^3A\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24a^3A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8a^3A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{44a^3A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{52a^3A(\tan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/7*a^3*A*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*
x+c)-2*a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+
c)+5/16*d*x+5/16*c)+2*a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+
3/8*d*x+3/8*c)-1/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.28, size = 157, normalized size = 1.12

$$-\frac{96\left(5\cos(dx+c)^7-21\cos(dx+c)^5+35\cos(dx+c)^3-35\cos(dx+c)\right)Aa^3-1120\left(\cos(dx+c)^3-3\cos(dx+c)\right)Aa^3+35\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)Aa^3-210\left(12dx+12c+\sin(4dx+4c)-8\sin(2dx+2c)\right)Aa^3}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*
cos(d*x + c))*A*a^3 - 1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 35*(4*
sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*
c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a
^3)/d
```

Fricas [A]
time = 0.37, size = 105, normalized size = 0.75

$$-\frac{120\,Aa^3\cos(dx+c)^7-504\,Aa^3\cos(dx+c)^5+560\,Aa^3\cos(dx+c)^3-105\,Aa^3dx-35\left(8\,Aa^3\cos(dx+c)^5-14\,Aa^3\cos(dx+c)^3+3\,Aa^3\cos(dx+c)\right)\sin(dx+c)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*cos
(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d*x
 + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(131) = 262$.
time = 0.66, size = 440, normalized size = 3.14

$$\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{8} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2} - \frac{5Aa^3x\cos^6(c+dx)}{8} + \frac{3Aa^3x\cos^4(c+dx)}{4} + \frac{Aa^3\sin^6(c+dx)\cos(c+dx)}{d} + \frac{11Aa^3\sin^5(c+dx)\cos(c+dx)}{8d} + \frac{2Aa^3\sin^4(c+dx)\cos^3(c+dx)}{d} + \frac{5Aa^3\sin^3(c+dx)\cos^3(c+dx)}{3d} - \frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{4d} + \frac{8Aa^3\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{Aa^3\sin^2(c+dx)\cos(c+dx)}{d} + \frac{5Aa^3\sin(c+dx)\cos^5(c+dx)}{8d} - \frac{3Aa^3\sin(c+dx)\cos^3(c+dx)}{4d} + \frac{16Aa^3\cos^7(c+dx)}{35d} - \frac{2Aa^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/8 - 15*A*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/8 + 3*A*a**3*x*sin(c + d*x)**4/4 - 15*A*a**3*x*sin(c + d*x)**2*
cos(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 - 5*A*a**3
*x*cos(c + d*x)**6/8 + 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*sin(c + d*x)**
6*cos(c + d*x)/d + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 2*A*a**3*
sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**
3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin(c + d*
x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 5*A*a
**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)
**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d*x)**3/(3*
d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, True))
```

Giac [A]
time = 0.47, size = 131, normalized size = 0.94

$$\frac{1}{8}Aa^3x - \frac{Aa^3\cos(7dx+7c)}{448d} + \frac{7Aa^3\cos(5dx+5c)}{320d} - \frac{5Aa^3\cos(3dx+3c)}{192d} - \frac{13Aa^3\cos(dx+c)}{64d} + \frac{Aa^3\sin(6dx+6c)}{96d} - \frac{Aa^3\sin(4dx+4c)}{32d} - \frac{Aa^3\sin(2dx+2c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c)
/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A*a
^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2*d*
x + 2*c)/d
```

Mupad [B]
time = 15.30, size = 300, normalized size = 2.14

$$\frac{A\,a^3\left(105\,c-210\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2464\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1400\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4032\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6790\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2240\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-14560\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-6790\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-3360\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+1400\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+210\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+105\,d\,x+735\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(c+d\,x\right)+2205\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(c+d\,x\right)+3675\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(c+d\,x\right)+3675\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(c+d\,x\right)+2205\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(c+d\,x\right)+735\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\,\left(c+d\,x\right)+105\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\,\left(c+d\,x\right)-352\right)}{840\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*(105*c - 210*tan(c/2 + (d*x)/2) - 2464*tan(c/2 + (d*x)/2)^2 - 1400*t
an(c/2 + (d*x)/2)^3 - 4032*tan(c/2 + (d*x)/2)^4 + 6790*tan(c/2 + (d*x)/2)^5
 + 2240*tan(c/2 + (d*x)/2)^6 - 14560*tan(c/2 + (d*x)/2)^8 - 6790*tan(c/2 +
(d*x)/2)^9 - 3360*tan(c/2 + (d*x)/2)^10 + 1400*tan(c/2 + (d*x)/2)^11 + 210*
tan(c/2 + (d*x)/2)^13 + 105*d*x + 735*tan(c/2 + (d*x)/2)^2*(c + d*x) + 2205
*tan(c/2 + (d*x)/2)^4*(c + d*x) + 3675*tan(c/2 + (d*x)/2)^6*(c + d*x) + 367
5*tan(c/2 + (d*x)/2)^8*(c + d*x) + 2205*tan(c/2 + (d*x)/2)^10*(c + d*x) + 7
35*tan(c/2 + (d*x)/2)^12*(c + d*x) + 105*tan(c/2 + (d*x)/2)^14*(c + d*x) -
352))/(840*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{3}{16}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{3a^3A\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d}$$

```
[Out] 3/16*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+2/5*a^3*A*cos(d*x+c)^5/d-3/16*a^3*A*c
os(d*x+c)*sin(d*x+c)/d+5/24*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^3*A*cos(d
*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3045, 2715, 8, 2713}

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)/
(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
```

```
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin^2(c+dx)+2a^3A\sin^3(c+dx)-2a\right.$$
$$= \left(a^3A\right)\int \sin^2(c+dx)\,dx-\left(a^3A\right)\int \sin^6(c+d$$
$$= -\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^3A\cos(c+d}{6}$$
$$= \frac{1}{2}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+}{5d}$$
$$= \frac{1}{2}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+}{5d}$$
$$= \frac{3}{16}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+}{5d}$$

Mathematica [A]

time = 0.08, size = 77, normalized size = 0.64

$$\frac{a^3A(180c+180dx-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx))-15\sin(2(c+dx))-45\sin(4(c+dx))+5\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5
*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

Maple [A]

time = 0.17, size = 136, normalized size = 1.12

method	result
risch	$\frac{3a^3xA}{16}-\frac{a^3\cos(dx+c)A}{4d}+\frac{\sin(6dx+6c)a^3A}{192d}+\frac{a^3\cos(5dx+5c)A}{40d}-\frac{3\sin(4dx+4c)a^3A}{64d}-\frac{a^3\cos(3dx+3c)A}{24d}-$
derivativedivides	$\frac{-a^3A\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^3A\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(d}{5}}{d}$

default	$\frac{-a^3 A\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^3A\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+ \cdots}{5}}{d}$
norman	$-\frac{8a^3A}{15d}+\frac{3a^3xA}{16}-\frac{16a^3A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a^3A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{8a^3A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{13a^3A\left(\tan^3\left(\frac{dx}{2} \cdots\right.\right.}{24d} \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c
)+5/16*d*x+5/16*c)+2/5*a^3*A*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
-2/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)+a^3*A*(-1/2*sin(d*x+c)*cos(d*x+c)+1/
2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 138, normalized size = 1.14

$$\frac{128\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)Aa^3+640\left(\cos(dx+c)^3-3\cos(dx+c)\right)Aa^3-5\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)Aa^3+240\left(2dx+2c-\sin(2dx+2c)\right)Aa^3}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 +
 640*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 + 60
*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x
+ 2*c - sin(2*d*x + 2*c))*A*a^3)/d
```

Fricas [A]
time = 0.51, size = 91, normalized size = 0.75

$$\frac{96\,Aa^3\cos(dx+c)^5-160\,Aa^3\cos(dx+c)^3+45\,Aa^3dx+5\left(8\,Aa^3\cos(dx+c)^5-26\,Aa^3\cos(dx+c)^3+9\,Aa^3\cos(dx+c)\right)\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x +
5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c))
*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(119) = 238$.
time = 0.45, size = 359, normalized size = 2.97

$$\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{16} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{Aa^3x\sin^2(c+dx)}{2} - \frac{5Aa^3x\cos^6(c+dx)}{16} + \frac{Aa^3x\cos^2(c+dx)}{2} + \frac{11Aa^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{2Aa^3\sin^4(c+dx)\cos(c+dx)}{d} + \frac{5Aa^3\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{8Aa^3\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{2Aa^3\sin^2(c+dx)\cos(c+dx)}{d} + \frac{5Aa^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{Aa^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{16Aa^3\cos^5(c+dx)}{15d} - \frac{4Aa^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/16 - 15*A*a**3*x*sin(c + d*x)**4*cos
(c + d*x)**2/16 - 15*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + A*a**3*x
*sin(c + d*x)**2/2 - 5*A*a**3*x*cos(c + d*x)**6/16 + A*a**3*x*cos(c + d*x)*
*2/2 + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 2*A*a**3*sin(c + d*x
)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*A*
a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*A*a**3*sin(c + d*x)**2*cos(c
 + d*x)/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a**3*sin(c + d
*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**3*cos(c
+ d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**2,
 True))
```

Giac [A]
time = 0.55, size = 113, normalized size = 0.93

$$\frac{3}{16}Aa^3x + \frac{Aa^3\cos(5dx+5c)}{40d} - \frac{Aa^3\cos(3dx+3c)}{24d} - \frac{Aa^3\cos(dx+c)}{4d} + \frac{Aa^3\sin(6dx+6c)}{192d} - \frac{3Aa^3\sin(4dx+4c)}{64d} - \frac{Aa^3\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)/
d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^3*
sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 15.29, size = 256, normalized size = 2.12

$$\frac{Aa^3\left(45c - 90\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - 768\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 130\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 1500\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 - 1280\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 1500\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - 1920\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 130\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 + 90\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + 45dx + 270\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(c+dx) + 675\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(c+dx) + 900\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(c+dx) + 675\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(c+dx) + 270\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(c+dx) + 45\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}(c+dx) - 128\right)}{240d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*(45*c - 90*tan(c/2 + (d*x)/2) - 768*tan(c/2 + (d*x)/2)^2 + 130*tan(c
/2 + (d*x)/2)^3 + 1500*tan(c/2 + (d*x)/2)^5 - 1280*tan(c/2 + (d*x)/2)^6 - 1
500*tan(c/2 + (d*x)/2)^7 - 1920*tan(c/2 + (d*x)/2)^8 - 130*tan(c/2 + (d*x)/
2)^9 + 90*tan(c/2 + (d*x)/2)^11 + 45*d*x + 270*tan(c/2 + (d*x)/2)^2*(c + d*
x) + 675*tan(c/2 + (d*x)/2)^4*(c + d*x) + 900*tan(c/2 + (d*x)/2)^6*(c + d*x
) + 675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 270*tan(c/2 + (d*x)/2)^10*(c + d*x
) + 45*tan(c/2 + (d*x)/2)^12*(c + d*x) - 128))/(240*d*(tan(c/2 + (d*x)/2)^2
 + 1)^6)
```

3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{1}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}$$

```
[Out] 1/4*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+1/5*a^3*A*cos(d*x+c)^5/d-1/4*a^3*A*cos
(d*x+c)*sin(d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3045, 2718, 2715, 8, 2713}

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*
x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin(c+dx)+2a^3A\sin^2(c+dx)-2a^3A\right.$$
$$= \left(a^3A\right)\int \sin(c+dx)\,dx - \left(a^3A\right)\int \sin^5(c+dx)$$
$$= -\frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cos(c+dx)\sin(c+dx}{d}$$
$$= a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d}$$
$$= \frac{1}{4}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx}{5d}$$

Mathematica [A]
time = 0.33, size = 55, normalized size = 0.57

$$\frac{a^3A(-90\cos(c+dx)-25\cos(3(c+dx))+3(20dx+\cos(5(c+dx))-5\sin(4(c+dx))))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*d*x + Cos[5*(c + d*x
)] - 5*Sin[4*(c + d*x)])))/(240*d)
```

Maple [A]
time = 0.14, size = 117, normalized size = 1.22

method	result
risch	$\frac{a^3xA}{4}-\frac{3a^3\cos(dx+c)A}{8d}+\frac{a^3\cos(5dx+5c)A}{80d}-\frac{\sin(4dx+4c)a^3A}{16d}-\frac{5a^3\cos(3dx+3c)A}{48d}$
derivativedivides	$\frac{\frac{a^3A\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2a^3A\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^3A\left(-\frac{\sin(\right.}{}}{d}$

default	$\frac{\frac{a^3 A\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2a^3 A\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^3 A\left(-\frac{\sin(dx}{}\right.}{d}$
norman	$-\frac{14a^3 A}{15d}+\frac{a^3 x A}{4}-\frac{4a^3 A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a^3 A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3 A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3 A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{a^3 A\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^3*A*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-2*a^3*A*(-1/4
*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a^3*A*(-1/2*sin(
d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-a^3*A*cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 112, normalized size = 1.17

$$\frac{16\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)Aa^3-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\sin(2\,dx+2\,c)\right)Aa^3+120\left(2\,dx+2\,c-\sin(2\,dx+2\,c)\right)Aa^3-240\,Aa^3\cos(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 -
15*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d
*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*cos(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 77, normalized size = 0.80

$$\frac{12\,Aa^3\cos(dx+c)^5-40\,Aa^3\cos(dx+c)^3+15\,Aa^3dx-15\left(2\,Aa^3\cos(dx+c)^3-Aa^3\cos(dx+c)\right)\sin(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 15
*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(88) = 176.
time = 0.30, size = 267, normalized size = 2.78

$$\begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{4}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2}+Aa^3x\sin^2(c+dx)-\frac{3Aa^3x\cos^4(c+dx)}{4}+Aa^3x\cos^2(c+dx)+\frac{Aa^3\sin^4(c+dx)\cos(c+dx)}{d}+\frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{4Aa^3\sin^2(c+dx)\cos^3(c+dx)}{3d}+\frac{3Aa^3\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{Aa^3\sin(c+dx)\cos(c+dx)}{d}+\frac{8Aa^3\cos^5(c+dx)}{15d}-\frac{Aa^3\cos(c+dx)}{d} & \text{for}\; d\neq 0 \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 + A*
a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**3*s
in(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*x)**
3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c + d*x)
*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*x)/d,
Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))
```

Giac [A]
time = 0.45, size = 77, normalized size = 0.80

$$\frac{1}{4}\,Aa^3x+\frac{Aa^3\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,Aa^3\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,Aa^3\cos\left(dx+c\right)}{8\,d}-\frac{Aa^3\sin\left(4\,dx+4\,c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/d
 - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 15.03, size = 292, normalized size = 3.04

$$\frac{A\,a^3\,x}{4}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{12}-\frac{A\,a^3\,\left(75\,c+75\,d\,x-120\right)}{60}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{12}-\frac{A\,a^3\,\left(75\,c+75\,d\,x-160\right)}{60}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{6}-\frac{A\,a^3\,\left(150\,c+150\,d\,x-80\right)}{60}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{6}-\frac{A\,a^3\,\left(150\,c+150\,d\,x-480\right)}{60}\right)+\frac{A\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-3\,A\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+3\,A\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-\frac{A\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{2}+\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{60}-\frac{A\,a^3\,\left(15\,c+15\,d\,x-56\right)}{60}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*x)/4 - (tan(c/2 + (d*x)/2)^8*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(7
5*c + 75*d*x - 120))/60) + tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/12
 - (A*a^3*(75*c + 75*d*x - 160))/60) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c +
 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 80))/60) + tan(c/2 + (d*x)/2)^6*((A
*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 480))/60) + (A*a^3*tan(
c/2 + (d*x)/2))/2 - 3*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c/2 + (d*x)/
2)^7 - (A*a^3*tan(c/2 + (d*x)/2)^9)/2 + (A*a^3*(15*c + 15*d*x))/60 - (A*a^3
*(15*c + 15*d*x - 56))/60)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.227 $\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$\frac{5}{8}a^3Ax-\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{A\cos^3(c+dx)\,(a^3+a^3\sin(c+dx))}{4d}$$

```
[Out] 5/8*a^3*A*x-5/12*a^3*A*cos(d*x+c)^3/d+5/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/4
*A*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2815, 2757, 2748, 2715, 8}

$$-\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\,(a^3\sin(c+dx)+a^3)}{4d}+\frac{5a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*x)/8 - (5*a^3*A*Cos[c + d*x]^3)/(12*d) + (5*a^3*A*Cos[c + d*x]*Sin
[c + d*x])/(8*d) - (A*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = (aA)\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$= -\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2A\right)\int \cos^2(c$$
$$= -\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}$$
$$= -\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{A\,\mathrm{c}}{}$$
$$= \frac{5}{8}a^3Ax-\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx}{8d}$$

Mathematica [A]
time = 0.24, size = 54, normalized size = 0.66

$$\frac{a^3A(60dx-48\cos(c+dx)-16\cos(3(c+dx))+24\sin(2(c+dx))-3\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
[Out] (a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x)
] - 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.12, size = 89, normalized size = 1.09

method	result

risch	$\frac{5a^3xA}{8}-\frac{a^3\cos(dx+c)A}{2d}-\frac{\sin(4dx+4c)a^3A}{32d}-\frac{a^3\cos(3dx+3c)A}{6d}+\frac{\sin(2dx+2c)a^3A}{4d}$
derivativedivides	$\frac{-a^3A\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^3A\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^3A\cos(dx+c)+a^3A(dx+c)}{d}$
default	$\frac{-a^3A\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^3A\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^3A\cos(dx+c)+a^3A(dx+c)}{d}$
norman	$-\frac{4a^3A}{3d}+\frac{5a^3xA}{8}-\frac{4a^3A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4a^3A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^3A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{11a^3A\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2
/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)-2*a^3*A*cos(d*x+c)+a^3*A*(d*x+c))
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.05

$$-\frac{64\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)Aa^3+3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\sin\left(2\,dx+2\,c\right)\right)Aa^3-96\left(dx+c\right)Aa^3+192\,Aa^3\cos\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3*c
os(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 63, normalized size = 0.77

$$-\frac{16\,Aa^3\cos\left(dx+c\right)^3-15\,Aa^3dx+3\left(2\,Aa^3\cos\left(dx+c\right)^3-5\,Aa^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3 -
 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.
time = 0.20, size = 196, normalized size = 2.39

$$\begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{8}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}-\frac{3Aa^3x\cos^4(c+dx)}{8}+Aa^3x+\frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{2Aa^3\sin^2(c+dx)\cos(c+dx)}{d}+\frac{3Aa^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{4Aa^3\cos^3(c+dx)}{3d}-\frac{2Aa^3\cos(c+dx)}{d} & \text{for } d\neq 0 \\ x\left(-A\sin(c)+A\right)\left(a\sin(c)+a\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c + d
*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*A*a
**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d) - 2
*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3, Tr
ue))
```

Giac [A]

time = 0.45, size = 77, normalized size = 0.94

$$\frac{5}{8}\,Aa^3x-\frac{Aa^3\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{Aa^3\cos\left(dx+c\right)}{2\,d}-\frac{Aa^3\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{Aa^3\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/3
2*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]

time = 15.19, size = 250, normalized size = 3.05

$$\frac{5\,A\,a^3\,x}{8}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{6}-\frac{A\,a^3\,\left(60\,c+60\,d\,x-32\right)}{24}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{6}-\frac{A\,a^3\,\left(60\,c+60\,d\,x-96\right)}{24}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{4}-\frac{A\,a^3\,\left(90\,c+90\,d\,x-96\right)}{24}\right)-\frac{3\,A\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{11\,A\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{4}+\frac{11\,A\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}+\frac{3\,A\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+\frac{A\,a^3\,\left(15\,c+15\,d\,x\right)}{24}-\frac{A\,a^3\,\left(15\,c+15\,d\,x-32\right)}{24}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*A*a^3*x)/8 - (tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(
60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/6
- (A*a^3*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c + 1
5*d*x))/4 - (A*a^3*(90*c + 90*d*x - 96))/24) - (3*A*a^3*tan(c/2 + (d*x)/2))
/4 - (11*A*a^3*tan(c/2 + (d*x)/2)^3)/4 + (11*A*a^3*tan(c/2 + (d*x)/2)^5)/4
+ (3*A*a^3*tan(c/2 + (d*x)/2)^7)/4 + (A*a^3*(15*c + 15*d*x))/24 - (A*a^3*(1
5*c + 15*d*x - 32))/24)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=76

$$a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3A\cos(c+dx)}{d}-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{d}$$

```
[Out] a^3*A*x-a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-1/3*a^3*A*cos(d*x+c)
^3/d+a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3045, 3855, 2715, 8, 2713}

$$-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*A
*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
```

```
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(2a^3A+a^3A\csc(c+dx)-2a^3A\sin^2(c+dx\right.$$

$$= 2a^3Ax+\left(a^3A\right)\int \csc(c+dx)\,dx-\left(a^3A\right)\int \sin$$

$$= 2a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3A\cos(c}{}$$

$$= a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3A\cos(c-}{d}$$

Mathematica [A]
time = 0.12, size = 74, normalized size = 0.97

$$\frac{a^3A\left(9\cos(c+dx)-\cos(3(c+dx))+6\left(-2c+2dx-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin(2(c+dx))\right)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
[Out] (a^3*A*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(-2*c + 2*d*x - 2*Log[Cos[(c
+ d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12*d)
```

Maple [A]
time = 0.23, size = 88, normalized size = 1.16

method	result
derivativedivides	$\frac{2a^3A(dx+c)+a^3A\ln(\csc(dx+c)-\cot(dx+c))-2a^3A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3A\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
default	$\frac{2a^3A(dx+c)+a^3A\ln(\csc(dx+c)-\cot(dx+c))-2a^3A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3A\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
risch	$a^3xA+\frac{3a^3A\,e^{i(dx+c)}}{8d}+\frac{3a^3A\,e^{-i(dx+c)}}{8d}+\frac{a^3A\ln(e^{i(dx+c)}-1)}{d}-\frac{a^3A\ln(e^{i(dx+c)}+1)}{d}-\frac{a^3\cos(3dx+3c)A}{12d}-$

norman	$\frac{a^3xA+a^3xA\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4a^3A}{3d}+\frac{4a^3A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^3A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^3*A*(d*x+c)+a^3*A*ln(csc(d*x+c)-cot(d*x+c))-2*a^3*A*(-1/2*sin(d*x+
c)*cos(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 85, normalized size = 1.12

$$-\frac{2\left(\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)\right)Aa^{3}+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-12\left(dx+c\right)Aa^{3}+6\,Aa^{3}\log\left(\cot\left(dx+c\right)+\csc\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(2*d*x + 2*c - sin(2*d*
x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*log(cot(d*x + c) + csc(d*x +
 c)))/d
```

Fricas [A]
time = 0.42, size = 92, normalized size = 1.21

$$-\frac{2\,Aa^{3}\cos\left(dx+c\right)^{3}-6\,Aa^{3}dx-6\,Aa^{3}\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,Aa^{3}\cos\left(dx+c\right)+3\,Aa^{3}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,Aa^{3}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x +
 c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^3*
log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-Aa^{3}\left(\int\left(-2\sin\left(c+dx\right)\csc\left(c+dx\right)\right)\,dx+\int 2\sin^{3}\left(c+dx\right)\csc\left(c+dx\right)\,dx+\int\sin^{4}\left(c+dx\right)\csc\left(c+dx\right)\,dx+\int\left(-\csc\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*x
)**3*csc(c + d*x), x) + Integral(sin(c + d*x)**4*csc(c + d*x), x) + Integra
l(-csc(c + d*x), x))
```

Giac [A]
time = 0.42, size = 107, normalized size = 1.41

$$\frac{3\,(dx+c)Aa^3+3\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Aa^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*A*a^
3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1/2
*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 13.23, size = 212, normalized size = 2.79

$$\frac{-2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,A\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{4\,A^2\,a^6}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] ((4*A*a^3)/3 + 2*A*a^3*tan(c/2 + (d*x)/2) + 4*A*a^3*tan(c/2 + (d*x)/2)^2 -
2*A*a^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x
)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*A*a^3*atan((4*A^2*a^6)/(4*A^2*a^6
- 4*A^2*a^6*tan(c/2 + (d*x)/2)) + (4*A^2*a^6*tan(c/2 + (d*x)/2))/(4*A^2*a^6
 - 4*A^2*a^6*tan(c/2 + (d*x)/2))))/d + (A*a^3*log(tan(c/2 + (d*x)/2)))/d
```

3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2}a^3Ax-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -1/2*a^3*A*x-2*a^3*A*arctanh(cos(d*x+c))/d+2*a^3*A*cos(d*x+c)/d-a^3*A*cot(d
*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3029, 2788, 3855, 3852, 8, 2718, 2715}

$$\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{1}{2}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a^3*A*x) - (2*a^3*A*ArcTanh[Cos[c + d*x]])/d + (2*a^3*A*Cos[c + d*x])
/d - (a^3*A*Cot[c + d*x])/d + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
 p/2, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = (aA)\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$$

$$= \frac{A\int (2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\,\text{si}}{a}$$

$$= (a^3A)\int \csc^2(c+dx)\,dx - (a^3A)\int \sin^2(c+$$

$$= -\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^3A\cos(c+d}{d}$$

$$= -\frac{1}{2}a^3Ax - \frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^3A}{}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.97

$$\frac{a^3A\left(-2c-2dx+8\cos(c)\cos(dx)-4\cot(c+dx)-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\sin(c)\sin(dx)+\sin(2(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-2*c - 2*d*x + 8*Cos[c]*Cos[d*x] - 4*Cot[c + d*x] - 8*Log[Cos[(c +
d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] - 8*Sin[c]*Sin[d*x] + Sin[2*(c + d*x)]))
/(4*d)
```

Maple [A]
time = 0.21, size = 80, normalized size = 1.01

method	result
derivativedivides	$\frac{2a^3A\ln(\csc(dx+c)-\cot(dx+c))-a^3A\cot(dx+c)+2a^3A\cos(dx+c)-a^3A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{2a^3A\ln(\csc(dx+c)-\cot(dx+c))-a^3A\cot(dx+c)+2a^3A\cos(dx+c)-a^3A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$-\frac{a^3xA}{2}-\frac{ia^3A\,e^{2i(dx+c)}}{8d}+\frac{a^3A\,e^{i(dx+c)}}{d}+\frac{a^3A\,e^{-i(dx+c)}}{d}+\frac{ia^3A\,e^{-2i(dx+c)}}{8d}-\frac{2ia^3A}{d\left(e^{2i(dx+c)}-1\right)}-\frac{2a^3A\ln(e^{i\cdots}}{d}$
norman	$\frac{\frac{4a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a^3A}{2d}+\frac{4a^3A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12a^3A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12a^3A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a^3A(t\cdots}{}}{t\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3*A*cot(d*x+c)+2*a^3*A*cos(d*x+c)-
a^3*A*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 83, normalized size = 1.05

$$-\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))Aa^3+4\,Aa^3(\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1))-8\,Aa^3\cos(dx+c)+\frac{4\,Aa^3}{\tan(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) - 8*A*a^3*cos(d*x + c) + 4*A*a^3/tan(d*x + c))/
d
```

Fricas [A]
time = 0.42, size = 111, normalized size = 1.41

$$-\frac{Aa^3\cos(dx+c)^3+2\,Aa^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,Aa^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+Aa^3\cos(dx+c)+(Aa^3dx-4\,Aa^3\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x + c)
 + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3\left(\int\left(-2\sin\left(c+dx\right)\csc^2\left(c+dx\right)\right)\,dx+\int 2\sin^3\left(c+dx\right)\csc^2\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\csc^2\left(c+dx\right)dx+\int\left(-\csc^2\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c +
d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**2, x)
+ Integral(-csc(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.
time = 0.44, size = 153, normalized size = 1.94

$$-\frac{(dx+c)Aa^3-4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,Aa^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*((d*x + c)*A*a^3 - 4*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - A*a^3*tan(
1/2*d*x + 1/2*c) + (4*A*a^3*tan(1/2*d*x + 1/2*c) + A*a^3)/tan(1/2*d*x + 1/2
*c) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - A*
a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 13.19, size = 226, normalized size = 2.86

$$\frac{A\,a^3\,\mathrm{atan}\left(\frac{A^2\,a^6}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+A\,a^3}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{2\,A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (A*a^3*atan((A^2*a^6)/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2)) - (4*A^2*a^6
*tan(c/2 + (d*x)/2))/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2))))/d - (A*a^3
- 8*A*a^3*tan(c/2 + (d*x)/2) - 8*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c
/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(
c/2 + (d*x)/2)^5)) + (2*A*a^3*log(tan(c/2 + (d*x)/2)))/d + (A*a^3*tan(c/2 +
 (d*x)/2))/(2*d)
```

3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-2a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -2*a^3*A*x-1/2*a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-2*a^3*A*cot(d
*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3045, 3852, 8, 3853, 3855, 2718}

$$\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d -
 (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-2a^3A+2a^3A\csc^2(c+dx)+a^3A\csc^3(c+\right.$$

$$= -2a^3Ax + \left(a^3A\right)\int \csc^3(c+dx)\,dx - \left(a^3A\right)$$

$$= -2a^3Ax + \frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cot(c+dx)}{2d}$$

$$= -2a^3Ax - \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3A\cos}{}$$

Mathematica [A]
time = 0.04, size = 142, normalized size = 1.82

$$-2a^3Ax+\frac{a^3A\cos(c)\cos(dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a^3A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3A\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A*
Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*Lo
g[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*Sin[
c]*Sin[d*x])/d
```

Maple [A]
time = 0.23, size = 78, normalized size = 1.00

method	result

derivativedivides	$\frac{-2a^3A\cot(dx+c)+a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2a^3A(dx+c)+a^3A\cos(dx+c)}{d}$
default	$\frac{-2a^3A\cot(dx+c)+a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2a^3A(dx+c)+a^3A\cos(dx+c)}{d}$
risch	$-2a^3xA+\frac{a^3A\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{a^3A\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{a^3A\left(\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{i(dx+c)}-4i\mathrm{e}^{2i(dx+c)}+4i\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}+\frac{a^3A\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{2d}$
norman	$\frac{\frac{a^3A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^3A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3A}{8d}+\frac{3a^3A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^3A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{27a^3A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a^3A}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2*a^3*A*cot(d*x+c)+a^3*A*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c
)-cot(d*x+c)))-2*a^3*A*(d*x+c)+a^3*A*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 90, normalized size = 1.15

$$-\frac{8\,(dx+c)Aa^3-Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-4\,Aa^3\cos\left(dx+c\right)+\frac{8\,Aa^3}{\tan(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*(8*(d*x + c)*A*a^3 - A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 4*A*a^3*cos(d*x + c) + 8*A*a^3
/tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(74) = 148$.
time = 0.40, size = 152, normalized size = 1.95

$$-\frac{8\,Aa^3dx\cos\left(dx+c\right)^2-4\,Aa^3\cos\left(dx+c\right)^3-8\,Aa^3dx-8\,Aa^3\cos\left(dx+c\right)\sin\left(dx+c\right)+2\,Aa^3\cos\left(dx+c\right)+\left(Aa^3\cos\left(dx+c\right)^2-Aa^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(Aa^3\cos\left(dx+c\right)^2-Aa^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x - 8
*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x +
c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^3)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3\left(\int\left(-2\sin\left(c+dx\right)\csc^3\left(c+dx\right)\right)\,dx+\int 2\sin^3\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int\left(-\csc^3\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**3, x) + Integral(2*sin(c +
d*x)**3*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**3, x)
+ Integral(-csc(c + d*x)**3, x))
```

Giac [A]
time = 0.46, size = 137, normalized size = 1.76

$$\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\left(dx+c\right)Aa^3+4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(ta
n(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x +
1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 13.50, size = 220, normalized size = 2.82

$$\frac{A\,a^3\,\left(\frac{\cos\left(c+d\,x\right)}{2}-4\,\mathrm{atan}\left(\frac{\sqrt{17}\,\left(4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{17\,\cos\left(\frac{c}{2}-\mathrm{atan}\left(4\right)+\frac{d\,x}{2}\right)}\right)-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\cos\left(2\,c+2\,d\,x\right)+\frac{\cos\left(3\,c+3\,d\,x\right)}{2}+2\,\sin\left(2\,c+2\,d\,x\right)+4\,\mathrm{atan}\left(\frac{\sqrt{17}\,\left(4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{17\,\cos\left(\frac{c}{2}-\mathrm{atan}\left(4\right)+\frac{d\,x}{2}\right)}\right)\,\cos\left(2\,c+2\,d\,x\right)+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(2\,c+2\,d\,x\right)}{2}-1\right)}{2\,d\,\left({\cos\left(c+d\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (A*a^3*(cos(c + d*x)/2 - 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 +
 (d*x)/2)))/(17*cos(c/2 - atan(4) + (d*x)/2))) - log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))/2 + cos(2*c + 2*d*x) + cos(3*c + 3*d*x)/2 + 2*sin(2*c + 2*
d*x) + 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(17*co
s(c/2 - atan(4) + (d*x)/2)))*cos(2*c + 2*d*x) + (log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 - 1))/(2*d*(cos(c + d*x)^2 - 1))
```

3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-a^3Ax+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}$$

```
[Out] -a^3*A*x+a^3*A*arctanh(cos(d*x+c))/d-a^3*A*cot(d*x+c)/d-1/3*a^3*A*cot(d*x+c
)^3/d-a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3045, 3855, 3853, 3852}

$$-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a^
3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A-2a^3A\csc(c+dx)+2a^3A\csc^3(c+\right.$$

$$= -a^3Ax+\left(a^3A\right)\int \csc^4(c+dx)\,dx-\left(2a^3A\right)\int$$

$$= -a^3Ax+\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\,\mathrm{co}}{}$$

$$= -a^3Ax+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(}{d}$$

Mathematica [A]
time = 0.36, size = 141, normalized size = 1.81

$$-\frac{a^3A\left(24c+24dx+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\sec^2\left(\frac{1}{2}(c+dx)\right)-8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-8\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -1/24*(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 2
4*Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 -
 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2
- 8*Tan[(c + d*x)/2]))/d
```

Maple [A]
time = 0.22, size = 101, normalized size = 1.29

method	result
derivativedivides	$\frac{2a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^3A\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)-2a^3A\ln(\csc(dx+c)-\cot}{d}$
default	$\frac{2a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^3A\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)-2a^3A\ln(\csc(dx+c)-\cot}{d}$
risch	$-a^3xA+\frac{2a^3A\left(3\,e^{5i(dx+c)}+6ie^{2i(dx+c)}-2i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{a^3A\ln\left(e^{i(dx+c)}+1\right)}{d}-\frac{a^3A\ln\left(e^{i(dx+c)}-1\right)}{d}$

norman	$\frac{\frac{2a^3A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3A}{24d}+\frac{6a^3A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11a^3A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{21a^3A\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{13a^3}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^3*A*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+a^3
*A*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)-2*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3
*A*(d*x+c))
```

Maxima [A]
time = 0.29, size = 117, normalized size = 1.50

$$-\frac{6\,(dx+c)Aa^3-3\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,Aa^3(\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right))+\frac{2\left(3\,\tan(dx+c)^2+1\right)Aa^3}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6*(6*(d*x + c)*A*a^3 - 3*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3
)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(76) = 152$.
time = 0.38, size = 175, normalized size = 2.24

$$-\frac{4\,Aa^3\cos\left(dx+c\right)^3-6\,Aa^3\cos\left(dx+c\right)-3\left(Aa^3\cos\left(dx+c\right)^2-Aa^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(Aa^3\cos\left(dx+c\right)^2-Aa^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+6\left(Aa^3dx\cos\left(dx+c\right)^2-Aa^3dx-Aa^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c)
^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x + c
)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*cos(d
*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3\left(\int\left(-2\sin\left(c+dx\right)\csc^4\left(c+dx\right)\right)\,dx+\int 2\sin^3\left(c+dx\right)\csc^4\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\csc^4\left(c+dx\right)dx+\int\left(-\csc^4\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**4, x) + Integral(2*sin(c +
d*x)**3*csc(c + d*x)**4, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**4, x)
+ Integral(-csc(c + d*x)**4, x))
```

Giac [A]
time = 0.44, size = 150, normalized size = 1.92

$$\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)Aa^3-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{44\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(d
*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2*d
*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1/2*
c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 13.32, size = 245, normalized size = 3.14

$$-\frac{\frac{A\,a^3\,\sin(2\,c+2\,d\,x)}{2}-\frac{A\,a^3\,\cos(3\,c+3\,d\,x)}{6}+\frac{A\,a^3\,\cos(c+d\,x)}{2}-\frac{A\,a^3\,\sin(3\,c+3\,d\,x)\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,A\,a^3\,\sin(c+d\,x)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}+\frac{3\,A\,a^3\,\sin(c+d\,x)\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{A\,a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(3\,c+3\,d\,x)}{4}}{\frac{3\,d\,\sin(c+d\,x)}{4}-\frac{d\,\sin(3\,c+3\,d\,x)}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] -((A*a^3*sin(2*c + 2*d*x))/2 - (A*a^3*cos(3*c + 3*d*x))/6 + (A*a^3*cos(c +
d*x))/2 - (A*a^3*sin(3*c + 3*d*x)*atan((2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c
/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x)/2))))/2 + (3*A*a^3*sin(c + d*x)*l
og(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*A*a^3*sin(c + d*x)*atan((
2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x
)/2))))/2 - (A*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/((3*d*sin(c + d*x))/4 - (d*sin(3*c + 3*d*x))/4)
```

3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] 5/8*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-3/8*a^3*A*cot(d*x+
c)*csc(d*x+c)/d-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3045, 3855, 3852, 8, 3853}

$$-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (3
*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x]^
3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A\csc(c+dx)-2a^3A\csc^2(c+dx)+2\right.$$
$$= -\left(\left(a^3A\right)\int \csc(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^5($$
$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\operatorname{cs}}{4d}$$
$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3A\cot^3(c+dx)}{3d}$$
$$= \frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2a^3A\cot^3(c+dx}{3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(86) = 172.
time = 0.05, size = 210, normalized size = 2.44

$$a^3A\left(\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d
*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[
(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2
]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

Maple [**A**]
time = 0.24, size = 114, normalized size = 1.33

method	result

derivativedivides	$\frac{2a^3A\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+a^3A\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2a}{d}$
default	$\frac{2a^3A\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+a^3A\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2a}{d}$
risch	$\frac{a^3A\left(9\,e^{7i(dx+c)}-33\,e^{5i(dx+c)}+48ie^{6i(dx+c)}-33\,e^{3i(dx+c)}-48ie^{4i(dx+c)}+9\,e^{i(dx+c)}+16ie^{2i(dx+c)}-16i\right)}{12d\left(e^{2i(dx+c)}-1\right)^4}-\frac{5a^3A\ln\left(e^{i(c}\right.}{8d}$
norman	$-\frac{a^3A}{64d}-\frac{57a^3A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{19a^3A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{27a^3A\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{49a^3A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}-\frac{3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^3*A*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+a^3*A*((-1/4*csc(d*x+c)^3-3
/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+2*a^3*A*cot(d*x+c)
-a^3*A*ln(csc(d*x+c)-cot(d*x+c)))
```

Maxima [A]
time = 0.28, size = 145, normalized size = 1.69

$$\frac{3\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+24\,Aa^3(\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right))+\frac{96\,Aa^3}{\tan(dx+c)}-\frac{32\left(3\,\tan(dx+c)^2+1\right)Aa^3}{\tan(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 24
*A*a^3*(log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 96*A*a^3/tan(d*x +
 c) - 32*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.
time = 0.50, size = 166, normalized size = 1.93

$$-\frac{32\,Aa^3\cos\left(dx+c\right)^3\sin\left(dx+c\right)-18\,Aa^3\cos\left(dx+c\right)^3+30\,Aa^3\cos\left(dx+c\right)-15\left(Aa^3\cos\left(dx+c\right)^4-2\,Aa^3\cos\left(dx+c\right)^2+Aa^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(Aa^3\cos\left(dx+c\right)^4-2\,Aa^3\cos\left(dx+c\right)^2+Aa^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30*
A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*
```

```
a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d
*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*co
s(d*x + c)^2 + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(78) = 156.
time = 0.60, size = 174, normalized size = 2.02

$$\frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-48\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{250\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2
4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 48*A*a^3*tan(1/2*d*x + 1/2*c) + (250*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 48*A*a
^3*tan(1/2*d*x + 1/2*c)^3 - 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [**B**]
time = 13.12, size = 244, normalized size = 2.84

$$-\frac{A\,a^3\,\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-16\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+120\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{192\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^8 - 3*sin(c/2 + (d*x)/2)^8 - 16*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^7 + 16*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 2
4*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 48*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^5 - 48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2
 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4))/(192*d*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^4)
```

3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot^5(c+dx)}{5d}+\frac{a^3A\cot(c+dx)\csc(c+dx)}{4d}-\frac{a^3A\cot(c+$$

```
[Out] 1/4*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-1/5*a^3*A*cot(d*x+
c)^5/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^3/
d
```

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3029, 2788, 3852, 8, 3853, 3855}

$$-\frac{a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3A\cot(c+dx)\csc(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (a^3
*A*Cot[c + d*x]^5)/(5*d) + (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^3*A
*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= \left(a^3A^3\right)\int \frac{\cot^6(c+dx)}{(A-A\sin(c+dx))^2}\,dx\\ &= \frac{a^3\int \left(-A^4\csc^2(c+dx)-2A^4\csc^3(c+dx)+2\right.}{A^3}\\ &= -\left(\left(a^3A\right)\int \csc^2(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^6\\ &= \frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-\frac{a^3A\cot(c+dx}{2d}\\ &= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3A\cot^3(c+dx)}{3d}\\ &= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(105) = 210.
time = 0.05, size = 268, normalized size = 2.55

$$a^3A\left(\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{16d}-\frac{19\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{480d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{160d}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{16d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{19\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{480d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{160d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((7*Cot[(c + d*x)/2])/(30*d) + Csc[(c + d*x)/2]^2/(16*d) - (19*Cot[(c
 + d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - Csc[(c + d*x)/2]^4/(32*d) - (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + Log[Cos[(c + d*x)/2]]/(4*d) - Log
[Sin[(c + d*x)/2]]/(4*d) - Sec[(c + d*x)/2]^2/(16*d) + Sec[(c + d*x)/2]^4/(
32*d) - (7*Tan[(c + d*x)/2])/(30*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d))
```

Maple [A]
time = 0.22, size = 140, normalized size = 1.33

method	result
derivativedivides	$\frac{2a^3A\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a^3A\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15} \cdots\right.}{d}$
default	$\frac{2a^3A\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a^3A\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15} \cdots\right.}{d}$
risch	$-\frac{a^3A\left(-60i e^{8i(dx+c)}+15\,e^{9i(dx+c)}+240i e^{6i(dx+c)}+90\,e^{7i(dx+c)}-40i e^{4i(dx+c)}+80i e^{2i(dx+c)}-90\,e^{3i(dx+c)}-28i-15\,e^{i(d \cdots}\right.}{30d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^3*A*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+
c)-cot(d*x+c)))+a^3*A*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)
-2*a^3*A*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+a^3*A*c
ot(d*x+c))
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.67

$$\frac{15\,Aa^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{120\,Aa^3}{\tan(dx+c)}-\frac{8\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)Aa^3}{\tan(dx+c)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 10*ta
n(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

time = 0.37, size = 201, normalized size = 1.91

$$\frac{56 Aa^3\cos(dx+c)^5 - 80 Aa^3\cos(dx+c)^3 + 15\left(Aa^3\cos(dx+c)^4 - 2Aa^3\cos(dx+c)^2 + Aa^3\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 15\left(Aa^3\cos(dx+c)^4 - 2Aa^3\cos(dx+c)^2 + Aa^3\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 30\left(Aa^3\cos(dx+c)^3 + Aa^3\cos(dx+c)\right)\sin(dx+c)}{120\left(d\cos(dx+c)^4 - 2d\cos(dx+c)^2 + d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(56*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 + 15*(A*a^3*cos(d*
x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(A*a^3*cos(d*x + c)^3 + A*a^3*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.47, size = 174, normalized size = 1.66

$$\frac{3 Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 15 Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 25 Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 120 Aa^3\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - 90 Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{274 Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 90 Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 25 Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 15 Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 3Aa^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{480 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 2
5*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a
^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 13.16, size = 244, normalized size = 2.32

$$-\frac{A a^3\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9 + 15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 25\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 90\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 90\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 25\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}{480\, d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) -
 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 90*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^6 - 90*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 25*cos(c
/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(480*d*cos(c/2 + (d*
x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=130

$$\frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{2a^3A\cot^5(c+dx)}{5d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}-\frac{5a^3A\,\mathrm{cc}}{}$$

```
[Out] 3/16*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-2/5*a^3*A*cot(d*x
+c)^5/d+3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-5/24*a^3*A*cot(d*x+c)*csc(d*x+c)
^3/d-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A]

time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3045, 3853, 3855, 3852}

$$-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d)
- (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc[c
+ d*x]^5)/(6*d)
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A\csc^3(c+dx)-2a^3A\csc^4(c+dx)+2a\right.$$

$$= -\left(\left(a^3A\right)\int \csc^3(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^7(c$$

$$= \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-\frac{a^3A\cot(c+dx)}{6d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^3A\cot^3(c+dx)}{3d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^3A\cot^3(c+dx)}{3d}$$

$$= \frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^3(c+dx)}{3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 306 vs. 2(130) = 260.
time = 0.06, size = 306, normalized size = 2.35

$$a^3A\left(\frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d}+\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{240d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{80d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{240d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{80d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (3*L
og[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Sec[(c
 + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/(384*
d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2
40*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

Maple [**A**]
time = 0.26, size = 163, normalized size = 1.25

method	result
derivativedivides	$\frac{2a^3A\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+a^3A\left(\left(-\frac{\left(\csc^5(dx+c)\right)}{6}-\frac{5\left(\csc^3(dx+c)\right)}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+\right.}{}$
default	$\frac{2a^3A\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+a^3A\left(\left(-\frac{\left(\csc^5(dx+c)\right)}{6}-\frac{5\left(\csc^3(dx+c)\right)}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+\right.}{}$
risch	$-\frac{a^3A\left(45\,e^{11i(dx+c)}+65\,e^{9i(dx+c)}-750\,e^{7i(dx+c)}+960ie^{8i(dx+c)}-750\,e^{5i(dx+c)}-640ie^{6i(dx+c)}+65\,e^{3i(dx+c)}+45\,e^{i(dx\right.}{120d\left(e^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a^3*A*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+a^3*A*((
-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/16*ln(csc
(d*x+c)-cot(d*x+c)))-2*a^3*A*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)-a^3*A*(-1/2
*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 207, normalized size = 1.59

$$\frac{5\,Aa^3\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-120\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{320\left(3\,\tan(dx+c)^2+1\right)Aa^3}{\tan(dx+c)^3}-\frac{64\left(15\,\tan(dx+c)^4+10\,\tan(dx+c)^2+3\right)Aa^3}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(5*A*a^3*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))
/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*
x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 120*A*a^3*(2*cos(d*x + c)/(cos(d*
x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 320*(3*tan
(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3 - 64*(15*tan(d*x + c)^4 + 10*tan(d*x
+ c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(118) = 236$.
time = 0.42, size = 240, normalized size = 1.85

$$-\frac{90\,Aa^3\cos\left(dx+c\right)^5-80\,Aa^3\cos\left(dx+c\right)^3-90\,Aa^3\cos\left(dx+c\right)-45\left(Aa^3\cos\left(dx+c\right)^6-3\,Aa^3\cos\left(dx+c\right)^4+3\,Aa^3\cos\left(dx+c\right)^2-Aa^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+45\left(Aa^3\cos\left(dx+c\right)^6-3\,Aa^3\cos\left(dx+c\right)^4+3\,Aa^3\cos\left(dx+c\right)^2-Aa^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+64\left(2\,Aa^3\cos\left(dx+c\right)^5-5\,Aa^3\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{480\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d*
x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*
x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6 -
3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x +
 c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(d*x +
 c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(118) = 236.
time = 0.45, size = 242, normalized size = 1.86

$$\frac{5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-360\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-240\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{882\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+240\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1920*(5*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
45*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^
3*tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 240*A
*a^3*tan(1/2*d*x + 1/2*c) + (882*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 240*A*a^3*t
an(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*tan(1/2*
d*x + 1/2*c)^3 - 45*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*tan(1/2*d*x + 1
/2*c) - 5*A*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 13.37, size = 340, normalized size = 2.62

$$-\frac{A\,a^3\,\left(5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-45\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-40\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+240\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-240\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+40\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+45\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+360\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right)}{1920\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(A*a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
 - 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)^3*
```

```
sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2
 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 +
 (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)
/2)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6
*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{41A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-$$

```
[Out] -19/2*A*x/a^3-4*A*cos(d*x+c)/a^3/d+1/2*A*cos(d*x+c)*sin(d*x+c)/a^3/d-2/5*A*
cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+41/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2
-199/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3045, 2718, 2715, 8, 2729, 2727}

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19Ax}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A*C
os[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*a^3
*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx &= \int \left(-\frac{9A}{a^3}+\frac{4A\sin(c+dx)}{a^3}-\frac{A\sin^2(c+dx)}{a^3}+\frac{2A}{a^3(1+\sin(c+dx \cdots}\right. \\
&= -\frac{9Ax}{a^3}-\frac{A\int \sin^2(c+dx)\,dx}{a^3}+\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(4A)\int \cdots}{\cdots} \\
&= -\frac{9Ax}{a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos \cdots}{5a^3d(1+\cdots} \\
&= -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos \cdots}{5a^3d(1+\cdots} \\
&= -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos \cdots}{5a^3d(1+\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.66, size = 254, normalized size = 1.97

$$\frac{A\left(-11400dx\cos\left(\frac{dx}{2}\right)+12060\cos\left(c+\frac{dx}{2}\right)-14090\cos\left(c+\frac{3dx}{2}\right)+5700dx\cos\left(2c+\frac{3dx}{2}\right)+1140dx\cos\left(2c+\frac{5dx}{2}\right)+1050\cos\left(3c+\frac{5dx}{2}\right)+165\cos\left(3c+\frac{7dx}{2}\right)+15\cos\left(5c+\frac{9dx}{2}\right)+19780\sin\left(\frac{dx}{2}\right)-11400dx\sin\left(c+\frac{dx}{2}\right)-5700dx\sin\left(c+\frac{3dx}{2}\right)+1830\sin\left(2c+\frac{3dx}{2}\right)-4234\sin\left(2c+\frac{5dx}{2}\right)+1140dx\sin\left(3c+\frac{5dx}{2}\right)+165\sin\left(4c+\frac{7dx}{2}\right)-15\sin\left(4c+\frac{9dx}{2}\right)\right)}{480a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-11400*d*x*Cos[(d*x)/2] + 12060*Cos[c + (d*x)/2] - 14090*Cos[c + (3*d*x
)/2] + 5700*d*x*Cos[2*c + (3*d*x)/2] + 1140*d*x*Cos[2*c + (5*d*x)/2] + 1050
*Cos[3*c + (5*d*x)/2] + 165*Cos[3*c + (7*d*x)/2] + 15*Cos[5*c + (9*d*x)/2]
```

```
+ 19780*Sin[(d*x)/2] - 11400*d*x*Sin[c + (d*x)/2] - 5700*d*x*Sin[c + (3*d*x
)/2] + 1830*Sin[2*c + (3*d*x)/2] - 4234*Sin[2*c + (5*d*x)/2] + 1140*d*x*Sin
[3*c + (5*d*x)/2] + 165*Sin[4*c + (7*d*x)/2] - 15*Sin[4*c + (9*d*x)/2]))/(4
80*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.40, size = 154, normalized size = 1.19

method	result
derivativedivides	$\frac{32A\left(-\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+4}{16\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{19\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}\right.}{d\,a^3}$
default	$\frac{32A\left(-\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+4}{16\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{19\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}\right.}{d\,a^3}$
risch	$-\frac{19Ax}{2a^3}-\frac{iA\,e^{2i(dx+c)}}{8d\,a^3}-\frac{2A\,e^{i(dx+c)}}{a^3d}-\frac{2A\,e^{-i(dx+c)}}{a^3d}+\frac{iA\,e^{-2i(dx+c)}}{8d\,a^3}-\frac{2\left(825iA\,e^{3i(dx+c)}+240A\,e^{4i(dx+c)}-75\right.}{15d\,a^3\left(e^{i(\right.}}$
norman	$\frac{-\frac{19A\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2232A\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1235Ax\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{665Ax\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{391A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{285Ax}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 32/d*A/a^3*(-1/16*(1/2*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)^2-1/2*tan(
1/2*d*x+1/2*c)+4)/(1+tan(1/2*d*x+1/2*c)^2)^2-19/32*arctan(tan(1/2*d*x+1/2*c
))-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/24/(tan(1/2
*d*x+1/2*c)+1)^3-5/16/(tan(1/2*d*x+1/2*c)+1)^2-9/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(119) = 238.
time = 0.52, size = 715, normalized size = 5.54

$$-\frac{A\left(\frac{\frac{1325\sin(dx+c)}{\cos(dx+c)+1}+\frac{2673\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4329\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3575\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2275\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{975\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{195\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+304}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{26a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{26a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{12a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{195\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+6A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{189\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{160\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{75\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+24}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{11a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/15*(A*((1325*sin(d*x + c)/(cos(d*x + c) + 1) + 2673*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4329*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 3575*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 227
5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 975*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + 195*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 304)/(a^3 + 5*a^3*sin(d*x
```

```
+ c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 20*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 26*a^3*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 26*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 12*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) +
1)^9) + 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 6*A*((105*sin(d*
x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 200*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(119) = 238$.
time = 0.38, size = 248, normalized size = 1.92

$$-\frac{15\,A\cos\left(dx+c\right)^{5}+90\,A\cos\left(dx+c\right)^{4}+\left(285\,Adx+683\,A\right)\cos\left(dx+c\right)^{3}-1140\,Adx+\left(855\,Adx-526\,A\right)\cos\left(dx+c\right)^{2}-6\left(95\,Adx+191\,A\right)\cos\left(dx+c\right)-\left(15\,A\cos\left(dx+c\right)^{4}-75\,A\cos\left(dx+c\right)^{3}+1140\,Adx-19\left(15\,Adx-32\,A\right)\cos\left(dx+c\right)^{2}+6\left(95\,Adx+189\,A\right)\cos\left(dx+c\right)-12\,A\right)\sin\left(dx+c\right)-12\,A}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos(
d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*x
+ 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 1140*A
*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d*x +
 c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos
(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3614 vs. $2(126) = 252$.
time = 27.94, size = 3614, normalized size = 28.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-285*A*d*x*tan(c/2 + d*x/2)**9/(30*a**3*d*tan(c/2 + d*x/2)**9 +
150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*
d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2
```

```
 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)
**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2
)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*
a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*ta
n(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d
*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 3
0*a**3*d) - 3420*A*d*x*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3
*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2
)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/
2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360
*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*t
an(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**9
+ 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c
/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/
2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x
/2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 36
0*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*
tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
 d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
 30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/2)**3/(30*a**3*d*tan(c/2 + d*x/2)**9
 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a*
*3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(
c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x
/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 3420*A*d*x*tan(c/2 + d*
x/2)**2/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 3
60*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d
*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2
+ d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2)
+ 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3
*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2
)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 285*A*d*x/(30*a**3*d*tan(
c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x
/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 +
780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*
d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 570*A*ta
n(c/2 + d*x/2)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x
/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 +
780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*
```

```
d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2
 + d*x/2) + 30*a**3*d) - 2850*A*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*
x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 +
 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3
*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/
2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 6650*A*tan(c/2 +
 d*x/2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8
+ 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**
3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c
/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/
2) + 30*a**3*d) - 10450*A*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**
9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a
**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan
(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*
x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 12846*A*tan(c/2 + d*x/
2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360
*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*t
an(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 15...
```

Giac [A]
time = 0.49, size = 156, normalized size = 1.21

$$-\frac{\frac{285\,(dx+c)A}{a^3}+\frac{30\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+615\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1025\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+685\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+164\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(285*(d*x + c)*A/a^3 + 30*(A*tan(1/2*d*x + 1/2*c)^3 + 8*A*tan(1/2*d*x
 + 1/2*c)^2 - A*tan(1/2*d*x + 1/2*c) + 8*A)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2
*a^3) + 4*(135*A*tan(1/2*d*x + 1/2*c)^4 + 615*A*tan(1/2*d*x + 1/2*c)^3 + 10
25*A*tan(1/2*d*x + 1/2*c)^2 + 685*A*tan(1/2*d*x + 1/2*c) + 164*A)/(a^3*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 17.08, size = 326, normalized size = 2.53

$$\frac{\left(\frac{95\,A\,(c+d\,x)}{2}-\frac{A\,(1425\,c+1425\,d\,x+570)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(114\,A\,(c+d\,x)-\frac{A\,(3420\,c+3420\,d\,x+2850)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(190\,A\,(c+d\,x)-\frac{A\,(5700\,c+5700\,d\,x+6650)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(247\,A\,(c+d\,x)-\frac{A\,(7410\,c+7410\,d\,x+10450)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(247\,A\,(c+d\,x)-\frac{A\,(7410\,c+7410\,d\,x+12846)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(190\,A\,(c+d\,x)-\frac{A\,(5700\,c+5700\,d\,x+11270)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(114\,A\,(c+d\,x)-\frac{A\,(3420\,c+3420\,d\,x+7902)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(\frac{95\,A\,(c+d\,x)}{2}-\frac{A\,(1425\,c+1425\,d\,x+3910)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{19\,A\,(c+d\,x)}{2}-\frac{A\,(285\,c+285\,d\,x+896)}{30}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}-\frac{19\,A\,x}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 3910))/30
) + tan(c/2 + (d*x)/2)^8*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 570)
```

```
)/30) + tan(c/2 + (d*x)/2)^7*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 285
0))/30) + tan(c/2 + (d*x)/2)^2*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 7
902))/30) + tan(c/2 + (d*x)/2)^6*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x +
 6650))/30) + tan(c/2 + (d*x)/2)^3*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x
 + 11270))/30) + tan(c/2 + (d*x)/2)^5*(247*A*(c + d*x) - (A*(7410*c + 7410*
d*x + 10450))/30) + tan(c/2 + (d*x)/2)^4*(247*A*(c + d*x) - (A*(7410*c + 74
10*d*x + 12846))/30) + (19*A*(c + d*x))/2 - (A*(285*c + 285*d*x + 896))/30)
/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (19*A*x)
/(2*a^3)
```

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{104A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

```
[Out] 4*A*x/a^3+A*cos(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-31/15*
A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+104/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c)
)
```

Rubi [A]
time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3045, 2718, 2729, 2727}

$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + S
in[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (10
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx &= \int \left(\frac{4A}{a^3}-\frac{A\sin(c+dx)}{a^3}-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{7A}{a^3(1+\sin(c+
\\
&= \frac{4Ax}{a^3}-\frac{A\int\sin(c+dx)\,dx}{a^3}-\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(7A)\int\frac{}{(1+\sin}}{a^3}
\\
&= \frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{7A\cos(c+d}{3a^3d(1+\sin(c+}
\\
&= \frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+}{15a^3d(1+\sin(c}
\\
&= \frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+}{15a^3d(1+\sin(c}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 228 vs. 2(103) = 206.
time = 0.54, size = 228, normalized size = 2.21

$$-\frac{A\left(-1200dx\cos\left(\frac{dx}{2}\right)+1665\cos\left(c+\frac{dx}{2}\right)-1675\cos\left(c+\frac{3dx}{2}\right)+600dx\cos\left(2c+\frac{3dx}{2}\right)+120dx\cos\left(2c+\frac{5dx}{2}\right)+75\cos\left(3c+\frac{5dx}{2}\right)+15\cos\left(3c+\frac{7dx}{2}\right)+2495\sin\left(\frac{dx}{2}\right)-1200dx\sin\left(c+\frac{dx}{2}\right)-600dx\sin\left(c+\frac{3dx}{2}\right)+405\sin\left(2c+\frac{3dx}{2}\right)-491\sin\left(2c+\frac{5dx}{2}\right)+120dx\sin\left(3c+\frac{5dx}{2}\right)+15\sin\left(4c+\frac{7dx}{2}\right)\right)}{120a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(A*(-1200*d*x*Cos[(d*x)/2] + 1665*Cos[c + (d*x)/2] - 1675*Cos[c + (3
*d*x)/2] + 600*d*x*Cos[2*c + (3*d*x)/2] + 120*d*x*Cos[2*c + (5*d*x)/2] + 75
*Cos[3*c + (5*d*x)/2] + 15*Cos[3*c + (7*d*x)/2] + 2495*Sin[(d*x)/2] - 1200*
d*x*Sin[c + (d*x)/2] - 600*d*x*Sin[c + (3*d*x)/2] + 405*Sin[2*c + (3*d*x)/2
] - 491*Sin[2*c + (5*d*x)/2] + 120*d*x*Sin[3*c + (5*d*x)/2] + 15*Sin[4*c +
(7*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^5)
```

Maple [A]
time = 0.35, size = 115, normalized size = 1.12

method	result

derivativedivides	$\frac{16A\left(\frac{1}{8+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{8(\tan(}\right.}{d\,a^3}$
default	$\frac{16A\left(\frac{1}{8+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{8(\tan(}\right.}{d\,a^3}$
risch	$\frac{4Ax}{a^3}+\frac{A\,\mathrm{e}^{i(dx+c)}}{2a^3d}+\frac{A\,\mathrm{e}^{-i(dx+c)}}{2a^3d}+\frac{2A\left(435i\mathrm{e}^{3i(dx+c)}+135\,\mathrm{e}^{4i(dx+c)}-385i\mathrm{e}^{i(dx+c)}-605\,\mathrm{e}^{2i(dx+c)}+104\right)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^5}$
norman	$\frac{5668A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{56Ax\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{20Ax\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{164A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{4Ax\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{800A(\tan}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 16/d*A/a^3*(1/8/(1+tan(1/2*d*x+1/2*c)^2)+1/2*arctan(tan(1/2*d*x+1/2*c))+1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan(1/2*d*x+1/2*c)+1)^4+1/12/(tan(1/2*d*x+1/2*c)+1)^3+3/8/(tan(1/2*d*x+1/2*c)+1)^2+1/2/(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(97) = 194$.
time $= 0.51$, size $= 543$, normalized size $= 5.27$

$$\frac{2\left(3A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{189\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{160\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{75\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+24}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{11a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+A\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(97) = 194$.
time = 0.38, size = 225, normalized size = 2.18

$$\frac{15\,A\cos\left(dx+c\right)^{4}+\left(60\,Adx+149\,A\right)\cos\left(dx+c\right)^{3}-240\,Adx+\left(180\,Adx-103\,A\right)\cos\left(dx+c\right)^{2}-3\,\left(40\,Adx+81\,A\right)\cos\left(dx+c\right)+\left(15\,A\cos\left(dx+c\right)^{3}-240\,Adx+2\,\left(30\,Adx-67\,A\right)\cos\left(dx+c\right)^{2}-3\,\left(40\,Adx+79\,A\right)\cos\left(dx+c\right)+6\,A\right)\sin\left(dx+c\right)-6\,A}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x +
 (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) + (1
5*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*(40
*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(100) = 200$.
time = 16.28, size = 2290, normalized size = 22.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**7/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75
*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*t
an(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 +
d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d
*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 1
65*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d
*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 +
 d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d
*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 +
 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3
*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*
x*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 +
d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4
 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**
3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/2)**3/(15*a**3*
d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2
+ d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)*
*3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*
d) + 660*A*d*x*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3
```

```
*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/
2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2
)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d*x/2)
/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*
d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2
 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
+ 15*a**3*d) + 60*A*d*x/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2
+ d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)*
*4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 120*A*tan(c/2 + d*x/2)**6/(15*a**3*d*
tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 +
d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3
 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 + 600*A*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan
(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*
x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 +
 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1280*A*tan(c/2 + d*x/2)**4/(15*a
**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(
c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x
/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a
**3*d) + 1540*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**
3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c
/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/
2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1468*A*tan(c/2 + d*x/2)**
2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3
*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/
2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
 + 15*a**3*d) + 820*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*
a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*ta
n(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d
*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 188*A/(15*a**3*d*tan(c
/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2
)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 16
5*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(
d, 0)), (x*(-A*sin(c) + A)*sin(c)**3/(a*sin(c) + a)**3, True))
```

Giac [A]

time = 0.45, size = 113, normalized size = 1.10

$$\frac{2\left(\frac{30\,(dx+c)A}{a^3}+\frac{15\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+285\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+505\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+335\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+79\,A}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A*
tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x +
 1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c) +
 1)^5))/d
```

Mupad [B]
time = 16.93, size = 261, normalized size = 2.53

$$\frac{4\,A\,x}{a^3}-\frac{\left(20\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(75\,c+75\,d\,x+30\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\left(44\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(165\,c+165\,d\,x+150\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(60\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(225\,c+225\,d\,x+320\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(60\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(225\,c+225\,d\,x+385\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(44\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(165\,c+165\,d\,x+367\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\left(20\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(75\,c+75\,d\,x+205\right)}{15}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,A\,\left(c+d\,x\right)-\frac{4\,A\,\left(15\,c+15\,d\,x+47\right)}{15}}{a^3\,d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^5\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (4*A*x)/a^3 - (tan(c/2 + (d*x)/2)*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 2
05))/15) + tan(c/2 + (d*x)/2)^6*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 30)
)/15) + tan(c/2 + (d*x)/2)^5*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 150)
)/15) + tan(c/2 + (d*x)/2)^2*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 367)
)/15) + tan(c/2 + (d*x)/2)^4*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 320)
)/15) + tan(c/2 + (d*x)/2)^3*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 385)
)/15) + 4*A*(c + d*x) - (4*A*(15*c + 15*d*x + 47))/15)/(a^3*d*(tan(c/2 + (d
*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$-\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

```
[Out] -A*x/a^3-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+7/5*A*cos(d*x+c)/a^3/d/(1+
sin(d*x+c))^2-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3045, 2729, 2727}

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{Ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{A}{a^3}+\frac{2A}{a^3(1+\sin(c+dx))^3}-\frac{5A}{a^3(1+\sin(c+dx))^2}+\frac{}{a^3(1+\mathrm{s}}\right.$$

$$= -\frac{Ax}{a^3}+\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(4A)\int\frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(5A)\int\frac{}{(1-}}{}$$

$$= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{5A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{4A}{a^3d(1+}$$

$$= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{7A}{3a^3d(1}$$

$$= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{13A}{5a^3d(1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(89) = 178.
time = 0.60, size = 189, normalized size = 2.12

$$\frac{A\left(-50dx\cos\left(\frac{dx}{2}\right)+110\cos\left(c+\frac{dx}{2}\right)-90\cos\left(c+\frac{3dx}{2}\right)+25dx\cos\left(2c+\frac{3dx}{2}\right)+5dx\cos\left(2c+\frac{5dx}{2}\right)+150\sin\left(\frac{dx}{2}\right)-50dx\sin\left(c+\frac{dx}{2}\right)-25dx\sin\left(c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{3dx}{2}\right)-26\sin\left(2c+\frac{5dx}{2}\right)+5dx\sin\left(3c+\frac{5dx}{2}\right)\right)}{20a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-50*d*x*Cos[(d*x)/2] + 110*Cos[c + (d*x)/2] - 90*Cos[c + (3*d*x)/2] + 2
5*d*x*Cos[2*c + (3*d*x)/2] + 5*d*x*Cos[2*c + (5*d*x)/2] + 150*Sin[(d*x)/2]
- 50*d*x*Sin[c + (d*x)/2] - 25*d*x*Sin[c + (3*d*x)/2] + 40*Sin[2*c + (3*d*x
)/2] - 26*Sin[2*c + (5*d*x)/2] + 5*d*x*Sin[3*c + (5*d*x)/2]))/(20*a^3*d*(Co
s[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.30, size = 96, normalized size = 1.08

method	result
risch	$-\frac{Ax}{a^3}-\frac{2\left(-75A\,\mathrm{e}^{2i(dx+c)}+55iA\,\mathrm{e}^{3i(dx+c)}+20A\,\mathrm{e}^{4i(dx+c)}-45iA\,\mathrm{e}^{i(dx+c)}+13A\right)}{5d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^5}$
derivativedivides	$\frac{8A\left(-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{4\left(\tan\left(\frac{dx}{2}\right.\right.}\right.}{d\,a^3}$
default	$\frac{8A\left(-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{4\left(\tan\left(\frac{dx}{2}\right.\right.}\right.}{d\,a^3}$

norman	$\frac{-\frac{Ax}{a}-\frac{16A}{5ad}-\frac{5Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{13Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{25Ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{38Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{46Ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/d*A/a^3*(-1/4*arctan(tan(1/2*d*x+1/2*c))-2/5/(tan(1/2*d*x+1/2*c)+1)^5+1/(
tan(1/2*d*x+1/2*c)+1)^4-1/2/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/2*d*x+1/2*c
)+1)^2-1/4/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(83) = 166$.
time = 0.49, size = 392, normalized size = 4.40

$$-\frac{2\left(A\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -2/15*(A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.
time = 0.36, size = 204, normalized size = 2.29

$$-\frac{(5\,Adx+13\,A)\cos(dx+c)^3-20\,Adx+3\,(5\,Adx-2\,A)\cos(dx+c)^2-(10\,Adx+21\,A)\cos(dx+c)-\left(20\,Adx-(5\,Adx-13\,A)\cos(dx+c)^2+(10\,Adx+19\,A)\cos(dx+c)-2\,A\right)\sin(dx+c)-2\,A}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d*
x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*co
s(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3
*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(85) = 170$.
time = 9.09, size = 1268, normalized size = 14.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-5*A*d*x*tan(c/2 + d*x/2)**5/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*
a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(
c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2
 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4
+ 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d
*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(
c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2
)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*
d) - 50*A*d*x*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d
*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 +
d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2 + d*x
/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3
*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2
+ d*x/2) + 5*a**3*d) - 5*A*d*x/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*ta
n(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x
/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 10*A*tan(c/2 + d*x/2)**4/
(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*t
an(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*
x/2) + 5*a**3*d) - 50*A*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(c/2 + d*x/2)**5 +
 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*
tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 110*A*tan(c/
2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4
 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*
d*tan(c/2 + d*x/2) + 5*a**3*d) - 70*A*tan(c/2 + d*x/2)/(5*a**3*d*tan(c/2 +
d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 +
 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 1
6*A/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3
*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2
+ d*x/2) + 5*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)**2/(a*sin(c) + a
)**3, True))
```

Giac [A]
time = 0.51, size = 93, normalized size = 1.04

$$-\frac{\frac{5\,(dx+c)A}{a^3}+\frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*tan(1/2*d*x + 1/2*c)^4 + 25*A*tan(1/2*d*x
+ 1/2*c)^3 + 55*A*tan(1/2*d*x + 1/2*c)^2 + 35*A*tan(1/2*d*x + 1/2*c) + 8*A)
/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 14.92, size = 178, normalized size = 2.00

$$\frac{\left(5\,A\,(c+d\,x)-\frac{A\,(25\,c+25\,d\,x+10)}{5}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(10\,A\,(c+d\,x)-\frac{A\,(50\,c+50\,d\,x+50)}{5}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(10\,A\,(c+d\,x)-\frac{A\,(50\,c+50\,d\,x+110)}{5}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(5\,A\,(c+d\,x)-\frac{A\,(25\,c+25\,d\,x+70)}{5}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+A\,(c+d\,x)-\frac{A\,(5\,c+5\,d\,x+16)}{5}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}-\frac{A\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 70))/5) + tan(c/2
+ (d*x)/2)^4*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 10))/5) + tan(c/2 + (d*x)
/2)^3*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 50))/5) + tan(c/2 + (d*x)/2)^2*
(10*A*(c + d*x) - (A*(50*c + 50*d*x + 110))/5) + A*(c + d*x) - (A*(5*c + 5*
d*x + 16))/5)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5) - (A*x)/a^3
```

3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

```
[Out] 2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-11/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x
+c))^2+4/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3045, 2729, 2727}

$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(15
*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*
x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{3A}{a^3(1+\sin(c+dx))^2}-\frac{A}{a^3(1+\sin(c\right.$$
$$= -\frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(3A)\int \frac{1}{(1+\sin(c+dx))}}{a^3}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))^2}+\frac{A\cos(c+dx}{a^3d(1+\sin(c+}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-\frac{(4A)\int \frac{1}{1+\sin(c}}{15a^3}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{4A\cos(c}{15a^3d(1+\sin}$$

Mathematica [A]
time = 0.32, size = 107, normalized size = 1.30

$$-\frac{A\left(15\cos\left(c+\frac{dx}{2}\right)-5\cos\left(c+\frac{3dx}{2}\right)+25\sin\left(\frac{dx}{2}\right)+15\sin\left(2c+\frac{3dx}{2}\right)-4\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 15
*Sin[2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.28, size = 71, normalized size = 0.87

method	result
derivativedivides	$\frac{4A\left(-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}\right)}{d\,a^3}$
default	$\frac{4A\left(-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}\right)}{d\,a^3}$
risch	$\frac{2A\left(15i\mathrm{e}^{3i(dx+c)}+15\,\mathrm{e}^{4i(dx+c)}-5i\mathrm{e}^{i(dx+c)}-25\,\mathrm{e}^{2i(dx+c)}+4\right)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^5}$
norman	$\frac{-\frac{2A}{15ad}-\frac{14A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{10A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{2A\left(\tan^6\left(\frac{dx}{2}\right.\right.}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d*A/a^3*(-2/(tan(1/2*d*x+1/2*c)+1)^4+4/5/(tan(1/2*d*x+1/2*c)+1)^5+5/3/(ta
n(1/2*d*x+1/2*c)+1)^3-1/2/(tan(1/2*d*x+1/2*c)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(76) = 152$.
time = 0.31, size = 348, normalized size = 4.24

$$\frac{2\left(\frac{2A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 2/15*(2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3 + 5*a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(76) = 152$.
time = 0.38, size = 156, normalized size = 1.90

$$\frac{4\,A\cos(dx+c)^3+7\,A\cos(dx+c)^2-3\,A\cos(dx+c)-\left(4\,A\cos(dx+c)^2-3\,A\cos(dx+c)-6\,A\right)\sin(dx+c)-6\,A}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*cos
(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos
(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(78) = 156$.

time = 5.05, size = 461, normalized size = 5.62

$$\begin{cases} -\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} + \frac{10A\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} - \frac{10A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} - \frac{2A}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} & \text{for}\ d \neq 0 \\ \frac{x(-A\sin(c)+A)\sin(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2 +
d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150
*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*ta
n(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)/(a*sin(c)
 + a)**3, True))
```

Giac [A]

time = 0.43, size = 63, normalized size = 0.77

$$-\frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+A\right)}{15a^3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(1
/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

Mupad [B]

time = 13.45, size = 110, normalized size = 1.34

$$-\frac{2A\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}{15a^3d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(2*A*cos(c/2 + (d*x)/2)^2*(cos(c/2 + (d*x)/2)^3 + 15*sin(c/2 + (d*x)/2)^3
- 5*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 5*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)))/(15*a^3*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=58

$$-\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}$$

```
[Out] -1/5*a*A*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^4-1/15*A*cos(d*x+c)^3/d/(a+a*sin(d
*x+c))^3
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2815, 2751, 2750}

$$-\frac{A\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^3}-\frac{aA\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*(a*A*Cos[c + d*x]^3)/(d*(a + a*Sin[c + d*x])^4) - (A*Cos[c + d*x]^3)/(
15*d*(a + a*Sin[c + d*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= (aA)\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx \\
&= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}+\frac{1}{5}A\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx \\
&= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 92, normalized size = 1.59

$$
\frac{A\left(-15\cos\left(c+\frac{dx}{2}\right)+5\cos\left(c+\frac{3dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c
+ (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^5)
```

Maple [A]
time = 0.25, size = 86, normalized size = 1.48

method	result
risch	$\frac{2iA\left(-5i\,e^{2i(dx+c)}+15\,e^{3i(dx+c)}-i-5\,e^{i(dx+c)}\right)}{15d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
derivativedivides	$\frac{2A\left(-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}\right)}{d\,a^3}$
default	$\frac{2A\left(-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}\right)}{d\,a^3}$
norman	$\frac{-\frac{8A}{15ad}-\frac{58A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{8A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{2A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2A\left(\tan^6\left(\frac{dx}{2}\right.\right.}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*A/a^3*(-1/(tan(1/2*d*x+1/2*c)+1)+3/(tan(1/2*d*x+1/2*c)+1)^2+4/(tan(1/2*
d*x+1/2*c)+1)^4-8/5/(tan(1/2*d*x+1/2*c)+1)^5-14/3/(tan(1/2*d*x+1/2*c)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(54) = 108$.
time = 0.30, size = 387, normalized size = 6.67

$$-\frac{2\left(\frac{A\left(\frac{20\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+7\right)}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(54) = 108$.
time = 0.38, size = 154, normalized size = 2.66

$$\frac{A\cos\left(dx+c\right)^3-2\,A\cos\left(dx+c\right)^2+3\,A\cos\left(dx+c\right)-\left(A\cos\left(dx+c\right)^2+3\,A\cos\left(dx+c\right)+6\,A\right)\sin\left(dx+c\right)+6\,A}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)-4\,a^3d+\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)-4\,a^3d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*x
 + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x
 + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(53) = 106$.
time = 2.81, size = 573, normalized size = 9.88

$$\begin{cases} -\frac{30A\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d}-\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d}-\frac{50A\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d}-\frac{10A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d}-\frac{8A}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} & \text{for}\ d\neq 0 \\ \frac{x\left(-A\sin\left(c\right)+A\right)}{\left(a\sin\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 30*A*tan(c/2 +
d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 50*A*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/
2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)
**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3
*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(
c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x
/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 8*A/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
, Ne(d, 0)), (x*(-A*sin(c) + A)/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.44, size = 79, normalized size = 1.36

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,A\right)}{15\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*tan
(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x +
 1/2*c) + 1)^5)
```

Mupad [B]
time = 13.31, size = 134, normalized size = 2.31

$$-\frac{2\,A\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+25\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{15\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(2*A*cos(c/2 + (d*x)/2)*(4*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
+ 15*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 5*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2) + 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*a^3*d*(c
os(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}+\frac{8A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

```
[Out] -A*arctanh(cos(d*x+c))/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+3/5*A*
cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+8/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3045, 3855, 2729, 2727}

$$\frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin
[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx &= \int \left(\frac{A\csc(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{A}{a^3(1+\sin(c+dx))^2} \right. \\
&= \frac{A\int \csc(c+dx)\,dx}{a^3} - \frac{A\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} - \\
&= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{A\cos(c+}{3a^3d(1+\sin} \\
&= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c}{5a^3d(1+\sin} \\
&= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c}{5a^3d(1+\sin}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(98) = 196.
time = 0.72, size = 313, normalized size = 3.19

$$\frac{\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(2\cos\left(\frac{c}{2}\right)-2\sin\left(\frac{c}{2}\right)+3\cos\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-3\sin\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\right)+2\sin\left(\frac{dx}{2}\right)(-17+4\cos(2(c+dx))-19\sin(c+dx))\right)(A-A\sin(c+dx))}{5a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2*Cos[c/2] - 2*Sin[c/2] + 3*Cos[c/
2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c/2]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2 - 5*Log[Cos[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + 5*Log[Sin[(c + d*x)/2]]*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 2*Sin[(d*x)/2]*(-17 + 4*C
os[2*(c + d*x)] - 19*Sin[c + d*x]))*(A - A*Sin[c + d*x]))/(5*a^3*d*(Cos[c/2
] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^5)
```

Maple [A]
time = 0.43, size = 95, normalized size = 0.97

method	result

derivativedivides	$\frac{A\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}\right)}{d\,a^{3}}$
default	$\frac{A\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}\right)}{d\,a^{3}}$
risch	$\frac{2A\left(25i\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{4i(dx+c)}-35i\,\mathrm{e}^{i(dx+c)}-55\,\mathrm{e}^{2i(dx+c)}+8\right)}{5d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}}-\frac{A\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d\,a^{3}}+\frac{A\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d\,a^{3}}$
norman	$\frac{\frac{8A\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{18A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{38A\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{26A}{5ad}+\frac{22A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{40A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{176A\left(\tan^{2}\left(\frac{dx}{2}\right.\right.}{5ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*A/a^3*(ln(tan(1/2*d*x+1/2*c))+16/5/(tan(1/2*d*x+1/2*c)+1)^5-8/(tan(1/2*
d*x+1/2*c)+1)^4+12/(tan(1/2*d*x+1/2*c)+1)^3-10/(tan(1/2*d*x+1/2*c)+1)^2+8/(
tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(92) = 184$.
time = 0.30, size = 433, normalized size = 4.42

$$\frac{A\left(\frac{2\left(\frac{115\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{135\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{45\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+32\right)}{a^{3}+\frac{5a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}\right)+\frac{2A\left(\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{30\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+7\right)}{a^{3}+\frac{5a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(92) = 184$.

time = 0.38, size = 310, normalized size = 3.16

$$\frac{16\,A\cos\left(dx+c\right)^{3}-22\,A\cos\left(dx+c\right)^{2}-42\,A\cos\left(dx+c\right)-5\,\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}-2\,A\cos\left(dx+c\right)+\left(A\cos\left(dx+c\right)^{2}-2\,A\cos\left(dx+c\right)-4\,A\right)\sin\left(dx+c\right)-4\,A\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+5\,\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}-2\,A\cos\left(dx+c\right)+\left(A\cos\left(dx+c\right)^{2}-2\,A\cos\left(dx+c\right)-4\,A\right)\sin\left(dx+c\right)-4\,A\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,\left(8\,A\cos\left(dx+c\right)^{2}+19\,A\cos\left(dx+c\right)-2\,A\right)\sin\left(dx+c\right)-4\,A}{10\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx+c\right)-4\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A*
cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2
- 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) +
 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x +
 c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c) - 4*A
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4
*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\right)dx+\int\frac{\sin\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
 d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*si
n(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.46, size = 99, normalized size = 1.01

$$\frac{\frac{5\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{2\,\left(20\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+75\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+45\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,A\right)}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)^
4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(1/
2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 14.77, size = 199, normalized size = 2.03

$$\frac{A\left(5\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+90\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+150\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+110\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+25\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+50\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+50\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+25\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+26\right)}{5\,a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(5*log(tan(c/2 + (d*x)/2)) + 90*tan(c/2 + (d*x)/2) + 150*tan(c/2 + (d*x)
/2)^2 + 110*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^4 + 25*log(tan(c/2
 + (d*x)/2))*tan(c/2 + (d*x)/2) + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x
)/2)^2 + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 25*log(tan(c/2 +
 (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 5*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)
/2)^5 + 26))/(5*a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=113

$$\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)}{a^3d}-\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3}+\frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2}-\frac{104A\cot(c}{15a^3d(1+\csc}$$

```
[Out] 4*A*arctanh(cos(d*x+c))/a^3/d-94/15*A*cot(d*x+c)/a^3/d+2/5*A*cot(d*x+c)/a^3
/d/(1+sin(d*x+c))^3+13/15*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))^2+4*A*cot(d*x+c
)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]

time = 0.29, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3029, 2788, 3855, 3852, 8, 3862, 4007, 4004, 3879}

$$-\frac{A\cot(c+dx)}{a^3d}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{104A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)}+\frac{31A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)^2}-\frac{2A\cot(c+dx)}{5a^3d(\csc(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (A*Cot[c + d*x])/(a^3*d) - (2*A*Cot[c
 + d*x])/(5*a^3*d*(1 + Csc[c + d*x])^3) + (31*A*Cot[c + d*x])/(15*a^3*d*(1
+ Csc[c + d*x])^2) - (104*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = (aA)\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$

$$= \frac{A\int \left(\frac{9}{a^2} - \frac{4\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a^2} - \frac{2}{a^2(1+\csc(c+dx))^3} + \frac{9}{a^2(1+\csc(c+dx))^2}\right.}{a}$$

$$= \frac{9Ax}{a^3} + \frac{A\int \csc^2(c+dx)\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\csc(c+dx))^3}\,dx}{a^3} - \frac{(4A)\int \mathrm{cs}}{}$$

$$= \frac{9Ax}{a^3} + \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{3}{a^3d(}$$

$$= \frac{2Ax}{a^3} + \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c-}{5a^3d(1+\csc(}$$

$$= \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))}$$

$$= \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))}$$

Mathematica [A]

time = 1.97, size = 167, normalized size = 1.48

$$-\frac{A\left(15\cot\left(\frac{1}{2}(c+dx)\right) - 120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{12}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + \frac{38}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-287+79\cos(2(c+dx))-354\sin(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5} - 15\tan\left(\frac{1}{2}(c+dx)\right)\right)}{30a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c
+ d*x)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*x)]
 - 354*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[(c +
 d*x)/2]))/(a^3*d)
```

Maple [A]

time = 0.46, size = 120, normalized size = 1.06

method	result
derivativedivides	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - 8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} + \frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} - \frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} + \frac{}{(\tan}\right.}{2d\,a^3}$
default	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - 8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5} + \frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4} - \frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} + \frac{}{(\tan}\right.}{2d\,a^3}$

risch	$-\frac{4\left(-320A\,e^{4i(dx+c)}+150iA\,e^{5i(dx+c)}+367A\,e^{2i(dx+c)}-385iA\,e^{3i(dx+c)}-47A+205iA\,e^{i(dx+c)}+30A\,e^{6i(dx+c)}\right)}{15\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^5 a^3 d}-\frac{4A\ldots}{\ldots}$
norman	$\frac{-\frac{A}{2ad}+\frac{A\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{3811A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}-\frac{893A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{413A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{161A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-805A\left(\ldots\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d*A/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c
))-32/5/(tan(1/2*d*x+1/2*c)+1)^5+16/(tan(1/2*d*x+1/2*c)+1)^4-88/3/(tan(1/2*
d*x+1/2*c)+1)^3+28/(tan(1/2*d*x+1/2*c)+1)^2-36/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(107) = 214$.
time = 0.29, size = 519, normalized size = 4.59

$$-\frac{3A\left(\frac{\frac{121\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\sin(dx+c)}{a^3(\cos(dx+c)+1)}\right)+2A\left(\frac{2\left(\frac{115\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{135\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+32\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/30*(3*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + 2*A*(2*(115*sin(d*x + c)/(cos
(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(107) = 214$.
time = 0.42, size = 406, normalized size = 3.59

$$\frac{94A\cos(dx+c)^4+222A\cos(dx+c)^3-115A\cos(dx+c)^2-237A\cos(dx+c)+30\left(A\cos(dx+c)^4-2A\cos(dx+c)^3-5A\cos(dx+c)^2+2A\cos(dx+c)-\left(A\cos(dx+c)^3+3A\cos(dx+c)^2-2A\cos(dx+c)-4A\right)\sin(dx+c)+4A\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-30\left(A\cos(dx+c)^4-2A\cos(dx+c)^3-5A\cos(dx+c)^2+2A\cos(dx+c)-\left(A\cos(dx+c)^3+3A\cos(dx+c)^2-2A\cos(dx+c)-4A\right)\sin(dx+c)+4A\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(94A\cos(dx+c)^3-128A\cos(dx+c)^2-243A\cos(dx+c)-6A\right)\sin(dx+c)+6A}{15\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^3-5a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+4a^3d-\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 - 2
37*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x
 + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*c
os(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30*(A*
cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c)
 - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x
 + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 128*A*co
s(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x
 + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Giac [A]

time = 0.50, size = 146, normalized size = 1.29

$$-\frac{\frac{120\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{15\,\left(8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\,\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+605\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+385\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+104\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c)
/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(13
5*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/2*d
*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^5))/d
```

Mupad [B]

time = 15.77, size = 210, normalized size = 1.86

$$\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^3\,d}-\frac{4\,A\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}-\frac{37\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+121\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{514\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{338\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{491\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15}+A}{d\,\left(2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*tan(c/2 + (d*x)/2))/(2*a^3*d) - (4*A*log(tan(c/2 + (d*x)/2)))/(a^3*d) -
(A + (491*A*tan(c/2 + (d*x)/2))/15 + (338*A*tan(c/2 + (d*x)/2)^2)/3 + (514*
A*tan(c/2 + (d*x)/2)^3)/3 + 121*A*tan(c/2 + (d*x)/2)^4 + 37*A*tan(c/2 + (d*
x)/2)^5)/(d*(10*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3 + 20
*a^3*tan(c/2 + (d*x)/2)^4 + 10*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (
d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)))
```

3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=138

$$-\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{29A}{15a^3d(1}$$

```
[Out] -19/2*A*arctanh(cos(d*x+c))/a^3/d+4*A*cot(d*x+c)/a^3/d-1/2*A*cot(d*x+c)*csc
(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+29/15*A*cos(d*x+c)/a^
3/d/(1+sin(d*x+c))^2+164/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3045, 3855, 3852, 8, 3853, 2729, 2727}

$$\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*C
ot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[
c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (164*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
```

```
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx &= \int \left(\frac{9A\csc(c+dx)}{a^3}-\frac{4A\csc^2(c+dx)}{a^3}+\frac{A\csc^3(c+dx)}{a^3}-\frac{\ldots}{a^3(1+\ldots}\right.\\ &= \frac{A\int\csc^3(c+dx)\,dx}{a^3}-\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}-\frac{(4A)\int\csc^2(c+dx\ldots}{a^3}\\ &= -\frac{9A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{2A\,\mathrm{co}\ldots}{5a^3d(1+\ldots}\\ &= -\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}-\frac{A\cot(c+dx)\csc(c\ldots}{2a^3d}\\ &= -\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}-\frac{A\cot(c+dx)\csc(c\ldots}{2a^3d}\end{aligned}$$

Mathematica [A]
time = 2.48, size = 245, normalized size = 1.78

$$\frac{A\left(240\cot\left(\frac{1}{2}(c+dx)\right)-15\csc^2\left(\frac{1}{2}(c+dx)\right)-1140\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1140\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+15\sec^2\left(\frac{1}{2}(c+dx)\right)-\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}+\frac{48}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{464\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{232}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{2624\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}-240\tan\left(\frac{1}{2}(c+dx)\right)\right)}{120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/2
]] + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^4 - (464*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*x)/2]
)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120*a^3*d
)
```

Maple [A]
time = 0.53, size = 148, normalized size = 1.07

method	result
derivativedivides	$\frac{A\left(\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+38\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{3\ldots}{\left(\tan\left(\frac{dx}{2}\ldots\right.\right.}\right.}{4d\,a^3}$
default	$\frac{A\left(\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+38\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{3\ldots}{\left(\tan\left(\frac{dx}{2}\ldots\right.\right.}\right.}{4d\,a^3}$
risch	$\frac{A\left(1425i\,e^{7i(dx+c)}+285\,e^{8i(dx+c)}-5225i\,e^{5i(dx+c)}-3325\,e^{6i(dx+c)}+5635i\,e^{3i(dx+c)}+6423\,e^{4i(dx+c)}-1955i\,e^{i(dx+c)}-395\ldots\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^5a^3d}$
norman	$\frac{\frac{57A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{A}{8ad}+\frac{11A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{11A\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{1943A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{2627A\left(\tan\ldots\right.}{60\ldots}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d*A/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-8*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+
1/2*c)^2+8/tan(1/2*d*x+1/2*c)+38*ln(tan(1/2*d*x+1/2*c))+64/5/(tan(1/2*d*x+1
/2*c)+1)^5-32/(tan(1/2*d*x+1/2*c)+1)^4+208/3/(tan(1/2*d*x+1/2*c)+1)^3-72/(t
an(1/2*d*x+1/2*c)+1)^2+128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(128) = 256$.
time = 0.31, size = 622, normalized size = 4.51

$$\frac{12\,A\left(\frac{\frac{121\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\sin(dx+c)}{a^3(\cos(dx+c)+1)}\right)+A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{2782\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{9410\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{13645\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9285\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2580\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-15}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}-\frac{15\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3}+\frac{780\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x +
```

```
c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a
^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)/(cos(d*
x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9285*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*
x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(128) = 256$.
time = 0.40, size = 498, normalized size = 3.61

$$\frac{896\,A\cos(dx+c)^5 - 1222\,A\cos(dx+c)^4 - 3218\,A\cos(dx+c)^3 + 1168\,A\cos(dx+c)^2 + 2292\,A\cos(dx+c) - 285\left(A\cos(dx+c)^5 + 3\,A\cos(dx+c)^4 - 3\,A\cos(dx+c)^3 - 7\,A\cos(dx+c)^2 + 2\,A\cos(dx+c) + \left(A\cos(dx+c)^4 - 2\,A\cos(dx+c)^3 - 5\,A\cos(dx+c)^2 + 2\,A\cos(dx+c) + 4\,A\right)\sin(dx+c) + 4\,A\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 285\left(A\cos(dx+c)^5 + 3\,A\cos(dx+c)^4 - 3\,A\cos(dx+c)^3 - 7\,A\cos(dx+c)^2 + 2\,A\cos(dx+c) + \left(A\cos(dx+c)^4 - 2\,A\cos(dx+c)^3 - 5\,A\cos(dx+c)^2 + 2\,A\cos(dx+c) + 4\,A\right)\sin(dx+c) + 4\,A\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 2\left(448\,A\cos(dx+c)^4 + 1059\,A\cos(dx+c)^3 - 550\,A\cos(dx+c)^2 - 1134\,A\cos(dx+c) + 12\,A\right)\sin(dx+c) + 24\,A}{60\left(a^3d\cos(dx+c)^5 + 3\,a^3d\cos(dx+c)^4 - 3\,a^3d\cos(dx+c)^3 - 7\,a^3d\cos(dx+c)^2 + 2\,a^3d\cos(dx+c) + 4\,a^3d + \left(a^3d\cos(dx+c)^4 - 2\,a^3d\cos(dx+c)^3 - 5\,a^3d\cos(dx+c)^2 + 2\,a^3d\cos(dx+c) + 4\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
+ 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c)^5 + 3*A
*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c
) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d
*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) + 285*(A*cos
(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2
+ 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x +
 c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*cos(d*x + c
)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos(d*x + c)^
5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*
x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x
 + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx + \int\frac{\sin(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.53, size = 180, normalized size = 1.30

$$\frac{\frac{1140\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,\left(114\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{15\,\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6}+\frac{16\,\left(240\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+825\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1165\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+755\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+199\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x +
1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 15
*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(2
40*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan(1/2
*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*x + 1
/2*c) + 1)^5))/d
```

Mupad [B]
time = 15.74, size = 288, normalized size = 2.09

$$\frac{A\,\left(165\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4234\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+14090\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+19780\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+12060\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+1830\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-1050\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-165\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+1140\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+5700\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+11400\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+11400\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+5700\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+1140\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-15\right)}{120\,a^3\,d\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(165*tan(c/2 + (d*x)/2) + 4234*tan(c/2 + (d*x)/2)^2 + 14090*tan(c/2 + (d
*x)/2)^3 + 19780*tan(c/2 + (d*x)/2)^4 + 12060*tan(c/2 + (d*x)/2)^5 + 1830*t
an(c/2 + (d*x)/2)^6 - 1050*tan(c/2 + (d*x)/2)^7 - 165*tan(c/2 + (d*x)/2)^8
+ 15*tan(c/2 + (d*x)/2)^9 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)
^2 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 11400*log(tan(c/2
+ (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 11400*log(tan(c/2 + (d*x)/2))*tan(c/2 +
(d*x)/2)^5 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 1140*log(t
an(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 - 15))/(120*a^3*d*tan(c/2 + (d*x)/2
)^2*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$\frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{2A\cos(c+dx}{5a^3d(1+\sin(c+}$$

```
[Out] 18*A*arctanh(cos(d*x+c))/a^3/d-10*A*cot(d*x+c)/a^3/d-1/3*A*cot(d*x+c)^3/a^3
/d+2*A*cot(d*x+c)*csc(d*x+c)/a^3/d-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-
13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2-93/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x
+c))
```

Rubi [A]
time = 0.17, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3045, 3855, 3852, 8, 3853, 2729, 2727}

$$-\frac{A\cot^3(c+dx)}{3a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Cot
[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx &= \int \left(-\frac{16A\csc(c+dx)}{a^3}+\frac{9A\csc^2(c+dx)}{a^3}-\frac{4A\csc^3(c+dx)}{a^3}+\frac{A\right.\\ &= \frac{A\int\csc^4(c+dx)\,dx}{a^3}+\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}-\frac{(4A)\int\csc^3(c+d}{a^3}\\ &= \frac{16A\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{2A}{5a^3d(1}\\ &= \frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}+\\ &= \frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(153) = 306.

time = 6.18, size = 348, normalized size = 2.27

$$\frac{A\left(-\frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{6d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{2d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}+\frac{18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{2d}+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}-\frac{2}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{26\sin\left(\frac{1}{2}(c+dx)\right)}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{13}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{186\sin\left(\frac{1}{2}(c+dx)\right)}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{29\tan\left(\frac{1}{2}(c+dx)\right)}{6d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[Si
n[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*Sin[(c
+ d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c + d*x)/
2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)))/a^3
```

Maple [A]
time = 0.52, size = 174, normalized size = 1.14

method	result
derivativedivides	$\frac{A\left(\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{39}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-144\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{8d\,a^{3}}$
default	$\frac{A\left(\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{39}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-144\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{8d\,a^{3}}$
risch	$-\frac{4\left(675iA\,e^{9i(dx+c)}+135A\,e^{10i(dx+c)}-3150iA\,e^{7i(dx+c)}-1710A\,e^{8i(dx+c)}+5180iA\,e^{5i(dx+c)}+4572A\,e^{6i(dx+c)}-3590iA\right.}{15\left(e^{2i(dx+c)}-1\right)^{3}\left(e^{i(dx+c)}+i\right)^{5}a^{3}d}$
norman	$\frac{-\frac{A}{24ad}+\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{17A\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{17A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{7A\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{A\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{66469A\left(\tan^{5}\right.}{120c}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d*A/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+39*tan(1/2*d*x
+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2-39/tan(1/2*d*x+1/2*
c)-144*ln(tan(1/2*d*x+1/2*c))-128/5/(tan(1/2*d*x+1/2*c)+1)^5+64/(tan(1/2*d*
x+1/2*c)+1)^4-160/(tan(1/2*d*x+1/2*c)+1)^3+176/(tan(1/2*d*x+1/2*c)+1)^2-400
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(145) = 290$.
time = 0.30, size = 706, normalized size = 4.61

$$-\frac{A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{2782\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{9410\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{13645\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{9285\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2580\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-15}{\frac{a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{5a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{10a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}-\frac{15\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{3}}+\frac{780\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}\right)-A\left(\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}-\frac{230\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4777\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{15785\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{22390\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{14940\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{4005\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-5}{\frac{a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{10a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{5\left(\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{3}}-\frac{1380\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 25
80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a
^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((20*si
n(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)/(a^3*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 9
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(145) = 290$.
time = 0.39, size = 594, normalized size = 3.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4 -
 2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135*(
A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x +
c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(
d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4
*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x + c)^6
 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A
*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c)
 - 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 578*A*cos(d*
x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A*cos(d*x +
c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*cos(d*x + c)^
5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*cos(d*x + c)^
```

```
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*
x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*
x + c) + 4*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.49, size = 213, normalized size = 1.39

$$-\frac{\frac{2160\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{5\,\left(792\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-117\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{48\,\left(125\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+445\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+635\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+415\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}-\frac{5\,\left(Aa^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,Aa^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+117\,Aa^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^9}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/(a
^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*tan(1
/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x + 1/2*
c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x + 1/2
*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2*c))/a
^9)/d
```

Mupad [B]
time = 15.28, size = 314, normalized size = 2.05

$$-\frac{A\left(335\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7559\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+24610\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+33170\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+18670\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+1310\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-2375\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-335\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+2160\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+10800\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+21600\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+21600\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10800\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+2160\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\right)}{120\,a^3\,d\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(A*(335*tan(c/2 + (d*x)/2)^2 - 35*tan(c/2 + (d*x)/2) + 7559*tan(c/2 + (d*x
)/2)^3 + 24610*tan(c/2 + (d*x)/2)^4 + 33170*tan(c/2 + (d*x)/2)^5 + 18670*ta
n(c/2 + (d*x)/2)^6 + 1310*tan(c/2 + (d*x)/2)^7 - 2375*tan(c/2 + (d*x)/2)^8
- 335*tan(c/2 + (d*x)/2)^9 + 35*tan(c/2 + (d*x)/2)^10 - 5*tan(c/2 + (d*x)/2
```

```
)^11 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 10800*log(tan(c/
2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2
+ (d*x)/2)^5 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 10800*l
og(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 + 2160*log(tan(c/2 + (d*x)/2))*
tan(c/2 + (d*x)/2)^8 + 5))/(120*a^3*d*tan(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)
/2) + 1)^5)
```

3.244 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=327

$$\frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A\left(8c^3+12c^2d+12cd^2+3d^3\right)\right)x-\frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-}{}$$

```
[Out] 1/8*a*(B*(4*c^3+12*c^2*d+9*c*d^2+3*d^3)+A*(8*c^3+12*c^2*d+12*c*d^2+3*d^3))*
x-1/30*a*(5*A*d*(3*c^3+16*c^2*d+12*c*d^2+4*d^3)-B*(3*c^4-15*c^3*d-52*c^2*d^
2-60*c*d^3-16*d^4))*cos(f*x+e)/d/f-1/120*a*(5*A*d*(6*c^2+20*c*d+9*d^2)-B*(6
*c^3-30*c^2*d-71*c*d^2-45*d^3))*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(4*(5*A+4*B)
*d^2-3*c*(B*c-5*(A+B)*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*a*(B*c-5*(
A+B)*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a*B*cos(f*x+e)*(c+d*sin(f*x+e
))^4/d/f
```

Rubi [A]
time = 0.40, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2832, 2813}

$$-\frac{a(5Ad(6c^2+20cd+9d^2)-B(6c^3-30c^2d-71cd^2-45d^3))\sin(e+fx)\cos(e+fx)}{120f}+\frac{1}{8}ax\left(A\left(8c^3+12c^2d+12cd^2+3d^3\right)+B\left(4c^3+12c^2d+9cd^2+3d^3\right)\right)-\frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-B(3c^4-15c^3d-52c^2d^2-60cd^3-16d^4))\cos(e+fx)}{30df}-\frac{a(4d^2(5A+4B)-3c(Bc-5d(A+B)))\cos(e+fx)(c+d\sin(e+fx))^2}{60df}+\frac{a(Bc-5d(A+B))\cos(e+fx)(c+d\sin(e+fx))^3}{20df}-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c*d^2
 + 3*d^3))*x)/8 - (a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) - B*(3*c^
4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/(30*d*f) - (a
*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45*d^3)
)*Cos[e + f*x]*Sin[e + f*x])/(120*f) - (a*(4*(5*A + 4*B)*d^2 - 3*c*(B*c - 5
*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d*f) + (a*(B*c - 5*(A
 + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a*B*Cos[e + f*x]*
(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \int (c+d\sin(e+fx))^3\left(aA+(aA+aB\right.$$
$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}+$$
$$= \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\,\mathrm{s}}{20df}$$
$$= -\frac{a(4(5A+4B)d^2-3c(Bc-5(A+B)}{60d}$$
$$= \frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A(8\right.$$

Mathematica [A]
time = 1.40, size = 267, normalized size = 0.82

$$\frac{a(1+\sin(e+fx))\left(-60(2A(4c^3+12c^2d+9cd^2+3d^3)+B(8c^3+18c^2d+18cd^2+5d^3))\cos(e+fx)+10d(4Ad(3c+d)+B(12c^2+12cd+5d^2))\cos(3(e+fx))-6Bd^3\cos(5(e+fx))+15(4(B(4c^3+12c^2d+9cd^2+3d^3)+A(8c^3+12c^2d+12cd^2+3d^3))fx-8(B(c+d)^3+Ad(3c^2+3cd+d^2))\sin(2(e+fx))+d^2(Ad+B(3c+d))\sin(4(e+fx)))\right)}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(8
*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d) +
 B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)] +
 15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c
*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2*(e +
 f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)
```

Maple [A]
time = 0.28, size = 422, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(A*c^3*a*(f*x+e)-3*A*c^2*d*a*cos(f*x+e)+3*A*c*d^2*a*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-1/3*A*d^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)-B*c^3*a*cos
(f*x+e)+3*B*c^2*d*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*c*d^2*a*(2
+sin(f*x+e)^2)*cos(f*x+e)+B*d^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f
*x+e)+3/8*f*x+3/8*e)-A*c^3*a*cos(f*x+e)+3*A*c^2*d*a*(-1/2*cos(f*x+e)*sin(f*
x+e)+1/2*f*x+1/2*e)-A*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+A*d^3*a*(-1/4*(si
n(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*c^3*a*(-1/2*cos(f*x+
e)*sin(f*x+e)+1/2*f*x+1/2*e)-B*c^2*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*c*d^
2*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*d^3
*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A]
time = 0.29, size = 438, normalized size = 1.34

$$\frac{480\,(fx+e)Aac^3 + 120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac^3 + 360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aac^2d + 480\,(\cos(fx+e)^3-3\,\cos(fx+e))Bac^2d + 360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac^2d + 480\,(\cos(fx+e)^3-3\,\cos(fx+e))Aacd^2 + 360\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aacd^2 + 480\,(\cos(fx+e)^3-3\,\cos(fx+e))Bacd^2 + 45\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Bacd^2 + 160\,(\cos(fx+e)^3-3\,\cos(fx+e))Aad^3 + 15\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Aad^3 - 32\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))Bad^3 + 15\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Bad^3 - 480\,Aac^3\cos(fx+e) - 480\,Bac^3\cos(fx+e) - 1440\,Aac^2d\cos(fx+e)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^3
 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*d +
 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d^2 +
45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2 + 160*
(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e
)^3 + 15*cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*cos(f*x +
 e) - 1440*A*a*c^2*d*cos(f*x + e))/f
```

Fricas [**A**]
time = 0.44, size = 245, normalized size = 0.75

$$-\frac{24\,Bad^3\cos\left(fx+e\right)^5-40\,\left(3\,Bac^2d+3\,\left(A+B\right)acd^2+\left(A+2\,B\right)ad^3\right)\cos\left(fx+e\right)^3-15\,\left(4\,\left(2\,A+B\right)ac^3+12\,\left(A+B\right)ac^2d+3\,\left(4\,A+3\,B\right)acd^2+3\,\left(A+B\right)ad^3\right)fx+120\,\left(\left(A+B\right)ac^3+3\,\left(A+B\right)ac^2d+3\,\left(A+B\right)acd^2+\left(A+B\right)ad^3\right)\cos\left(fx+e\right)-15\,\left(2\,\left(3\,Bacd^2+\left(A+B\right)ad^3\right)\cos\left(fx+e\right)^3-\left(4\,Bac^3+12\,\left(A+B\right)ac^2d+3\,\left(4\,A+5\,B\right)acd^2+5\,\left(A+B\right)ad^3\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 + (
A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3*(A
 + B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2*(3*
B*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(311) = 622$.
time = 0.42, size = 996, normalized size = 3.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f*
x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*x)*
*2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*cos(
e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d**2*co
s(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*
A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f*x)**3/(3
*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2 - B*a*c**
3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*B*a*c**2*d*
x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*c**2*d*sin(e
 + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) -
2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)**4/8 + 9*B*a*c
*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*cos(e + f*x)**4/
8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a*c*d**2*sin(e +
 f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) -
 2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*x)**4/8 + 3*B*a*d*
*3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*cos(e + f*x)**4/8 - B
*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*sin(e + f*x)**3*cos(e +
```

```
f*x)/(8*f) - 4*B*a*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*d**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a*d**3*cos(e + f*x)**5/(15*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a), True))
```

Giac [A]
time = 0.74, size = 314, normalized size = 0.96

$$-\frac{Bad^3\cos(5\,fx+5\,e)}{80\,f}+\frac{1}{8}\,(8\,Aac^3+4\,Bac^3+12\,Aac^2d+12\,Bac^2d+12\,Aacd^2+9\,Bacd^2+3\,Aad^3+3\,Bad^3)x+\frac{(12\,Bac^2d+12\,Aacd^2+12\,Bacd^2+4\,Aad^3+5\,Bad^3)\cos(3\,fx+3\,e)}{48\,f}-\frac{(8\,Aac^3+8\,Bac^3+24\,Aac^2d+18\,Bac^2d+18\,Aacd^2+18\,Bacd^2+6\,Aad^3+5\,Bad^3)\cos(fx+e)}{8\,f}+\frac{(3\,Bacd^2+Aad^3+Bad^3)\sin(4\,fx+4\,e)}{32\,f}-\frac{(Bac^3+3\,Aac^2d+3\,Bac^2d+3\,Aacd^2+3\,Bacd^2+Aad^3+Bad^3)\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2*
d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x +
1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3)*c
os(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*c^2*
d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/f + 1
/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c^3 + 3
*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*s
in(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.58, size = 830, normalized size = 2.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A
*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*(2*A*a*c^3 + (3*A*a*d^3)/
4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 +
 3*B*a*c^2*d)))*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 12*A*
c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*f) - (tan(e/2 + (f*x)/2)*((3*A*a*d^3)/4
 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 +
3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^3 + 2*B*a*c^3 + 6*A*a*c^2*d) +
 tan(e/2 + (f*x)/2)^2*(8*A*a*c^3 + (20*A*a*d^3)/3 + 8*B*a*c^3 + (16*B*a*d^3
)/3 + 20*A*a*c*d^2 + 24*A*a*c^2*d + 20*B*a*c*d^2 + 20*B*a*c^2*d) + tan(e/2
+ (f*x)/2)^4*(12*A*a*c^3 + (28*A*a*d^3)/3 + 12*B*a*c^3 + (32*B*a*d^3)/3 + 2
8*A*a*c*d^2 + 36*A*a*c^2*d + 28*B*a*c*d^2 + 28*B*a*c^2*d) + tan(e/2 + (f*x)
/2)^6*(8*A*a*c^3 + 4*A*a*d^3 + 8*B*a*c^3 + 12*A*a*c*d^2 + 24*A*a*c^2*d + 12
*B*a*c*d^2 + 12*B*a*c^2*d) - tan(e/2 + (f*x)/2)^9*((3*A*a*d^3)/4 + B*a*c^3
+ (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 + 3*B*a*c^2*d
) + tan(e/2 + (f*x)/2)^3*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a
*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) - tan(e/2 + (f*x)/2)
```

```
^7*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d +
 (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) + 2*A*a*c^3 + (4*A*a*d^3)/3 + 2*B*a*c^3 +
(16*B*a*d^3)/15 + 4*A*a*c*d^2 + 6*A*a*c^2*d + 4*B*a*c*d^2 + 4*B*a*c^2*d)/(f
*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^
6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.245 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=213

$$\frac{1}{8}a\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)x-\frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e$$

```
[Out] 1/8*a*(4*A*(2*c^2+2*c*d+d^2)+B*(4*c^2+8*c*d+3*d^2))*x-1/6*a*(4*A*d*(c^2+3*c
*d+d^2)-B*(c^3-4*c^2*d-8*c*d^2-4*d^3))*cos(f*x+e)/d/f-1/24*a*(3*(4*A+3*B)*d
^2-2*c*(B*c-4*(A+B)*d))*cos(f*x+e)*sin(f*x+e)/f+1/12*a*(B*c-4*(A+B)*d)*cos(
f*x+e)*(c+d*sin(f*x+e))^2/d/f-1/4*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A]
time = 0.25, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2832, 2813}

$$\frac{a(-8cd(A+B)-3d^2(4A+3B)+2Bc^2)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}ax\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)-\frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e+fx)}{6df}+\frac{a(Bc-4d(A+B))\cos(e+fx)(c+d\sin(e+fx))^2}{12df}-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/8 - (a*(4*A*d
*(c^2 + 3*c*d + d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/(
6*d*f) + (a*(2*B*c^2 - 8*(A + B)*c*d - 3*(4*A + 3*B)*d^2)*Cos[e + f*x]*Sin[
e + f*x])/(24*f) + (a*(B*c - 4*(A + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*d*f) - (a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (c+d\sin(e+fx))^2\left(aA+(aA+aB\right.$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df} +$$

$$= \frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\,\mathrm{s}}{12df}$$

$$= \frac{1}{8}a\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd\right.\right.$$

Mathematica [A]
time = 1.07, size = 185, normalized size = 0.87

$$\frac{a(1+\sin(e+fx))\left(-24\left(B\left(4c^2+6cd+3d^2\right)+A\left(4c^2+8cd+3d^2\right)\right)\cos(e+fx)+8d(Ad+B(2c+d))\cos(3(e+fx))+3\left(4\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)fx-8\left(B(c+d)^2+Ad(2c+d)\right)\sin(2(e+fx))+Bd^2\sin(4(e+fx))\right)\right)}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d +
3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4*A
*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2 +
A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)])))/(96*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A]
time = 0.19, size = 274, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(A*c^2*a*(f*x+e)-2*A*c*d*a*cos(f*x+e)+A*d^2*a*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-B*c^2*a*cos(f*x+e)+2*B*c*d*a*(-1/2*cos(f*x+e)*sin(f*x+e)+
1/2*f*x+1/2*e)-1/3*B*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)-A*c^2*a*cos(f*x+e)+2
*A*c*d*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*A*d^2*a*(2+sin(f*x+
e)^2)*cos(f*x+e)+B*c^2*a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/3*B*c
*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^2*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e)
)*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A]
time = 0.29, size = 285, normalized size = 1.34

$$\frac{96\,(fx+e)Aac^2+24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac^2+48\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aacd+64\,(\cos(fx+e)^3-3\,\cos(fx+e))Bacd+48\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bacd+32\,(\cos(fx+e)^3-3\,\cos(fx+e))Aad^2+24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aad^2+32\,(\cos(fx+e)^3-3\,\cos(fx+e))Bad^2+3\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Bad^2-96\,Aac^2\cos(fx+e)-96\,Bac^2\cos(fx+e)-192\,Aacd\cos(fx+e)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a
*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e) - 96
*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

Fricas [A]
time = 0.41, size = 165, normalized size = 0.77

$$\frac{8\,(2\,Bacd+(A+B)ad^2)\cos(fx+e)^3+3\,(4\,(2\,A+B)ac^2+8\,(A+B)acd+(4\,A+3\,B)ad^2)fx-24\,((A+B)ac^2+2\,(A+B)acd+(A+B)ad^2)\cos(fx+e)+3\,(2\,Bad^2\cos(fx+e)^3-(4\,Bac^2+8\,(A+B)acd+(4\,A+5\,B)ad^2)\cos(fx+e))\sin(fx+e)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2 +
 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)*a
*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*a*c
^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(201) = 402$.
time = 0.28, size = 571, normalized size = 2.68

$$\begin{cases} Aac^2x - \frac{Aac^2\cos(e+fx)}{f} + Aacdx\sin^2(e+fx) + Aacdx\cos^2(e+fx) - \frac{Aacd\sin(e+fx)\cos(e+fx)}{f} - \frac{2Aacd\cos(e+fx)}{f} + \frac{Aad^2x\sin^2(e+fx)}{2} + \frac{Aad^2x\cos^2(e+fx)}{2} - \frac{Aad^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{Aad^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aad^2\cos^3(e+fx)}{3f} + \frac{Bac^2x\sin^2(e+fx)}{2} + \frac{Bac^2x\cos^2(e+fx)}{2} - \frac{Bac^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{Bac^2\cos(e+fx)}{f} + Bacdx\sin^2(e+fx) + Bacdx\cos^2(e+fx) - \frac{2Bacd\sin^2(e+fx)\cos(e+fx)}{f} - \frac{Bacd\sin(e+fx)\cos(e+fx)}{f} - \frac{4Bacd\cos^3(e+fx)}{3f} + \frac{3Bad^2x\sin^4(e+fx)}{8} + \frac{3Bad^2x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3Bad^2x\cos^4(e+fx)}{8} - \frac{5Bad^2\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{Bad^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Bad^2\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{2Bad^2\cos^3(e+fx)}{3f} & \text{for}\: f \neq 0 \\ x(A+B\sin(e))\,(c+d\sin(e))^2\,(a\sin(e)+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**2
 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a*
c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f*x)
**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)*cos
(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e + f*x
)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e + f*x)/
(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*d*x*cos
(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*sin(e + f
*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*x*sin(e +
 f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**2*x*
cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - B*a*d**
2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
 d*sin(e))**2*(a*sin(e) + a), True))
```

Giac [A]
time = 0.53, size = 198, normalized size = 0.93

$$\frac{Bad^2\sin(4fx+4e)}{32f}+\frac{1}{8}(8Aac^2+4Bac^2+8Aacd+8Bacd+4Aad^2+3Bad^2)x+\frac{(2Bacd+Aad^2+Bad^2)\cos(3fx+3e)}{12f}-\frac{(4Aac^2+4Bac^2+8Aacd+6Bacd+3Aad^2+3Bad^2)\cos(fx+e)}{4f}-\frac{(Bac^2+2Aacd+2Bacd+Aad^2+Bad^2)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d +
8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^2)
*cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d +
3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*a*c*
d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.41, size = 547, normalized size = 2.57

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8Ac^2+4Ad^2+4Bc^2+3Bd^2+8Acd+8Bcd)}{4\left(2Aac^2+Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2Bacd\right)}\right)(8Ac^2+4Ad^2+4Bc^2+3Bd^2+8Acd+8Bcd)}{4f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(2Aac^2+2Bac^2+4Aacd)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2Bacd\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6Aac^2+4Aad^2+6Bac^2+4Bad^2+12Aacd+8Bacd)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(6Aac^2+\frac{16Aad^2}{3}+6Bac^2+\frac{16Bad^2}{3}+12Aacd+\frac{32Bacd}{3}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2Bacd\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(Aad^2+Bac^2+\frac{11Bad^2}{4}+2Aacd+2Bacd\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(Aad^2+Bac^2+\frac{11Bad^2}{4}+2Aacd+2Bacd\right)+2Aac^2+\frac{4Aad^2}{3}+2Bac^2+\frac{4Bad^2}{3}+4Aacd+\frac{8Bacd}{3}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*
c*d + 8*B*c*d))/(4*(2*A*a*c^2 + A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c
*d + 2*B*a*c*d)))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*c*d + 8*B*c*
d))/(4*f) - (tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 + 2*B*a*c^2 + 4*A*a*c*d) + tan
```

```
(e/2 + (f*x)/2)*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
 + tan(e/2 + (f*x)/2)^4*(6*A*a*c^2 + 4*A*a*d^2 + 6*B*a*c^2 + 4*B*a*d^2 + 12
*A*a*c*d + 8*B*a*c*d) + tan(e/2 + (f*x)/2)^2*(6*A*a*c^2 + (16*A*a*d^2)/3 +
6*B*a*c^2 + (16*B*a*d^2)/3 + 12*A*a*c*d + (32*B*a*c*d)/3) - tan(e/2 + (f*x)
/2)^7*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) + tan(e/2
 + (f*x)/2)^3*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
- tan(e/2 + (f*x)/2)^5*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*
B*a*c*d) + 2*A*a*c^2 + (4*A*a*d^2)/3 + 2*B*a*c^2 + (4*B*a*d^2)/3 + 4*A*a*c*
d + (8*B*a*c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*
tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=111

$$\frac{1}{2}a(B(c+d)+A(2c+d))x-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Bc+3Ad-Bd)\cos(e+fx)\sin(e+$$

```
[Out] 1/2*a*(B*(c+d)+A*(2*c+d))*x-1/3*a*(3*A*(c+d)+B*(3*c+d))*cos(f*x+e)/f-1/6*a*
(3*A*d+3*B*c-B*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e
))^2/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3047, 3102, 2813}

$$-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}ax(A(2c+d)+B(c+d))-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(B*(c + d) + A*(2*c + d))*x)/2 - (a*(3*A*(c + d) + B*(3*c + d))*Cos[e +
f*x])/(3*f) - (a*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (
B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(3*a*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))\left(Ac+(Bc+Ad)\sin\right.$$
$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af}+\int$$
$$= \frac{1}{2}a(B(c+d)+A(2c+d))x-\frac{a(3A(c+d}{}$$

Mathematica [A]
time = 0.30, size = 104, normalized size = 0.94

$$\frac{a(12Acfx+6Bcfx+6Adfx+6Bdfx-3(4A(c+d)+B(4c+3d))\cos(e+fx)+Bd\cos(3(e+fx))-3Bc\sin(2(e+fx))-3Ad\sin(2(e+fx))-3Bd\sin(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4*
c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] - 3*
A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)]))/(12*f)
```

Maple [A]
time = 0.14, size = 147, normalized size = 1.32

method	result
derivativedivides	$\frac{-\frac{Bad\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aad\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bad\left(-\cos(\right.}{f}$
default	$\frac{-\frac{Bad\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aad\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bad\left(-\cos(\right.}{f}$
risch	$aAcx+\frac{Aadx}{2}+\frac{Bacx}{2}+\frac{Badx}{2}-\frac{a\cos(fx+e)Ac}{f}-\frac{a\cos(fx+e)Ad}{f}-\frac{Bac\cos(fx+e)}{f}-\frac{3a\cos(fx+e)Bd}{4f}+$
norman	$\frac{(Aac+\frac{1}{2}Aad+\frac{1}{2}Bac+\frac{1}{2}Bad)x+(Aac+\frac{1}{2}Aad+\frac{1}{2}Bac+\frac{1}{2}Bad)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3Aac+\frac{3}{2}Aad+\frac{3}{2}Bac+\frac{3}{2}Bad)x\left(\tan^2\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f*(-1/3*B*a*d*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a*d*(-1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)+B*a*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+B*a*d*(-
1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-A*a*c*cos(f*x+e)-A*a*d*cos(f*x+e)-
B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

Maxima [A]
time = 0.29, size = 155, normalized size = 1.40

$$\frac{12\,(fx+e)Aac+3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac+3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aad+4\,\left(\cos(fx+e)^{3}-3\,\cos(fx+e)\right)Bad+3\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bad-12\,Aac\cos(fx+e)-12\,Bac\cos(fx+e)-12\,Aad\cos(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(f*x + e)*A*a*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*A*a*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*B
*a*d + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*d - 12*A*a*c*cos(f*x + e) - 1
2*B*a*c*cos(f*x + e) - 12*A*a*d*cos(f*x + e))/f
```

Fricas [A]
time = 0.38, size = 88, normalized size = 0.79

$$\frac{2\,Bad\cos(fx+e)^{3}+3\,((2\,A+B)ac+(A+B)ad)fx-3\,(Bac+(A+B)ad)\cos(fx+e)\sin(fx+e)-6\,((A+B)ac+(A+B)ad)\cos(fx+e)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a*
c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d)*
cos(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(100) = 200$.
time = 0.15, size = 277, normalized size = 2.50

$$\begin{cases} Aacx-\frac{Aac\cos(e+fx)}{f}+\frac{Aadx\sin^{2}(e+fx)}{2}+\frac{Aadx\cos^{2}(e+fx)}{2}-\frac{Aad\sin(e+fx)\cos(e+fx)}{2f}-\frac{Aad\cos(e+fx)}{f}+\frac{Bacx\sin^{2}(e+fx)}{2}+\frac{Bacx\cos^{2}(e+fx)}{2}-\frac{Bac\sin(e+fx)\cos(e+fx)}{2f}-\frac{Bac\cos(e+fx)}{f}+\frac{Badx\sin^{2}(e+fx)}{2}+\frac{Badx\cos^{2}(e+fx)}{2}-\frac{Bad\sin^{2}(e+fx)\cos(e+fx)}{f}-\frac{Bad\sin(e+fx)\cos(e+fx)}{2f}-\frac{2Bad\cos^{3}(e+fx)}{3f} & \text{for}\ f\neq 0 \\ x(A+B\sin(e))\,(c+d\sin(e))\,(a\sin(e)+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*a
*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*cos(
e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a*c*
sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e + f*
x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*x)/f
```

```
- B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))
```

Giac [A]
time = 0.50, size = 101, normalized size = 0.91

$$\frac{Bad\cos(3fx+3e)}{12f}+\frac{1}{2}(2Aac+Bac+Aad+Bad)x-\frac{(4Aac+4Bac+4Aad+3Bad)\cos(fx+e)}{4f}-\frac{(Bac+Aad+Bad)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x - 1
/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c + A*
a*d + B*a*d)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 13.31, size = 134, normalized size = 1.21

$$-\frac{\frac{3Aad\sin(2e+2fx)}{2}-\frac{Bad\cos(3e+3fx)}{2}+\frac{3Bac\sin(2e+2fx)}{2}+\frac{3Bad\sin(2e+2fx)}{2}+6Aac\cos(e+fx)+6Aad\cos(e+fx)+6Bac\cos(e+fx)+\frac{9Bad\cos(e+fx)}{2}-6Aacfx-3Aadfx-3Bacfx-3Badfx}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*A*a*d*sin(2*e + 2*f*x))/2 - (B*a*d*cos(3*e + 3*f*x))/2 + (3*B*a*c*sin(
2*e + 2*f*x))/2 + (3*B*a*d*sin(2*e + 2*f*x))/2 + 6*A*a*c*cos(e + f*x) + 6*A
*a*d*cos(e + f*x) + 6*B*a*c*cos(e + f*x) + (9*B*a*d*cos(e + f*x))/2 - 6*A*a
*c*f*x - 3*A*a*d*f*x - 3*B*a*c*f*x - 3*B*a*d*f*x)/(6*f)
```

3.247 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}a(2A+B)x-\frac{a(A+B)\cos(e+fx)}{f}-\frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*A+B)*x-a*(A+B)*cos(f*x+e)/f-1/2*a*B*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2813\}$

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx=\frac{1}{2}a(2A+B)x-\frac{a(A+B)\cos(e+fx)}{f}-\frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.94

$$\frac{a(2Be+4Afx+2Bfx-4(A+B)\cos(e+fx)-B\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

[Out] (a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*Cos[e + f*x] - B*Sin[2*(e + f*x)]
))/(4*f)

Maple [A]
time = 0.12, size = 59, normalized size = 1.23

method	result
risch	$axA+\frac{aBx}{2}-\frac{a\cos(fx+e)A}{f}-\frac{a\cos(fx+e)B}{f}-\frac{aB\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{aB\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-aA\cos(fx+e)-aB\cos(fx+e)+aA(fx+e)}{f}$
default	$\frac{aB\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-aA\cos(fx+e)-aB\cos(fx+e)+aA(fx+e)}{f}$
norman	$\frac{(aA+\frac{1}{2}aB)x+(aA+\frac{1}{2}aB)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2aA+aB)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(2aA+2aB)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{aB\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/f*(a*B*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a*A*cos(f*x+e)-a*B*cos(
f*x+e)+a*A*(f*x+e))

Maxima [A]
time = 0.28, size = 62, normalized size = 1.29

$$\frac{4\,(fx+e)Aa+(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))Ba-4\,Aa\cos\left(fx+e\right)-4\,Ba\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] 1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*x
 + e) - 4*B*a*cos(f*x + e))/f

Fricas [A]
time = 0.37, size = 46, normalized size = 0.96

$$\frac{(2\,A+B)afx-Ba\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A+B)a\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/2*((2*A + B)*a*f*x - B*a*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*a*cos(f*x
+ e))/f

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 0.09, size = 94, normalized size = 1.96

$$\begin{cases} Aax - \frac{Aa\cos(e+fx)}{f} + \frac{Bax\sin^2(e+fx)}{2} + \frac{Bax\cos^2(e+fx)}{2} - \frac{Ba\sin(e+fx)\cos(e+fx)}{2f} - \frac{Ba\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A+B\sin(e))(a\sin(e)+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*cos
(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f,
Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))
```

Giac [A]
time = 0.47, size = 48, normalized size = 1.00

$$\frac{1}{2}(2Aa + Ba)x - \frac{Ba\sin(2fx + 2e)}{4f} - \frac{(Aa + Ba)\cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e)
/f
```

Mupad [B]
time = 13.26, size = 100, normalized size = 2.08

$$A\,a\,x - \frac{-B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 + (2\,A\,a + 2\,B\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) + 2\,A\,a + 2\,B\,a}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 1\right)} + \frac{B\,a\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

```
[Out] A*a*x - (2*A*a + 2*B*a + tan(e/2 + (f*x)/2)^2*(2*A*a + 2*B*a) - B*a*tan(e/2
 + (f*x)/2)^3 + B*a*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/
2 + (f*x)/2)^4 + 1)) + (B*a*x)/2
```

3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$-\frac{a(Bc-(A+B)d)x}{d^2}+\frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{aB\cos(e+fx)}{df}$$

```
[Out] -a*(B*c-(A+B)*d)*x/d^2-a*B*cos(f*x+e)/d/f+2*a*(c-d)*(-A*d+B*c)*arctan((d+c*
tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3047, 3102, 2814, 2739, 632, 210}

$$\frac{2a(c-d)(Bc-Ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d^2}-\frac{aB\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a*(B*c - (A + B)*d)*x)/d^2) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a*B*Cos[e + f*x
])/(d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx$$

$$= -\frac{aB\cos(e+fx)}{df}+\frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2}-\frac{aB\cos(e+fx)}{df}+\frac{(a(c-d)(Bc-$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2}-\frac{aB\cos(e+fx)}{df}+\frac{(2a(c-d)(Bc$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2}-\frac{aB\cos(e+fx)}{df}-\frac{(4a(c-d)(Bc$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2}+\frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan}{\sqrt{}}\right.}{d^2\sqrt{c^2-d^2}\,f}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.45, size = 196, normalized size = 2.00

$$\frac{a\left(Adx+B(-c+d)x-\frac{Bd\cos(e)\cos(fx)}{f}+\frac{2(c-d)(Bc-Ad)\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{Bd\sin(e)\sin(fx)}{f}\right)(1+\sin(e+fx))}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x])/f + (2*(c - d)*(B*c - A*d)
*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)
/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(
Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/f)*(
1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A]
time = 0.30, size = 120, normalized size = 1.22

method	result
derivativedivides	$\frac{2a\left(\frac{\left(-Acd+A\,d^2+B\,c^2-Bcd\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}+\frac{-\frac{Bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(Ad-Bc+Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$
default	$\frac{2a\left(\frac{\left(-Acd+A\,d^2+B\,c^2-Bcd\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}+\frac{-\frac{Bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(Ad-Bc+Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$
risch	$\frac{axA}{d}-\frac{axBc}{d^2}+\frac{axB}{d}-\frac{Ba\,e^{i(fx+e)}}{2df}-\frac{Ba\,e^{-i(fx+e)}}{2df}+\frac{\sqrt{-(c-d)(c+d)}\,a\ln\left(e^{i(fx+e)}-\frac{-ic+\sqrt{-(c}}{}\right.}{(c+d)fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*a*((-A*c*d+A*d^2+B*c^2-B*c*d)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+1/d^2*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A
*d-B*c+B*d)*arctan(tan(1/2*f*x+1/2*e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 303, normalized size = 3.09

$$\left[-\frac{2\,Bad\cos\left(fx+e\right)+2\left(Bac-(A+B)ad\right)fx-\left(Bac-Aad\right)\sqrt{-\frac{c-d}{c+d}}\log\left(-\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2-2\left(\left(c^2+cd\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2}\right)}{2\,d^2f},\ -\frac{Bad\cos\left(fx+e\right)+\left(Bac-(A+B)ad\right)fx+\left(Bac-Aad\right)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{\left(c\sin\left(fx+e\right)+d\right)\sqrt{\frac{c-d}{c+d}}}{\left(c-d\right)\cos\left(fx+e\right)}\right)}{d^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x - (B*a*c - A*a*d)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d)*f*x
 + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt(
(c - d)/(c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5508 vs. $2(82) = 164$.
time = 172.70, size = 5508, normalized size = 56.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*sin(e))*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0)
& Eq(f, 0)), (A*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3
+ d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)*
*(3/2)) + A*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*
f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2))
 + 2*A*a*d**2*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(
e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A
*a*d**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(
3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x*sqrt(d**2)*tan(e/2
```

```
+ f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2
)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x*sqrt(d**2)/(d**
3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2
 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d
**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e
/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*
x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f
*(d**2)**(3/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)
**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d*
*2)**(3/2)) - B*a*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3
+ d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)*
*(3/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*
f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2))
 - B*a*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(
d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*B*a*d**2*tan(e/2 +
f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)*
*(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - 2*B*a*d**2*tan(e/2 + f*x/2)
/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*ta
n(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*B*a*d**2/(d**3*f*tan(e/2 + f*x/2)*
*3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**
2)**(3/2)) + B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x
/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*
(d**2)**(3/2)) - B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 +
 f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2
- f*(d**2)**(3/2)) + B*a*d*f*x*sqrt(d**2)*tan(e/2 + f*x/2)/(d**3*f*tan(e/2
+ f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2
 - f*(d**2)**(3/2)) - B*a*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d*
*3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/
2)) + 2*B*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(
3/2)) + 4*B*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f
*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)), Eq(c, -sqrt
(d**2))), (A*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3 + d
**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3
/2)) + A*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*t
an(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) +
2*A*a*d**2*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2
 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*A*a*
d**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2
)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + A*a*d*f*x*sqrt(d**2)*tan(e/2 + f
*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**
(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + A*a*d*f*x*sqrt(d**2)/(d**3*f
*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 +
f*x/2)**2 + f*(d**2)**(3/2)) - 2*A*a*d*sqrt(d**2)*tan(e/2 + f*x/2)**2/(d**3
*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2
```

```
+ f*x/2)**2 + f*(d**2)**(3/2)) - 2*A*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2
)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d
**2)**(3/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x/2)**3
 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)
**(3/2)) - B*a*d**2*f*x*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d
**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3
/2)) + B*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*t
an(e/2 + f*x/2) + f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) -
B*a*d**2*f*x/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**
2)**(3/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) + 2*B*a*d**2*tan(e/2 + f*x
/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3
/2)*tan(e/2 + f*x/2)**2 + f*(d**2)**(3/2)) - 2*B*a*d**2*tan(e/2 + f*x/2)/(d
**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*...
```

Giac [A]
time = 0.56, size = 141, normalized size = 1.44

$$-\frac{\frac{(Bac-Aad-Bad)(fx+e)}{d^2}+\frac{2\,Ba}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\,(Bac^2-Aacd-Bacd+Aad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 +
1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^2))/f
```

Mupad [B]
time = 16.58, size = 2500, normalized size = 25.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*A*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*B*a*at
an(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (B*a*cos(e + f*x))
/(f*(c + d)) + (2*A*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(
c + d)) - (A*a*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i + A^2*
d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2)*(d
^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*
d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2)*(d
```

^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + A^
2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (f*x)
/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1
i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^3*d
^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x)/2)
*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*3i
- A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*sin(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c
^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^2*c^
4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*x)/2
)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*6i
+ A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2 + (
f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/
2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d^4*c
os(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)*(d^
2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A
*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2) + 2
*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*A^2*
c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5*d^2
*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*sin(
e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f*x)/2) + 2*B^2*c^4*d^3*sin(e/2 +
(f*x)/2) + 4*A*B*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c*d^6*cos(e/2 + (f*x)/2) +
B^2*c*d^6*cos(e/2 + (f*x)/2) + 4*A^2*c*d^6*sin(e/2 + (f*x)/2) - 4*A*B*c^3*
d^4*cos(e/2 + (f*x)/2) + 2*A*B*c^5*d^2*cos(e/2 + (f*x)/2) - 8*A*B*c^2*d^5*s
in(e/2 + (f*x)/2) + 4*A*B*c^4*d^3*sin(e/2 + (f*x)/2) + 2*A*B*c*d^6*cos(e/2
+ (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(d*f*(c + d)) - (2*B*a*c^2*atan(sin(e/2
+ (f*x)/2)/cos(e/2 + (f*x)/2)))/(d^2*f*(c + d)) - (B*a*c*cos(e + f*x))/(d*f
*(c + d)) + (B*a*c*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i +
A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2
)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i +
B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2
)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i
+ A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (
f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/
2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e
/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 -
c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c
^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x
)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2
)*3i - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*si
n(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2

```
 - c^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^
2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*
x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)
*6i + A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2
 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d
^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)
*(d^2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i
 - A*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 -
c^2)^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2)
 + 2*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*
A^2*c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5
*d^2*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*
sin(e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f...
```

3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{aBx}{d^2}+\frac{2a((A+B)(c-d)d^2-Bc(c^2-d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f}+\frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

```
[Out] a*B*x/d^2+2*a*((A+B)*(c-d)*d^2-B*c*(c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e
))/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f
/(c+d*sin(f*x+e))
```

Rubi [A]
time = 0.23, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3047, 3100, 2814, 2739, 632, 210}

$$\frac{2a(d^2(A+B)(c-d)-Bc(c^2-d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{aBx}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/d^2 + (2*a*((A + B)*(c - d)*d^2 - B*c*(c^2 - d^2))*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d
)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx &= \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^2}\,dx \\
&= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\int \frac{-a(A+B)(c-d)d-aB\left(c^2-d^2\right)\sin}{c+d\sin(e+fx)}}{d\left(c^2-d^2\right)} \\
&= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{(a(Ad^2-B(c^2+c}{d} \\
&= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{(2a(Ad^2-B(c^2+}{} \\
&= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{(4a(Ad^2-B(c^2+}{} \\
&= \frac{aBx}{d^2} + \frac{2a(Ad^2-B(c^2+cd-d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}\,f}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.93, size = 217, normalized size = 1.75

$$\frac{a(1+\sin(e+fx))\left(Bx+\frac{2(Ad^2-B(c^2+cd-d^2))\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{(c+d)\sqrt{c^2-d^2}\,f\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{(-Bc+Ad)\csc(e)(c\cos(e)+d\sin(fx))}{(c+d)f(c+d\sin(e+fx))}\right)}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[(
f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c
^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-(B*c) + A*d)*Csc[e]*(c*Cos[e]
 + d*Sin[f*x]))/((c + d)*f*(c + d*Sin[e + f*x]))))/(d^2*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2)
```

Maple [A]

time = 0.43, size = 174, normalized size = 1.40

method	result
derivativedivides	$\frac{2a\left(\frac{\frac{-\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(Ad-Bc)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-Bcd+B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}+\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$
default	$\frac{2a\left(\frac{\frac{-\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(Ad-Bc)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-Bcd+B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}+\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}\right)}{f}$
risch	$\frac{aBx}{d^2}-\frac{2ia(-Ad+Bc)\left(id+c\,e^{i(fx+e)}\right)}{d^2(c+d)f\left(id-id\,e^{2i(fx+e)}+2c\,e^{i(fx+e)}\right)}-\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)A}{\sqrt{-c^2+d^2}\,(c+d)f}+\frac{a\ln\left(e^{i(fx+e)}+i\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a*(1/d^2*((-d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(A*d-B*c)/(c+d))
/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(A*d^2-B*c^2-B*c*d+B*d^2
```

```
)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(
1/2)))+B/d^2*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(122) = 244.
time = 0.46, size = 672, normalized size = 5.42

$$\left[\frac{2\left(Bac^3d+Bac^2d^2-Bacd^3-Bad^4\right)fx\sin\left(fx+e\right)+2\left(Bac^4+Bac^3d-Bac^2d^2-Bacd^3\right)fx+\left(Bac^3+Bac^2d-(A+B)acd^2+\left(Bac^2d+Bacd^2-(A+B)ad^3\right)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2\left(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e)\right)\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2\left(Bac^3d-Aac^2d^2-Bacd^3+Aad^4\right)\cos\left(fx+e\right)}{2\left(\left(c^3d^3+c^2d^4-cd^5-d^6\right)f\sin\left(fx+e\right)+\left(c^4d^2+c^3d^3-c^2d^4-cd^5\right)f\right)}, \frac{\left(Bac^3d+Bac^2d^2-Bacd^3-Bad^4\right)fx\sin\left(fx+e\right)+\left(Bac^4+Bac^3d-Bac^2d^2-Bacd^3\right)fx+\left(Bac^3+Bac^2d-(A+B)acd^2+\left(Bac^2d+Bacd^2-(A+B)ad^3\right)\sin\left(fx+e\right)\right)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+\left(Bac^3d-Aac^2d^2-Bacd^3+Aad^4\right)\cos\left(fx+e\right)}{\left(c^3d^3+c^2d^4-cd^5-d^6\right)f\sin\left(fx+e\right)+\left(c^4d^2+c^3d^3-c^2d^4-cd^5\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) +
2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*
d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c^3*d -
 A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c*d
^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f), ((B*a*
c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a*c^4 + B*
a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (A + B)*a*c
*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqrt(c^2 - d^2
)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (B*a*c^3*d
 - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c
*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.68, size = 204, normalized size = 1.65

$$\frac{\frac{(fx+e)Ba}{d^2} - \frac{2\left(Bac^2+Bacd-Aad^2-Bad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^2+d^3)\sqrt{c^2-d^2}} + \frac{2\left(Bacd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Aad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Bac^2-Aacd\right)}{(c^2d+cd^2)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] ((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2*e)
 - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

Mupad [B]
time = 20.32, size = 2500, normalized size = 20.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] (2*B*a*atan(((B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d
))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 +
2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B
^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2
*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a
*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c
^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d
^3 + d^4 + c^2*d^2) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 +
 d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4
*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2))/d^2 +
(B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 +
 d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3
*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d
+ 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5
+ c^2*d^3) + (B*a*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5
```

))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*
c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^
2*d^3) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^
2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*
c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2))/d^2)/((64*(B^3*a^3*
c^3 + A*B^2*a^3*c^3 - B^3*a^3*c*d^2 + B^3*a^3*c^2*d - 2*A*B^2*a^3*c*d^2 + A
*B^2*a^3*c^2*d - A^2*B*a^3*c*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (64*tan(e/2
+ (f*x)/2)*(2*B^3*a^3*c*d^3 - 2*B^3*a^3*c^4 - 4*B^3*a^3*c^3*d + 2*A*B^2*a^3
*c*d^3 + 2*A*B^2*a^3*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (B*a*((32*(B^2*a
^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2)
+ (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2
*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*
d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (B*
a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*
a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*
a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (B*a*((
32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2
+ (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d
^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2)*1i)/d^2 + (B*a*((32*(B^2*a^2*c^2*d^3
+ 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(
e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 -
A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*
B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(B*
a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^
2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*
B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(c^2*d^
7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2
)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 +
c^2*d^3))*1i)/d^2)*1i)/d^2)*1i)/d^2)))/(d^2*f) - ((2*(A*a*d - B*a*c))/(d*(
c + d)) + (2*a*tan(e/2 + (f*x)/2)*(A*d - B*c))/(c*(c + d)))/(f*(c + 2*d*tan
(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (a*atan(((a*(-(c + d)^3*(c - d
))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d
^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2
*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^
5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 +
d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A
*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*
c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^
5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8
+ c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3
(c - d))^(1/2)(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3
- c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 -
c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*1i)/(2*c*d^5 + d^6 - 2*c^3*d^3 -
c^4*d^2) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*

```
c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2
)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^
5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3
 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1
/2)*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 +
d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*
c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*...
```

3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=176

$$\frac{a(2Ac+Bc-Ad-2Bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\left(c^2-d^2\right)^{3/2}f}+\frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a(A(c-2d)d+B(c^2}{2(c-d)d(c+d)}$$

```
[Out] a*(2*A*c-A*d+B*c-2*B*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c
+d)/(c^2-d^2)^(3/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e)
)^2-1/2*a*(A*(c-2*d)*d+B*(c^2+2*c*d-2*d^2))*cos(f*x+e)/(c-d)/d/(c+d)^2/f/(c
+d*sin(f*x+e))
```

Rubi [A]
time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3047, 3100, 2833, 12, 2739, 632, 210}

$$\frac{a(2Ac-Ad+Bc-2Bd)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{2df(c-d)(c+d)^2(c+d\sin(e+fx))}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c + d)*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2
))*Cos[e + f*x])/(2*(c - d)*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a(A+B)(c-d)d-a(c-d)(A}{(c+d\sin(e+}}{2d\,(c^2-}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+}{2(c-d)d(c+d)^2f(}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+}{2(c-d)d(c+d)^2f(}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+}{2(c-d)d(c+d)^2f(}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+}{2(c-d)d(c+d)^2f(}$$

$$= \frac{a(2Ac+Bc-Ad-2Bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{a}{2d(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.27, size = 345, normalized size = 1.96

$$\frac{a(1+\sin(e+fx))\left(\frac{4(2Ac+Bc-Ad-2Bd)\tan^{-1}\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{(2c^2+d^2)(A(c-2d)d+B(c^2+2cd-2d^2))\cot(e)+d\csc(e)(-d(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+2fx)+(Bc(2c^2+6cd-5d^2)-Ad(-4c^2+6cd+d^2))\sin(fx)+(Ad^2(-2c+d)+Bc(2c^2+2cd-3d^2))\sin(2e+fx))}{d^2(c+d\sin(e+fx))^2}\right)}{4(c-d)(c+d)^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]*
(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]
*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(
Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2
*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2))*Cos
[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d^2))*S
in[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e + f*x]))
/(d^2*(c + d*Sin[e + f*x])^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(167) = 334.
time = 0.65, size = 424, normalized size = 2.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a*((-1/2*(3*A*c^2*d-2*A*c*d^2-2*A*d^3-B*c^3+2*B*c^2*d)/c/(c^3+c^2*d-c*d
^2-d^3)*tan(1/2*f*x+1/2*e)^3-1/2*(2*A*c^4-2*A*c^3*d+3*A*c^2*d^2-4*A*c*d^3-2
*A*d^4+2*B*c^4-B*c^3*d+4*B*c^2*d^2-2*B*c*d^3)/(c^3+c^2*d-c*d^2-d^3)/c^2*tan
(1/2*f*x+1/2*e)^2-1/2*(5*A*c^2*d-6*A*c*d^2-2*A*d^3+B*c^3+6*B*c^2*d-4*B*c*d^
2)/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)-1/2*(2*A*c^2-2*A*c*d-A*d^2+2*
B*c^2-B*c*d)/(c^3+c^2*d-c*d^2-d^3))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x
+1/2*e)+c)^2+1/2*(2*A*c-A*d+B*c-2*B*d)/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(172) = 344$.
time = 0.42, size = 990, normalized size = 5.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*
c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A
+ 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 -
 (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2
)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))
*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)
```

```
*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d
^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3
*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 -
 c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)*a*c^3*d - (
2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*si
n(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A
+ 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2
*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-
(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*(A + B)*a*c^4 - (
2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*d^3 + A*a*d^4)*cos
(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos
(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)
*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*
d^6 + d^7)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(172) = 344.
time = 0.59, size = 594, normalized size = 3.38

$$\frac{\frac{(2Aac+Bac-Aad-2Bad)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^3+c^2d-cd^2-d^3)\sqrt{c^2-d^2}}+\frac{Bac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3Aac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2Bac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2Aac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2Aacd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2Aac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2Bac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Aac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Aac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4Bac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4Aacd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Bacd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Aad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-5Aac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6Bac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+6Aac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4Bac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Aacd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2Aac^4-2Bac^4+2Aac^3d+Bac^3d+Aac^2d^2}{(c^5+c^4d-c^3d^2-c^2d^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d -
c*d^2 - d^3)*sqrt(c^2 - d^2)) + (B*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 3*A*a*c^3
*d*tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c^2*
d^2*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a*c^4
*tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^3*d*ta
n(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*d^2*tan
(1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*A*a*c*d^3*ta
n(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*d^4*tan(1
/2*f*x + 1/2*e)^2 - B*a*c^4*tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*tan(1/2*f*x
+ 1/2*e) - 6*B*a*c^3*d*tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*tan(1/2*f*x + 1
```

```
/2*e) + 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*
e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2*d^2)/((c^5 +
 c^4*d - c^3*d^2 - c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1
/2*e) + c)^2))/f
```

Mupad [B]
time = 15.59, size = 554, normalized size = 3.15

$$-\frac{\frac{A\,a\,d^2-2\,A\,a\,c^2-2\,B\,a\,c^2+2\,A\,a\,c\,d+B\,a\,c\,d}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,d^3-B\,c^3+6\,A\,c\,d^2-5\,A\,c^2\,d+4\,B\,c\,d^2-6\,B\,c^2\,d\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,A\,d^3+B\,c^3+2\,A\,c\,d^2-3\,A\,c^2\,d-2\,B\,c^2\,d\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(c^2+2\,d^2\right)\,\left(A\,d^2-2\,A\,c^2-2\,B\,c^2+2\,A\,c\,d+B\,c\,d\right)}{c^2\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2+4\,d^2\right)+c^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+c^2+4\,c\,d\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+4\,c\,d\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,\left(2\,A\,c-A\,d+B\,c-2\,B\,d\right)\,\left(-2\,c^3\,d-2\,c^2\,d^2+2\,c\,d^3+2\,d^4\right)}{2\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,c\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,c-A\,d+B\,c-2\,B\,d\right)}{{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}}\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}{2\,A\,a\,c-A\,a\,d+B\,a\,c-2\,B\,a\,d}\right)\,\left(2\,A\,c-A\,d+B\,c-2\,B\,d\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((A*a*d^2 - 2*A*a*c^2 - 2*B*a*c^2 + 2*A*a*c*d + B*a*c*d)/(c*d^2 - c^2*d -
 c^3 + d^3) + (a*tan(e/2 + (f*x)/2)*(2*A*d^3 - B*c^3 + 6*A*c*d^2 - 5*A*c^2*
d + 4*B*c*d^2 - 6*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 +
(f*x)/2)^3*(2*A*d^3 + B*c^3 + 2*A*c*d^2 - 3*A*c^2*d - 2*B*c^2*d))/(c*(c*d^2
 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*d^2 - 2*A
*c^2 - 2*B*c^2 + 2*A*c*d + B*c*d))/(c^2*(c*d^2 - c^2*d - c^3 + d^3)))/(f*(t
an(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*
d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (a*atan((((a*(2*A*c -
 A*d + B*c - 2*B*d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2*(c + d)^(5/
2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 + (f*x)/2)*(2*
A*c - A*d + B*c - 2*B*d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2 - c^2*d - c
^3 + d^3))/(2*A*a*c - A*a*d + B*a*c - 2*B*a*d))*(2*A*c - A*d + B*c - 2*B*d)
)/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=464

$$\frac{1}{16}a^2\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c^2d+36cd^2+11d^3\right)\right)x+\frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-4$$

```
[Out] 1/16*a^2*(6*A*(4*c^3+8*c^2*d+7*c*d^2+2*d^3)+B*(16*c^3+42*c^2*d+36*c*d^2+11*
d^3))*x+1/60*a^2*(6*A*d*(c^4-10*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)-B*(2*c^5-
12*c^4*d+47*c^3*d^2+208*c^2*d^3+216*c*d^4+64*d^5))*cos(f*x+e)/d^2/f+1/240*a
^2*(6*A*d*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)-B*(4*c^4-24*c^3*d+96*c^2*d^2+284
*c*d^3+165*d^4))*cos(f*x+e)*sin(f*x+e)/d/f+1/120*a^2*(6*A*d*(c^2-10*c*d-12*
d^2)-B*(2*c^3-12*c^2*d+51*c*d^2+64*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/
f+1/120*a^2*(6*A*(c-10*d)*d-B*(2*c^2-12*c*d+55*d^2))*cos(f*x+e)*(c+d*sin(f*
x+e))^3/d^2/f+1/30*a^2*(-6*A*d+2*B*c-7*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d
^2/f-1/6*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^4/d/f
```

Rubi [A]
time = 0.65, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\frac{a^2(6Ad(c-10d)-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}+\frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f}+\frac{1}{16}a^2x\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c^2d+36cd^2+11d^3\right)\right)+\frac{a^2(6Ad(2c^3-20c^2d-57cd^2-30d^3)-B(4c^4-24c^3d+96c^2d^2+284cd^3+165d^4))\sin(e+fx)\cos(e+fx)}{240df}+\frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)-B(2c^5-12c^4d+47c^3d^2+208c^2d^3+216cd^4+64d^5))\cos(e+fx)}{60d^2f}+\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c
*d^2 + 11*d^3))*x)/16 + (a^2*(6*A*d*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3
 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 + 208*c^2*d^3 + 216*c*d^4 + 6
4*d^5))*Cos[e + f*x])/(60*d^2*f) + (a^2*(6*A*d*(2*c^3 - 20*c^2*d - 57*c*d^2
 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c*d^3 + 165*d^4))*Cos[e
 + f*x]*Sin[e + f*x])/(240*d*f) + (a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2) - B*(
2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)
/(120*d^2*f) + (a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^2*(2*B*c - 6*A*d - 7*B*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
```

```
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(}{6df}$$
$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(}{6df}$$
$$= \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+}{30d^2f}$$
$$= \frac{a^2(6A(c-10d)d-B(2c^2-12cd+55}{120d}$$
$$= \frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-}{}$$
$$= \frac{1}{16}a^2\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+\right.$$

Mathematica [A]
time = 2.07, size = 437, normalized size = 0.94

$$-\frac{a^2\cos(e+fx)\left(60(6A(4c^3+8c^2d+7cd^2+2d^3)+B(16c^3+42c^2d+36cd^2+11d^3))\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(960Ac^3+880Bc^3+2640Ac^2d+2400Bc^2d+2400Acd^2+2268Bcd^2+756Ad^3+712Bd^3-16(3Ad(5c^2+10cd+4d^2)+B(5c^3+30c^2d+36cd^2+14d^3))\cos(2(e+fx))+12d^2(3Bc+Ad+2Bd)\cos(4(e+fx))+240Ac^3\sin(e+fx)+480Bc^3\sin(e+fx)+1440Ac^2d\sin(e+fx)+1530Bc^2d\sin(e+fx)+1530Acd^2\sin(e+fx)+1620Bcd^2\sin(e+fx)+540Ad^3\sin(e+fx)+545Bd^3\sin(e+fx)-90Bc^2d\sin(3(e+fx))-90Acd^2\sin(3(e+fx))-180Bcd^2\sin(3(e+fx))-60Ad^3\sin(3(e+fx))-80Bd^3\sin(3(e+fx))+5Bd^3\sin(5(e+fx)))\right)}{480f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/480*(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(
16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*B*c
^2*d + 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*(5*c
^2 + 10*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2*(e +
 f*x)] + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Sin[e +
f*x] + 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c^2*d*Si
n[e + f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x] + 540*A*
d^3*Sin[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e + f*x)] - 9
0*A*c*d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*d^3*Sin[3*
(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)])))/(f*Sqr
t[Cos[e + f*x]^2])
```

Maple [A]
time = 0.36, size = 745, normalized size = 1.61 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^2*A*c^3*(f*x+e)-3*a^2*A*c^2*d*cos(f*x+e)+3*a^2*A*c*d^2*(-1/2*cos(f*x
+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a
^2*c^3*cos(f*x+e)+3*B*a^2*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-
B*a^2*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d^3*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^2*A*c^3*cos(f*x+e)+6*a^2*A*c^2*d*(
-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a^2*A*c*d^2*(2+sin(f*x+e)^2)*co
s(f*x+e)+2*a^2*A*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)+2*B*a^2*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*B*a^2*c^2*
d*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x
+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^2*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+
e)^2)*cos(f*x+e)+a^2*A*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^2*A
*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^2*A*d^3*(8/3+sin(f*x+e)^4+4/3*sin
(f*x+e)^2)*cos(f*x+e)-1/3*B*a^2*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^2*c^2
*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^2*
c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d^3*(-1/6*(sin(f
*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [A]
time = 0.31, size = 778, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2
*c^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c^2*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(cos(f*x +
 e)^3 - 3*cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*
a^2*c^2*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d^2 + 90*(12*f*x
 + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*(2*f*x +
 2*e - sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x +
 e)^3 + 15*cos(f*x + e))*B*a^2*c*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e)
)*B*a^2*c*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*B*a^2*c*d^2 - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*
A*a^2*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^3 + 60*(12*f*x +
12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^3 - 128*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^3 + 5*(4*sin(2*f*x + 2*
e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*d^3
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^3 - 19
20*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e) - 2880*A*a^2*c^2*d*c
os(f*x + e))/f
```

Fricas [A]
time = 0.43, size = 371, normalized size = 0.80

$$-\frac{48\,(3\,Ba^2cd^2+(A+2\,B)a^2d^3)\cos(fx+e)^5-80\,(Ba^2c^3+3\,(A+2\,B)a^2c^2d+3\,(2\,A+3\,B)a^2cd^2+(3\,A+4\,B)a^2d^3)\cos(fx+e)^3-15\,(8\,(3\,A+2\,B)a^2c^3+6\,(8\,A+7\,B)a^2c^2d+6\,(7\,A+6\,B)a^2cd^2+(12\,A+11\,B)a^2d^3)fx+480\,((A+B)a^2c^3+3\,(A+B)a^2c^2d+3\,(A+B)a^2cd^2+(A+B)a^2d^3)\cos(fx+e)+5\,(8\,Ba^2d^3\cos(fx+e)^5-2\,(18\,Ba^2c^2d+18\,(A+2\,B)a^2cd^2+(12\,A+19\,B)a^2d^3)\cos(fx+e)^3+3\,(8\,(A+2\,B)a^2c^3+6\,(8\,A+9\,B)a^2c^2d+6\,(9\,A+10\,B)a^2cd^2+(20\,A+21\,B)a^2d^3)\cos(fx+e))\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*c
^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^3)
*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d + 6*(
7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^3 +
3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x + e) +
 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2*c*d^2
 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 + 6*(8*A
+ 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3)*cos(f*
x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1865 vs. 2(450) = 900.
time = 0.67, size = 1865, normalized size = 4.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a**2*c**3*x*sin(e + f*x)**2/2 + A*a**2*c**3*x*cos(e + f*x)**2/
2 + A*a**2*c**3*x - A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**3*cos(e + f*x)/f + 3*A*a**2*c**2*d*x*sin(e + f*x)**2 + 3*A*a**2*c**2*d*x
*cos(e + f*x)**2 - 3*A*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**
2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a**2*c**2*d*cos(e + f*x)**3/f -
3*A*a**2*c**2*d*cos(e + f*x)/f + 9*A*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*A*
a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**2*c*d**2*x*sin(e +
 f*x)**2/2 + 9*A*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**2*c*d**2*x*cos(e
+ f*x)**2/2 - 15*A*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*A*a**
2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**2*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 3*A*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a
**2*c*d**2*cos(e + f*x)**3/f + 3*A*a**2*d**3*x*sin(e + f*x)**4/4 + 3*A*a**2
*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**2*d**3*x*cos(e + f*x)**4
/4 - A*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*d**3*sin(e + f*x
)**3*cos(e + f*x)/(4*f) - 4*A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*
f) - A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*d**3*sin(e + f*x
)*cos(e + f*x)**3/(4*f) - 8*A*a**2*d**3*cos(e + f*x)**5/(15*f) - 2*A*a**2*d
```

```
**3*cos(e + f*x)**3/(3*f) + B*a**2*c**3*x*sin(e + f*x)**2 + B*a**2*c**3*x*c
os(e + f*x)**2 - B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c**3*s
in(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c**3*cos(e + f*x)**3/(3*f) - B*a**2*c
**3*cos(e + f*x)/f + 9*B*a**2*c**2*d*x*sin(e + f*x)**4/8 + 9*B*a**2*c**2*d*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**2*c**2*d*x*sin(e + f*x)**2/2 +
 9*B*a**2*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**2*c**2*d*x*cos(e + f*x)**2/2
- 15*B*a**2*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*B*a**2*c**2*d*sin
(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 3*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*c**2*d*c
os(e + f*x)**3/f + 9*B*a**2*c*d**2*x*sin(e + f*x)**4/4 + 9*B*a**2*c*d**2*x*
sin(e + f*x)**2*cos(e + f*x)**2/2 + 9*B*a**2*c*d**2*x*cos(e + f*x)**4/4 - 3
*B*a**2*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**2*c*d**2*sin(e + f*
x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 3*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(4*f) - 8*B*a**2*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*
a**2*c*d**2*cos(e + f*x)**3/f + 5*B*a**2*d**3*x*sin(e + f*x)**6/16 + 15*B*a
**2*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**2*d**3*x*sin(e + f*x
)**4/8 + 15*B*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*d**
3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*B*a**2*d**3*x*cos(e + f*x)**6/16
+ 3*B*a**2*d**3*x*cos(e + f*x)**4/8 - 11*B*a**2*d**3*sin(e + f*x)**5*cos(e
+ f*x)/(16*f) - 2*B*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**
3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*B*a**2*d**3*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 8*B*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*B
*a**2*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*d**3*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 16*B*a**2*d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a)**2, True))
```

Giac [A]
time = 0.81, size = 474, normalized size = 1.02

$$-\frac{Ba^2d^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(24\,Aa^2c^3+16\,Ba^2c^3+48\,Aa^2c^2d+42\,Ba^2c^2d+42\,Aa^2cd^2+36\,Ba^2cd^2+12\,Aa^2d^3+11\,Ba^2d^3\right)x-\frac{\left(3\,Ba^2cd^2+Aa^2d^3+2\,Ba^2d^3\right)\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{\left(4\,Ba^2c^3+12\,Aa^2c^2d+24\,Ba^2c^2d+24\,Aa^2cd^2+27\,Ba^2cd^2+9\,Aa^2d^3+10\,Ba^2d^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(16\,Aa^2c^3+14\,Ba^2c^3+42\,Aa^2c^2d+36\,Ba^2c^2d+36\,Aa^2cd^2+33\,Ba^2cd^2+11\,Aa^2d^3+10\,Ba^2d^3\right)\cos\left(fx+e\right)}{8\,f}+\frac{\left(6\,Ba^2c^2d+6\,Aa^2cd^2+12\,Ba^2cd^2+4\,Aa^2d^3+5\,Ba^2d^3\right)\sin\left(4\,fx+4\,e\right)}{64\,f}-\frac{\left(16\,Aa^2c^3+32\,Ba^2c^3+96\,Aa^2c^2d+96\,Ba^2c^2d+96\,Aa^2cd^2+96\,Ba^2cd^2+32\,Aa^2d^3+31\,Ba^2d^3\right)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 + 4
8*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a^2
*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*cos
(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d + 24*
A*a^2*c*d^2 + 27*B*a^2*c*d^2 + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x + 3*e)
/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2*d + 3
6*A*a^2*c*d^2 + 33*B*a^2*c*d^2 + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*x + e)/
f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c*d^2 + 12*B*a^2*c*d^2 + 4*A*a^2*d^3 + 5*
B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3 + 96*A*a^
```

```
2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c*d^2 + 96*B*a^2*c*d^2 + 32*A*a^2*d^3 +
 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.99, size = 1291, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(24*A*c^3 + 12*A*d^3 + 16*B*c^3 + 11*B*d^
3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))/(8*(3*A*a^2*c^3 + (
3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*
a^2*c^2*d + (9*B*a^2*c*d^2)/2 + (21*B*a^2*c^2*d)/4)))*(24*A*c^3 + 12*A*d^3
+ 16*B*c^3 + 11*B*d^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))
/(8*f) - (tan(e/2 + (f*x)/2)*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (
11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 +
(21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*(20*A*a^2*c^3 + 4*A*a^2*d^3 + 14
*B*a^2*c^3 + 24*A*a^2*c*d^2 + 42*A*a^2*c^2*d + 12*B*a^2*c*d^2 + 24*B*a^2*c^
2*d) - tan(e/2 + (f*x)/2)^11*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (
11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 +
(21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^5*(2*A*a^2*c^3 + 7*A*a^2*d^3 + 4*B
*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^2*d + 21*B*a^
2*c*d^2 + (33*B*a^2*c^2*d)/2) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^3 + 7*A*a^2
*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^2*d
 + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) + tan(e/2 + (f*x)/2)^3*(3*A*a^2*c^3
 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24 + (87*A*a^2*c*d^2)/4
 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2*d)/4) - tan(e/2 + (f
*x)/2)^9*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24
 + (87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2
*d)/4) + tan(e/2 + (f*x)/2)^4*(40*A*a^2*c^3 + 32*A*a^2*d^3 + 36*B*a^2*c^3 +
 32*B*a^2*d^3 + 96*A*a^2*c*d^2 + 108*A*a^2*c^2*d + 96*B*a^2*c*d^2 + 96*B*a^
2*c^2*d) + tan(e/2 + (f*x)/2)^2*(20*A*a^2*c^3 + (72*A*a^2*d^3)/5 + 18*B*a^2
*c^3 + (64*B*a^2*d^3)/5 + 48*A*a^2*c*d^2 + 54*A*a^2*c^2*d + (216*B*a^2*c*d^
2)/5 + 48*B*a^2*c^2*d) + tan(e/2 + (f*x)/2)^6*(40*A*a^2*c^3 + 24*A*a^2*d^3
+ (100*B*a^2*c^3)/3 + (64*B*a^2*d^3)/3 + 80*A*a^2*c*d^2 + 100*A*a^2*c^2*d +
 72*B*a^2*c*d^2 + 80*B*a^2*c^2*d) + tan(e/2 + (f*x)/2)^10*(4*A*a^2*c^3 + 2*
B*a^2*c^3 + 6*A*a^2*c^2*d) + 4*A*a^2*c^3 + (12*A*a^2*d^3)/5 + (10*B*a^2*c^3
)/3 + (32*B*a^2*d^3)/15 + 8*A*a^2*c*d^2 + 10*A*a^2*c^2*d + (36*B*a^2*c*d^2)
/5 + 8*B*a^2*c^2*d)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 +
20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10
 + tan(e/2 + (f*x)/2)^12 + 1))
```

3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=336

$$\frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad^2+6Bd^2\right)x+\frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d}{30d^2f}$$

```
[Out] 1/8*a^2*(12*A*c^2+16*A*c*d+7*A*d^2+8*B*c^2+14*B*c*d+6*B*d^2)*x+1/30*a^2*(5*
A*d*(c^3-8*c^2*d-20*c*d^2-8*d^3)-2*B*(c^4-5*c^3*d+16*c^2*d^2+40*c*d^3+18*d^
4))*cos(f*x+e)/d^2/f+1/120*a^2*(5*A*d*(2*c^2-16*c*d-21*d^2)-B*(4*c^3-20*c^2
*d+66*c*d^2+90*d^3))*cos(f*x+e)*sin(f*x+e)/d/f+1/60*a^2*(5*A*(c-8*d)*d-2*B*
(c^2-5*c*d+18*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/20*a^2*(2*B*(c-3*
d)-5*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f-1/5*B*cos(f*x+e)*(a^2+a^2*sin
(f*x+e))*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A]
time = 0.48, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\frac{a^2(5Ad(c-8d)-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f}+\frac{1}{8}a^2x\left(12Ac^2+16Acd+7Ad^2+8Bc^2+14Bcd+6Bd^2\right)+\frac{a^2(5Ad(2c^2-16cd-21d^2)-B(4c^3-20c^2d+66cd^2+90d^3))\sin(e+fx)\cos(e+fx)}{120df}+\frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d+16c^2d^2+40cd^3+18d^4))\cos(e+fx)}{30d^2f}+\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*A*c^2 + 8*B*c^2 + 16*A*c*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/8 +
(a^2*(5*A*d*(c^3 - 8*c^2*d - 20*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^
2*d^2 + 40*c*d^3 + 18*d^4))*Cos[e + f*x])/(30*d^2*f) + (a^2*(5*A*d*(2*c^2 -
 16*c*d - 21*d^2) - B*(4*c^3 - 20*c^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*
Sin[e + f*x])/(120*d*f) + (a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2
))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d^2*f) + (a^2*(2*B*(c - 3*d) -
5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d^2*f) - (B*Cos[e + f*x]*(a
^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(5*d*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c}{5df}$$
$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c}{5df}$$
$$= \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+}{20d^2f}$$
$$= \frac{a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))}{60d^2f}$$
$$= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+\right.$$

Mathematica [A]
time = 1.02, size = 296, normalized size = 0.88

$$-\frac{a^2\cos(e+fx)\left(60(2B(4c^2+7cd+3d^2)+A(12c^2+16cd+7d^2))\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(480Ac^2+440Bc^2+880Acd+800Bcd+400Ad^2+378Bd^2-8(10Ad(c+d)+B(5c^2+20cd+12d^2))\cos(2(e+fx))+6Bd^2\cos(4(e+fx))+120Ac^2\sin(e+fx)+240Bc^2\sin(e+fx)+480Acd\sin(e+fx)+510Bcd\sin(e+fx)+255Ad^2\sin(e+fx)+270Bd^2\sin(e+fx)-30Bcd\sin(3(e+fx))-15Ad^2\sin(3(e+fx))-30Bd^2\sin(3(e+fx)))\right)}{240f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -1/240*(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16*
c*d + 7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]
*(480*A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2 - 8
*(10*A*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*d^2*
Cos[4*(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 480*A*
c*d*Sin[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] + 270*B*
d^2*Sin[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f*x)] -
30*B*d^2*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A]
time = 0.27, size = 496, normalized size = 1.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^2*A*c^2*(f*x+e)-2*a^2*A*c*d*cos(f*x+e)+a^2*A*d^2*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^2*cos(f*x+e)+2*B*a^2*c*d*(-1/2*cos(f*x+e)*s
in(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*
c^2*cos(f*x+e)+4*a^2*A*c*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/3*a
^2*A*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*c^2*B*(-1/2*cos(f*x+e)*sin(f*x+e
)+1/2*f*x+1/2*e)-4/3*B*a^2*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+2*B*a^2*d^2*(-1/
```

```
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*A*c^2*(-1/2*c
os(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*A*c*d*(2+sin(f*x+e)^2)*cos(f*x+
e)+a^2*A*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
1/3*B*a^2*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*B*a^2*c*d*(-1/4*(sin(f*x+e)^3+3
/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^2*d^2*(8/3+sin(f*x+e)^4+4/
3*sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [**A**]
time = 0.29, size = 514, normalized size = 1.53

$$\frac{120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2c^2+480\,(fx+e)Aa^2c^2+160\,(\cos(fx+e)^3-3\,\cos(fx+e))Ba^2c^2+240\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Ba^2c^2+320\,(\cos(fx+e)^3-3\,\cos(fx+e))Aa^2cd+480\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2cd+640\,(\cos(fx+e)^3-3\,\cos(fx+e))Ba^2cd+30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Ba^2cd+240\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Ba^2cd+320\,(\cos(fx+e)^3-3\,\cos(fx+e))Aa^2d^2+15\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Aa^2d^2+120\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2d^2-32\,(3\,\cos(fx+e)^5-10\,\cos(fx+e)^3+15\,\cos(fx+e))Ba^2d^2+160\,(\cos(fx+e)^3-3\,\cos(fx+e))Ba^2d^2+30\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Ba^2d^2-960\,Aa^2c^2\cos(fx+e)-480\,Ba^2c^2\cos(fx+e)-960\,Aa^2cd\cos(fx+e)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2
*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e)^3
- 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(
2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c*d +
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
 e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2 + 30*(12*f
*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 960*A*a^2*c^
2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos(f*x + e))/f
```

Fricas [**A**]
time = 0.42, size = 251, normalized size = 0.75

$$-\frac{24\,Ba^2d^2\cos(fx+e)^5-40\,(Ba^2c^2+2\,(A+2\,B)a^2cd+(2\,A+3\,B)a^2d^2)\cos(fx+e)^3-15\,(4\,(3\,A+2\,B)a^2c^2+2\,(8\,A+7\,B)a^2cd+(7\,A+6\,B)a^2d^2)fx+240\,((A+B)a^2c^2+2\,(A+B)a^2cd+(A+B)a^2d^2)\cos(fx+e)-15\,(2\,(2\,Ba^2cd+(A+2\,B)a^2d^2)\cos(fx+e)^3-(4\,(A+2\,B)a^2c^2+2\,(8\,A+9\,B)a^2cd+(9\,A+10\,B)a^2d^2)\cos(fx+e))\sin(fx+e)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/120*(24*B*a^2*d^2*cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d +
 (2*A + 3*B)*a^2*d^2)*cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A +
 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A + B)
*a^2*c*d + (A + B)*a^2*d^2)*cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*B)*a
^2*d^2)*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d + (9*
A + 10*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(330) = 660$.
time = 0.44, size = 1129, normalized size = 3.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a**2*c**2*x*sin(e + f*x)**2/2 + A*a**2*c**2*x*cos(e + f*x)**2/
2 + A*a**2*c**2*x - A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**2*cos(e + f*x)/f + 2*A*a**2*c*d*x*sin(e + f*x)**2 + 2*A*a**2*c*d*x*cos(e
 + f*x)**2 - 2*A*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A*a**2*c*d*sin
(e + f*x)*cos(e + f*x)/f - 4*A*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*A*a**2*c*
d*cos(e + f*x)/f + 3*A*a**2*d**2*x*sin(e + f*x)**4/8 + 3*A*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*d**2*x*sin(e + f*x)**2/2 + 3*A*a**2*
d**2*x*cos(e + f*x)**4/8 + A*a**2*d**2*x*cos(e + f*x)**2/2 - 5*A*a**2*d**2*
sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a**2*d**2*sin(e + f*x)**2*cos(e +
f*x)/f - 3*A*a**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*d**2*sin
(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a**2*d**2*cos(e + f*x)**3/(3*f) + B*a**2
*c**2*x*sin(e + f*x)**2 + B*a**2*c**2*x*cos(e + f*x)**2 - B*a**2*c**2*sin(e
 + f*x)**2*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a
**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e + f*x)/f + 3*B*a**2*c*d*
x*sin(e + f*x)**4/4 + 3*B*a**2*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*
a**2*c*d*x*sin(e + f*x)**2 + 3*B*a**2*c*d*x*cos(e + f*x)**4/4 + B*a**2*c*d*
x*cos(e + f*x)**2 - 5*B*a**2*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a
**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*c*d*sin(e + f*x)*cos(e +
f*x)**3/(4*f) - B*a**2*c*d*sin(e + f*x)*cos(e + f*x)/f - 8*B*a**2*c*d*cos(e
 + f*x)**3/(3*f) + 3*B*a**2*d**2*x*sin(e + f*x)**4/4 + 3*B*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/2 + 3*B*a**2*d**2*x*cos(e + f*x)**4/4 - B*a**2*
d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - 4*B*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*d**2*sin(e + f*x)*cos(e + f*
x)**3/(4*f) - 8*B*a**2*d**2*cos(e + f*x)**5/(15*f) - 2*B*a**2*d**2*cos(e +
f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a
)**2, True))
```

Giac [**A**]
time = 0.62, size = 311, normalized size = 0.93

$$-\frac{Ba^2d^2\cos(5fx+5e)}{80f} + \frac{1}{8}\left(12Aa^2c^2 + 8Ba^2c^2 + 16Aa^2cd + 14Ba^2cd + 7Aa^2d^2 + 6Ba^2d^2\right)x + \frac{(4Ba^2c^2 + 8Aa^2cd + 16Ba^2cd + 8Aa^2d^2 + 9Ba^2d^2)\cos(3fx+3e)}{48f} - \frac{(16Aa^2c^2 + 14Ba^2c^2 + 28Aa^2cd + 24Ba^2cd + 12Aa^2d^2 + 11Ba^2d^2)\cos(fx+e)}{8f} + \frac{(2Ba^2cd + Aa^2d^2 + 2Ba^2d^2)\sin(4fx+4e)}{32f} - \frac{(Aa^2c^2 + 2Ba^2c^2 + 4Aa^2cd + 4Ba^2cd + 2Aa^2d^2 + 2Ba^2d^2)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/80*B*a^2*d^2*cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*A
*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^2
+ 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*cos(3*f*x + 3*e)/
f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12*A*a
^2*d^2 + 11*B*a^2*d^2)*cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2 + 2*B
*a^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d +
 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.71, size = 765, normalized size = 2.28

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(12Ac^2+7Ad^2+8Bc^2+6Bd^2+16Acd+14Bcd\right)}{4\left(3Aa^2c^2+\frac{7Aa^2d^2}{4}+2Ba^2c^2+\frac{3Ba^2d^2}{2}+4Aa^2cd+\frac{7Ba^2cd}{2}\right)}\right)\left(12Ac^2+7Ad^2+8Bc^2+6Bd^2+16Acd+14Bcd\right)}{4f}-\frac{{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8\left(4Aa^2c^2+2Ba^2c^2+4Aa^2cd\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(Aa^2c^2+\frac{7Aa^2d^2}{4}+2Ba^2c^2+\frac{3Ba^2d^2}{2}+4Aa^2cd+\frac{7Ba^2cd}{2}\right)-{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^9\left(Aa^2c^2+\frac{7Aa^2d^2}{4}+2Ba^2c^2+\frac{3Ba^2d^2}{2}+4Aa^2cd+\frac{7Ba^2cd}{2}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3\left(2Aa^2c^2+\frac{11Aa^2d^2}{2}+4Ba^2c^2+7Ba^2d^2+8Aa^2cd+11Ba^2cd\right)-{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7\left(2Aa^2c^2+\frac{11Aa^2d^2}{2}+4Ba^2c^2+7Ba^2d^2+8Aa^2cd+11Ba^2cd\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6\left(16Aa^2c^2+8Aa^2d^2+12Ba^2c^2+4Ba^2d^2+24Aa^2cd+16Ba^2cd\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2\left(16Aa^2c^2+\frac{40Aa^2d^2}{3}+\frac{44Ba^2c^2}{3}+12Ba^2d^2+\frac{88Aa^2cd}{3}+\frac{80Ba^2cd}{3}\right)+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4\left(24Aa^2c^2+\frac{56Aa^2d^2}{3}+\frac{64Ba^2c^2}{3}+20Ba^2d^2+\frac{128Aa^2cd}{3}+\frac{112Ba^2cd}{3}\right)+4Aa^2c^2+\frac{8Aa^2d^2}{3}+\frac{10Ba^2c^2}{3}+\frac{12Ba^2d^2}{5}+\frac{20Aa^2cd}{3}+\frac{16Ba^2cd}{3}}{f\left(5{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+10{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4+10{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6+5{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^8+{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^{10}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2 +
 16*A*c*d + 14*B*c*d))/(4*(3*A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3
*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)))*(12*A*c^2 + 7*A*d^2 + 8*B*
c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^
2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d) + tan(e/2 + (f*x)/2)*(A*a^2*c^2 + (7*A*a
^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)
- tan(e/2 + (f*x)/2)^9*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^
2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) + tan(e/2 + (f*x)/2)^3*(2*A*a^2*c
^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*
c*d) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 +
 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(16*A*a^2
*c^2 + 8*A*a^2*d^2 + 12*B*a^2*c^2 + 4*B*a^2*d^2 + 24*A*a^2*c*d + 16*B*a^2*c
*d) + tan(e/2 + (f*x)/2)^2*(16*A*a^2*c^2 + (40*A*a^2*d^2)/3 + (44*B*a^2*c^2
)/3 + 12*B*a^2*d^2 + (88*A*a^2*c*d)/3 + (80*B*a^2*c*d)/3) + tan(e/2 + (f*x)
/2)^4*(24*A*a^2*c^2 + (56*A*a^2*d^2)/3 + (64*B*a^2*c^2)/3 + 20*B*a^2*d^2 +
(128*A*a^2*c*d)/3 + (112*B*a^2*c*d)/3) + 4*A*a^2*c^2 + (8*A*a^2*d^2)/3 + (1
0*B*a^2*c^2)/3 + (12*B*a^2*d^2)/5 + (20*A*a^2*c*d)/3 + (16*B*a^2*c*d)/3)/(f
*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^
6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=166

$$\frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)}{6f}-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\,\mathrm{c}}{24f}$$

```
[Out] 1/8*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*x-1/6*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos
(f*x+e)/f-1/24*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/12*
(4*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*d*cos(f*x+e)*(a+a*s
in(f*x+e))^3/a/f
```

Rubi [A]
time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2830, 2723}

$$-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\cos(e+fx)}{6f}-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x(12Ac+8Ad+8Bc+7Bd)-\frac{(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{12f}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*x)/8 - (a^2*(12*A*c + 8*B*c + 8*A*d +
 7*B*d)*Cos[e + f*x])/(6*f) - (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*Cos[e +
 f*x]*Sin[e + f*x])/(24*f) - ((4*B*c + 4*A*d - B*d)*Cos[e + f*x]*(a + a*Sin
[e + f*x])^2)/(12*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (a+a\sin(e+fx))^2\left(Ac+(Bc+Ad\right.$$

$$=-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}+$$

$$=-\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a}{12f}$$

$$=\frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x-\frac{a^2(}{}$$

Mathematica [A]
time = 0.52, size = 160, normalized size = 0.96

$$-\frac{a^2\cos(e+fx)\left(6(12Ac+8Bc+8Ad+7Bd)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8(6Ac+5Bc+5Ad+4Bd)+3(4Ac+8Bc+8Ad+7Bd)\sin(e+fx)+8(Ad+B(c+2d))\sin^2(e+fx)+6Bd\sin^3(e+fx)\right)\right)}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -1/24*(a^2*Cos[e + f*x]*(6*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*ArcSin[Sqrt[1 -
 Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(6*A*c + 5*B*c + 5*A*d +
4*B*d) + 3*(4*A*c + 8*B*c + 8*A*d + 7*B*d)*Sin[e + f*x] + 8*(A*d + B*(c + 2
*d))*Sin[e + f*x]^2 + 6*B*d*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A]
time = 0.18, size = 278, normalized size = 1.67

method	result

risch	$\frac{3A\,a^2cx}{2}+A\,a^2dx+B\,a^2cx+\frac{7B\,a^2dx}{8}-\frac{2a^2\cos(fx+e)Ac}{f}-\frac{7a^2\cos(fx+e)Ad}{4f}-\frac{7a^2\cos(fx+e)Bc}{4f}-\frac{3a}{}$
derivativedivides	$\frac{a^2Ac(fx+e)-a^2Ad\cos(fx+e)-B\,a^2c\cos(fx+e)+B\,a^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2a^2Ac\cos(fx+e)+2a^2Ad\left(-\right.}{}$
default	$\frac{a^2Ac(fx+e)-a^2Ad\cos(fx+e)-B\,a^2c\cos(fx+e)+B\,a^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2a^2Ac\cos(fx+e)+2a^2Ad\left(-\right.}{}$
norman	$\frac{\left(\frac{3}{2}a^2Ac+a^2Ad+B\,a^2c+\frac{7}{8}B\,a^2d\right)x+\left(6a^2Ac+4a^2Ad+4B\,a^2c+\frac{7}{2}B\,a^2d\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(6a^2Ac+4a^2Ad+4B\,a^2c+\frac{7}{2}B\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(a^2*A*c*(f*x+e)-a^2*A*d*cos(f*x+e)-B*a^2*c*cos(f*x+e)+B*a^2*d*(-1/2*co
s(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a^2*A*c*cos(f*x+e)+2*a^2*A*d*(-1/2*cos
(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2*B*a^2*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)-2/3*B*a^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c*(-1/2*cos(f*x+e
)*sin(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*B*a
^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e)
)*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A]

time = 0.29, size = 289, normalized size = 1.74

$$\frac{24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2c+96\,(fx+e)Aa^2c+32\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)Ba^2c+48\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Ba^2c+32\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)Aa^2d+48\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^2d+64\,\left(\cos(fx+e)^3-3\,\cos(fx+e)\right)Ba^2d+3\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))Ba^2d+24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Ba^2d-192\,Aa^2c\cos(fx+e)-96\,Ba^2c\cos(fx+e)-96\,Aa^2d\cos(fx+e)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c +
32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*
a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d +
 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x + e) - 9
6*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f
```

Fricas [A]

time = 0.39, size = 149, normalized size = 0.90

$$\frac{8\,(Ba^2c+(A+2\,B)a^2d)\cos(fx+e)^3+3\,(4\,(3\,A+2\,B)a^2c+(8\,A+7\,B)a^2d)fx-48\,((A+B)a^2c+(A+B)a^2d)\cos(fx+e)+3\,\left(2\,Ba^2d\cos(fx+e)^3-(4\,(A+2\,B)a^2c+(8\,A+9\,B)a^2d)\cos(fx+e)\right)\sin(fx+e)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*c
 + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x + e)
 + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)*co
s(f*x + e))*sin(f*x + e))/f
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(163) = 326$.
time = 0.27, size = 571, normalized size = 3.44

$$\begin{cases} \frac{Aa^2cx\sin^2(e+fx)}{2} + \frac{Aa^2cx\cos^2(e+fx)}{2} + Aa^2cx - \frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^2c\cos(e+fx)}{f} + Aa^2dx\sin^2(e+fx) + Aa^2dx\cos^2(e+fx) - \frac{Aa^2d\sin^2(e+fx)\cos(e+fx)}{f} - \frac{Aa^2d\sin(e+fx)\cos(e+fx)}{f} - \frac{2Aa^2d\cos^3(e+fx)}{3f} - \frac{Aa^2d\cos(e+fx)}{f} + Ba^2cx\sin^2(e+fx) + Ba^2cx\cos^2(e+fx) - \frac{Ba^2c\sin^2(e+fx)\cos(e+fx)}{f} - \frac{Ba^2c\sin(e+fx)\cos(e+fx)}{f} - \frac{2Ba^2c\cos^3(e+fx)}{3f} - \frac{Ba^2c\cos(e+fx)}{f} + \frac{3Ba^2dx\sin^4(e+fx)}{8} + \frac{3Ba^2dx\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{Ba^2dx\sin^2(e+fx)}{2} + \frac{3Ba^2dx\cos^4(e+fx)}{8} + \frac{Ba^2dx\cos^2(e+fx)}{2} - \frac{5Ba^2d\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{2Ba^2d\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Ba^2d\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{Ba^2d\sin(e+fx)\cos(e+fx)}{2f} - \frac{4Ba^2d\cos^3(e+fx)}{3f} & \text{for}\ f \neq 0 \\ x(A+B\sin(e))(c+d\sin(e))(a\sin(e)+a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A*
a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e + f*
x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*d*s
in(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*
a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*sin(e +
 f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(e + f*
x)/f - B*a**2*c*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x)**3/(3
*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*B*a**2*d
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/2 + 3*B*a
**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a**2*d*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e + f*x)/f -
 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin
(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))
```

Giac [**A**]
time = 0.64, size = 172, normalized size = 1.04

$$\frac{Ba^2d\sin(4fx+4e)}{32f} + \frac{1}{8}(12Aa^2c + 8Ba^2c + 8Aa^2d + 7Ba^2d)x + \frac{(Ba^2c + Aa^2d + 2Ba^2d)\cos(3fx+3e)}{12f} - \frac{(8Aa^2c + 7Ba^2c + 7Aa^2d + 6Ba^2d)\cos(fx+e)}{4f} - \frac{(Aa^2c + 2Ba^2c + 2Aa^2d + 2Ba^2d)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d +
 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f - 1
/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*(A*
a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.60, size = 492, normalized size = 2.96

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d\right)}{4\,\left(3\,A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B\,a^2\,d}{4}\right)}\right)\,\left(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d\right)}{4\,f}-\frac{a^2\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\,\left(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{15\,B\,a^2\,d}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B\,a^2\,d}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{15\,B\,a^2\,d}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(12\,A\,a^2\,c+10\,A\,a^2\,d+10\,B\,a^2\,c+8\,B\,a^2\,d\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(12\,A\,a^2\,c+\frac{34\,A\,a^2\,d}{3}+\frac{34\,B\,a^2\,c}{3}+\frac{32\,B\,a^2\,d}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(4\,A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B\,a^2\,d}{4}\right)+4\,A\,a^2\,c+\frac{10\,A\,a^2\,d}{3}+\frac{10\,B\,a^2\,c}{3}+\frac{8\,B\,a^2\,d}{3}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*(3*A
*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4)))*(12*A*c + 8*A*d + 8*B*c +
 7*B*d))/(4*f) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*A*c + 8*A*d
+ 8*B*c + 7*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(A*a^2*c + 2*A*a^2*d + 2*B*
a^2*c + (15*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^7*(A*a^2*c + 2*A*a^2*d + 2*B*a
^2*c + (7*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^5*(A*a^2*c + 2*A*a^2*d + 2*B*a^2
*c + (15*B*a^2*d)/4) + tan(e/2 + (f*x)/2)^4*(12*A*a^2*c + 10*A*a^2*d + 10*B
*a^2*c + 8*B*a^2*d) + tan(e/2 + (f*x)/2)^2*(12*A*a^2*c + (34*A*a^2*d)/3 + (
34*B*a^2*c)/3 + (32*B*a^2*d)/3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c + 2*A*a^2
*d + 2*B*a^2*c) + tan(e/2 + (f*x)/2)*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*
B*a^2*d)/4) + 4*A*a^2*c + (10*A*a^2*d)/3 + (10*B*a^2*c)/3 + (8*B*a^2*d)/3)/
(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^
6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.254 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}a^2(3A+2B)x-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\cos(e+fx)\sin(e+fx)}{6f}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}$$

```
[Out] 1/2*a^2*(3*A+2*B)*x-2/3*a^2*(3*A+2*B)*cos(f*x+e)/f-1/6*a^2*(3*A+2*B)*cos(f*
x+e)*sin(f*x+e)/f-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2830, 2723}

$$-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3A+2B)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 - (2*a^2*(3*A + 2*B)*Cos[e + f*x])/(3*f) - (a^2*(3*A
+ 2*B)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx=-\frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}+\frac{1}{3}(3A+2B)\int (a+$$

$$=\frac{1}{2}a^2(3A+2B)x-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+$$

Mathematica [A]
time = 0.22, size = 106, normalized size = 1.13

$$-\frac{a^2\cos(e+fx)\left(6(3A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(2(6A+5B)+3(A+2B)\sin(e+fx)+2B\sin^2(e+fx))\right)}{6f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/6*(a^2*Cos[e + f*x]*(6*(3*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*Sin[e + f*x] + 2*B*Si
n[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A]
time = 0.13, size = 117, normalized size = 1.24

method	result
risch	$\frac{3a^2xA}{2}+B\,a^2x-\frac{2a^2\cos(fx+e)A}{f}-\frac{7a^2\cos(fx+e)B}{4f}+\frac{B\,a^2\cos(3fx+3e)}{12f}-\frac{\sin(2fx+2e)a^2A}{4f}-\frac{\sin(2fx+2e}{2f}$
derivativedivides	$\frac{a^2A(fx+e)-B\,a^2\cos(fx+e)-2a^2A\cos(fx+e)+2B\,a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2A\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}\right.}{f}$
default	$\frac{a^2A(fx+e)-B\,a^2\cos(fx+e)-2a^2A\cos(fx+e)+2B\,a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2A\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}\right.}{f}$
norman	$\frac{\left(\frac{3}{2}a^2A+B\,a^2\right)x+\left(\frac{3}{2}a^2A+B\,a^2\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{9}{2}a^2A+3B\,a^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{9}{2}a^2A+3B\,a^2\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\left(1+\mathrm{ta}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*A*(f*x+e)-B*a^2*cos(f*x+e)-2*a^2*A*cos(f*x+e)+2*B*a^2*(-1/2*cos(f*
x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^2*A*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/
2*e)-1/3*B*a^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A]
time = 0.28, size = 123, normalized size = 1.31

$$\frac{3\,(2\,fx+2\,e-\sin\,(2\,fx+2\,e))Aa^2+12\,(fx+e)Aa^2+4\,(\cos\,(fx+e)^3-3\,\cos\,(fx+e))Ba^2+6\,(2\,fx+2\,e-\sin\,(2\,fx+2\,e))Ba^2-24\,Aa^2\cos\,(fx+e)-12\,Ba^2\cos\,(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*B
*a^2 - 24*A*a^2*cos(f*x + e) - 12*B*a^2*cos(f*x + e))/f
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.79

$$\frac{2\,Ba^{2}\cos\left(fx+e\right)^{3}+3\,(3\,A+2\,B)a^{2}fx-3\,(A+2\,B)a^{2}\cos\left(fx+e\right)\sin\left(fx+e\right)-12\,(A+B)a^{2}\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*a^2*cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*cos(f
*x + e)*sin(f*x + e) - 12*(A + B)*a^2*cos(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(85) = 170$.
time = 0.15, size = 199, normalized size = 2.12

$$\begin{cases} \frac{Aa^2x\sin^2(e+fx)}{2}+\frac{Aa^2x\cos^2(e+fx)}{2}+Aa^2x-\frac{Aa^2\sin(e+fx)\cos(e+fx)}{2f}-\frac{2Aa^2\cos(e+fx)}{f}+Ba^2x\sin^2(e+fx)+Ba^2x\cos^2(e+fx)-\frac{Ba^2\sin^2(e+fx)\cos(e+fx)}{f}-\frac{Ba^2\sin(e+fx)\cos(e+fx)}{f}-\frac{2Ba^2\cos^3(e+fx)}{3f}-\frac{Ba^2\cos(e+fx)}{f} & \text{for } f\neq 0 \\ x(A+B\sin(e))\,(a\sin(e)+a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**2
*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B*a
**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**2*c
os(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f*x)*
*3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
a)**2, True))
```

Giac [A]
time = 0.61, size = 88, normalized size = 0.94

$$\frac{Ba^{2}\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\left(3\,Aa^{2}+2\,Ba^{2}\right)x-\frac{\left(8\,Aa^{2}+7\,Ba^{2}\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(Aa^{2}+2\,Ba^{2}\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 +
7*B*a^2)*cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 13.16, size = 91, normalized size = 0.97

$$-\frac{\frac{3\,A\,a^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{B\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{2}+3\,B\,a^2\,\sin\left(2\,e+2\,f\,x\right)+12\,A\,a^2\,\cos\left(e+f\,x\right)+\frac{21\,B\,a^2\,\cos\left(e+f\,x\right)}{2}-9\,A\,a^2\,f\,x-6\,B\,a^2\,f\,x}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] -((3*A*a^2*sin(2*e + 2*f*x))/2 - (B*a^2*cos(3*e + 3*f*x))/2 + 3*B*a^2*sin(2
*e + 2*f*x) + 12*A*a^2*cos(e + f*x) + (21*B*a^2*cos(e + f*x))/2 - 9*A*a^2*f
*x - 6*B*a^2*f*x)/(6*f)
```

3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$-\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3}-\frac{2a^2(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f}+\frac{a^2(2Bc-2Ad$$

```
[Out] -1/2*a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x/d^3+1/2*a^2*(-2*A*d+2*B*c-
3*B*d)*cos(f*x+e)/d^2/f-1/2*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/f-2*a^2*(c-
d)^2*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2
-d^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3055, 3047, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2(c-d)^2(Bc-Ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}-\frac{a^2x(2Ad(c-2d)-B(2c^2-4cd+3d^2))}{2d^3}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{2d^2f}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/2*(a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/d^3 - (2*a^2*(c
- d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*S
qrt[c^2 - d^2]*f) + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(2*d^2*f) -
(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} + \int \frac{(a+a\sin(e+fx))(a(B \cdots}{\cdots}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} + \int \frac{a^2(Bc+2Ad)+(a^2(Bc \cdots}{\cdots}$$

$$= \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\left(a^2+ \cdots\right)}{2df}$$

$$= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} + \frac{a^2(2Bc-2A \cdots}{\cdots}$$

$$= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} + \frac{a^2(2Bc-2A \cdots}{\cdots}$$

$$= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} + \frac{a^2(2Bc-2A \cdots}{\cdots}$$

$$= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} - \frac{2a^2(c-d)^2(\cdots}{\cdots}$$

Mathematica [A]
time = 0.44, size = 177, normalized size = 1.04

$$\frac{a^2(1+\sin(e+fx))^2\left(2(2Ad(-c+2d)+B(2c^2-4cd+3d^2))(e+fx)-\frac{8(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-4d(-Bc+Ad+2Bd)\cos(e+fx)-Bd^2\sin(2(e+fx))\right)}{4d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2))
*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] - B*
d^2*Sin[2*(e + f*x)]))/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A]
time = 0.36, size = 235, normalized size = 1.37

method	result

derivativedivides	$\frac{2a^2\left(\frac{\left(A\,c^2d-2Ac\,d^2+A\,d^3-B\,c^3+2B\,c^2d-Bc\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}}-\frac{-\frac{B\,d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(A\,d^2-Bcd+2B\,d^2\right)\left(\right.}{\left(1+\mathrm{ta}\right.}\right.}{f}$
default	$\frac{2a^2\left(\frac{\left(A\,c^2d-2Ac\,d^2+A\,d^3-B\,c^3+2B\,c^2d-Bc\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}}-\frac{-\frac{B\,d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(A\,d^2-Bcd+2B\,d^2\right)\left(\right.}{\left(1+\mathrm{ta}\right.}\right.}{f}$
risch	$-\frac{a^2xAc}{d^2}+\frac{2a^2xA}{d}+\frac{a^2xB\,c^2}{d^3}-\frac{2a^2xBc}{d^2}+\frac{3a^2xB}{2d}-\frac{a^2\mathrm{e}^{i(fx+e)}A}{2df}+\frac{a^2\mathrm{e}^{i(fx+e)}Bc}{2d^2f}-\frac{a^2\mathrm{e}^{i(fx+e)}B}{df}-a^2\mathrm{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a^2*((A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2*d-B*c*d^2)/d^3/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1/d^3*((-1/2*B
*d^2*tan(1/2*f*x+1/2*e)^3+(A*d^2-B*c*d+2*B*d^2)*tan(1/2*f*x+1/2*e)^2+1/2*B*
d^2*tan(1/2*f*x+1/2*e)+A*d^2-B*c*d+2*B*d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2+1/2*
(2*A*c*d-4*A*d^2-2*B*c^2+4*B*c*d-3*B*d^2)*arctan(tan(1/2*f*x+1/2*e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.48, size = 467, normalized size = 2.73

$$\left[-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e)-(2\,Ba^2c^2-2\,(A+2\,B)a^2cd+(4\,A+3\,B)a^2d^2)fx+(Ba^2c^2-(A+B)a^2cd+Aa^2d^2)\sqrt{-\frac{c-d}{c+d}}\log\left(-\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2-2\,((c^2+cd)\cos(fx+e)\sin(fx+e)+(cd+d^2)\cos(fx+e))\sqrt{-\frac{c-d}{c+d}}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2}\right)-2\,(Ba^2cd-(A+2\,B)a^2d^2)\cos(fx+e)}{2\,d^3f},\ -\frac{Ba^2d^2\cos(fx+e)\sin(fx+e)-(2\,Ba^2c^2-2\,(A+2\,B)a^2cd+(4\,A+3\,B)a^2d^2)fx-2\,(Ba^2c^2-(A+B)a^2cd+Aa^2d^2)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)-2\,(Ba^2cd-(A+2\,B)a^2d^2)\cos(fx+e)}{2\,d^3f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x + (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f),
 -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^
2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))
/((c - d)*cos(f*x + e))) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/
(d^3*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 314, normalized size = 1.84

$$\frac{\frac{(2\,Ba^2c^2-2\,Aa^2cd-4\,Ba^2cd+4\,Aa^2d^2+3\,Ba^2d^2)(fx+e)}{d^3}-\frac{4\,(Ba^2c^3-Aa^2c^2d-2\,Ba^2c^2d+2\,Aa^2cd^2+Ba^2cd^2-Aa^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^3}+\frac{2\left(Ba^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,Ba^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Aa^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,Ba^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ba^2c-2\,Aa^2d-4\,Ba^2d\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)*
(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^2
+ B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3) + 2*
(B*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^
2*d*tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*tan(1/2*f*x + 1/2*e)^2 - B*a^2*d*tan
(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((tan(1/2*f*x + 1/2*
e)^2 + 1)^2*d^2))/f
```

Mupad [**B**]
time = 20.07, size = 2500, normalized size = 14.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x)),x)

[Out] (atan(((((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*1i)/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*1i)/d^3)/((16*(2*B^3*a^6*c^7 + 20*A^3*a^6*c^2*d^5 - 16*A^3*a^6*c^3*d^4 + 4*A^3*a^6*c^4*d^3 + 3*B^3*a^6*c^3*d^4 - 10*B^3*a^6*c^4*d^3 + 13*B^3*a^6*c^5*d^2 - 8*A^3*a^6*c*d^6 - 8*B^3*a^6*c^6*d - 6*A^2*B*a^6*c*d^6 + 3*A*B^2*a^6*c^2*d^5 - 6*A*B^2*a^6*c^3*d^4 + 3*A*B^2*a^6*c^4*d^3 + 24*A^2*B*a^6*c^2*d^5 - 36*A^2*B*a^6*c^3*d^4 + 24*A^2*B*a^6*c^4*d^3 - 6*A^2*B*a^6*c^5*d^2))/d^5 - (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 + (f

x)/2)(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1
i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d
^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*tan(e/2
 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c
^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2
*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x
)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*
a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 -
 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c
*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a
^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d
^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)
/2))/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^
4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^
4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 3
2*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8*(8*A*a^2*c*d^8 + 6*B*a^2
*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + ((32*c
^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i
+ (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*tan(
e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^
2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*
d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (
f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A
^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^
5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^
4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7...

3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=198

$$-\frac{a^2(2Bc-Ad-2Bd)x}{d^3}-\frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}\,f}+\frac{a^2(Ad-$$

```
[Out] -a^2*(-A*d+2*B*c-2*B*d)*x/d^3+a^2*(A*d-B*(2*c+d))*cos(f*x+e)/d^2/(c+d)/f+(-
A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))-2*a^2*(
c-d)*(A*d*(c+2*d)-B*(2*c^2+2*c*d-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2
-d^2)^(1/2))/d^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3054, 3047, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{a^2x(-Ad+2Bc-2Bd)}{d^3}+\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2f(c+d)}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*(2*B*c - A*d - 2*B*d)*x)/d^3) - (2*a^2*(c - d)*(A*d*(c + 2*d) - B*(2
*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3
*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(A*d - B*(2*c + d))*Cos[e + f*x])/(d^2*(
c + d)*f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(d*(c + d)*
f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e}{}}{}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{-a^2(B(c-}{}}{}$$

$$= \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx}{d(c+d)f(c+}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+f}{d^2(c+d)f}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+f}{d^2(c+d)f}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+f}{d^2(c+d)f}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} - \frac{2a^2(c-d)\left(Ad(c+2d)-B(2\right.}{d^3(c}$$

Mathematica [A]
time = 0.69, size = 192, normalized size = 0.97

$$\frac{a^2(1+\sin(e+fx))^2\left((-2Bc+Ad+2Bd)(e+fx)+\frac{2(c-d)\left(-Ad(c+2d)+B\left(2c^2+2cd-d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}-Bd\cos(e+fx)-\frac{d(-c+d)(-Bc+Ad)\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}\right)}{d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(-
(A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/
Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c + d
)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^3*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A]
time = 0.52, size = 251, normalized size = 1.27

method	result

derivativedivides	$\frac{2a^2\left(-\frac{\frac{-\frac{d^2\left(Acd-A\,d^2-B\,c^2+Bcd\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-A\,d^2-B\,c^2+Bcd\right)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,c^2d+Ac\,d^2-2A\,d^3-2B\,c^3+3Bc\,d^2-B\,d^3\right)\operatorname{arctan}}{(c+d)\sqrt{c^2-d^2}}}{d^3}\right.}{f}$
default	$\frac{2a^2\left(-\frac{\frac{-\frac{d^2\left(Acd-A\,d^2-B\,c^2+Bcd\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-A\,d^2-B\,c^2+Bcd\right)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,c^2d+Ac\,d^2-2A\,d^3-2B\,c^3+3Bc\,d^2-B\,d^3\right)\operatorname{arctan}}{(c+d)\sqrt{c^2-d^2}}}{d^3}\right.}{f}$
risch	$\frac{a^2xA}{d^2}-\frac{2a^2xBc}{d^3}+\frac{2a^2xB}{d^2}-\frac{B\,a^2\mathrm{e}^{i(fx+e)}}{2d^2f}-\frac{B\,a^2\mathrm{e}^{-i(fx+e)}}{2d^2f}+\frac{2ia^2\left(-Acd+A\,d^2+B\,c^2-Bcd\right)\left(id+c\,\mathrm{e}^{i(fx+e)}\right)}{d^3(c+d)f\left(id-id\,\mathrm{e}^{2i(fx+e)}+2c\,\mathrm{e}^{i(fx+e)}\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2*(-1/d^3*((-d^2*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d)/c*tan(1/2*f*x+1/2*e)
-d*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x
+1/2*e)+c)+(A*c^2*d+A*c*d^2-2*A*d^3-2*B*c^3+3*B*c*d^2-B*d^3)/(c+d)/(c^2-d^2
)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))+1/d^3*(-B
*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-2*B*c+2*B*d)*arctan(tan(1/2*f*x+1/2*e))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.43, size = 750, normalized size = 3.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c
^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)
*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((
2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*
d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c +
 d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B*a^2*c
^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A*a^2*c*
d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(
f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*B*a^2*
c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A - 2*B)*a^2
*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*d^2 - (2*A
+ B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
 d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c^2*d - A*a^2*
c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d^2 - (A + 2*B)
*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/((c*d
^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(199) = 398.
time = 0.79, size = 498, normalized size = 2.52

$$\frac{\frac{2\,(2\,Ba^2c^3-Aa^2c^2d-Aa^2cd^2-3\,Ba^2cd^2+2\,Aa^2d^3+Ba^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^3+d^4)\sqrt{c^2-d^2}}-\frac{2\left(Ba^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Aa^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Ba^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+Aa^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,Ba^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Aa^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+Aa^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Ba^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Aa^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ba^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ba^2c^3-Aa^2c^2d+Aa^2cd^2\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)(c^2d^2+cd^3)}-\frac{(2\,Ba^2c-Aa^2d-2\,Ba^2d)(fx+e)}{d^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3 +
 B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*
x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B*a^
2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B*a^2
```

```
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a^2*c*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a^2*c*d^
2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d^3*tan(1
/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan(1/2*f*x
+ 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*
tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^2*d - 2*B*a
^2*d)*(f*x + e)/d^3)/f
```

Mupad [B]
time = 21.58, size = 2500, normalized size = 12.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (
f*x)/2)^2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2
 + (f*x)/2)*(A*a^2*d^2 + 3*B*a^2*c^2 - A*a^2*c*d + B*a^2*c*d))/(c*d*(c + d)
) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2*d^2 + B*a^2*c^2 - A*a^2*c*d - B*a^2*c*d)
)/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2
+ c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) - (atan((((B*a^2*c*2i
 - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*
c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B
*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/
(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((((32*(c^2*
d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x
)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 +
 d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9
+ 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c
^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10
 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4
*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c
*d^7 + d^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4
*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^
7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4
*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*
a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 +
4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3 + ((B*a^2*c*2i - a^2*d
*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4
+ 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2
*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6
 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 +
2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4
```

```
*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/
(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 +
 c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i
- a^2*d*(A + 2*B)*1i))/d^3 - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a
^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c
^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d
^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4
*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B
^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2
 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*
d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^
4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3)/((64*(4*B^3*a^6*c^6 - 2*A^3*a
^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*d^3 -
2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5*d + 9
*A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2*a^6*c
^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c^2*d^4
 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(
A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 +
4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d
^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 +
 d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((((32*(c^2*d^10 + 2*c
^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*
d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2
*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9 + 2*B*a^2*
c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(
2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2
*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2
*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8
 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c
^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2
*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 -
 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^
6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*
c*d^8))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 - ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i
)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c
^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*
a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*
d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9
 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*...
```

3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{a^2Bx}{d^3}+\frac{a^2(3Ad^3-B(2c^3+4c^2d+cd^2-4d^3))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}\,f}+\frac{(Bc-Ad)\cos(e+fx)\,(a^2+a^2\sin(}{2d(c+d)f(c+d\sin(e+fx)}$$

```
[Out] a^2*B*x/d^3+1/2*(-A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*s
in(f*x+e))^2-1/2*a^2*(3*A*d^2-B*(2*c^2+3*c*d-2*d^2))*cos(f*x+e)/d^2/(c+d)^2
/f/(c+d*sin(f*x+e))+a^2*(3*A*d^3-B*(2*c^3+4*c^2*d+c*d^2-4*d^3))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3054, 3047, 3100, 2814, 2739, 632, 210}

$$\frac{a^2(3Ad^3-B(2c^3+4c^2d+cd^2-4d^3))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(Bc-Ad)\cos(e+fx)\,(a^2\sin(e+fx)+a^2)}{2df(c+d)(c+d\sin(e+fx))^2}+\frac{a^2Bx}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*B*x)/d^3 + (a^2*(3*A*d^3 - B*(2*c^3 + 4*c^2*d + c*d^2 - 4*d^3))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*
f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - (a^2*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f
*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx &= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}+\frac{\int \frac{(a+a\sin(e+}{}}{} \\ &= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}+\frac{\int \frac{-a^2(Bc-3A}{}}{} \\ &= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a^2(3Ad^2-}{2d^2(c} \\ &= \frac{a^2Bx}{d^3}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a^2}{} \\ &= \frac{a^2Bx}{d^3}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a^2}{} \\ &= \frac{a^2Bx}{d^3}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2}-\frac{a^2}{} \\ &= \frac{a^2Bx}{d^3}-\frac{a^2\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\tan^{-1}\left(\frac{d}{}\right.}{d^3(c+d)^2\sqrt{c^2-d^2}\,f}\end{aligned}$$

Mathematica [A]
time = 0.94, size = 226, normalized size = 1.05

$$\frac{a^2(1+\sin(e+fx))^2\left(2B(e+fx)-\frac{2\left(-3Ad^3+B\left(2c^3+4c^2d+cd^2-4d^3\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}}-\frac{d(-c+d)(-Bc+Ad)\cos(e+fx)}{(c+d)(c+d\sin(e+fx))^2}-\frac{d\left(Ad(c+4d)+B\left(-3c^2-4cd+2d^2\right)\right)\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))}\right)}{2d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2*
d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c +
 d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*
(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2))*C
os[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(206) = 412.
time = 0.70, size = 431, normalized size = 2.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^2*(1/d^3*((1/2*d^2*(A*c^2*d-4*A*c*d^2-2*A*d^3+B*c^3+4*B*c^2*d)/(c^2+2
*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(4*A*c^3*d^2+A*c^2*d^3+8*A*c*d^4+2*A
*d^5-2*B*c^5-4*B*c^4*d-3*B*c^3*d^2-8*B*c^2*d^3+2*B*c*d^4)/(c^2+2*c*d+d^2)/c
^2*tan(1/2*f*x+1/2*e)^2-1/2*d^2*(A*c^2*d+12*A*c*d^2+2*A*d^3-7*B*c^3-12*B*c^
2*d+4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-1/2*d*(4*A*c*d^2+A*d^3-
2*B*c^3-4*B*c^2*d+B*c*d^2)/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan
(1/2*f*x+1/2*e)+c)^2+1/2*(3*A*d^3-2*B*c^3-4*B*c^2*d-B*c*d^2+4*B*d^3)/(c^2+2
*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)
^(1/2)))+B/d^3*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(212) = 424$.
time = 0.46, size = 1508, normalized size = 7.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*c
os(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*d^
4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*a^2
*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a^2*c
^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x + e)^
2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*a^2*c*
d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c
*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)
```

```
) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3 - (A + 4*B
)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) - 2*(4*(B*a^2
*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x + (3*B*a^2*c^
4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A - 4*B)*a^2*c*d
^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^5 + 2*c^3*d^6
 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5 - 2*c^2*d^7 - c
*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2*d^7 - 2*c*d^8 -
 d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d
^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a
^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d
+ 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (
2*B*a^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(
f*x + e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*
B)*a^2*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(s
qrt(c^2 - d^2)*cos(f*x + e))) - (2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3
*B)*a^2*c^3*d^3 - (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*
cos(f*x + e) - (4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*
c*d^5)*f*x + (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2
*d^4 + (A - 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^4*d^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2
*c^4*d^5 - 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d
^5 - c^2*d^7 - 2*c*d^8 - d^9)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(212) = 424.
time = 0.73, size = 703, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^2
*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*ta
n(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c
```

```
^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/2*f*
x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^3*tan
(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^2*c^3*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 8
*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2
- 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e) -
 A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)
 - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan(1/2*f*x + 1/
2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B*a^2*c^4*d - 4
*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2
*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

Mupad [B]
time = 22.50, size = 2500, normalized size = 11.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (2*B*a^2*atan(((B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*
a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c
^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 +
 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*
c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4
*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*
c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2 + (f*x
)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*
d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5
*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6)
 - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^
7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))
/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12
 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*
c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*
d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(
4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3 +
 (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 +
16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3
*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^
3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^
2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*
A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 +
```

6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3)/((16*(2*B^3*a^6*c^5 + 17*B^3*a^6*c^3*d^2 - 16*B^3*a^6*c*d^4 + 12*B^3*a^6*c^4*d - 24*A*B^2*a^6*c*d^4 + 6*A*B^2*a^6*c^4*d - 9*A^2*B*a^6*c*d^4 + 12*A*B^2*a^6*c^3*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (16*tan(e/2 + (f*x)/2)*(28*B^3*a^6*c^2*d^4 - 8*B^3*a^6*c^6 - 8*B^3*a^6*c^3*d^3 - 44*B^3*a^6*c^4*d^2 + 16*B^3*a^6*c*d^5 - 32*B^3*a^6*c^5*d + 12*A*B^2*a^6*c*d^5 + 24*A*B^2*a^6*c^2*d^4 + 12*A*B^2*a^6*c^3*d^3))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3 + (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A...

3.258 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=604

$$\frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A\left(40c^3+90c^2d+78cd^2+23d^3\right)\right)x-\frac{a^3(7Ad(2c^5-18c^4d+107c^3}{}$$

```
[Out] 1/16*a^3*(3*B*(10*c^3+26*c^2*d+23*c*d^2+7*d^3)+A*(40*c^3+90*c^2*d+78*c*d^2+
23*d^3))*x-1/420*a^3*(7*A*d*(2*c^5-18*c^4*d+107*c^3*d^2+472*c^2*d^3+456*c*d
^4+136*d^5)-3*B*(2*c^6-14*c^5*d+51*c^4*d^2-189*c^3*d^3-920*c^2*d^4-952*c*d^
5-288*d^6))*cos(f*x+e)/d^3/f-1/1680*a^3*(7*A*d*(4*c^4-36*c^3*d+216*c^2*d^2+
626*c*d^3+345*d^4)-3*B*(4*c^5-28*c^4*d+104*c^3*d^2-392*c^2*d^3-1263*c*d^4-7
35*d^5))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/840*a^3*(7*A*d*(2*c^3-18*c^2*d+111*c
*d^2+136*d^3)-B*(6*c^4-42*c^3*d+165*c^2*d^2-651*c*d^3-864*d^4))*cos(f*x+e)*
(c+d*sin(f*x+e))^2/d^3/f-1/840*a^3*(7*A*d*(2*c^2-18*c*d+115*d^2)-B*(6*c^3-4
2*c^2*d+177*c*d^2-735*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f-1/210*a^3*(
-14*A*c*d+91*A*d^2+6*B*c^2-27*B*c*d+87*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4
/d^3/f-1/7*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4/d/f+1/42*(3
*B*(c-3*d)-7*A*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^4/d^2/f
```

Rubi [A]
time = 0.98, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3047, 3102, 2832, 2813}

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 7
8*c*d^2 + 23*d^3))*x)/16 - (a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 189
*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/(420*d^3*f) -
(a^3*(7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4
*c^5 - 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e
+ f*x]*Sin[e + f*x])/(1680*d^2*f) - (a^3*(7*A*d*(2*c^3 - 18*c^2*d + 111*c*d
^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*d^3 - 864*d^4))*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(840*d^3*f) - (a^3*(7*A*d*(2*c^2 - 18*c
*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3))*Cos[e + f*x]*(c
 + d*Sin[e + f*x])^3)/(840*d^3*f) - (a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 9
1*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(210*d^3*f) - (a*B
*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(7*d*f) + ((3*
```

```
B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f
*x])^4)/(42*d^2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(\ldots}{7df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(\ldots}{7df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(\ldots}{7df}$$

$$= -\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2\ldots}{2\ldots}$$

$$= -\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c\ldots}{\ldots}$$

$$= -\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136\ldots}{\ldots}$$

$$= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)\ldots\right.$$

Mathematica [A]
time = 3.12, size = 528, normalized size = 0.87

$$-\frac{a^3\cos(e+fx)\left(420(3B(10c^3+26c^2d+23cd^2+7d^3)+A(40c^3+90c^2d+78cd^2+23d^3))\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(12880Ac^3+11760Bc^3+35280Ac^2d+32676Bc^2d+32676Acd^2+30828Bcd^2+10276Ad^3+9762Bd^3-(112A(5c^3+45c^2d+66cd^2+26d^3)+3B(560c^3+2464c^2d+2912cd^2+1083d^3))\cos(2(e+fx))+18d(14Ad(c+d)+B(14c^2+42cd+23d^2))\cos(4(e+fx))-15Bd^3\cos(6(e+fx))+5040Ac^3\sin(e+fx)+6930Bc^3\sin(e+fx)+\ldots)\right)}{3360f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/3360*(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3)
+ A*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/
Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2*d
+ 32676*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*d^3
- (112*A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*c^2*d
 + 2912*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) + B*(14*
c^2 + 42*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)] + 5040
*A*c^3*Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e + f*x]
+ 22050*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*B*c*d^2*S
in[e + f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] - 210*B*c^3
*Sin[3*(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*Sin[3*(e +
f*x)] - 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e + f*x)] - 980
*A*d^3*Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*c*d^2*Sin[5*(
e + f*x)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e + f*x)])))/(f*Sq
rt[Cos[e + f*x]^2])
```

Maple [A]
time = 0.50, size = 1077, normalized size = 1.78

method	result
risch	$-\frac{63a^3\cos(fx+e)Bcd^2}{8f}+\frac{5a^3Ac^3x}{2}-\frac{39a^3\cos(fx+e)Ac^2d}{4f}-\frac{69a^3\cos(fx+e)Acd^2}{8f}-\frac{69a^3\cos(fx+e)Bc^2d}{8f}+$
derivativedivides	Expression too large to display
default	Expression too large to display
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(-B*a^3*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+9*B*a^3*c*d^2*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*B*a^3*c*d^2*(-1/6*(sin(f*x
+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-3*a^3*A
*c^2*d*cos(f*x+e)+9*a^3*A*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x
+e)+3/8*f*x+3/8*e)+9*B*a^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*
x+e)+3/8*f*x+3/8*e)+3*a^3*A*c*d^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e
)+3*B*a^3*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/7*B*a^3*d^3*(1
6/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-1/3*a^3*A*d^
3*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a^3*A*c^2*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)-3/5*B*a^3*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*
a^3*A*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)-3*B*a^3*c^2*d*(2+sin(f*x+e)^2)*cos(
f*x+e)-3*a^3*A*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)-9/5*B*a^3*c*d^2*(8/3+sin(f
*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3/5*a^3*A*c*d^2*(8/3+sin(f*x+e)^4+4/3*
sin(f*x+e)^2)*cos(f*x+e)-3/5*B*a^3*c^2*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2
)*cos(f*x+e)+3*a^3*A*c^3*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^3*A*c
^3*(f*x+e)+3*a^3*A*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)+3*a^3*c^3*B*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3*B*a^3*d^
3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x
+5/16*e)+a^3*A*d^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*co
s(f*x+e)+5/16*f*x+5/16*e)-a^3*c^3*B*cos(f*x+e)+B*a^3*d^3*(-1/4*(sin(f*x+e)^
3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*A*c^3*cos(f*x+e)+a^3*c^3*
B*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*A*d
^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^3*c^3*B*(2+sin(f*x+e)^2
)*cos(f*x+e)-1/3*a^3*A*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c^2*d*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A]
time = 0.32, size = 1136, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^3 + 5040*(2*f*x + 2*
e - sin(2*f*x + 2*e))*A*a^3*c^3 + 6720*(f*x + e)*A*a^3*c^3 + 6720*(cos(f*x
+ e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 210*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a
^3*c^3 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2*d + 630*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2*d + 15120*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^3*c^2*d - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a^3*c^2*d + 20160*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x +
2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2*d - 134
4*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c*d^2 + 20
160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*B*a^3*c*d^2 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*
c*d^2 + 105*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
 + 15*cos(f*x + e))*A*a^3*d^3 + 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^
3*d^3 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*d^3 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin
(2*f*x + 2*e))*A*a^3*d^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*c
os(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*d^3 - 1344*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^3 + 105*(4*sin(2*f*x + 2*e)^3 + 60*
f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^3 + 210*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^3 - 20160*A*a^3
*c^3*cos(f*x + e) - 6720*B*a^3*c^3*cos(f*x + e) - 20160*A*a^3*c^2*d*cos(f*x
 + e))/f
```

<u>Fricas</u> [**A**]
time = 0.44, size = 440, normalized size = 0.73

$$\frac{240 Ba^3d^3\cos(fx+e)^7 - 1008(Ba^3c^2d + (A+3B)a^3cd^2 + (A+2B)a^3d^3)\cos(fx+e)^5 + 560((A+3B)a^3c^3 + 3(3A+5B)a^3c^2d + 3(5A+7B)a^3cd^2 + (7A+9B)a^3d^3)\cos(fx+e)^3 + 105(10(4A+3B)a^3c^3 + 6(15A+13B)a^3c^2d + 3(26A+23B)a^3cd^2 + (23A+21B)a^3d^3)fx - 6720((A+B)a^3c^3 + 3(A+B)a^3c^2d + 3(A+B)a^3cd^2 + (A+B)a^3d^3)\cos(fx+e) - 35(8(3Ba^3cd^2 + (A+3B)a^3d^3)\cos(fx+e)^5 - 2(6Ba^3c^3 + 18(A+3B)a^3c^2d + 3(18A+31B)a^3cd^2 + (31A+45B)a^3d^3)\cos(fx+e)^3 + 3(2(12A+17B)a^3c^3 + 6(17A+19B)a^3c^2d + 3(38A+41B)a^3cd^2 + (41A+43B)a^3d^3)\cos(fx+e))\sin(fx+e)}{1680f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c*
d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A +
 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x +
e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*A +
23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 + 3*(A
```

```
 + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e) - 35*
(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c^3 + 18
*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^3*d^3)*c
os(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c^2*d + 3*
(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*sin(f*x + e)
)/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. $2(598) = 1196$.
time = 0.96, size = 2878, normalized size = 4.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x)
**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9
*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x*sin(
e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d*x*cos
(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*A*
a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*
A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f + 27*A*a**
3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d**2*x*cos(e +
 f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*d**2*sin(e +
f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*a**3*c*d**2*sin
(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*c*d**2*
cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f + 5*A*a**3*d**3*x
*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 +
 9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*x*sin(e + f*x)**2*cos(e
 + f*x)**4/16 + 9*A*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*A*a**
3*d**3*x*cos(e + f*x)**6/16 + 9*A*a**3*d**3*x*cos(e + f*x)**4/8 - 11*A*a**3
*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*A*a**3*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 5*A*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*
a**3*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**3*sin(e + f*x)**
2*cos(e + f*x)**3/f - A*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**3
*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*A*a**3*d**3*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 8*A*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*A*a**3*d**3*cos
(e + f*x)**3/(3*f) + 3*B*a**3*c**3*x*sin(e + f*x)**4/8 + 3*B*a**3*c**3*x*si
```

```
n(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**3*x*sin(e + f*x)**2/2 + 3*B*a
**3*c**3*x*cos(e + f*x)**4/8 + 3*B*a**3*c**3*x*cos(e + f*x)**2/2 - 5*B*a**3
*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3*c**3*sin(e + f*x)**2*co
s(e + f*x)/f - 3*B*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*
c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c**3*cos(e + f*x)**3/f - B*
a**3*c**3*cos(e + f*x)/f + 27*B*a**3*c**2*d*x*sin(e + f*x)**4/8 + 27*B*a**3
*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**2*d*x*sin(e + f*x
)**2/2 + 27*B*a**3*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**3*c**2*d*x*cos(e + f
*x)**2/2 - 3*B*a**3*c**2*d*sin(e + f*x)**4*cos(e + f*x)/f - 45*B*a**3*c**2*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a**3*c**2*d*sin(e + f*x)**2*cos(
e + f*x)**3/f - 9*B*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 27*B*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*d*sin(e + f*x)*co
s(e + f*x)/(2*f) - 8*B*a**3*c**2*d*cos(e + f*x)**5/(5*f) - 6*B*a**3*c**2*d*
cos(e + f*x)**3/f + 15*B*a**3*c*d**2*x*sin(e + f*x)**6/16 + 45*B*a**3*c*d**
2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 27*B*a**3*c*d**2*x*sin(e + f*x)**4
/8 + 45*B*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 27*B*a**3*c*d*
*2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*c*d**2*x*cos(e + f*x)**6
/16 + 27*B*a**3*c*d**2*x*cos(e + f*x)**4/8 - 33*B*a**3*c*d**2*sin(e + f*x)*
*5*cos(e + f*x)/(16*f) - 9*B*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5
*B*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 45*B*a**3*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 12*B*a**3*c*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/f - 3*B*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 15*B*a**3*c*d*
*2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 27*B*a**3*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 24*B*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*a**3*c*d**
2*cos(e + f*x)**3/f + 15*B*a**3*d**3*x*sin(e + f*x)**6/16 + 45*B*a**3*d**3*
x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**3*d**3*x*sin(e + f*x)**4/8 +
45*B*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**3*d**3*x*sin(e
 + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*d**3*x*cos(e + f*x)**6/16 + 3*B*a*
*3*d**3*x*cos(e + f*x)**4/8 - B*a**3*d**3*sin(e + f*x)**6*cos(e + f*x)/f -
33*B*a**3*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*B*a**3*d**3*sin(e +
f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*B*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 5*B*a**3*d**3*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) - 8*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) - 4*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 15*B*a**3*d**3*s
in(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**3*d**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 16*B*a**3*d**3*cos(e + f*x)**7/(35*f) - 8*B*a**3*d**3*cos(e +
f*x)**5/(5*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a
)**3, True))
```

Giac [A]
time = 0.82, size = 566, normalized size = 0.94

$$\frac{Ba^3d^3\cos(7fx+7e)}{448f}+\frac{1}{16}(40Aa^3c^3+30Ba^3c^3+90Aa^3c^2d+78Ba^3c^2d+78Aa^3cd^2+69Ba^3cd^2+23Aa^3d^3+21Ba^3d^3)x-\frac{(12Ba^3c^2d+12Aa^3cd^2+36Ba^3cd^2+12Aa^3d^3+19Ba^3d^3)\cos(5fx+5e)}{320f}+\frac{(16Aa^3c^3+48Ba^3c^3+144Aa^3c^2d+204Ba^3c^2d+204Aa^3cd^2+228Ba^3cd^2+76Aa^3d^3+81Ba^3d^3)\cos(3fx+3e)}{192f}-\frac{(240Aa^3c^3+208Ba^3c^3+624Aa^3c^2d+552Ba^3c^2d+552Aa^3cd^2+504Ba^3cd^2+168Aa^3d^3+155Ba^3d^3)\cos(fx+e)}{64f}-\frac{(3Ba^3cd^2+Aa^3d^3+3Ba^3d^3)\sin(6fx+6e)}{192f}+\frac{(2Ba^3c^3+6Aa^3c^2d+18Ba^3c^2d+18Aa^3cd^2+27Ba^3cd^2+9Aa^3d^3+11Ba^3d^3)\sin(4fx+4e)}{64f}-\frac{(48Aa^3c^3+64Ba^3c^3+192Aa^3c^2d+192Ba^3c^2d+192Aa^3cd^2+189Ba^3cd^2+63Aa^3d^3+61Ba^3d^3)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 90
*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^3*
d^3 + 21*B*a^3*d^3)*x - 1/320*(12*B*a^3*c^2*d + 12*A*a^3*c*d^2 + 36*B*a^3*c
*d^2 + 12*A*a^3*d^3 + 19*B*a^3*d^3)*cos(5*f*x + 5*e)/f + 1/192*(16*A*a^3*c^
3 + 48*B*a^3*c^3 + 144*A*a^3*c^2*d + 204*B*a^3*c^2*d + 204*A*a^3*c*d^2 + 22
8*B*a^3*c*d^2 + 76*A*a^3*d^3 + 81*B*a^3*d^3)*cos(3*f*x + 3*e)/f - 1/64*(240
*A*a^3*c^3 + 208*B*a^3*c^3 + 624*A*a^3*c^2*d + 552*B*a^3*c^2*d + 552*A*a^3*
c*d^2 + 504*B*a^3*c*d^2 + 168*A*a^3*d^3 + 155*B*a^3*d^3)*cos(f*x + e)/f - 1
/192*(3*B*a^3*c*d^2 + A*a^3*d^3 + 3*B*a^3*d^3)*sin(6*f*x + 6*e)/f + 1/64*(2
*B*a^3*c^3 + 6*A*a^3*c^2*d + 18*B*a^3*c^2*d + 18*A*a^3*c*d^2 + 27*B*a^3*c*d
^2 + 9*A*a^3*d^3 + 11*B*a^3*d^3)*sin(4*f*x + 4*e)/f - 1/64*(48*A*a^3*c^3 +
64*B*a^3*c^3 + 192*A*a^3*c^2*d + 192*B*a^3*c^2*d + 192*A*a^3*c*d^2 + 189*B*
a^3*c*d^2 + 63*A*a^3*d^3 + 61*B*a^3*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 16.18, size = 1395, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d^
3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 + 78*B*c^2*d))/(8*(5*A*a^3*c^3 + (
23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4
+ (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4)))*(40*A*c^3
 + 23*A*d^3 + 30*B*c^3 + 21*B*d^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 +
78*B*c^2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3*c^3 + (23*A*a^3*d^3)/8 +
(15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (45*A*a^3*c^2*d)
/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^10*(40*A
*a^3*c^3 + 4*A*a^3*d^3 + 24*B*a^3*c^3 + 36*A*a^3*c*d^2 + 72*A*a^3*c^2*d + 1
2*B*a^3*c*d^2 + 36*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^13*(3*A*a^3*c^3 + (23*
A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (
45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f
*x)/2)^3*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 + (35*B*a^3*d^3)/
2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 + 57*B*a^3*c^2*d)
 - tan(e/2 + (f*x)/2)^11*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 +
 (35*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 +
 57*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^8*((322*A*a^3*c^3)/3 + (148*A*a^3*d^3
)/3 + 82*B*a^3*c^3 + 32*B*a^3*d^3 + 188*A*a^3*c*d^2 + 246*A*a^3*c^2*d + 148
*B*a^3*c*d^2 + 188*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*((448*A*a^3*c^3)/3 +
 (328*A*a^3*d^3)/3 + 128*B*a^3*c^3 + 112*B*a^3*d^3 + 344*A*a^3*c*d^2 + 384*
A*a^3*c^2*d + 328*B*a^3*c*d^2 + 344*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^2*((1
```

36*A*a^3*c^3)/3 + (476*A*a^3*d^3)/15 + 40*B*a^3*c^3 + (144*B*a^3*d^3)/5 + (532*A*a^3*c*d^2)/5 + 120*A*a^3*c^2*d + (476*B*a^3*c*d^2)/5 + (532*B*a^3*c^2*d)/5) + tan(e/2 + (f*x)/2)^5*(15*A*a^3*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 + (841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*(15*A*a^3*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 + (841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^4*(114*A*a^3*c^3 + (456*A*a^3*d^3)/5 + 102*B*a^3*c^3 + (432*B*a^3*d^3)/5 + (1416*A*a^3*c*d^2)/5 + 306*A*a^3*c^2*d + (1368*B*a^3*c*d^2)/5 + (1416*B*a^3*c^2*d)/5) + tan(e/2 + (f*x)/2)^12*(6*A*a^3*c^3 + 2*B*a^3*c^3 + 6*A*a^3*c^2*d) + (22*A*a^3*c^3)/3 + (68*A*a^3*d^3)/15 + 6*B*a^3*c^3 + (144*B*a^3*d^3)/35 + (76*A*a^3*c*d^2)/5 + 18*A*a^3*c^2*d + (68*B*a^3*c*d^2)/5 + (76*B*a^3*c^2*d)/5)/(f*(7*tan(e/2 + (f*x)/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 + 1))

3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=463

$$\frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60cd+26d^2\right)\right)x-\frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)}{}$$

```
[Out] 1/16*a^3*(B*(30*c^2+52*c*d+23*d^2)+A*(40*c^2+60*c*d+26*d^2))*x-1/60*a^3*(2*
A*d*(2*c^4-15*c^3*d+72*c^2*d^2+180*c*d^3+76*d^4)-B*(2*c^5-12*c^4*d+37*c^3*d
^2-112*c^2*d^3-304*c*d^4-136*d^5))*cos(f*x+e)/d^3/f-1/240*a^3*(2*A*d*(4*c^3
-30*c^2*d+146*c*d^2+195*d^3)-B*(4*c^4-24*c^3*d+76*c^2*d^2-236*c*d^3-345*d^4
))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/120*a^3*(2*A*d*(2*c^2-15*c*d+76*d^2)-B*(2*
c^3-12*c^2*d+41*c*d^2-136*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^3/f+1/40*a^
3*(2*A*(2*c-11*d)*d-B*(2*c^2-8*c*d+21*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d
^3/f-1/6*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3/d/f+1/30*(-6*
A*d+3*B*c-8*B*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^3/d^2/f
```

Rubi [A]
time = 0.76, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3055, 3047, 3102, 2832, 2813}

$$\frac{1}{16}a^3x\left(A\left(40c^2+60cd+26d^2\right)+B\left(30c^2+52cd+23d^2\right)\right)+\frac{a^3(2Ad(2c-11d)-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f}-\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^3f}-\frac{a^3(2Ad(4c^3-30c^2d+146cd^2+195d^3)-B(4c^4-24c^3d+76c^2d^2-236cd^3-345d^4))\sin(e+fx)\cos(e+fx)}{240d^2f}-\frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)-B(2c^5-12c^4d+37c^3d^2-112c^2d^3-304cd^4-136d^5))\cos(e+fx)}{60d^3f}+\frac{(-6Ad+3Bc-8Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^3}{30d^2f}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/16 -
(a^3*(2*A*d*(2*c^4 - 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5
 - 12*c^4*d + 37*c^3*d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x]
)/(60*d^3*f) - (a^3*(2*A*d*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*
c^4 - 24*c^3*d + 76*c^2*d^2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*
x])/(240*d^2*f) - (a^3*(2*A*d*(2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2
*d + 41*c*d^2 - 136*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^3*f)
+ (a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*d + 21*d^2))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^3)/(40*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c
 + d*Sin[e + f*x])^3)/(6*d*f) + ((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3
+ a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(30*d^2*f)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
```

```
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c}{6df} \\
&= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c}{6df} \\
&= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c}{6df} \\
&= \frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d}{40d^3f} \\
&= -\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-}{} \\
&= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2\right.\right.
\end{aligned}
$$

Mathematica [A]
time = 1.61, size = 355, normalized size = 0.77

$$
-\frac{a^3\cos(e+fx)\left(60\left(B(30c^2+52cd+23d^2)+A(40c^2+60cd+26d^2)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(1840Ac^2+1680Bc^2+3360Acd+3112Bcd+1556Ad^2+1468Bd^2-16(A(5c^2+30cd+22d^2)+B(15c^2+44cd+26d^2))\cos(2(e+fx))+12d(2Bc+Ad+3Bd)\cos(4(e+fx))+720Ac^2\sin(e+fx)+990Bc^2\sin(e+fx)+1980Acd\sin(e+fx)+2100Bcd\sin(e+fx)+1050Ad^2\sin(e+fx)+1085Bd^2\sin(e+fx)-30Bc^2\sin(3(e+fx))-60Acd\sin(3(e+fx))-180Bcd\sin(3(e+fx))-90Ad^2\sin(3(e+fx))-140Bd^2\sin(3(e+fx))+5Bd^2\sin(5(e+fx))\right)\right)}{480f\sqrt{\cos^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -1/480*(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60
*c*d + 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^
2]*(1840*A*c^2 + 1680*B*c^2 + 3360*A*c*d + 3112*B*c*d + 1556*A*d^2 + 1468*B
*d^2 - 16*(A*(5*c^2 + 30*c*d + 22*d^2) + B*(15*c^2 + 44*c*d + 26*d^2))*Cos[
2*(e + f*x)] + 12*d*(2*B*c + A*d + 3*B*d)*Cos[4*(e + f*x)] + 720*A*c^2*Sin[
e + f*x] + 990*B*c^2*Sin[e + f*x] + 1980*A*c*d*Sin[e + f*x] + 2100*B*c*d*Si
n[e + f*x] + 1050*A*d^2*Sin[e + f*x] + 1085*B*d^2*Sin[e + f*x] - 30*B*c^2*S
in[3*(e + f*x)] - 60*A*c*d*Sin[3*(e + f*x)] - 180*B*c*d*Sin[3*(e + f*x)] -
90*A*d^2*Sin[3*(e + f*x)] - 140*B*d^2*Sin[3*(e + f*x)] + 5*B*d^2*Sin[5*(e +
 f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A]
time = 0.36, size = 725, normalized size = 1.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(a^3*A*c^2*(f*x+e)-2*a^3*A*c*d*cos(f*x+e)+a^3*A*d^2*(-1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^2*cos(f*x+e)+2*B*a^3*c*d*(-1/2*cos(f*x+e)*s
```

```
in(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*
c^2*cos(f*x+e)+6*a^3*A*c*d*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-a^3*A
*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/
2*f*x+1/2*e)-2*B*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*d^2*(-1/4*(sin
(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*A*c^2*(-1/2*cos(f
*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2*a^3*A*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a
^3*A*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-B*a^
3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^3*c*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^3*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*
x+e)^2)*cos(f*x+e)-1/3*a^3*A*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c*d*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*A*d^2*(
8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^2*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^3*c*d*(8/3+sin(f*x+e)^4+
4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3
+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [A]
time = 0.30, size = 758, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e)^
3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2
+ 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(
f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c*d + 18
0*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d + 480*(
2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5 - 10*cos(f
*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x +
e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 192*(3*cos(f*x
 + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 320*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e +
 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*a^3*c^2*cos(f*x +
 e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x + e))/f
```

Fricas [A]

time = 0.42, size = 306, normalized size = 0.66

$$-\frac{48\,(2\,Ba^3cd+(A+3\,B)a^3d^2)\cos(fx+e)^5-80\,((A+3\,B)a^3c^2+2\,(3\,A+5\,B)a^3cd+(5\,A+7\,B)a^3d^2)\cos(fx+e)^3-15\,(10\,(4\,A+3\,B)a^3c^2+4\,(15\,A+13\,B)a^3cd+(26\,A+23\,B)a^3d^2)fx+960\,((A+B)a^3c^2+2\,(A+B)a^3cd+(A+B)a^3d^2)\cos(fx+e)+5\,(8\,Ba^3d^2\cos(fx+e)^5-2\,(6\,Ba^3c^2+12\,(A+3\,B)a^3cd+(18\,A+31\,B)a^3d^2)\cos(fx+e)^3+3\,(2\,(12\,A+17\,B)a^3c^2+4\,(17\,A+19\,B)a^3cd+(38\,A+41\,B)a^3d^2)\cos(fx+e))\sin(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B)
*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 - 15
*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d^2)
*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(f*x
+ e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a^3*c*
d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^2 + 4*
(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/
f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. 2(449) = 898.
time = 0.65, size = 1804, normalized size = 3.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x)
**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 + 3*A*
a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*sin(e + f*x)*
*2 + 3*A*a**3*c*d*x*cos(e + f*x)**4/4 + 3*A*a**3*c*d*x*cos(e + f*x)**2 - 5*
A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e + f*x)**
2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*A*a*
*3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/f - 2*A*a
**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*sin(e + f*x)**2/2 + 9*A
*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/2 - A*a**3*d
**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*A*a**
3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*d**2
*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f + 3*B*a**3*c**2*x
*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*
B*a**3*c**2*x*sin(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e + f*x)**4/8 + 3*B*a
```

```
**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 3*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f)
 - 2*B*a**3*c**2*cos(e + f*x)**3/f - B*a**3*c**2*cos(e + f*x)/f + 9*B*a**3*
c*d*x*sin(e + f*x)**4/4 + 9*B*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2
+ B*a**3*c*d*x*sin(e + f*x)**2 + 9*B*a**3*c*d*x*cos(e + f*x)**4/4 + B*a**3*
c*d*x*cos(e + f*x)**2 - 2*B*a**3*c*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*
a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 8*B*a**3*c*d*sin(e + f*x)**2*
cos(e + f*x)**3/(3*f) - 6*B*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a
**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*d*sin(e + f*x)*cos(e
+ f*x)/f - 16*B*a**3*c*d*cos(e + f*x)**5/(15*f) - 4*B*a**3*c*d*cos(e + f*x)
**3/f + 5*B*a**3*d**2*x*sin(e + f*x)**6/16 + 15*B*a**3*d**2*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*B*a**3*d**2*x*sin(e + f*x)**4/8 + 15*B*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*d**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 5*B*a**3*d**2*x*cos(e + f*x)**6/16 + 9*B*a**3*d**2*x*cos(e
+ f*x)**4/8 - 11*B*a**3*d**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*B*a**3
*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)**3*cos(e
+ f*x)**3/(6*f) - 15*B*a**3*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a
**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - B*a**3*d**2*sin(e + f*x)**2*co
s(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*B*a**3
*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a**3*d**2*cos(e + f*x)**5/(5
*f) - 2*B*a**3*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c
+ d*sin(e))**2*(a*sin(e) + a)**3, True))
```

Giac [A]
time = 0.65, size = 380, normalized size = 0.82

$$-\frac{Ba^3d^2\sin(6fx+6e)}{192f}+\frac{1}{16}\left(40Aa^3c^2+30Ba^3c^2+60Aa^3cd+52Ba^3cd+26Aa^3d^2+23Ba^3d^2\right)x-\frac{(2Ba^3cd+Aa^3d^2+3Ba^3d^2)\cos(5fx+5e)}{80f}+\frac{(4Aa^3c^2+12Ba^3c^2+24Aa^3cd+34Ba^3cd+17Aa^3d^2+19Ba^3d^2)\cos(3fx+3e)}{48f}-\frac{(30Aa^3c^2+26Ba^3c^2+52Aa^3cd+46Ba^3cd+23Aa^3d^2+21Ba^3d^2)\cos(fx+e)}{8f}+\frac{(2Ba^3c^2+4Aa^3cd+12Ba^3cd+6Aa^3d^2+9Ba^3d^2)\sin(4fx+4e)}{64f}-\frac{(48Aa^3c^2+64Ba^3c^2+128Aa^3cd+128Ba^3cd+64Aa^3d^2+63Ba^3d^2)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 + 6
0*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a^3
*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 + 12
*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)*cos
(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46*B*a^
3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c^2 + 4
*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4*e)/f -
 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d + 64*A*a
^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.63, size = 976, normalized size = 2.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^
2 + 60*A*c*d + 52*B*c*d))/(8*(5*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^
2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2)))*(40*A*c^2
+ 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*f) - (tan(e/2 +
 (f*x)/2)^10*(6*A*a^3*c^2 + 2*B*a^3*c^2 + 4*A*a^3*c*d) + tan(e/2 + (f*x)/2)
*(3*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (1
5*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^11*(3*A*a^3*c^2 + (
13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 +
(13*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^8*(34*A*a^3*c^2 + 12*A*a^3*d^2 + 22*
B*a^3*c^2 + 4*B*a^3*d^2 + 44*A*a^3*c*d + 24*B*a^3*c*d) + tan(e/2 + (f*x)/2)
^5*(6*A*a^3*c^2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 +
19*A*a^3*c*d + 25*B*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*A*a^3*c^2 + (25*A*a^
3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 + 19*A*a^3*c*d + 25*B*a^3*c*
d) + tan(e/2 + (f*x)/2)^4*(76*A*a^3*c^2 + 64*A*a^3*d^2 + 68*B*a^3*c^2 + 64*
B*a^3*d^2 + 136*A*a^3*c*d + 128*B*a^3*c*d) + tan(e/2 + (f*x)/2)^3*(9*A*a^3*
c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^3*
c*d)/2 + (63*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^9*(9*A*a^3*c^2 + (63*A*a^3*
d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^3*c*d)/2 + (63*B*a
^3*c*d)/2) + tan(e/2 + (f*x)/2)^2*(38*A*a^3*c^2 + (152*A*a^3*d^2)/5 + 34*B*
a^3*c^2 + (136*B*a^3*d^2)/5 + 68*A*a^3*c*d + (304*B*a^3*c*d)/5) + tan(e/2 +
 (f*x)/2)^6*((220*A*a^3*c^2)/3 + (152*A*a^3*d^2)/3 + 60*B*a^3*c^2 + (136*B*
a^3*d^2)/3 + 120*A*a^3*c*d + (304*B*a^3*c*d)/3) + (22*A*a^3*c^2)/3 + (76*A*
a^3*d^2)/15 + 6*B*a^3*c^2 + (68*B*a^3*d^2)/15 + 12*A*a^3*c*d + (152*B*a^3*c
*d)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 +
 (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
 (f*x)/2)^12 + 1))
```

3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=201

$$\frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x-\frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{5f}+\frac{a^3(20Ac+15Bc+15A}{6}$$

```
[Out] 1/8*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*x-1/5*a^3*(20*A*c+15*A*d+15*B*c+13*B*
d)*cos(f*x+e)/f+1/60*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)^3/f-3/40*
a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/20*(5*A*d+5*B*c
-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^3/f-1/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^4
/a/f
```

Rubi [A]
time = 0.23, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3047, 3102, 2830, 2724, 2718, 2715, 8, 2713}

$$\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos^3(e+fx)}{60f}-\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{5f}-\frac{3a^3(20Ac+15Ad+15Bc+13Bd)\sin(e+fx)\cos(e+fx)}{40f}+\frac{1}{8}a^3x(20Ac+15Ad+15Bc+13Bd)-\frac{(5Ad+5Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^3}{20f}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*x)/8 - (a^3*(20*A*c + 15*B*c + 15*
A*d + 13*B*d)*Cos[e + f*x])/(5*f) + (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d
)*Cos[e + f*x]^3)/(60*f) - (3*a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e
 + f*x]*Sin[e + f*x])/(40*f) - ((5*B*c + 5*A*d - B*d)*Cos[e + f*x]*(a + a*S
in[e + f*x])^3)/(20*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^3 \left(Ac+(Bc+Ad\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} +$$

$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a}{20f}$$

$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a}{20f}$$

$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x -$$

$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x -$$

$$= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x -$$

Mathematica [A]
time = 0.79, size = 156, normalized size = 0.78

$$\frac{\cos(e+fx)\left(-\frac{1}{4}a^4(5Bc+5Ad-Bd)(1+\sin(e+fx))^3-Bd(a+a\sin(e+fx))^4-\frac{a^4(20Ac+15Bc+15Ad+13Bd)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(22+9\sin(e+fx)+2\sin^2(e+fx))\right)}{24\sqrt{\cos^2(e+fx)}}\right)}{5af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] (Cos[e + f*x]*(-1/4*(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3) - B*d*
(a + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*ArcSi
n[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(22 + 9*Sin[e + f*
x] + 2*Sin[e + f*x]^2)))/(24*Sqrt[Cos[e + f*x]^2])))/(5*a*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(189) = 378$.
time = 0.26, size = 414, normalized size = 2.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/f*(a^3*A*c*(f*x+e)-a^3*A*d*cos(f*x+e)-B*a^3*c*cos(f*x+e)+B*a^3*d*(-1/2*co
s(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3*a^3*A*c*cos(f*x+e)+3*a^3*A*d*(-1/2*cos
(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3*B*a^3*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
```

```
*f*x+1/2*e)-B*a^3*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c*(-1/2*cos(f*x+e)*
sin(f*x+e)+1/2*f*x+1/2*e)-a^3*A*d*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c*(2+si
n(f*x+e)^2)*cos(f*x+e)+3*B*a^3*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*
x+e)+3/8*f*x+3/8*e)-1/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*A*d*(-1/4*(
sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*a^3*c*(-1/4*(sin(f
*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^3*d*(8/3+sin(f*x+
e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(197) = 394$.
time = 0.32, size = 430, normalized size = 2.14

$$\frac{160 \left(\cos \left(fx+e\right)^3-3 \cos \left(fx+e\right)\right) A a^3 c+360 \left(2 fx+2 e-\sin \left(2 fx+2 e\right)\right) A a^3 c+480 \left(fx+e\right) A a^3 c+480 \left(\cos \left(fx+e\right)^3-3 \cos \left(fx+e\right)\right) B a^3 c+15 \left(12 fx+12 e+\sin \left(4 fx+4 e\right)-8 \sin \left(2 fx+2 e\right)\right) B a^3 c+360 \left(2 fx+2 e-\sin \left(2 fx+2 e\right)\right) B a^3 c+480 \left(\cos \left(fx+e\right)^3-3 \cos \left(fx+e\right)\right) A a^3 d+15 \left(12 fx+12 e+\sin \left(4 fx+4 e\right)-8 \sin \left(2 fx+2 e\right)\right) A a^3 d+360 \left(2 fx+2 e-\sin \left(2 fx+2 e\right)\right) A a^3 d-32 \left(3 \cos \left(fx+e\right)^5-10 \cos \left(fx+e\right)^3+15 \cos \left(fx+e\right)\right) B a^3 d+480 \left(\cos \left(fx+e\right)^3-3 \cos \left(fx+e\right)\right) B a^3 d+45 \left(12 fx+12 e+\sin \left(4 fx+4 e\right)-8 \sin \left(2 fx+2 e\right)\right) B a^3 d+120 \left(2 fx+2 e-\sin \left(2 fx+2 e\right)\right) B a^3 d-1440 A a^3 c \cos \left(fx+e\right)-480 B a^3 c \cos \left(fx+e\right)-480 A a^3 d \cos \left(fx+e\right)}{480 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*A*a^3*c + 480*(f*x + e)*A*a^3*c + 480*(cos(f*x + e)^3 - 3*
cos(f*x + e))*B*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*B*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 480*(cos(f
*x + e)^3 - 3*cos(f*x + e))*A*a^3*d + 15*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*A*a^3*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*
d - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d + 4
80*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d + 120*(2*f*x + 2*e - sin(2*f*x + 2*e
))*B*a^3*d - 1440*A*a^3*c*cos(f*x + e) - 480*B*a^3*c*cos(f*x + e) - 480*A*a
^3*d*cos(f*x + e))/f
```

Fricas [**A**]
time = 0.40, size = 184, normalized size = 0.92

$$-\frac{24 B a^3 d \cos \left(fx+e\right)^5-40 \left(\left(A+3 B\right) a^3 c+\left(3 A+5 B\right) a^3 d\right) \cos \left(fx+e\right)^3-15 \left(5 \left(4 A+3 B\right) a^3 c+\left(15 A+13 B\right) a^3 d\right) fx+480 \left(\left(A+B\right) a^3 c+\left(A+B\right) a^3 d\right) \cos \left(fx+e\right)-15 \left(2 \left(B a^3 c+\left(A+3 B\right) a^3 d\right) \cos \left(fx+e\right)^3-\left(\left(12 A+17 B\right) a^3 c+\left(17 A+19 B\right) a^3 d\right) \cos \left(fx+e\right)\right) \sin \left(fx+e\right)}{120 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a^3*d*cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*d
)*cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 480
*((A + B)*a^3*c + (A + B)*a^3*d)*cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*B)*
a^3*d)*cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d)*cos(f*x
 + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(201) = 402$.
time = 0.41, size = 960, normalized size = 4.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*c*x*sin(e + f*x)**2/2 + 3*A*a**3*c*x*cos(e + f*x)**2/2
+ A*a**3*c*x - A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*sin(e +
 f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c*cos(e + f*x)**3/(3*f) - 3*A*a**3*c*co
s(e + f*x)/f + 3*A*a**3*d*x*sin(e + f*x)**4/8 + 3*A*a**3*d*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*A*a**3*d*x*sin(e + f*x)**2/2 + 3*A*a**3*d*x*cos(e +
 f*x)**4/8 + 3*A*a**3*d*x*cos(e + f*x)**2/2 - 5*A*a**3*d*sin(e + f*x)**3*co
s(e + f*x)/(8*f) - 3*A*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*d*sin(e + f*x)*cos(e + f*x)/(2
*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B*a**3*c*x
*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a
**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B*a**3*c*x*c
os(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c*cos(e + f*
x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x)**4/8 + 9*B*a*
*3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e + f*x)**2/2 + 9
*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2/2 - B*a**3*d*sin
(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a**3*d*sin(e + f
*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a
**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*d*cos(e + f*x)**5/(15*f) -
 2*B*a**3*d*cos(e + f*x)**3/f, Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*
(a*sin(e) + a)**3, True))
```

Giac [A]
time = 0.76, size = 217, normalized size = 1.08

$$-\frac{Ba^3d\cos(5\,fx+5\,e)}{80\,f}+\frac{1}{8}\,(20\,Aa^3c+15\,Ba^3c+15\,Aa^3d+13\,Ba^3d)x+\frac{(4\,Aa^3c+12\,Ba^3c+12\,Aa^3d+17\,Ba^3d)\cos(3\,fx+3\,e)}{48\,f}-\frac{(30\,Aa^3c+26\,Ba^3c+26\,Aa^3d+23\,Ba^3d)\cos(fx+e)}{8\,f}+\frac{(Ba^3c+Aa^3d+3\,Ba^3d)\sin(4\,fx+4\,e)}{32\,f}-\frac{(3\,Aa^3c+4\,Ba^3c+4\,Aa^3d+4\,Ba^3d)\sin(2\,fx+2\,e)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/80*B*a^3*d*cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3*
d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*d)
*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a^3*
```

```
d)*cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d)*sin(4*f*x + 4*e)/f
 - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.62, size = 550, normalized size = 2.74

$$\frac{a^3\,\mathrm{atan}\left(\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(20\,A\,c+15\,A\,d+15\,B\,c+13\,B\,d)}{4\,\left(5\,A\,a^3\,c+\frac{15\,A\,a^3\,d}{4}+\frac{15\,B\,a^3\,c}{4}+\frac{13\,B\,a^3\,d}{4}\right)}\right)\,(20\,A\,c+15\,A\,d+15\,B\,c+13\,B\,d)}{4\,f}-\frac{a^3\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\,(20\,A\,c+15\,A\,d+15\,B\,c+13\,B\,d)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(6\,A\,a^3\,c+\frac{19\,A\,a^3\,d}{2}+\frac{19\,B\,a^3\,c}{2}+\frac{25\,B\,a^3\,d}{2}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\,\left(3\,A\,a^3\,c+\frac{15\,A\,a^3\,d}{4}+\frac{15\,B\,a^3\,c}{4}+\frac{13\,B\,a^3\,d}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(6\,A\,a^3\,c+\frac{19\,A\,a^3\,d}{2}+\frac{19\,B\,a^3\,c}{2}+\frac{25\,B\,a^3\,d}{2}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(28\,A\,a^3\,c+20\,A\,a^3\,d+20\,B\,a^3\,c+12\,B\,a^3\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{92\,A\,a^3\,c}{3}+28\,A\,a^3\,d+28\,B\,a^3\,c+\frac{76\,B\,a^3\,d}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{136\,A\,a^3\,c}{3}+40\,A\,a^3\,d+40\,B\,a^3\,c+\frac{116\,B\,a^3\,d}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,(6\,A\,a^3\,c+2\,A\,a^3\,d+2\,B\,a^3\,c)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,A\,a^3\,c+\frac{15\,A\,a^3\,d}{4}+\frac{15\,B\,a^3\,c}{4}+\frac{13\,B\,a^3\,d}{4}\right)+\frac{22\,A\,a^3\,c}{3}+6\,A\,a^3\,d+6\,B\,a^3\,c+\frac{76\,B\,a^3\,d}{15}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*(
5*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4)))*(20*A*c + 1
5*A*d + 15*B*c + 13*B*d))/(4*f) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(6*A*a^
3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/2) - tan(e/2 + (f*x)/2
)^9*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4) - tan(e/
2 + (f*x)/2)^7*(6*A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/
2) + tan(e/2 + (f*x)/2)^6*(28*A*a^3*c + 20*A*a^3*d + 20*B*a^3*c + 12*B*a^3*
d) + tan(e/2 + (f*x)/2)^2*((92*A*a^3*c)/3 + 28*A*a^3*d + 28*B*a^3*c + (76*B
*a^3*d)/3) + tan(e/2 + (f*x)/2)^4*((136*A*a^3*c)/3 + 40*A*a^3*d + 40*B*a^3*
c + (116*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^8*(6*A*a^3*c + 2*A*a^3*d + 2*B*a^
3*c) + tan(e/2 + (f*x)/2)*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (1
3*B*a^3*d)/4) + (22*A*a^3*c)/3 + 6*A*a^3*d + 6*B*a^3*c + (76*B*a^3*d)/15)/(
f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)
^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=127

$$\frac{5}{8}a^3(4A+3B)x-\frac{5a^3(4A+3B)\cos(e+fx)}{6f}-\frac{5a^3(4A+3B)\cos(e+fx)\sin(e+fx)}{24f}-\frac{a(4A+3B)\cos(e+}{1}$$

```
[Out] 5/8*a^3*(4*A+3*B)*x-5/6*a^3*(4*A+3*B)*cos(f*x+e)/f-5/24*a^3*(4*A+3*B)*cos(f
*x+e)*sin(f*x+e)/f-1/12*a*(4*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*c
os(f*x+e)*(a+a*sin(f*x+e))^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2830, 2724, 2718, 2715, 8, 2713}

$$\frac{a^3(4A+3B)\cos^3(e+fx)}{12f}-\frac{a^3(4A+3B)\cos(e+fx)}{f}-\frac{3a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4A+3B)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 - (a^3*(4*A + 3*B)*Cos[e + f*x])/f + (a^3*(4*A + 3*
B)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*A + 3*B)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int (a+\\ &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int \left(a^3+\right.\\ &= \frac{1}{4}a^3(4A+3B)x-\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(\\ &= \frac{1}{4}a^3(4A+3B)x-\frac{3a^3(4A+3B)\cos(e+fx)}{4f}-\frac{3a^3(4A+3}{}\\ &= \frac{5}{8}a^3(4A+3B)x-\frac{a^3(4A+3B)\cos(e+fx)}{f}+\frac{a^3(4A+3B}{1}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 120, normalized size = 0.94

$$-\frac{a^3\cos(e+fx)\left(30(4A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,(88A+72B+9(4A+5B)\sin(e+fx)+8(A+3B)\sin^2(e+fx)+6B\sin^3(e+fx))\right)}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/24*(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*(A
 + 3*B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [A]
time = 0.18, size = 178, normalized size = 1.40

method	result
risch	$\frac{5a^3xA}{2}+\frac{15a^3Bx}{8}-\frac{15a^3\cos(fx+e)A}{4f}-\frac{13a^3\cos(fx+e)B}{4f}+\frac{B\,a^3\sin(4fx+4e)}{32f}+\frac{a^3\cos(3fx+3e)A}{12f}+\frac{a^3\cos}{}$
derivativedivides	$\frac{a^3A(fx+e)-B\,a^3\cos(fx+e)-3a^3A\cos(fx+e)+3B\,a^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+3a^3A\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.}{}$
default	$\frac{a^3A(fx+e)-B\,a^3\cos(fx+e)-3a^3A\cos(fx+e)+3B\,a^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+3a^3A\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.}{}$
norman	$\frac{\left(\frac{5}{2}a^3A+\frac{15}{8}B\,a^3\right)x+\left(10a^3A+\frac{15}{2}B\,a^3\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(10a^3A+\frac{15}{2}B\,a^3\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(15a^3A+\frac{45}{4}B\,a^3\right)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^3*A*(f*x+e)-B*a^3*cos(f*x+e)-3*a^3*A*cos(f*x+e)+3*B*a^3*(-1/2*cos(f*
x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3*a^3*A*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+
1/2*e)-B*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*a^3*A*(2+sin(f*x+e)^2)*cos(f*x
+e)+B*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A]

time = 0.33, size = 185, normalized size = 1.46

$$\frac{32\,(\cos(fx+e)^3-3\cos(fx+e))Aa^3+72\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aa^3+96\,(fx+e)Aa^3+96\,(\cos(fx+e)^3-3\cos(fx+e))Ba^3+3\,(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\sin(2\,fx+2\,e))Ba^3+72\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Ba^3-288\,Aa^3\cos(fx+e)-96\,Ba^3\cos(fx+e)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3 + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3 + 96*(f*x + e)*A*a^3 + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*B*a^3 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3 - 288*A*a^3*cos(f*x + e) - 96*B
*a^3*cos(f*x + e))/f
```

Fricas [A]

time = 0.38, size = 98, normalized size = 0.77

$$\frac{8\,(A+3\,B)a^3\cos(fx+e)^3+15\,(4\,A+3\,B)a^3fx-96\,(A+B)a^3\cos(fx+e)+3\left(2\,Ba^3\cos(fx+e)^3-(12\,A+17\,B)a^3\cos(fx+e)\right)\sin(fx+e)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)*
a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e)^3 - (12*A + 17*B)*a^3*cos(f*x +
e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(119) = 238$.
time = 0.24, size = 371, normalized size = 2.92

$$\begin{cases} \frac{3Aa^3x\sin^2(e+fx)}{2} + \frac{3Aa^3x\cos^2(e+fx)}{2} + Aa^3x - \frac{Aa^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Aa^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^3\cos^3(e+fx)}{3f} - \frac{3Aa^3\cos(e+fx)}{f} + \frac{3Ba^3x\sin^4(e+fx)}{8} + \frac{3Ba^3x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3Ba^3x\sin^2(e+fx)}{2} + \frac{3Ba^3x\cos^4(e+fx)}{8} + \frac{3Ba^3x\cos^2(e+fx)}{2} - \frac{5Ba^3\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3Ba^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Ba^3\sin(e+fx)\cos^3(e+fx)}{8f} - \frac{3Ba^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Ba^3\cos^3(e+fx)}{f} - \frac{Ba^3\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A+B\sin(e))(a\sin(e)+a)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A*
a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/f +
 3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*a**3*
x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**
3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f*x)**3/f
 - B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3, T
rue))
```

Giac [A]
time = 0.57, size = 116, normalized size = 0.91

$$\frac{Ba^3\sin(4fx+4e)}{32f} + \frac{5}{8}\left(4Aa^3+3Ba^3\right)x + \frac{\left(Aa^3+3Ba^3\right)\cos(3fx+3e)}{12f} - \frac{\left(15Aa^3+13Ba^3\right)\cos(fx+e)}{4f} - \frac{\left(3Aa^3+4Ba^3\right)\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1/4
*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.26, size = 330, normalized size = 2.60

$$\frac{5a^3\,\mathrm{atan}\left(\frac{5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4A+3B)}{4\left(5Aa^3+\frac{15Ba^3}{4}\right)}\right)(4A+3B)}{4f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3Aa^3+\frac{15Ba^3}{4}\right) + \frac{22Aa^3}{3} + 6Ba^3 + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(6Aa^3+2Ba^3\right) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(3Aa^3+\frac{15Ba^3}{4}\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3Aa^3+\frac{23Ba^3}{4}\right) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(3Aa^3+\frac{23Ba^3}{4}\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(22Aa^3+18Ba^3\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{70Aa^3}{3}+22Ba^3\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)} - \frac{5a^3(4A+3B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*A + 3*B))/(4*(5*A*a^3 + (15*B*a^3)
/4)))*(4*A + 3*B))/(4*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3 + (15*B*a^3)/4) + (
22*A*a^3)/3 + 6*B*a^3 + tan(e/2 + (f*x)/2)^6*(6*A*a^3 + 2*B*a^3) - tan(e/2
+ (f*x)/2)^7*(3*A*a^3 + (15*B*a^3)/4) + tan(e/2 + (f*x)/2)^3*(3*A*a^3 + (23
*B*a^3)/4) - tan(e/2 + (f*x)/2)^5*(3*A*a^3 + (23*B*a^3)/4) + tan(e/2 + (f*x
)/2)^4*(22*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^2*((70*A*a^3)/3 + 22*B*a^
3))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)
/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (5*a^3*(4*A + 3*B)*(atan(tan(e/2 + (f*
x)/2)) - (f*x)/2))/(4*f)
```

3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4}+\frac{2a^3(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}\,f}+$$

```
[Out] 1/2*a^3*(A*d*(2*c^2-6*c*d+7*d^2)-B*(2*c^3-6*c^2*d+7*c*d^2-5*d^3))*x/d^4+1/2
*a^3*(A*(2*c-5*d)*d-B*(2*c^2-5*c*d+5*d^2))*cos(f*x+e)/d^3/f-1/3*a*B*cos(f*x
+e)*(a+a*sin(f*x+e))^2/d/f+1/6*(-3*A*d+3*B*c-5*B*d)*cos(f*x+e)*(a^3+a^3*sin
(f*x+e))/d^2/f+2*a^3*(c-d)^3*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^
2-d^2)^(1/2))/d^4/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3055, 3047, 3102, 2814, 2739, 632, 210}

$$\frac{2a^3(c-d)^3(Bc-Ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f\sqrt{c^2-d^2}}+\frac{a^3(Ad(2c-5d)-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f}+\frac{a^3x(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))}{2d^4}+\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{6d^2f}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(A*d*(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*
x)/(2*d^4) + (2*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(d^4*Sqrt[c^2 - d^2]*f) + (a^3*(A*(2*c - 5*d)*d - B*(2*c^2
 - 5*c*d + 5*d^2))*Cos[e + f*x])/(2*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^2)/(3*d*f) + ((3*B*c - 3*A*d - 5*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[
e + f*x]))/(6*d^2*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{\int \frac{(a+a\sin(e+fx))^2(a}{}}{}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5.}{}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5.}{}$$

$$= \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} - \frac{aB}{}$$

$$= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4}$$

$$= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4}$$

$$= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4}$$

$$= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4}$$

Mathematica [A]
time = 0.67, size = 233, normalized size = 0.95

$$\frac{a^3(1+\sin(e+fx))^3\left(6(Ad(2c^2-6cd+7d^2)+B(-2c^3+6c^2d-7cd^2+5d^3))(e+fx)+\frac{24(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-3d(4Ad(-c+3d)+B(4c^2-12cd+15d^2))\cos(e+fx)+Bd^3\cos(3(e+fx))-3d^2(-Bc+Ad+3Bd)\sin(2(e+fx))\right)}{12d^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*c
^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d +
c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c + 3*
d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)] - 3
*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)]))/(12*d^4*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6)
```

Maple [A]
time = 0.42, size = 378, normalized size = 1.54

method	result
derivativedivides	$2a^3\left(\frac{\left(-A\,c^3d+3A\,c^2d^2-3Ac\,d^3+A\,d^4+B\,c^4-3B\,c^3d+3B\,c^2d^2-Bc\,d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A\,d^3-\frac{1}{2}Bc\,d^2+\frac{3}{2}B\,d^3\right)(}{}\right.$
default	$2a^3\left(\frac{\left(-A\,c^3d+3A\,c^2d^2-3Ac\,d^3+A\,d^4+B\,c^4-3B\,c^3d+3B\,c^2d^2-Bc\,d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A\,d^3-\frac{1}{2}Bc\,d^2+\frac{3}{2}B\,d^3\right)(}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f*a^3*((-A*c^3*d+3*A*c^2*d^2-3*A*c*d^3+A*d^4+B*c^4-3*B*c^3*d+3*B*c^2*d^2-
B*c*d^3)/d^4/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))+1/d^4*(((1/2*A*d^3-1/2*B*c*d^2+3/2*B*d^3)*tan(1/2*f*x+1/2*e)^5+(
A*c*d^2-3*A*d^3-B*c^2*d+3*B*c*d^2-3*B*d^3)*tan(1/2*f*x+1/2*e)^4+(2*A*c*d^2-
6*A*d^3-2*B*c^2*d+6*B*c*d^2-8*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3+1/2*B
*c*d^2-3/2*B*d^3)*tan(1/2*f*x+1/2*e)+A*c*d^2-3*A*d^3-B*c^2*d+3*B*c*d^2-11/3
*B*d^3)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(2*A*c^2*d-6*A*c*d^2+7*A*d^3-2*B*c^3
+6*B*c^2*d-7*B*c*d^2+5*B*d^3)*arctan(tan(1/2*f*x+1/2*e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.50, size = 644, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d +
 (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3
*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) + 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d
 + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d
^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*
x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^
2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d - (A +
 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f), 1/6*(2*B*a^3*
d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a
^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*c
os(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a
^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sq
rt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 6*(B*a^3*c^2*d - (A + 3*B)*a^
3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(239) = 478.
time = 0.60, size = 617, normalized size = 2.51

$$-\frac{\frac{3\,(2\,Ba^3c^3-2\,Aa^3c^2d-6\,Ba^3c^2d+6\,Aa^3cd^2+7\,Ba^3cd^2-7\,Aa^3d^3-5\,Ba^3d^3)(fx+e)}{d^4}-\frac{12\,(Ba^3c^4-Aa^3c^3d-3\,Ba^3c^3d+3\,Aa^3c^2d^2+3\,Ba^3c^2d^2-3\,Aa^3cd^3-Ba^3cd^3+Aa^3d^4)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^4}+\frac{2\left(3\,Ba^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,Aa^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-9\,Ba^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,Ba^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-6\,Aa^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-18\,Ba^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+18\,Aa^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+18\,Ba^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+12\,Ba^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,Aa^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-36\,Ba^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,Aa^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+48\,Ba^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ba^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Aa^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Ba^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,Ba^3c^2-6\,Aa^3cd-18\,Ba^3cd+18\,Aa^3d^2+22\,Ba^3d^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3d^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B*
a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*a^
3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*d^3
 - B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4) + 2
*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 -
```

```
9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^4 -
6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 +
18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 +
 12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^2
- 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^2
 + 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*x + 1/2*e) +
 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e) + 6*B*
a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^3*d^2)/((tan(1
/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

Mupad [B]
time = 21.76, size = 2500, normalized size = 10.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x)),x)
```

```
[Out] - ((2*(9*A*a^3*d^2 + 3*B*a^3*c^2 + 11*B*a^3*d^2 - 3*A*a^3*c*d - 9*B*a^3*c*d
))/(3*d^3) - (tan(e/2 + (f*x)/2)^5*(A*a^3*d - B*a^3*c + 3*B*a^3*d))/d^2 + (
4*tan(e/2 + (f*x)/2)^2*(3*A*a^3*d^2 + B*a^3*c^2 + 4*B*a^3*d^2 - A*a^3*c*d -
 3*B*a^3*c*d))/d^3 + (2*tan(e/2 + (f*x)/2)^4*(3*A*a^3*d^2 + B*a^3*c^2 + 3*B
*a^3*d^2 - A*a^3*c*d - 3*B*a^3*c*d))/d^3 + (tan(e/2 + (f*x)/2)*(A*a^3*d - B
*a^3*c + 3*B*a^3*d))/d^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)
^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*
a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 +
 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^
6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 7
0*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6
*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x
)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116
*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2
*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 13
6*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^
9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*
A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^
6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9
+ ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B*
a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 -
(a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d
^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3
*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A
*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24
*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i +
 (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*
```

```
c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2
- (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4 +
(((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2
*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8
+ 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a
^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 +
 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6
*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^
2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8
*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6
*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 +
 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*
d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 25
2*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c
^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (((8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11
 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^
9 - 2*B*a^3*c^4*d^8))/d^8 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13
 - 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*
d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*tan(e/2
 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a
^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*
a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^
3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i
 + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*
A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4)/((16*(2*B^3*a^9*c^10 - 47*A^3*a^9*c^2*d^8
+ 55*A^3*a^9*c^3*d^7 - 21*A^3*a^9*c^4*d^6 - 7*A^3*a^9*c^5*d^5 + 8*A^3*a^9*c
^6*d^4 - 2*A^3*a^9*c^7*d^3 - 15*B^3*a^9*c^3*d^7 + 71*B^3*a^9*c^4*d^6 - 148*
B^3*a^9*c^5*d^5 + 180*B^3*a^9*c^6*d^4 - 139*B^3*a^9*c^7*d^3 + 67*B^3*a^9*c^
8*d^2 + 14*A^3*a^9*c*d^9 - 18*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B*
a^9*c*d^9 + 5*A*B^2*a^9*c^2*d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4*
d^6 - 280*A*B^2*a^9*c^5*d^5 + 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3
 + 44*A*B^2*a^9*c^8*d^2 - 32*A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45*
A^2*B*a^9*c^4*d^6 - 97*A^2*B*a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B*
a^9*c^7*d^3 + 6*A^2*B*a^9*c^8*d^2))/d^8 + (((8*(49*A^2*a^6*c^2*d^9 - 84*A^2
*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5
+ 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a
^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 +
70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^
6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*
x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^1...
```

3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=283

$$-\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4}+\frac{2a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2))\tan^{-1}\left(\frac{d+c\tan}{\sqrt{c^2}}\right.}{d^4(c+d)\sqrt{c^2-d^2}\,f}$$

```
[Out] -1/2*a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/d^4-1/2*a^3*(4*A*c*d-B*
(6*c^2-3*c*d-5*d^2))*cos(f*x+e)/d^3/(c+d)/f+1/2*(2*A*d-B*(3*c+d))*cos(f*x+e
)*(a^3+a^3*sin(f*x+e))/d^2/(c+d)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))
^2/d/(c+d)/f/(c+d*sin(f*x+e))+2*a^3*(c-d)^2*(A*d*(2*c+3*d)-B*(3*c^2+3*c*d-d
^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^4/(c+d)/f/(c^2-d^2)
^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3054, 3055, 3047, 3102, 2814, 2739, 632, 210}

$$\frac{2a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2))\,\text{ArcTan}\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)\sqrt{c^2-d^2}}-\frac{a^3x(2Ad(2c-3d)-B(6c^2-12cd+7d^2))}{2d^4}-\frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3f(c+d)}+\frac{(2Ad-B(3c+d))\cos(e+fx)\,(a^3\sin(e+fx)+a^3)}{2d^2f(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*(a^3*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*d^2))*x)/d^4 + (2*a^3*
(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)*Sqrt[c^2 - d^2]*f) - (a^3*(4*A*c*
d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(2*d^3*(c + d)*f) + ((2*A*d -
B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d^2*(c + d)*f) + (a*
(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Sin[e
+ f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+}{}}{}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(}{}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(}{}$$

$$= -\frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B}{}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} - \frac{a^3(4Acd-}{}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} - \frac{a^3(4Acd-}{}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} - \frac{a^3(4Acd-}{}$$

$$= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} + \frac{2a^3(c-d)}{}$$

Mathematica [A]
time = 1.03, size = 244, normalized size = 0.86

$$\frac{a^3(1+\sin(e+fx))^3\left(2(2Ad(-2c+3d)+B(6c^2-12cd+7d^2))(e+fx)-\frac{8(c-d)^2(-Ad(2c+3d)+B(3c^2+3cd-d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}-4d(-2Bc+Ad+3Bd)\cos(e+fx)+\frac{4(c-d)^2d(Bc-Ad)\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}-Bd^2\sin(2(e+fx))\right)}{4d^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d^
2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2))
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]
) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d)*Co
s[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)]))/(4*d^
4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [A]

time = 0.60, size = 406, normalized size = 1.43

method	result
derivativedivides	$\frac{2a^3\left(\frac{\frac{-\frac{d^2\left(Ac^2d-2Acd^2+Ad^3-Bc^3+2Bc^2d-Bcd^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Ac^2d-2Acd^2+Ad^3-Bc^3+2Bc^2d-Bcd^2\right)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(2Ac^3d-\cdots}{\cdots}}{d^4}\cdots\right.}{\cdots}$
default	$\frac{2a^3\left(\frac{\frac{-\frac{d^2\left(Ac^2d-2Acd^2+Ad^3-Bc^3+2Bc^2d-Bcd^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Ac^2d-2Acd^2+Ad^3-Bc^3+2Bc^2d-Bcd^2\right)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(2Ac^3d-\cdots}{\cdots}}{d^4}\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3*(1/d^4*((-d^2*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2*d-B*c*d^2)/(c+
d)/c*tan(1/2*f*x+1/2*e)-d*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2*d-B*c*d^2)
/(c+d))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(2*A*c^3*d-A*c^2*
d^2-4*A*c*d^3+3*A*d^4-3*B*c^4+3*B*c^3*d+4*B*c^2*d^2-5*B*c*d^3+B*d^4)/(c+d)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1
/d^4*((-1/2*B*d^2*tan(1/2*f*x+1/2*e)^3+(A*d^2-2*B*c*d+3*B*d^2)*tan(1/2*f*x+
1/2*e)^2+1/2*B*d^2*tan(1/2*f*x+1/2*e)+A*d^2-2*B*c*d+3*B*d^2)/(1+tan(1/2*f*x
+1/2*e)^2)^2+1/2*(4*A*c*d-6*A*d^2-6*B*c^2+12*B*c*d-7*B*d^2)*arctan(tan(1/2*
f*x+1/2*e))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.46, size = 1048, normalized size = 3.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*
B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*B*
a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3*
B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*si
n(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*
d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (
2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*c^3*d -
2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3*d^4)*f*
x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)*cos(f*x
 + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f),
1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*B
)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + 2*(3*B
*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3
*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*s
in(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d
)/(c + d))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2
*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*
c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3
*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)
*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d
^5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(278) = 556.
time = 0.53, size = 588, normalized size = 2.08

$$-\frac{\frac{4\left(3Ba^3c^4-2Aa^3c^3d-3Ba^3c^3d+Aa^3c^2d^2-4Ba^3c^2d^2+4Aa^3cd^3+5Ba^3cd^3-3Aa^3d^4-Ba^3d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^4+d^5)\sqrt{c^2-d^2}}-\frac{4\left(Ba^3c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Aa^3c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2Ba^3c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Aa^3cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ba^3cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Aa^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ba^3c^4-Aa^3c^3d-2Ba^3c^3d+2Aa^3c^2d^2+Ba^3c^2d^2-Aa^3cd^3\right)}{(c^2d^3+cd^4)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}-\frac{(6Ba^3c^2-4Aa^3cd-12Ba^3cd+6Aa^3d^2+7Ba^3d^2)(fx+e)}{d^4}-\frac{2\left(Ba^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4Ba^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2Aa^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Ba^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ba^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4Ba^3c-2Aa^3d-6Ba^3d\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2d^3}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B*
a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(1/2*
f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(1/2*f*
x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*d - 2*B
*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d^3 + c*d
^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (6*B*a^3*c
^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x + e)/d^4
- 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*
A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)^2 - B*a^3*d
*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((tan(1/2*f*x +
1/2*e)^2 + 1)^2*d^3))/f
```

Mupad [B]
time = 23.88, size = 2500, normalized size = 8.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^
2 + 3*B*a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^4*(A*a^3*d^3 - 3*
B*a^3*c^3 - B*a^3*d^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + B*a^3*c*d^2 + 3*B*a^3
*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^2*(2*A*a^3*d^3 - 6*B*a^3*c^3
 + B*a^3*d^3 - 2*A*a^3*c*d^2 + 4*A*a^3*c^2*d + 5*B*a^3*c*d^2 + 6*B*a^3*c^2*
d))/(d^3*(c + d)) + (4*tan(e/2 + (f*x)/2)^3*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^
3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) +
 (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^3 - 3*B*a^3*c^3 - 4*A*a^3*c*d^2 + 2*A*a^3
*c^2*d - 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + (tan(e/2 + (f*x)
```

/2)*(2*A*a^3*d^3 - 9*B*a^3*c^3 + 6*A*a^3*c^2*d + 10*B*a^3*c*d^2 + 9*B*a^3*c ^2*d))/(c*d^2*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 + 3*c*tan(e/2 + (f*x)/2)^4 + c*tan(e/2 + (f*x)/2)^6 + 4*d*tan(e/2 + (f*x)/2)^3 + 2*d*tan(e/2 + (f*x)/2)^5)) - (atan(((((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6* c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 14 4*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^ 8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88* A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3 *d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32* A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6* c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c *d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 5 72*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6 *c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((((8*(4*c^2 *d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^1 2 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d ^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)* (24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^ 3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/ 2)*1i)/d^4 + (((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4 *d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^ 2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d ^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B* a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)* (144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2 *a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^ 10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B ^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9 *d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A* B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6 *d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c *d^10 + d^11 + c^2*d^9) + (((8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3 *c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a ^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (((8*(4*c^2*d^13 + 8*c^3*d^12 + 4 *c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 +

```
 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c
^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*
c)*1i)/2))/d^4 - (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 -
8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9
- 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d
^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1
i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*
B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2)*1i)/d^4)/((16*(132*A^3*a^9*c^3*d^
6 - 252*A^3*a^9*c^2*d^7 - 54*B^3*a^9*c^9 + 76*A^3*a^9*c^4*d^5 - 80*A^3*a^9*
c^5*d^4 + 16*A^3*a^9*c^6*d^3 - 115*B^3*a^9*c^2*d^7 + 350*B^3*a^9*c^3*d^6 -
537*B^3*a^9*c^4*d^5 + 387*B^3*a^9*c^5*d^4 + 36*B^3*a^9*c^6*d^3 - 297*B^3*a^
9*c^7*d^2 + 108*A^3*a^9*c*d^8 + 14*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 96*A
*B^2*a^9*c*d^8 + 108*A*B^2*a^9*c^8*d + 198*A^2*...
```

3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=305

$$-\frac{a^3(3Bc-Ad-3Bd)x}{d^4}-\frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\tan^{-1}\left(\frac{d+c\tan(\frac{1}{2}(e+}{\sqrt{c^2-d}}\right.}{d^4(c+d)^2\sqrt{c^2-d^2}\,f}$$

```
[Out] -a^3*(-A*d+3*B*c-3*B*d)*x/d^4-1/2*a^3*(3*B*c*(2*c+3*d)-A*d*(2*c+5*d))*cos(f
*x+e)/d^3/(c+d)^2/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/(c+d)/
f/(c+d*sin(f*x+e))^2-1/2*(A*d*(c+4*d)-B*(3*c^2+4*c*d-2*d^2))*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))/d^2/(c+d)^2/f/(c+d*sin(f*x+e))-a^3*(c-d)*(A*d*(2*c^2+6*c*
d+7*d^2)-3*B*(2*c^3+4*c^2*d+c*d^2-2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(
c^2-d^2)^(1/2))/d^4/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3054, 3047, 3102, 2814, 2739, 632, 210}

$$-\frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\text{ArcTan}\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)^2\sqrt{c^2-d^2}}-\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2d^2f(c+d)^2(c+d\sin(e+fx))}-\frac{a^3x(-Ad+3Bc-3Bd)}{d^4}-\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3f(c+d)^2}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(3*B*c - A*d - 3*B*d)*x)/d^4) - (a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7
*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/
2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (a^3*(3*B*c*(2*c
+ 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x])/(2*d^3*(c + d)^2*f) + (a*(B*c - A*d
)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^
2) - ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e + f*x]*(a^3 + a^3*S
in[e + f*x]))/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+}{}}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d}{}$$

$$= -\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-}{2d}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d}{2d^3(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d}{2d^3(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d}{2d^3(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 830 vs. 2(305) = 610.
time = 2.14, size = 830, normalized size = 2.72

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*B
*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^
2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e + 8*
A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4*e +
 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x - 12*B*
c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B*c^2*d
^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x - d*(2*A
*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^2*d^2 +
4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B*d)*(e +
```

```
f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Cos[3*(e + f
*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*c^3*d^2*e*S
in[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[e + f*x] + 2
4*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c*d^4*e*Sin[e
+ f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e + f*x] - 24*B*
c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 24*B*c^2*d^3*f*x
*Sin[e + f*x] + 8*A*c*d^4*f*x*Sin[e + f*x] + 24*B*c*d^4*f*x*Sin[e + f*x] -
9*B*c^3*d^2*Sin[2*(e + f*x)] + 3*A*c^2*d^3*Sin[2*(e + f*x)] - 9*B*c^2*d^3*S
in[2*(e + f*x)] + 3*A*c*d^4*Sin[2*(e + f*x)] + 4*B*c*d^4*Sin[2*(e + f*x)] -
 6*A*d^5*Sin[2*(e + f*x)] - 2*B*d^5*Sin[2*(e + f*x)])/(c + d*Sin[e + f*x])^
2))/(4*d^4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [A]
time = 0.88, size = 586, normalized size = 1.92

method	result
derivativedivides	$2a^3\left(-\frac{-\frac{d^2\left(A\,c^3d+5A\,c^2d^2-4Ac\,d^3-2A\,d^4-3B\,c^4-3B\,c^3d+6B\,c^2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(2A\,c^5d+4A\,c^4d^2-A\,c^3d^3+7A\,c^2d^4-\cdots\right.}{\cdots}}{\cdots}\right.$
default	$2a^3\left(-\frac{-\frac{d^2\left(A\,c^3d+5A\,c^2d^2-4Ac\,d^3-2A\,d^4-3B\,c^4-3B\,c^3d+6B\,c^2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(2A\,c^5d+4A\,c^4d^2-A\,c^3d^3+7A\,c^2d^4-\cdots\right.}{\cdots}}{\cdots}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f*a^3*(-1/d^4*((-1/2*d^2*(A*c^3*d+5*A*c^2*d^2-4*A*c*d^3-2*A*d^4-3*B*c^4-3
*B*c^3*d+6*B*c^2*d^2)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(2*A*c^5
*d+4*A*c^4*d^2-A*c^3*d^3+7*A*c^2*d^4-10*A*c*d^5-2*A*d^6-4*B*c^6-2*B*c^5*d-B
*c^4*d^2-5*B*c^3*d^3+14*B*c^2*d^4-2*B*c*d^5)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*
x+1/2*e)^2-1/2*d^2*(7*A*c^3*d+11*A*c^2*d^2-16*A*c*d^3-2*A*d^4-13*B*c^4-5*B*
c^3*d+22*B*c^2*d^2-4*B*c*d^3)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-1/2*d*(2
*A*c^3*d+4*A*c^2*d^2-5*A*c*d^3-A*d^4-4*B*c^4-2*B*c^3*d+7*B*c^2*d^2-B*c*d^3)
/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(
2*A*c^3*d+4*A*c^2*d^2+A*c*d^3-7*A*d^4-6*B*c^4-6*B*c^3*d+9*B*c^2*d^2+9*B*c*d
```

```
^3-6*B*d^4)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2
*e)+2*d)/(c^2-d^2)^(1/2)))+1/d^4*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-3*B*c+
3*B*d)*arctan(tan(1/2*f*x+1/2*e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(302) = 604.
time = 0.47, size = 1697, normalized size = 5.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 -
 (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 +
 B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3
*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x
 - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*
A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a
^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*
a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2
*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) -
 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(
A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*B*a^3*c^4*d -
(A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x
+ (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3
*A + B)*a^3*d^5)*cos(f*x + e))*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*c
os(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 +
```

```
 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B*a^3*c^3*d^2 - (A -
 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x
+ e)^2 + 2*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 + B*a^3*d^5)*cos(f*x + e)^3 - 2*(
3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*
A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^
3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A -
B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2
*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B
*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B
)*a^3*c*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (6*B*a^3*c^4*d - 2*(A -
3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B
)*a^3*d^5)*cos(f*x + e) - (4*(3*B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A
+ 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3
*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e)
)*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 +
2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*
d^7 + d^8)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(302) = 604.
time = 0.54, size = 986, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*c
^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(
c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((tan(1/
2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A*a^3*
c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 5
*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e
```

```
)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/2*f*x +
1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(1/2*f*x +
 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^2*tan(1/2*
f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^3*d^3*tan(1
/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*A*a^3*c^2*d^
4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 10*A*a
^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*
A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e) - 7*
A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) -
 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2
*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2*d^4*tan(1/2*f*x +
 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^6 - 2*A*a^3*c^5*d
+ 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 + 5*A*a^3*c^3*d^3 + B*a
^3*c^3*d^3 + A*a^3*c^2*d^4)/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*(c*tan(1/2*f*x
 + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - (3*B*a^3*c - A*a^3*d - 3*B
*a^3*d)*(f*x + e)/d^4)/f
```

Mupad [B]
time = 25.41, size = 2500, normalized size = 8.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3 +
 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2)/(d^3*(c + d)^2) + (4*ta
n(e/2 + (f*x)/2)^3*(A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d
 + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2))/(c*d^2
*(c + d)^2) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^4 + 3*B*a^3*c^4 + 4*A*a^3*c*
d^3 - A*a^3*c^3*d + 3*B*a^3*c^3*d - 5*A*a^3*c^2*d^2 - 6*B*a^3*c^2*d^2))/(c*
d^2*(c + d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*a^3*d^6 + 6*B*a^3*c^6 + 5*A*a^3
*c*d^5 - 2*A*a^3*c^5*d + B*a^3*c*d^5 + 6*B*a^3*c^5*d - 3*A*a^3*c^2*d^4 + 3*
A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 3*B*a^3*c^2*d^4 + 11*B*a^3*c^3*d^3 + 3*B*
a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)^4*(2*A*a^3*d^6 + 6*
B*a^3*c^6 + 10*A*a^3*c*d^5 - 2*A*a^3*c^5*d + 2*B*a^3*c*d^5 + 6*B*a^3*c^5*d
- 7*A*a^3*c^2*d^4 + A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 14*B*a^3*c^2*d^4 + 5*
B*a^3*c^3*d^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)
*(2*A*a^3*d^4 + 21*B*a^3*c^4 + 16*A*a^3*c*d^3 - 7*A*a^3*c^3*d + 4*B*a^3*c*d
^3 + 21*B*a^3*c^3*d - 11*A*a^3*c^2*d^2 - 14*B*a^3*c^2*d^2))/(c*d^2*(c + d)^
2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c^2 + 4*d^2) + tan(e/2 + (f*x)/2)^4*(3*c^2
+ 4*d^2) + c^2*tan(e/2 + (f*x)/2)^6 + c^2 + 8*c*d*tan(e/2 + (f*x)/2)^3 + 4*
c*d*tan(e/2 + (f*x)/2)^5 + 4*c*d*tan(e/2 + (f*x)/2))) - (atan((((B*a^3*c*3i
 - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2
*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9
```

+ 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))/d^4)*1i)/d^4 + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 3

```
2*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12
- 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*...
```

3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=220

$$\frac{(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))\,x}{2a}+\frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e$$

```
[Out] 1/2*(3*A*d*(2*c^2-2*c*d+d^2)+B*(2*c^3-6*c^2*d+9*c*d^2-3*d^3))*x/a+2/3*d*(3*
A*(c^2-3*c*d+d^2)-B*(7*c^2-9*c*d+4*d^2))*cos(f*x+e)/a/f+1/6*d^2*(6*A*c-9*A*
d-11*B*c+9*B*d)*cos(f*x+e)*sin(f*x+e)/a/f+1/3*(3*A-4*B)*d*cos(f*x+e)*(c+d*s
in(f*x+e))^2/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))
```

Rubi [A]
time = 0.24, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3056, 2832, 2813}

$$\frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{3af}+\frac{x(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))}{2a}+\frac{d^2(6Ac-9Ad-11Bc+9Bd)\sin(e+fx)\cos(e+fx)}{6af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}+\frac{d(3A-4B)\cos(e+fx)(c+d\sin(e+fx))^2}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((3*A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/(
2*a) + (2*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f
*x])/(3*a*f) + (d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f
*x])/(6*a*f) + ((3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a*f)
- ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} + \frac{\int(c+d\sin(e}{}$$
$$= \frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(A-B)\cos}{f}$$
$$= \frac{(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))\,x}{2a} + \frac{2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 788 vs. 2(220) = 440.
time = 0.90, size = 788, normalized size = 3.58

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*(
1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d*(
1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x)))*C
os[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos[(3*
(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f*x))/
2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A*c^3*S
in[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f*x)/2]
+ 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180*B*c*d^2
*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e + f*x)/2] +
 24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72*B*c^2*d*e
*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*e*Sin[(e +
f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x)/2] + 24*B*
```

```
c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 72*B*c^2*d*f*x
*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c*d^2*f*x*Sin[(
e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*Sin[(e + f*x)/2]
 - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e + f*x))/2] + 27*B
*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2] - 18*B*d^3*Sin[(
3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^3*Sin[(5*(e + f*x)
)/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e + f*x))/2]))/(24*a*f*
(1 + Sin[e + f*x]))
```

Maple [**A**]
time = 0.30, size = 337, normalized size = 1.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-(A*c^3-3*A*c^2*d+3*A*c*d^2-A*d^3-B*c^3+3*B*c^2*d-3*B*c*d^2+B*d^3)/(
tan(1/2*f*x+1/2*e)+1)+((1/2*A*d^3+3/2*B*c*d^2-1/2*B*d^3)*tan(1/2*f*x+1/2*e)
^5+(-3*A*c*d^2+A*d^3-3*B*c^2*d+3*B*c*d^2-B*d^3)*tan(1/2*f*x+1/2*e)^4+(-6*A*
c*d^2+2*A*d^3-6*B*c^2*d+6*B*c*d^2-4*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3
-3/2*B*c*d^2+1/2*B*d^3)*tan(1/2*f*x+1/2*e)-3*A*c*d^2+A*d^3-3*B*c^2*d+3*B*c*
d^2-5/3*B*d^3)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(6*A*c^2*d-6*A*c*d^2+3*A*d^3+
2*B*c^3-6*B*c^2*d+9*B*c*d^2-3*B*d^3)*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. 2(220) = 440.
time = 0.59, size = 1226, normalized size = 5.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*d^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 9*B*c*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
```

```
e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*d^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a) + 18*B*c^2*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 18*A*c*d^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*B*
c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1))) - 18*A*c^2*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 6*A*c^3/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(220) = 440$.
time = 0.47, size = 480, normalized size = 2.18

$$\frac{2 Bd^3 \cos(fx+e)^4 - 6(A-B)c^3 + 18(A-B)c^2 d - 18(A-B)cd^2 + 6(A-B)d^3 + (9Bcd^2 + (3A-B)d^3)\cos(fx+e)^3 + 3(2Bc^3 + 6(A-B)c^2 d - 3(2A-3B)cd^2 + 3(A-B)d^3)fx - 6(3Bc^2 d + 3(A-B)cd^2 - (A-2B)d^3)\cos(fx+e)^2 - 3(2(A-B)c^3 - 6(A-2B)c^2 d + 3(4A-3B)cd^2 - (3A-5B)d^3 - (2Bc^3 + 6(A-B)c^2 d - 3(2A-3B)cd^2 + 3(A-B)d^3)fx)\cos(fx+e) + (2Bd^3\cos(fx+e)^3 + 6(A-B)c^3 - 18(A-B)c^2 d + 18(A-B)cd^2 - 6(A-B)d^3 + 3(2Bc^3 + 6(A-B)c^2 d - 3(2A-3B)cd^2 + 3(A-B)d^3)fx - 3(3Bcd^2 + (A-B)d^3)\cos(fx+e)^2 - 3(6Bc^2 d + 3(2A-B)cd^2 - (A-3B)d^3)\cos(fx+e))\sin(fx+e)}{6(af\cos(fx+e) + af\sin(fx+e) + af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B)
*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(2*
B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(3*B
*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)*c^3
 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^3 + 6
*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x + e) + (
2*B*d^3*cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A - B)*c*d^
2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*
(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^2 - 3*(6*B*c^2*
d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14644 vs. $2(204) = 408$.
time = 4.99, size = 14644, normalized size = 66.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)

[Out] Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 12*A*c**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*A*c**2*d*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan

```
(e/2 + f*x/2) + 6*a*f) + 108*A*c**2*d*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 108*A*c**2*d*tan(e/2 + f*x/2)**2/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*A*c**2*d/(6*a*f*t
an(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*A*c*d**2*f*x*tan(e/2 +
f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*t
an(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**
3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*A*c*d
**2*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*
x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*
tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
 6*a*f) - 54*A*c*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 54*A*c*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c*d**2*f*x*tan(e/2 + f*x
/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(220) = 440.

time = 0.54, size = 479, normalized size = 2.18

$$\frac{\frac{3\,(2\,Bc^3+6\,Ac^2d-6\,Bc^2d-6\,Acd^2+9\,Bcd^2+3\,Ad^3-3\,Bd^3)(fx+e)}{a}-\frac{12\,(Ac^3-Bc^3-3\,Ac^2d+3\,Bc^2d+3\,Acd^2-3\,Bcd^2-Ad^3+Bd^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(9\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-18\,Bc^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-18\,Acd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+18\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+6\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-6\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-36\,Bc^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-36\,Acd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-24\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-18\,Bc^2d-18\,Acd^2+18\,Bcd^2+6\,Ad^3-10\,Bd^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3 -
 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c*d
^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*c*d
^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*tan(1/
2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(1/2*f
*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x + 1/2*e)^
2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 +
```

```
 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c*
d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3*tan(1/2*f
*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 10*B*d^3)/((
tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

Mupad [B]
time = 14.05, size = 839, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(12*A*c^3*cos(e/2 + (f*x)/2) - 18*A*d^3*cos(e/2 + (f*x)/2) - 12*B*c^3*cos(
e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2) + 6*A*d^3*cos(e/2 + (f*x)/2)^3
 - 12*A*d^3*cos(e/2 + (f*x)/2)^5 - 6*B*d^3*cos(e/2 + (f*x)/2)^3 + 36*B*d^3*
cos(e/2 + (f*x)/2)^5 - 16*B*d^3*cos(e/2 + (f*x)/2)^7 - 9*A*d^3*cos(e/2 + (f
*x)/2)*(e + f*x) - 6*B*c^3*cos(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*cos(e/2 +
 (f*x)/2)*(e + f*x) - 9*A*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*sin(e/
2 + (f*x)/2)*(e + f*x) + 9*B*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*d^3*co
s(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*A*d^3*cos(e/2 + (f*x)/2)^4*sin(e
/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*B*d^3
*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2) - 16*B*d^3*cos(e/2 + (f*x)/2)^6*si
n(e/2 + (f*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2) - 36*A*c^2*d*cos(e/2 + (f*
x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f*x)/2) + 36*
A*c*d^2*cos(e/2 + (f*x)/2)^3 + 18*B*c*d^2*cos(e/2 + (f*x)/2)^3 + 36*B*c^2*d
*cos(e/2 + (f*x)/2)^3 - 36*B*c*d^2*cos(e/2 + (f*x)/2)^5 + 18*A*c*d^2*cos(e/
2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*cos(e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d
^2*cos(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*cos(e/2 + (f*x)/2)*(e + f*x) +
 18*A*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*sin(e/2 + (f*x)/2)*(e
 + f*x) - 27*B*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*sin(e/2 + (f
*x)/2)*(e + f*x) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) - 54*
B*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f*x
)/2)^2*sin(e/2 + (f*x)/2) + 36*B*c*d^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)
/2))/(6*a*f*cos(e/2 + (f*x)/2) + 6*a*f*sin(e/2 + (f*x)/2))
```

3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=143

$$\frac{(2A(2c-d)d+B(2c^2-4cd+3d^2))\,x}{2a}+\frac{2(A(c-d)-B(2c-d))d\cos(e+fx)}{af}+\frac{(2A-3B)d^2\cos(e+fx)\,\mathrm{si}}{2af}$$

```
[Out] 1/2*(2*A*(2*c-d)*d+B*(2*c^2-4*c*d+3*d^2))*x/a+2*(A*(c-d)-B*(2*c-d))*d*cos(f
*x+e)/a/f+1/2*(2*A-3*B)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(A-B)*cos(f*x+e)*(c+d
*sin(f*x+e))^2/f/(a+a*sin(f*x+e))
```

Rubi [A]
time = 0.14, antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3056, 2813}

$$\frac{x(-(d^2(2A-3B))+4Acd+2Bc(c-2d))}{2a}+\frac{2d(A(c-d)-B(2c-d))\cos(e+fx)}{af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}+\frac{d^2(2A-3B)\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/(2*a) + (2*(A*(c - d) - B
*(2*c - d))*d*Cos[e + f*x])/(a*f) + ((2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e + f
*x])/(2*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[
e + f*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))} + \frac{\int (c+d\sin(\cdots}{\cdots}$$

$$= \frac{(2Bc(c-2d)+4Acd-(2A-3B)d^2)\,x}{2a} + \frac{2(A(c-d)-\cdots}{\cdots}$$

Mathematica [A]

time = 0.34, size = 200, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)+2(2A(2c-d)d+B(2c^2-4cd+3d^2))(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+4d(-Ad+B(-2c+d))\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-Bd^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(2(e+fx))\right)}{4af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[2*(e + f*x)]))/(4*a*f*(1 + Sin[e + f*x]))
```

Maple [A]

time = 0.25, size = 193, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{2\left(A c^2-2Acd+A d^2-B c^2+2Bcd-B d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(-A d^2-2Bcd+B d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}\cdots\right)}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}\cdots}{af}$
default	$\frac{-\frac{2\left(A c^2-2Acd+A d^2-B c^2+2Bcd-B d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\left(-A d^2-2Bcd+B d^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}\cdots\right)}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}\cdots}{af}$
risch	$\frac{2xAcd}{a}-\frac{xA\,d^2}{a}+\frac{c^2xB}{a}-\frac{2xBcd}{a}+\frac{3xB\,d^2}{2a}-\frac{d^2\mathrm{e}^{i(fx+e)}A}{2af}-\frac{d\,\mathrm{e}^{i(fx+e)}Bc}{af}+\frac{d^2\mathrm{e}^{i(fx+e)}B}{2af}-\frac{d^2\mathrm{e}^{-i(fx+e)}\cdots}{2af}$
norman	$\frac{\frac{\left(2A c^2-4Acd-2B c^2-2B d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(2A c^2-4Acd+2A d^2-2B c^2+4Bcd-3B d^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(6A c^2-12Acd+2A d^2\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-(A*c^2-2*A*c*d+A*d^2-B*c^2+2*B*c*d-B*d^2)/(tan(1/2*f*x+1/2*e)+1)+(1
/2*B*d^2*tan(1/2*f*x+1/2*e)^3+(-A*d^2-2*B*c*d+B*d^2)*tan(1/2*f*x+1/2*e)^2-1
```

```
/2*B*d^2*tan(1/2*f*x+1/2*e)-A*d^2-2*B*c*d+B*d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2
+1/2*(4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+3*B*d^2)*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(145) = 290$.
time = 0.53, size = 660, normalized size = 4.62

$$\frac{Bd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-4Bcd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-2Ad^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+2Bc^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+4Acd\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{2Ac^2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4*B*c*d*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) + 2*B*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 4*A*c*d*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(145) = 290$.
time = 0.38, size = 311, normalized size = 2.17

$$\frac{Bd^2\cos(fx+e)^3-2(A-B)c^2+4(A-B)cd-2(A-B)d^2+(2Bc^2+4(A-B)cd-(2A-3B)d^2)fx-2(2Bcd+(A-B)d^2)\cos(fx+e)^2-(2(A-B)c^2-4(A-2B)cd+(4A-3B)d^2-(2Bc^2+4(A-B)cd-(2A-3B)d^2)fx)\cos(fx+e)-(Bd^2\cos(fx+e)^2-2(A-B)c^2+4(A-B)cd-2(A-B)d^2-(2Bc^2+4(A-B)cd-(2A-3B)d^2)fx+(4Bcd+(2A-B)d^2)\cos(fx+e))\sin(fx+e)}{2(af\cos(fx+e)+af\sin(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 +
 (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d^2
)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 - (2*
B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos(f*x
 + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4*(A -
```

```
B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x + e))*si
n(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5763 vs. $2(117) = 234$.
time = 2.55, size = 5763, normalized size = 40.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-4*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*t
an(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c
*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*
x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan
(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d*f*x*tan(e/2 +
f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) + 8*A*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*
c*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 16*A*c*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 +
f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*f*x*tan(e/2 + f*x/2)**3/(2*a*
```

```
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2
*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/
2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*tan(e/2
 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f
*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
 2*a*f) - 4*A*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*A*d**2*tan(e/2 + f*x/2)**2/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 8*A*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2
)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
+ 4*B*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e
/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*
a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*
tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4
*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/
2) + 2*a*f) + 2*B*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(...
```

Giac [A]

time = 0.50, size = 222, normalized size = 1.55

$$\frac{\frac{(2Bc^2+4Acd-4Bcd-2Ad^2+3Bd^2)(fx+e)}{a}-\frac{4(Ac^2-Bc^2-2Acd+2Bcd+Ad^2-Bd^2)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4Bcd-2Ad^2+2Bd^2\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*c
^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e) +
```

```
1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 - 2*
A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*tan(1
/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*a))/f
```

Mupad [B]
time = 16.67, size = 297, normalized size = 2.08

$$\frac{x\,\left(2\,B\,c^2-2\,A\,d^2+3\,B\,d^2+4\,A\,c\,d-4\,B\,c\,d\right)}{2\,a}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,A\,d^2-3\,B\,d^2+4\,B\,c\,d\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,A\,c^2+2\,A\,d^2-2\,B\,c^2-3\,B\,d^2-4\,A\,c\,d+4\,B\,c\,d\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(4\,A\,c^2+6\,A\,d^2-4\,B\,c^2-5\,B\,d^2-8\,A\,c\,d+12\,B\,c\,d\right)+2\,A\,c^2+4\,A\,d^2-2\,B\,c^2-4\,B\,d^2+\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,d^2-B\,d^2+4\,B\,c\,d\right)-4\,A\,c\,d+8\,B\,c\,d}{f\,\left(a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+2\,a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+2\,a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

```
[Out] (x*(2*B*c^2 - 2*A*d^2 + 3*B*d^2 + 4*A*c*d - 4*B*c*d))/(2*a) - (tan(e/2 + (f
*x)/2)^3*(2*A*d^2 - 3*B*d^2 + 4*B*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 + 2*
A*d^2 - 2*B*c^2 - 3*B*d^2 - 4*A*c*d + 4*B*c*d) + tan(e/2 + (f*x)/2)^2*(4*A*
c^2 + 6*A*d^2 - 4*B*c^2 - 5*B*d^2 - 8*A*c*d + 12*B*c*d) + 2*A*c^2 + 4*A*d^2
 - 2*B*c^2 - 4*B*d^2 + tan(e/2 + (f*x)/2)*(2*A*d^2 - B*d^2 + 4*B*c*d) - 4*A
*c*d + 8*B*c*d)/(f*(a + a*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2
*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2)^4 + a*tan(e/2 + (f*x)/2)^5))
```

3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=67

$$\frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}-\frac{(A-B)(c-d)\cos(e+fx)}{af(1+\sin(e+fx))}$$

```
[Out] (B*(c-d)+A*d)*x/a-B*d*cos(f*x+e)/a/f-(A-B)*(c-d)*cos(f*x+e)/a/f/(1+sin(f*x+
e))
```

Rubi [A]
time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3102, 2814, 2727}

$$-\frac{(A-B)(c-d)\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{x(Ad+B(c-d))}{a}-\frac{Bd\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B*(c - d) + A*d)*x)/a - (B*d*Cos[e + f*x])/(a*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(a*f*(1 + Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx &= \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{a+a\sin(e+fx)}\,dx\\ &= -\frac{Bd\cos(e+fx)}{af}+\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}\\ &= \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}+((A-B)(c-d))\int\\ &= \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}-\frac{(A-B)(c-d)\cos}{f(a+a\sin(e+}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 126, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left((B(c-d)+Ad)(e+fx)-Bd\cos(e+fx)\right)+(2Ac+B(c-d)(-2+e+fx)+Ad(-2+e+fx)-Bd\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d)
*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*(-
2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])
)
```

Maple [A]
time = 0.22, size = 81, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{2(Ac-Ad-Bc+Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2Bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+2(Ad+Bc-Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
default	$\frac{-\frac{2(Ac-Ad-Bc+Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2Bd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+2(Ad+Bc-Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
risch	$\frac{xAd}{a}+\frac{cxB}{a}-\frac{xBd}{a}-\frac{Bd\,e^{i(fx+e)}}{2af}-\frac{Bd\,e^{-i(fx+e)}}{2af}-\frac{2cA}{fa(e^{i(fx+e)}+i)}+\frac{2Ad}{fa(e^{i(fx+e)}+i)}+\frac{2cB}{fa(e^{i(fx+e)}+i)}$

norman	$\frac{\frac{(Ad+Bc-Bd)x}{a}+\frac{(Ad+Bc-Bd)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{(Ad+Bc-Bd)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(Ad+Bc-Bd)x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{2Ac-2Ad-2Bc}{af}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f/a*(-(A*c-A*d-B*c+B*d)/(tan(1/2*f*x+1/2*e)+1)-B*d/(1+tan(1/2*f*x+1/2*e)^
2)+(A*d+B*c-B*d)*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(70) = 140$.
time = 0.51, size = 278, normalized size = 4.15

$$-\frac{2\left(Bd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Bc\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-Ad\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{Ac}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*d*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + A*c/(a + a
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(70) = 140$.
time = 0.37, size = 160, normalized size = 2.39

$$-\frac{Bd\cos(fx+e)^2-(Bc+(A-B)d)fx+(A-B)c-(A-B)d-((Bc+(A-B)d)fx-(A-B)c+(A-2B)d)\cos(fx+e)-((Bc+(A-B)d)fx-Bd\cos(fx+e)+(A-B)c-(A-B)d)\sin(fx+e)}{af\cos(fx+e)+af\sin(fx+e)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B*
c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A - B
)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1307 vs. $2(49) = 98$.
time = 1.25, size = 1307, normalized size = 19.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/
2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*tan(e/2 + f*x/2)**
3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2
 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e
/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 +
 a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 +
f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x
/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*
x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*ta
n(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
 B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**
2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*ta
n(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2
+ f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/
2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f
*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*B*d*
tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*
f*tan(e/2 + f*x/2) + a*f) - 2*B*d*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*B*d/(a*f*tan(e
/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(
f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(70) = 140$.
time = 0.55, size = 160, normalized size = 2.39

$$\frac{\frac{(Bc+Ad-Bd)(fx+e)}{a}-\frac{2\left(Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac-Bc-Ad+2Bd\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] ((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1/
2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^2
+ B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/2*e
)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

Mupad [B]
time = 13.68, size = 122, normalized size = 1.82

$$\frac{x\,(A\,d+B\,c-B\,d)}{a}-\frac{(2\,A\,c-2\,A\,d-2\,B\,c+2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,B\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,c-2\,A\,d-2\,B\,c+4\,B\,d}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] (x*(A*d + B*c - B*d))/a - (2*A*c - 2*A*d - 2*B*c + 4*B*d + tan(e/2 + (f*x)/
2)^2*(2*A*c - 2*A*d - 2*B*c + 2*B*d) + 2*B*d*tan(e/2 + (f*x)/2))/(f*(a + a*
tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=35

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}$$

```
[Out] B*x/a-(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2814, 2727}

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] (B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= \frac{Bx}{a}-(-A+B)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= \frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.
time = 0.12, size = 79, normalized size = 2.26

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(B(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2A+B(-2+e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*A
 + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.12, size = 42, normalized size = 1.20

method	result	size
derivativedivides	$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$	42
default	$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$	42
risch	$\frac{Bx}{a}-\frac{2A}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2B}{fa\left(e^{i(fx+e)}+i\right)}$	54
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{Bx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{Bx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(2A-2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{(2A-2B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(37) = 74$.
time = 0.51, size = 84, normalized size = 2.40

$$\frac{2\left(B\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{A}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 0.37, size = 70, normalized size = 2.00

$$\frac{Bfx+(Bfx-A+B)\cos(fx+e)+(Bfx+A-B)\sin(fx+e)-A+B}{af\cos(fx+e)+af\sin(fx+e)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A +
B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.
time = 0.62, size = 109, normalized size = 3.11

$$\begin{cases} -\frac{2A}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{Bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{Bfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{2B}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f*
tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*tan
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True))
```

Giac [A]
time = 0.43, size = 40, normalized size = 1.14

$$\frac{\frac{(fx+e)B}{a} - \frac{2(A-B)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

Mupad [B]
time = 12.92, size = 35, normalized size = 1.00

$$\frac{B\,x}{a} - \frac{2\,A-2\,B}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x)),x)
```

```
[Out] (B*x)/a - (2*A - 2*B)/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=101

$$\frac{2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f}-\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

```
[Out] -(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))+2*(-A*d+B*c)*arctan((d+c*tan(1/2
*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 12, 2739, 632, 210}

$$\frac{2(Bc-Ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)
*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx &= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{\int \frac{a(Bc-Ad)}{c+d\sin(e+fx)}\,dx}{a^2(c-d)}\\ &= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)}\\ &= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(2(Bc-Ad))\mathrm{Subst}\left(\int \frac{}{c+2d}\right.}{a(c}\\ &= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}-\frac{(4(Bc-Ad))\mathrm{Subst}\left(\int \frac{}{-4(c}\right.}{}\\ &= \frac{2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f}-\frac{(A-B)\cos(e+}{(c-d)f(a+a\sin(e}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 148, normalized size = 1.47

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\sqrt{c^2-d^2}\,\sin\left(\frac{1}{2}(e+fx)\right)+(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{a(c-d)\sqrt{c^2-d^2}\,f(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e +
f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Sin[e
 + f*x]))
```

Maple [A]
time = 0.49, size = 94, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{2(-Ad+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{af}$
default	$\frac{\frac{2(-Ad+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{af}$
risch	$-\frac{2A}{f(c-d)a\left(e^{i(fx+e)}+i\right)}+\frac{2B}{f(c-d)a\left(e^{i(fx+e)}+i\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}+c^2-d^2}{\sqrt{-c^2+d^2}\,d}\right)Ad}{\sqrt{-c^2+d^2}\,(c-d)fa}+\frac{\ln\left(e^{i(fx+e)}+\frac{ic}{}\right.}{\sqrt{-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f/a*(1/(c-d)*(-A*d+B*c)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))-(A-B)/(c-d)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(99) = 198.
time = 0.40, size = 616, normalized size = 6.10

$$\left[-\frac{2(A-B)c^2-2(A-B)d^2+(Bc-Ad+(Bc-Ad)\cos(fx+e)+(Bc-Ad)\sin(fx+e))\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2+2(c\cos(fx+e)\sin(fx+e)+d\cos(fx+e))\sqrt{-c^2+d^2}}{d^2\cos(fx+e)^2-2cd\sin(fx+e)-c^2-d^2}\right)+2((A-B)c^2-(A-B)d^2)\cos(fx+e)-2((A-B)c^2-(A-B)d^2)\sin(fx+e)}{2((ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f\sin(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f)},-\frac{(A-B)c^2-(A-B)d^2+(Bc-Ad+(Bc-Ad)\cos(fx+e)+(Bc-Ad)\sin(fx+e))\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(fx+e)+d}{\sqrt{c^2-d^2}\cos(fx+e)}\right)+((A-B)c^2-(A-B)d^2)\cos(fx+e)-((A-B)c^2-(A-B)d^2)\sin(fx+e)}{(ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f\sin(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x + e
) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d
*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)*c^2
- (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x
 + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2
*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d + (B*c -
 A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*s
in(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 - (A - B)*d
^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^
2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)
*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 113, normalized size = 1.12

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(Bc-Ad)}{(ac-ad)\sqrt{c^2-d^2}}-\frac{A-B}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A - B
)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

Mupad [B]
time = 13.32, size = 154, normalized size = 1.52

$$\frac{2\,\mathrm{atan}\left(\frac{\frac{(A\,d-B\,c)\,\left(2\,a\,d^2-2\,a\,c\,d\right)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}-\frac{2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,(a\,c-a\,d)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}}{2\,A\,d-2\,B\,c}\right)\,(A\,d-B\,c)}{a\,f\,\sqrt{c+d}\,(c-d)^{3/2}}-\frac{2\,(A-B)}{f\,\left(a+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(c-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*atan((((A*d - B*c)*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2))
- (2*c*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a*c - a*d))/(a*(c + d)^(1/2)*(c - d)
^(3/2)))/(2*A*d - 2*B*c))*(A*d - B*c))/(a*f*(c + d)^(1/2)*(c - d)^(3/2)) -
(2*(A - B))/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=181

$$-\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{3/2}f}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))}-\frac{}{(c-}$$

```
[Out] -2*(A*d*(2*c+d)-B*(c^2+c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^
(1/2))/a/(c-d)/(c^2-d^2)^(3/2)/f+d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)
^2/(c+d)/f/(c+d*sin(f*x+e))-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*
sin(f*x+e))
```

Rubi [A]
time = 0.24, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$-\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(3/2)*f) + (d*(B*(2*c + d) - A*(c +
 2*d))*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])) - ((A - B
)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{\int \frac{a(2Ad}{}}{}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-}{(c-d)f(a+a\sin}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-}{(c-d)f(a+a\sin}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-}{(c-d)f(a+a\sin}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-}{(c-d)f(a+a\sin}$$

$$= -\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}\,f}$$

Mathematica [A]
time = 0.87, size = 209, normalized size = 1.15

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2(-Ad(2c+d)+B(c^2+cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)\sqrt{c^2-d^2}}+\frac{d(Bc-Ad)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))}\right)}{a(c-d)^2f(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-(
A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d^2]
) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c
+ d)*(c + d*Sin[e + f*x]))))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))
```

Maple [A]
time = 0.53, size = 197, normalized size = 1.09

method	result

derivativedivides	$\frac{-\frac{2(A-B)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2Acd+A\,d^2-B\,c^2-Bcd-B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}{2\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}}{(c-d)^2}}{af}$
default	$\frac{-\frac{2(A-B)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2Acd+A\,d^2-B\,c^2-Bcd-B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}{2\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}}{(c-d)^2}}{af}$
risch	$-\frac{2i\left(-3iAcd\,e^{i(fx+e)}-iA\,d^2e^{i(fx+e)}-2Acd\,e^{2i(fx+e)}+3iB\,c^2e^{i(fx+e)}+3iBcd\,e^{i(fx+e)}+B\,c^2e^{2i(fx+e)}+2A\,d^2-2Bcd-2\right.}{(c+d)\left(id-id\,e^{2i(fx+e)}+2c\,e^{i(fx+e)}\right)\left(e^{i(fx+e)}+i\right)(c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-(A-B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)-1/(c-d)^2*((d^2*(A*d-B*c)/(c+d
)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)/(c+d))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1
/2*f*x+1/2*e)+c)+(2*A*c*d+A*d^2-B*c^2-B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(182) = 364$.
time = 0.42, size = 1571, normalized size = 8.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3*
d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (
B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A
 - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A -
 B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A
 - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((A -
B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*x + e
) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d + (
2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x +
 e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f
*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4
- a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f -
 ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos
(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (
(A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B)*c^3*d + (2*A - B
)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (B*c^3 - 2*(A
 - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A - B)*c*d^2
- (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A - B)*c*d^2)*c
os(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 +
(B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (
(A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*
x + e) - ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d
+ (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*
x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6
)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d
^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*
f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*
cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(182) = 364$.
time = 0.48, size = 443, normalized size = 2.45

$$\frac{2\left(\frac{(Bc^2-2\,Acd+Bcd-Ad^2+Bd^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{c^2-d^2}}-\frac{Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+Ac^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,Ac^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,Bc^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Acd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ac^3-Bc^3+Ac^2d-2\,Bc^2d+Acd^2}{(ac^4-ac^3d-ac^2d^2+acd^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 2*((B*c^2 - 2*A*c*d + B*c*d - A*d^2 + B*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^3
- a*c^2*d - a*c*d^2 + a*d^3)*sqrt(c^2 - d^2)) - (A*c^3*tan(1/2*f*x + 1/2*e)
^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c^2*
d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + A*d^3*tan(1/2*f
*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x + 1/2*e) - 3*B*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e) - 3*B*c*d^2*tan(1/2*f*x + 1/2*e) + A*
d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*c^3 + A*c^2*d - 2*B*c^2*d + A*c*d^2)/(
(a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1
/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2
*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

Mupad [B]
time = 15.53, size = 437, normalized size = 2.41

$$\frac{2\,\mathrm{atan}\left(\frac{\frac{\left(2\,a\,c^3\,d-2\,a\,c^2\,d^2-2\,a\,c\,d^3+2\,a\,d^4\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}+\frac{2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^3-a\,c^2\,d-a\,c\,d^2+a\,d^3\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}}{2\,B\,c^2-2\,A\,d^2+2\,B\,d^2-4\,A\,c\,d+2\,B\,c\,d}\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}-\frac{\frac{2\,\left(A\,c^2+A\,d^2-B\,c^2+A\,c\,d-2\,B\,c\,d\right)}{\left(c+d\right)\,{\left(c-d\right)}^2}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d^2+2\,A\,c\,d-3\,B\,c\,d\right)}{c\,{\left(c-d\right)}^2}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(A\,c^3+A\,d^3-B\,c^3+A\,c^2\,d-B\,c\,d^2-B\,c^2\,d\right)}{c\,\left(c+d\right)\,{\left(c-d\right)}^2}}{f\,\left(a\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(a\,c+2\,a\,d\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(a\,c+2\,a\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*atan((((2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d)*(B*c^2 - A*d^2 +
B*d^2 - 2*A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)) + (2*c*tan(e/2 +
(f*x)/2)*(a*c^3 + a*d^3 - a*c*d^2 - a*c^2*d)*(B*c^2 - A*d^2 + B*d^2 - 2*A*c
*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(2*B*c^2 - 2*A*d^2 + 2*B*d^2
- 4*A*c*d + 2*B*c*d))*(B*c^2 - A*d^2 + B*d^2 - 2*A*c*d + B*c*d))/(a*f*(c +
d)^(3/2)*(c - d)^(5/2)) - ((2*(A*c^2 + A*d^2 - B*c^2 + A*c*d - 2*B*c*d))/((
c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*d^2 + 2*A*c*d - 3*B*c*d))/(c*(
c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*c^3 + A*d^3 - B*c^3 + A*c^2*d - B*c*
d^2 - B*c^2*d))/(c*(c + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2)^2*(a*c
+ 2*a*d) + tan(e/2 + (f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)/2)^3))
```

3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=283

$$-\frac{\left(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{5/2}f}-\frac{d(2Ac-3Bc+3Ad-2B}{2a(c-d)^2(c+d)f(c+a}$$

```
[Out] -(3*A*d*(2*c^2+2*c*d+d^2)-B*(2*c^3+4*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(
1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*A*c+3*A
*d-3*B*c-2*B*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2-(A-B)*cos(f
*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2*A*c^2+9*A*c*d+4*
A*d^2-5*B*c^2-6*B*c*d-4*B*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+
e))
```

Rubi [A]

time = 0.38, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$-\frac{\left(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d(2Ac^2+9Acd+4Ad^2-5Bc^2-6Bcd-4Bd^2)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{d(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2af(c-d)^2(c+d)(c+d\sin(e+fx))^2}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -(((3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(5/2
)*f)) - (d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*S
in[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*A*c^2 - 5*B*c^2 + 9*A*c*d - 6*
B*c*d + 4*A*d^2 - 4*B*d^2)*Cos[e + f*x])/(2*a*(c - d)^3*(c + d)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\int \frac{a(3A\cdots}{}}{}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{}{(c-d)f(a+\cdots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{}{(c-d)f(a+\cdots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{}{(c-d)f(a+\cdots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{}{(c-d)f(a+\cdots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{}{(c-d)f(a+\cdots}$$

$$= -\frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\tan\cdots}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [A]
time = 1.16, size = 313, normalized size = 1.11

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2(-3Ad(2c^2+2cd+d^2)+B(2c^3+4c^2d+7cd^2+2d^3))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2\sqrt{c^2-d^2}}+\frac{(c-d)d(Bc-Ad)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))^2}+\frac{d(-Ad(5c+2d)+B(3c^2+2cd+2d^2))\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}\right)}{2a(c-d)^3f(1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-3
*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[
(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) + (d*(
-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*a*(c - d)^3*
f*(1 + Sin[e + f*x]))
```

Maple [A]
time = 1.14, size = 482, normalized size = 1.70

method	result
derivativedivides	$-\frac{2(A-B)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{d^2\left(7A\,c^2d+2Ac\,d^2-2A\,d^3-5B\,c^3-2B\,c^2d\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(6A\,c^4d+2A\,c^3d^2+11A\,c^2d^3+4Ac\,d^4\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2(A-B)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{d^2\left(7A\,c^2d+2Ac\,d^2-2A\,d^3-5B\,c^3-2B\,c^2d\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(6A\,c^4d+2A\,c^3d^2+11A\,c^2d^3+4Ac\,d^4\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a*(-(A-B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)-1/(c-d)^3*((1/2*d^2*(7*A*c^2*d
+2*A*c*d^2-2*A*d^3-5*B*c^3-2*B*c^2*d)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^
3+1/2*d*(6*A*c^4*d+2*A*c^3*d^2+11*A*c^2*d^3+4*A*c*d^4-2*A*d^5-4*B*c^5-2*B*c
^4*d-9*B*c^3*d^2-4*B*c^2*d^3-2*B*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2
*e)^2+1/2*d^2*(17*A*c^2*d+6*A*c*d^2-2*A*d^3-11*B*c^3-6*B*c^2*d-4*B*c*d^2)/c
/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1/2*d*(6*A*c^2*d+2*A*c*d^2-A*d^3-4*B*c^
3-2*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f
*x+1/2*e)+c)^2+1/2*(6*A*c^2*d+6*A*c*d^2+3*A*d^3-2*B*c^3-4*B*c^2*d-7*B*c*d^2
-2*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1628 vs. $2(282) = 564$.

time = 0.46, size = 3348, normalized size = 11.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(4*(A - B)*c^6 - 12*(A - B)*c^4*d^2 + 12*(A - B)*c^2*d^4 - 4*(A - B)*d
^6 - 2*((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3
*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + 2*(4*(A - 2*B)*c^5*d +
 4*(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A
 - B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 - (2*B*c^5 - 2*(3*A - 4*B)*c^
4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 -
 (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4
 - (3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 1
8*(A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 +
 (2*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2
*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*
A - 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11
*B)*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A -
 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B
)*c^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*(A
- B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (7
*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(f*x + e) - 2*
(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*d^6 - ((
2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*(3*A - 2
*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d + (14*A - 13
*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A - 7*B)*c*d^
5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
)), 1/2*(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*
```

```
d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*
(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + (4*(A - 2*B)*c^5*d + 4*
(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A -
B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 + (2*B*c^5 - 2*(3*A - 4*B)*c^4*d
 - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3
*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4 -
(3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 18*(
A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + (2
*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2*d^
3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*A -
 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)
*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*
B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B)*c
^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x + e
))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x +
e))) + (2*(A - B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)
*c^3*d^3 - (7*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(
f*x + e) - (2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A -
B)*d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 -
 3*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d +
 (14*A - 13*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A
- 7*B)*c*d^5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c
^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 +
a*d^9)*f*cos(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^
4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x +
e)^2 - (a*c^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2
*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4
*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^
8 + a*d^9)*f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^
3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(282) = 564.
time = 0.55, size = 753, normalized size = 2.66

$$\frac{\frac{(2Bc^3-6Ac^2d+4Bc^2d-6Acd^2+7Bcd^2-3Ad^3+2Bd^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5)\sqrt{c^2-d^2}}-\frac{2(A-B)}{(ac^3-3ac^2d+3acd^2-ad^3)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{5Bc^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-7Ac^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2Bc^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2Ac^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2Acd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4Bc^5d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Ac^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Bc^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2Ac^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+9Bc^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-11Ac^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4Bc^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4Acd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Bcd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Ad^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+11Bc^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-17Ac^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+6Bc^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6Ac^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4Bc^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Acd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+4Bc^5d-6Ac^4d^2+2Bc^4d^2-2Ac^3d^3+Bc^3d^3+Ac^2d^4}{(ac^7-ac^6d-2ac^5d^2+2ac^4d^3+ac^3d^4-ac^2d^5)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*B*c^3 - 6*A*c^2*d + 4*B*c^2*d - 6*A*c*d^2 + 7*B*c*d^2 - 3*A*d^3 + 2*B*d
^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a
*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) - 2*(A - B)/((a*c^3 - 3*a*c^2*d + 3*a*c*d^
2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)) + (5*B*c^4*d^2*tan(1/2*f*x + 1/2*e)^
3 - 7*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*c^3*d^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^5*tan(1/2*f*x + 1/2*e)^3 +
4*B*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B
*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*
c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 11*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*B*
c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 4*A*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d
^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^6*tan(1/2*f*x + 1/2*e)^2 + 11*B*c^4*d^2*t
an(1/2*f*x + 1/2*e) - 17*A*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*B*c^3*d^3*tan(1
/2*f*x + 1/2*e) - 6*A*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^4*tan(1/2*f*
x + 1/2*e) + 2*A*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^5*d - 6*A*c^4*d^2 + 2*B
*c^4*d^2 - 2*A*c^3*d^3 + B*c^3*d^3 + A*c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5
*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 - a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d
*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

Mupad [B]
time = 17.44, size = 1076, normalized size = 3.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

```
[Out] ((A*d^4 - 2*A*c^4 + 2*B*c^4 - 8*A*c^2*d^2 + 4*B*c^2*d^2 - 2*A*c*d^3 - 4*A*c
^3*d + B*c*d^3 + 8*B*c^3*d)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (ta
n(e/2 + (f*x)/2)^3*(2*A*d^6 - 13*A*c^2*d^4 - 17*A*c^3*d^3 - 22*A*c^4*d^2 +
4*B*c^2*d^4 + 19*B*c^3*d^3 + 23*B*c^4*d^2 - 2*A*c*d^5 - 8*A*c^5*d + 2*B*c*d
^5 + 12*B*c^5*d))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (
tan(e/2 + (f*x)/2)^2*(2*A*d^5 - 4*A*c^5 + 4*B*c^5 - 21*A*c^2*d^3 - 14*A*c^3
*d^2 + 14*B*c^2*d^3 + 17*B*c^3*d^2 - 4*A*c*d^4 - 4*A*c^4*d + 2*B*c*d^4 + 8*
B*c^4*d))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) + (tan(e/2 + (f*x)/2)^4*(2*
A*c^5 - 2*A*d^5 - 2*B*c^5 + 7*A*c^2*d^3 + 2*A*c^3*d^2 - 2*B*c^2*d^3 - 7*B*c
^3*d^2 + 2*A*c*d^4 + 4*A*c^4*d - 4*B*c^4*d))/(c*(c^2 - 2*c*d + d^2)*(c*d^2
- c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(2*A*d^5 - 27*A*c^2*d^3 - 22*A*
c^3*d^2 + 15*B*c^2*d^3 + 29*B*c^3*d^2 - 5*A*c*d^4 - 8*A*c^4*d + 4*B*c*d^4 +
 12*B*c^4*d))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f
*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3*(2*a*c^2 + 4*
```

```
a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c*d) + tan(e/2 +
 (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5)) - (atan((((2*a
*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5 - 2*a*c^5*d)*(2*
B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))
/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (c*tan(e/2 + (f*x)/2)*(a*c^5 - a*d^5 +
 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^
3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))/(a*(c + d)^(5/2)*(c - d
)^(7/2)))/(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2
+ 4*B*c^2*d))*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*
d^2 + 4*B*c^2*d))/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=228

$$\frac{d(2A(3c-2d)d+B(6c^2-12cd+7d^2))\,x}{2a^2}+\frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f}+\frac{d^2}{}$$

```
[Out] 1/2*d*(2*A*(3*c-2*d)*d+B*(6*c^2-12*c*d+7*d^2))*x/a^2+2/3*d*(A*(c^2+6*c*d-5*
d^2)+B*(2*c^2-15*c*d+8*d^2))*cos(f*x+e)/a^2/f+1/6*d^2*(B*(4*c-21*d)+2*A*(c+
6*d))*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*(2*B*(c-4*d)+A*(c+5*d))*cos(f*x+e)*(c
+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))
^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A]

time = 0.35, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3056, 2813}

$$\frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f}+\frac{dx(2Ad(3c-2d)+B(6c^2-12cd+7d^2))}{2a^2}+\frac{d^2(2A(c+6d)+B(4c-21d))\sin(e+fx)\cos(e+fx)}{6a^2f}-\frac{(A(c+5d)+2B(c-4d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(\sin(e+fx)+1)}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*a^2) + (2*d*(A*(c
^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d + 8*d^2))*Cos[e + f*x])/(3*a^2*f) +
 (d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f)
 - ((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a
^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3
*f*(a + a*Sin[e + f*x])^2)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx &= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx)}{}}{} \\ &= -\frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))}{3a^2f(1+\sin(e+fx))} \\ &= \frac{d(2A(3c-2d)d+B(6c^2-12cd+7d^2))\,x}{2a^2} + \frac{2d(A(c^2+6c}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 547 vs. 2(228) = 456.
time = 2.39, size = 547, normalized size = 2.40

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\left(8Ad\left(6c^2+d^2(5-6e-6fx)+3cd(-4+3e+3fx)\right)+B\left(16c^3+24c^2d(-4+3e+3fx)-24cd^2(-5+6e+6fx)+7d^3(-7+12e+12fx)\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)-\left(4A\left(4c^3+24c^2d+d^3(41-12e-12fx)+6cd^2(-10+3e+3fx)\right)+B\left(32c^3+24c^2d(-10+3e+3fx)-12cd^2(-41+12e+12fx)+d^3(-239+84e+84fx)\right)\right)\cos\left(\frac{3}{2}(e+fx)\right)+3\left(d^2(12Bc+4Ad-5Bd)\cos\left(\frac{5}{2}(e+fx)\right)+Bd^3\cos\left(\frac{7}{2}(e+fx)\right)+2\left(8Ac^3+8Bc^3+24Ac^2d-72Bc^2d-72Acd^2+108Bcd^2+36Ad^3-50Bd^3+48Bc^2de+48Acd^2e-96Bcd^2e-32Ad^3e+56Bd^3e+48Bc^2dfx+48Acd^2fx-96Bcd^2fx-32Ad^3fx+56Bd^3fx+d\left(8Ad(3c(e+fx)-2d(1+e+fx))+B\left(24c^2(e+fx)-48cd(1+e+fx)+d^2(27+28e+28fx)\right)\right)\right)\cos(e+fx)+2d^2(-6Bc-2Ad+3Bd)\cos(2(e+fx))+Bd^3\cos(3(e+fx))\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{48a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(8*A*d*(6*c^2 + d^2*(5 - 6*e - 6*
f*x) + 3*c*d*(-4 + 3*e + 3*f*x)) + B*(16*c^3 + 24*c^2*d*(-4 + 3*e + 3*f*x)
- 24*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x)))*Cos[(e + f*x)/
2] - (4*A*(4*c^3 + 24*c^2*d + d^3*(41 - 12*e - 12*f*x) + 6*c*d^2*(-10 + 3*e
 + 3*f*x)) + B*(32*c^3 + 24*c^2*d*(-10 + 3*e + 3*f*x) - 12*c*d^2*(-41 + 12*
e + 12*f*x) + d^3*(-239 + 84*e + 84*f*x)))*Cos[(3*(e + f*x))/2] + 3*(d^2*(1
2*B*c + 4*A*d - 5*B*d)*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] +
2*(8*A*c^3 + 8*B*c^3 + 24*A*c^2*d - 72*B*c^2*d - 72*A*c*d^2 + 108*B*c*d^2 +
 36*A*d^3 - 50*B*d^3 + 48*B*c^2*d*e + 48*A*c*d^2*e - 96*B*c*d^2*e - 32*A*d^
3*e + 56*B*d^3*e + 48*B*c^2*d*f*x + 48*A*c*d^2*f*x - 96*B*c*d^2*f*x - 32*A*
d^3*f*x + 56*B*d^3*f*x + d*(8*A*d*(3*c*(e + f*x) - 2*d*(1 + e + f*x)) + B*(
24*c^2*(e + f*x) - 48*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x)))*Cos[e
+ f*x] + 2*d^2*(-6*B*c - 2*A*d + 3*B*d)*Cos[2*(e + f*x)] + B*d^3*Cos[3*(e +
 f*x)])*Sin[(e + f*x)/2])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.34, size = 340, normalized size = 1.49

method	result

derivativedivides	$\frac{-\frac{2\left(A c^3-3Ac d^2+2A d^3-3B c^2 d+6Bc d^2-3B d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A c^3+6A c^2 d-6Ac d^2+2A d^3+2B c^3-6B c^2 d+6Bc d^2-2B d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A c^3-6A\right.}{}}{}$
default	$\frac{-\frac{2\left(A c^3-3Ac d^2+2A d^3-3B c^2 d+6Bc d^2-3B d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A c^3+6A c^2 d-6Ac d^2+2A d^3+2B c^3-6B c^2 d+6Bc d^2-2B d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A c^3-6A\right.}{}}{}$
risch	$\frac{3d^2xAc}{a^2}-\frac{2d^3xA}{a^2}+\frac{3dxB c^2}{a^2}-\frac{6d^2xBc}{a^2}+\frac{7d^3xB}{2a^2}+\frac{iB d^3 e^{2i(fx+e)}}{8a^2 f}-\frac{d^3 e^{i(fx+e)}A}{2a^2 f}-\frac{3d^2 e^{i(fx+e)}Bc}{2a^2 f}+d^3$
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(-(A*c^3-3*A*c*d^2+2*A*d^3-3*B*c^2*d+6*B*c*d^2-3*B*d^3)/(tan(1/2*f*
x+1/2*e)+1)-1/2*(-2*A*c^3+6*A*c^2*d-6*A*c*d^2+2*A*d^3+2*B*c^3-6*B*c^2*d+6*B
*c*d^2-2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c^3-6*A*c^2*d+6*A*c*d^2-2
*A*d^3-2*B*c^3+6*B*c^2*d-6*B*c*d^2+2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^3+d*((1/
2*B*d^2*tan(1/2*f*x+1/2*e)^3+(-A*d^2-3*B*c*d+2*B*d^2)*tan(1/2*f*x+1/2*e)^2-
1/2*B*d^2*tan(1/2*f*x+1/2*e)-A*d^2-3*B*c*d+2*B*d^2)/(1+tan(1/2*f*x+1/2*e)^2
)^2+1/2*(6*A*c*d-4*A*d^2+6*B*c^2-12*B*c*d+7*B*d^2)*arctan(tan(1/2*f*x+1/2*e
))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. $2(227) = 454$.
time = 0.56, size = 1500, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*d^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 12*B*c*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
```

```
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) - 4*A*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^2) + 6*B*c^2*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*A*c*d^2*((9*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^2) - 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3) - 2*B*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 6*A*c^2*d*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(227) = 454$.
time = 0.40, size = 596, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/6*(3*B*d^3*cos(f*x + e)^4 - 2*(A - B)*c^3 + 6*(A - B)*c^2*d - 6*(A - B)*
c*d^2 + 2*(A - B)*d^3 + 6*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^3 + 6*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - (2*(A + 2*B)*c^3 + 6*(2
*A - 5*B)*c^2*d - 6*(5*A - 11*B)*c*d^2 + (22*A - 31*B)*d^3 + 3*(6*B*c^2*d +
 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e)^2 - (2*(2*A + B)*c^
3 + 6*(A - 4*B)*c^2*d - 6*(4*A - 13*B)*c*d^2 + 2*(13*A - 19*B)*d^3 - 3*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e) + (3*B*d^3*
cos(f*x + e)^3 + 2*(A - B)*c^3 - 6*(A - B)*c^2*d + 6*(A - B)*c*d^2 - 2*(A -
```

```
B)*d^3 + 6*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - 3*(6*B*
c*d^2 + (2*A - 3*B)*d^3)*cos(f*x + e)^2 - (2*(A + 2*B)*c^3 + 6*(2*A - 5*B)*
c^2*d - 6*(5*A - 14*B)*c*d^2 + 4*(7*A - 10*B)*d^3 - 3*(6*B*c^2*d + 6*(A - 2
*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14612 vs. $2(216) = 432$.

time = 9.25, size = 14612, normalized size = 64.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 1
8*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*ta
n(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x
/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c**3*tan(e/2 + f*x/2
)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**
2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
 6*a**2*f) - 32*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*A*c**3*tan(e/2 + f*x/
2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a*
*2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/
2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) - 28*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c**3*tan(e/2 + f*x
/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2
*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
+ f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) - 8*A*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 36*A*c**2*d*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c
**2*d*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
```

```
 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 72*A*c**2*d*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2
4*A*c**2*d*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*ta
n(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x
/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18
*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 36*A*c**2*d*tan(e/2 + f*x/2)/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 12*A*c**2*d/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6
+ 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f
*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
f*x/2) + 6*a**2*f) + 18*A*c*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2
+ f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5
 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 54*A*c*d**
2*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a*
*2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/
2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)*
*3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f)
 + 126*A*c*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 126*A*c*d**2*f*x*tan(e/2 +
f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) + 90*A*c*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f
*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*t
an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(227) = 454.
time = 0.47, size = 494, normalized size = 2.17

$$\frac{\frac{3\,(6\,Bc^2d+6\,Acd^2-12\,Bcd^2-4\,Ad^3+7\,Bd^3)(fx+e)}{a^2}+\frac{6\left(Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,Bcd^2-2\,Ad^3+4\,Bd^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}-\frac{4\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,Bc^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,Acd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+18\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Ac^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-27\,Bc^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-27\,Acd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+45\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-21\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac^3+Bc^3+3\,Ac^2d-12\,Bc^2d-12\,Acd^2+21\,Bcd^2+7\,Ad^3-10\,Bd^3\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/a
^2 + 6*(B*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2
*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*tan(1/2*f*x + 1/2*e)^2 - B*d^3*tan(
1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((tan(1/2*f*x + 1/2*e)^2
+ 1)^2*a^2) - 4*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*tan(1/2*f*x + 1
/2*e)^2 - 9*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e
)^2 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*A
*c^3*tan(1/2*f*x + 1/2*e) + 3*B*c^3*tan(1/2*f*x + 1/2*e) + 9*A*c^2*d*tan(1/
2*f*x + 1/2*e) - 27*B*c^2*d*tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*tan(1/2*f*x +
 1/2*e) + 45*B*c*d^2*tan(1/2*f*x + 1/2*e) + 15*A*d^3*tan(1/2*f*x + 1/2*e) -
 21*B*d^3*tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 12*B*c^2*d -
 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) +
 1)^3))/f
```

Mupad [B]
time = 16.35, size = 663, normalized size = 2.91

$$\frac{d\,\mathrm{atan}\left(\frac{d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,B\,c^2-4\,A\,d^2+7\,B\,d^2+6\,A\,c\,d-12\,B\,c\,d\right)}{7\,B\,d^3-4\,A\,d^3+6\,A\,c\,d^2-12\,B\,c\,d^2+6\,B\,c^2\,d}\right)\,\left(6\,B\,c^2-4\,A\,d^2+7\,B\,d^2+6\,A\,c\,d-12\,B\,c\,d\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,c^3+16\,A\,d^3+2\,B\,c^3-25\,B\,d^3-18\,A\,c\,d^2+6\,A\,c^2\,d+48\,B\,c\,d^2-18\,B\,c^2\,d\right)+\frac{4\,A\,c^3}{3}+\frac{20\,A\,d^3}{3}+\frac{2\,B\,c^3}{3}-\frac{32\,B\,d^3}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(2\,A\,c^3+4\,A\,d^3-7\,B\,d^3-6\,A\,c\,d^2+12\,B\,c\,d^2-6\,B\,c^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(2\,A\,c^3+12\,A\,d^3+2\,B\,c^3-21\,B\,d^3-18\,A\,c\,d^2+6\,A\,c^2\,d+36\,B\,c\,d^2-18\,B\,c^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(4\,A\,c^3+28\,A\,d^3+4\,B\,c^3-42\,B\,d^3-36\,A\,c\,d^2+12\,A\,c^2\,d+84\,B\,c\,d^2-36\,B\,c^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{16\,A\,c^3}{3}+\frac{56\,A\,d^3}{3}+\frac{2\,B\,c^3}{3}-\frac{98\,B\,d^3}{3}-20\,A\,c\,d^2+2\,A\,c^2\,d+56\,B\,c\,d^2-20\,B\,c^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{14\,A\,c^3}{3}+\frac{64\,A\,d^3}{3}+\frac{4\,B\,c^3}{3}-\frac{97\,B\,d^3}{3}-22\,A\,c\,d^2+4\,A\,c^2\,d+64\,B\,c\,d^2-22\,B\,c^2\,d\right)-8\,A\,c\,d^2+2\,A\,c^2\,d+20\,B\,c\,d^2-8\,B\,c^2\,d}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+5\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+7\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+7\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+5\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d*atan((d*tan(e/2 + (f*x)/2)*(6*B*c^2 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B
*c*d))/(7*B*d^3 - 4*A*d^3 + 6*A*c*d^2 - 12*B*c*d^2 + 6*B*c^2*d))*(6*B*c^2 -
 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B*c*d))/(a^2*f) - (tan(e/2 + (f*x)/2)*(2*
A*c^3 + 16*A*d^3 + 2*B*c^3 - 25*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 48*B*c*d^2
 - 18*B*c^2*d) + (4*A*c^3)/3 + (20*A*d^3)/3 + (2*B*c^3)/3 - (32*B*d^3)/3 +
tan(e/2 + (f*x)/2)^6*(2*A*c^3 + 4*A*d^3 - 7*B*d^3 - 6*A*c*d^2 + 12*B*c*d^2
- 6*B*c^2*d) + tan(e/2 + (f*x)/2)^5*(2*A*c^3 + 12*A*d^3 + 2*B*c^3 - 21*B*d^
3 - 18*A*c*d^2 + 6*A*c^2*d + 36*B*c*d^2 - 18*B*c^2*d) + tan(e/2 + (f*x)/2)^
3*(4*A*c^3 + 28*A*d^3 + 4*B*c^3 - 42*B*d^3 - 36*A*c*d^2 + 12*A*c^2*d + 84*B
*c*d^2 - 36*B*c^2*d) + tan(e/2 + (f*x)/2)^4*((16*A*c^3)/3 + (56*A*d^3)/3 +
(2*B*c^3)/3 - (98*B*d^3)/3 - 20*A*c*d^2 + 2*A*c^2*d + 56*B*c*d^2 - 20*B*c^2
*d) + tan(e/2 + (f*x)/2)^2*((14*A*c^3)/3 + (64*A*d^3)/3 + (4*B*c^3)/3 - (97
*B*d^3)/3 - 22*A*c*d^2 + 4*A*c^2*d + 64*B*c*d^2 - 22*B*c^2*d) - 8*A*c*d^2 +
 2*A*c^2*d + 20*B*c*d^2 - 8*B*c^2*d)/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2
*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/
2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*
tan(e/2 + (f*x)/2)))
```

3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=132

$$\frac{d(2B(c-d)+Ad)x}{a^2}+\frac{(A-4B)d^2\cos(e+fx)}{3a^2f}-\frac{(c-d)(2B(c-3d)+A(c+3d))\cos(e+fx)}{3a^2f(1+\sin(e+fx))}-\frac{(A-B)\cos\ldots}{3f(\ldots}$$

```
[Out] d*(2*B*(c-d)+A*d)*x/a^2+1/3*(A-4*B)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)*(2*B*(c-
3*d)+A*(c+3*d))*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*s
in(f*x+e))^2/f/(a+a*sin(f*x+e))^2
```

Rubi [A]

time = 0.34, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3056, 3047, 3102, 2814, 2727}

$$-\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{dx(Ad+2B(c-d))}{a^2}+\frac{d^2(A-4B)\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*B*(c - d) + A*d)*x)/a^2 + ((A - 4*B)*d^2*Cos[e + f*x])/(3*a^2*f) - ((
c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}+\frac{\int \frac{(c+d\sin(e+f}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}+\frac{\int \frac{ac(2B(c-d)+}{}}{}$$

$$= \frac{(A-4B)d^2\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)(c+d\sin}{3f(a+a\sin(e+fx))}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2}+\frac{(A-4B)d^2\cos(e+fx)}{3a^2f}-\frac{(A-}{}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2}+\frac{(A-4B)d^2\cos(e+fx)}{3a^2f}-\frac{(c-d}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(132) = 264.
time = 1.14, size = 338, normalized size = 2.56

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(Ad(4c+d(-4+3e+3fx))+B(2c^2+d^2(5-6e-6fx)+2cd(-4+3e+3fx)))\cos\left(\frac{1}{2}(e+fx)\right)-(B(8c^2+d^2(41-12e-12fx)+4cd(-10+3e+3fx))+2A(2c^2+8cd+d^2(-10+3e+3fx)))\cos\left(\frac{3}{2}(e+fx)\right)+3Bd^2\cos\left(\frac{5}{2}(e+fx)\right)+6(2Ac^2+2Bc^2+4Acd-12Bcd-6Ad^2+9Bd^2+8Bcde+4Ad^2e-8Bd^2e+8Bcdfx+4Ad^2fx-8Bd^2fx-2d(-2Bc(e+fx)-Ad(e+fx)+2Bd(1+e+fx))\cos(e+fx)-Bd^2\cos(2(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{12a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x))
 + B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e + f
*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x))
+ 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] + 3*B
*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d - 6*A
*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*A*d^2*
f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1 + e +
f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12*a^2*f*(
1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.32, size = 193, normalized size = 1.46

method	result
derivativedivides	$\frac{-\frac{2\left(A\,c^2-A\,d^2-2Bcd+2B\,d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^2+4Acd-2A\,d^2+2B\,c^2-4Bcd+2B\,d^2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A\,c^2-4Acd+2A\,d^2-2B\,c^2+4Bcd-2B\,d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2d\left(-\cdots\right.}{a^2 f}$
default	$\frac{-\frac{2\left(A\,c^2-A\,d^2-2Bcd+2B\,d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^2+4Acd-2A\,d^2+2B\,c^2-4Bcd+2B\,d^2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A\,c^2-4Acd+2A\,d^2-2B\,c^2+4Bcd-2B\,d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2d\left(-\cdots\right.}{a^2 f}$
risch	$\frac{d^2xA}{a^2}+\frac{2dxBc}{a^2}-\frac{2d^2xB}{a^2}-\frac{B\,d^2\mathrm{e}^{i(fx+e)}}{2a^2f}-\frac{B\,d^2\mathrm{e}^{-i(fx+e)}}{2a^2f}-\frac{2\left(-4Acd+3B\,c^2\mathrm{e}^{2i(fx+e)}+3iA\,c^2\mathrm{e}^{i(fx+e)}-2B\,c^2\cdots\right.}{\cdots}$
norman	$\frac{\frac{d(Ad+2Bc-2Bd)x}{a}+\frac{d(Ad+2Bc-2Bd)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4A\,c^2+4Acd-8A\,d^2+2B\,c^2-16Bcd+20B\,d^2}{3af}-\frac{\left(2A\,c^2-2A\,d^2-4Bcd+4B\,d^2\cdots\right.}{af}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(-(A*c^2-A*d^2-2*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c^
2+4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*
A*c^2-4*A*c*d+2*A*d^2-2*B*c^2+4*B*c*d-2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^3+d*(
-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+2*B*c-2*B*d)*arctan(tan(1/2*f*x+1/2*e)))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(132) = 264.
time = 0.52, size = 901, normalized size = 6.83

$$-\frac{2\left(2\,Bd^2\left(\frac{\frac{12\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+5}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4\,a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-2\,Bcd\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-Ad^2\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{Ac^2\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{Bc^2\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{2\,Acd\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*
B*c*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*d^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*c*d*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(132) = 264$.
time = 0.40, size = 385, normalized size = 2.92

$$-\frac{3\,Bd^2\cos\left(fx+e\right)^3-(A-B)c^2+2\,(A-B)cd-(A-B)d^2+6\,(2\,Bcd+(A-2\,B)d^2)fx-((A+2\,B)c^2+2\,(2\,A-5\,B)cd-(5\,A-11\,B)d^2+3\,(2\,Bcd+(A-2\,B)d^2)fx)\cos\left(fx+e\right)^2-((2\,A+B)c^2+2\,(A-4\,B)cd-(4\,A-13\,B)d^2-3\,(2\,Bcd+(A-2\,B)d^2)fx)\cos\left(fx+e\right)-(3\,Bd^2\cos\left(fx+e\right)^2-(A-B)c^2+2\,(A-B)cd-(A-B)d^2-6\,(2\,Bcd+(A-2\,B)d^2)fx+((A+2\,B)c^2+2\,(2\,A-5\,B)cd-(5\,A-14\,B)d^2-3\,(2\,Bcd+(A-2\,B)d^2)fx)\cos\left(fx+e\right))\sin\left(fx+e\right)}{3\,\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5*A
 - 11*B)*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e)^2 - ((2*A + B)
*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x
)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d - (A
 - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A - 5*B
)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e))*s
in(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*
cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5358 vs. $2(121) = 242$.

time = 4.91, size = 5358, normalized size = 40.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e/2
+ f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c**2/(3*a**2*f*tan(e/2 + f*
x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*A*
c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d*tan(e/2 + f*x/2)**2/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 12*A*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d/(3*a**2*f*tan
(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2
)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 3*A*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 3*A*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
```

```
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*A*d**2*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
 14*A*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 8*A*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
+ f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/2 + f*x/2)**3/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) - 2*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 2*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B*c*d*f*x*tan(e/2 +
 f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(132) = 264$.
time = 0.53, size = 277, normalized size = 2.10

$$\frac{\frac{3\,(2\,Bcd+Ad^2-2\,Bd^2)(fx+e)}{a^2}-\frac{6\,Bd^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{2\left(3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-18\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac^2+Bc^2+2\,Acd-8\,Bcd-4\,Ad^2+7\,Bd^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(2*B*c*d + A*d^2 - 2*B*d^2)*(f*x + e)/a^2 - 6*B*d^2/((tan(1/2*f*x +
1/2*e)^2 + 1)*a^2) - 2*(3*A*c^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c*d*tan(1/2*f*
x + 1/2*e)^2 - 3*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*d^2*tan(1/2*f*x + 1/2*e
)^2 + 3*A*c^2*tan(1/2*f*x + 1/2*e) + 3*B*c^2*tan(1/2*f*x + 1/2*e) + 6*A*c*d
*tan(1/2*f*x + 1/2*e) - 18*B*c*d*tan(1/2*f*x + 1/2*e) - 9*A*d^2*tan(1/2*f*x
```

```
 + 1/2*e) + 15*B*d^2*tan(1/2*f*x + 1/2*e) + 2*A*c^2 + B*c^2 + 2*A*c*d - 8*B
*c*d - 4*A*d^2 + 7*B*d^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 16.04, size = 365, normalized size = 2.77

$$\frac{2\,d\,\mathrm{atan}\left(\frac{2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d+2\,B\,c-2\,B\,d\right)}{2\,A\,d^2-4\,B\,d^2+4\,B\,c\,d}\right)\,\left(A\,d+2\,B\,c-2\,B\,d\right)}{a^2\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,A\,c^2-6\,A\,d^2+2\,B\,c^2+12\,B\,d^2+4\,A\,c\,d-12\,B\,c\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{10\,A\,c^2}{3}-\frac{14\,A\,d^2}{3}+\frac{2\,B\,c^2}{3}+\frac{44\,B\,d^2}{3}+\frac{4\,A\,c\,d}{3}-\frac{28\,B\,c\,d}{3}\right)+\frac{4\,A\,c^2}{3}-\frac{8\,A\,d^2}{3}+\frac{2\,B\,c^2}{3}+\frac{20\,B\,d^2}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,A\,c^2-2\,A\,d^2+4\,B\,d^2-4\,B\,c\,d\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,c^2-6\,A\,d^2+2\,B\,c^2+16\,B\,d^2+4\,A\,c\,d-12\,B\,c\,d\right)+\frac{4\,A\,c\,d}{3}-\frac{16\,B\,c\,d}{3}}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+4\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+4\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*d*atan((2*d*tan(e/2 + (f*x)/2)*(A*d + 2*B*c - 2*B*d))/(2*A*d^2 - 4*B*d^2
 + 4*B*c*d))*(A*d + 2*B*c - 2*B*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(2*A*c^
2 - 6*A*d^2 + 2*B*c^2 + 12*B*d^2 + 4*A*c*d - 12*B*c*d) + tan(e/2 + (f*x)/2)
^2*((10*A*c^2)/3 - (14*A*d^2)/3 + (2*B*c^2)/3 + (44*B*d^2)/3 + (4*A*c*d)/3
- (28*B*c*d)/3) + (4*A*c^2)/3 - (8*A*d^2)/3 + (2*B*c^2)/3 + (20*B*d^2)/3 +
tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*A*d^2 + 4*B*d^2 - 4*B*c*d) + tan(e/2 + (f
*x)/2)*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 16*B*d^2 + 4*A*c*d - 12*B*c*d) + (4*A
*c*d)/3 - (16*B*c*d)/3)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(e/2 + (f
*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 + a^2 + 3*
a^2*tan(e/2 + (f*x)/2)))
```

3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{Bdx}{a^2}-\frac{(Ac+2Bc+2Ad-5Bd)\cos(e+fx)}{3a^2f(1+\sin(e+fx))}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

```
[Out] B*d*x/a^2-1/3*(A*c+2*A*d+2*B*c-5*B*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(
A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2
```

Rubi [A]

time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.121$, Rules used = {3047, 3098, 2814, 2727}

$$-\frac{(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (B*d*x)/a^2 - ((A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((A - B)*(c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
```

```
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2B(c-d)+A(c+2d))-3aBd\sin\cdots}{a+a\sin(e+fx)}}{3a^2}$$

$$= \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(Ac+2Bc+2Ad-\cdots}{3\cdots}$$

$$= \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{(Ac+2Bc+2Ad-\cdots}{3f\left(a^2+a^2\sin\cdots\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.
time = 0.24, size = 180, normalized size = 2.12

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(Ac+2Bc+2Ad-5Bd)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+3Bd(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\right)}{3a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c +
2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3
*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[
e + f*x])^2)
```

Maple [A]
time = 0.28, size = 110, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{2(Ac-Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2Ad+2Bc-2Bd}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2Ad-2Bc+2Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2Bd\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}$
default	$\frac{-\frac{2(Ac-Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2Ad+2Bc-2Bd}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2Ad-2Bc+2Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2Bd\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}$

risch	$\frac{Bdx}{a^2} - \frac{2(-Ac-2Ad+3iAc\,\mathrm{e}^{i(fx+e)}+3id\,\mathrm{e}^{i(fx+e)}A-2Bc+5Bd+3iBc\,\mathrm{e}^{i(fx+e)}-9iBd\,\mathrm{e}^{i(fx+e)}+3Ad\,\mathrm{e}^{2i(fx+e)}+3Bc\,\mathrm{e}}{3f\,a^2(\mathrm{e}^{i(fx+e)}+i)^3}$
norman	$\frac{\frac{xBd}{a}+\frac{xBd\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4Ac+2Ad+2Bc-8Bd}{3af}-\frac{(2Ac-2Bd)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(16Ac+2Ad+2Bc-20Bd)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-(1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^2*(-(A*c-B*d)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c+2*A*d+2*B*c-2*B*d)/(
tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c-2*A*d-2*B*c+2*B*d)/(tan(1/2*f*x+1/2*e)+1
)^3+B*d*arctan(tan(1/2*f*x+1/2*e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(85) = 170$.
time = 0.51, size = 492, normalized size = 5.79

$$\frac{2\left(Bd\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{Bc\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{Ad\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*c*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - B*c*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*d*(3
*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(85) = 170$.
time = 0.40, size = 216, normalized size = 2.54

$$-\frac{6\,Bdfx-(3\,Bdfx+(A+2\,B)c+(2\,A-5\,B)d)\cos(fx+e)^2-(A-B)c+(A-B)d+(3\,Bdfx-(2\,A+B)c-(A-4\,B)d)\cos(fx+e)+(6\,Bdfx+(A-B)c-(A-B)d+(3\,Bdfx-(A+2\,B)c-(2\,A-5\,B)d)\cos(fx+e))\sin(fx+e)}{3\left(a^2f\cos(fx+e)^2-a^2f\cos(fx+e)-2\,a^2f-(a^2f\cos(fx+e)+2\,a^2f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2
- (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x +
 e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2*A
- 5*B)*d)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
 + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(83) = 166$.
time = 2.43, size = 1062, normalized size = 12.49

$$\begin{cases} -\frac{6Ac\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6Ac\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{4Ac}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6Ad\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{2Ad}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6Bc\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{2Bc}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{3Bdfx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{9Bdfx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{9Bdfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{3Bdfx}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{6Bd\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{18Bd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} + \frac{8Bd}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} & \text{for}\: f \neq 0 \\ \frac{x(A+B\sin(e))(c+d\sin(e))}{(a\sin(e)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c*tan
(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*d/(3*a**2*f*tan(e/2
 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B*c/(3*a**2
*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/
2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*B*d*f*x*tan
(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/(3*a**2*f*tan(e/2 + f*x
/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 6*B*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan
(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*B*d*tan(e/2 +
 f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*B*d/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f,
0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a)**2, True))
```

Giac [A]
time = 0.51, size = 141, normalized size = 1.66

$$\frac{\frac{3(fx+e)Bd}{a^2} - \frac{2\left(3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 9Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 2Ac + Bc + Ad - 4Bd\right)}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*d/a^2 - 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 3*B*d*tan(1/2*
f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) +
3*A*d*tan(1/2*f*x + 1/2*e) - 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c + A*d
 - 4*B*d)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 13.74, size = 94, normalized size = 1.11

$$\frac{B\,d\,x}{a^2}-\frac{\left(2\,A\,c-2\,B\,d\right)\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(2\,A\,c+2\,A\,d+2\,B\,c-6\,B\,d\right)\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{4\,A\,c}{3}+\frac{2\,A\,d}{3}+\frac{2\,B\,c}{3}-\frac{8\,B\,d}{3}}{a^2\,f\,{\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (B*d*x)/a^2 - ((4*A*c)/3 + (2*A*d)/3 + (2*B*c)/3 - (8*B*d)/3 + tan(e/2 + (f
*x)/2)*(2*A*c + 2*A*d + 2*B*c - 6*B*d) + tan(e/2 + (f*x)/2)^2*(2*A*c - 2*B*
d))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3)
```

3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(A+2B)\cos(e+fx)}{3f\,(a^2+a^2\sin(e+fx))}$$

```
[Out] -1/3*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(A+2*B)*cos(f*x+e)/f/(a^2+a^
2*sin(f*x+e))
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2829, 2727}

$$-\frac{(A+2B)\cos(e+fx)}{3f\,(a^2\sin(e+fx)+a^2)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e +
 f*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx &= -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(A+2B)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(A+2B)\cos(e+fx)}{3f\,(a^2+a^2\sin(e+fx))}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.66

$$-\frac{\cos(e+fx)(2A+B+(A+2B)\sin(e+fx))}{3a^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.20, size = 70, normalized size = 1.08

method	result	size
risch	$-\frac{2\left(-A+3iA\,{\mathrm e}^{i(fx+e)}+3iB\,{\mathrm e}^{i(fx+e)}+3B\,{\mathrm e}^{2i(fx+e)}-2B\right)}{3f\,a^{2}\left({\mathrm e}^{i(fx+e)}+i\right)^{3}}$	68
derivativedivides	$\frac{-\frac{-2A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A-2B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{2}f}$	70
default	$\frac{-\frac{-2A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A-2B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{2}f}$	70
norman	$\frac{-\frac{4A+2B}{3af}-\frac{2A\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2(5A+B)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(2A+2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2(A+B)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{a\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^2*(-1/2*(-2*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2-A/(tan(1/2*f*x+1/2*e)+1)-1/3*(2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(65) = 130.
time = 0.31, size = 232, normalized size = 3.57

$$-\frac{2\left(\frac{A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{B\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
```

```
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [A]
time = 0.36, size = 125, normalized size = 1.92

$$\frac{(A+2\,B)\cos\left(fx+e\right)^{2}+(2\,A+B)\cos\left(fx+e\right)+((A+2\,B)\cos\left(fx+e\right)-A+B)\sin\left(fx+e\right)+A-B}{3\,\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2\,a^{2}f-(a^{2}f\cos\left(fx+e\right)+2\,a^{2}f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*x
 + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(56) = 112$.
time = 1.29, size = 372, normalized size = 5.72

$$\begin{cases} -\frac{6A\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{6A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{4A}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{6B\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{2B}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f} & \text{for}\: f\neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*
a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f),
Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**2, True))
```

Giac [A]
time = 0.48, size = 68, normalized size = 1.05

$$-\frac{2\,\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,A+B\right)}{3\,a^{2}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*f*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)

Mupad [B]
time = 13.39, size = 97, normalized size = 1.49

$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,A}{2}+\frac{B}{2}-\frac{A\,\cos(e+f\,x)}{2}+\frac{B\,\cos(e+f\,x)}{2}+\frac{3\,A\,\sin(e+f\,x)}{2}+\frac{3\,B\,\sin(e+f\,x)}{2}\right)}{3\,a^2\,f\,\left(\frac{3\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}-\frac{\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)

[Out] -(2*cos(e/2 + (f*x)/2)*((5*A)/2 + B/2 - (A*cos(e + f*x))/2 + (B*cos(e + f*x))/2 + (3*A*sin(e + f*x))/2 + (3*B*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/2))

3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=152

$$-\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}\,f}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

```
[Out] -1/3*(A*(c-4*d)+B*(2*c+d))*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))-1/3*(A-B
)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2-2*d*(-A*d+B*c)*arctan((d+c*tan(1/2*
f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 12, 2739, 632, 210}

$$-\frac{2d(Bc-Ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c
 - d)^2*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/(3*
a^2*(c - d)^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(
a + a*Sin[e + f*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2Bc+A(c-3d))-a(A-B)}{(a+a\sin(e+fx))(c+d\sin}}{3a^2(c-d)}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos}{3(c-d)f(a+a}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos}{3(c-d)f(a+a}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos}{3(c-d)f(a+a}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos}{3(c-d)f(a+a}$$

$$= -\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}\,f} - \frac{(A(c-4d)+B}{3a^2(c-d)^2f}$$

Mathematica [A]
time = 0.44, size = 229, normalized size = 1.51

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(A(c-4d)+B(2c+d))\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{6d(-Bc+Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{\sqrt{c^2-d^2}}\right)}{3a^2(c-d)^2f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d) +
 B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6
*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]))/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])^2)
```

Maple [A]
time = 0.61, size = 159, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{-2A+2B}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(2A-2B)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(Ac-2Ad+Bd)}{(c-d)^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d(Ad-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{(c-d)^{2}\sqrt{c^{2}-d^{2}}}}{a^{2}f}$
default	$\frac{-\frac{-2A+2B}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(2A-2B)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(Ac-2Ad+Bd)}{(c-d)^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d(Ad-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{(c-d)^{2}\sqrt{c^{2}-d^{2}}}}{a^{2}f}$
risch	$\frac{\frac{2Ac}{3}-\frac{8Ad}{3}-2iAc\,e^{i(fx+e)}+6id\,e^{i(fx+e)}A+2Ad\,e^{2i(fx+e)}+\frac{4Bc}{3}+\frac{2Bd}{3}-2iBc\,e^{i(fx+e)}-2iBd\,e^{i(fx+e)}-2Bc\,e^{2i(fx+e)}}{\left(e^{i(fx+e)}+i\right)^{3}(c-d)^{2}f\,a^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^2*(-1/2*(-2*A+2*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A-2*B)/(c-d)
/(tan(1/2*f*x+1/2*e)+1)^3-(A*c-2*A*d+B*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)+d*
(A*d-B*c)/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(148) = 296$.
time = 0.41, size = 1318, normalized size = 8.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 + 2
*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*
x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d -
 A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin
(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sq
rt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*
((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos
(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 -
((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x
 + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos
(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e
) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2
*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d +
 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 - (A - B)*c^2*d
- (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B
)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A
*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 +
(B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(
f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^3 - (5*A - 2*B
)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos(f*x + e) - ((A - B)*c^3 -
(A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^
2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^
4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^
2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^
4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*si
n(f*x + e))]
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 259, normalized size = 1.70

$$-\frac{2\left(\frac{3\,(Bcd-Ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^2-2a^2cd+a^2d^2)\sqrt{c^2-d^2}}+\frac{3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Ac+Bc-5Ad+2Bd}{(a^2c^2-2a^2cd+a^2d^2)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((
c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x +
1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*
c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*x + 1
/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(tan(
1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 14.78, size = 302, normalized size = 1.99

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,(A\,d-B\,c)\,(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}+\frac{2\,c\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d-B\,c)\,(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}}{2\,A\,d^2-2\,B\,c\,d}\right)(A\,d-B\,c)}{a^2\,f\,\sqrt{c+d}\,(c-d)^{5/2}}-\frac{\frac{2\,(2\,A\,c-5\,A\,d+B\,c+2\,B\,d)}{3\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,c-3\,A\,d+B\,c+B\,d)}{(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(A\,c-2\,A\,d+B\,d)}{(c-d)^2}}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d*atan(((d*(A*d - B*c)*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c
+ d)^(1/2)*(c - d)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^2*c^2
+ a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*A*d^2 - 2*B*c
*d))*(A*d - B*c))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*A*c - 5*A*d
+ B*c + 2*B*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*c - 3*A*d + B*c +
B*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(A*c - 2*A*d + B*d))/(c - d)^2)/(
f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a^2*tan(
e/2 + (f*x)/2)))
```

3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=275

$$\frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f}-\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2}{3a^2(c-d)^3(c+d)f(c+d\sin(e+}$$

```
[Out] -1/3*d*(A*(c^2-6*c*d-10*d^2)+B*(2*c^2+9*c*d+4*d^2))*cos(f*x+e)/a^2/(c-d)^3/
(c+d)/f/(c+d*sin(f*x+e))-1/3*(A*c-6*A*d+2*B*c+3*B*d)*cos(f*x+e)/a^2/(c-d)^2
/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*
x+e))^2/(c+d*sin(f*x+e))+2*d*(A*d*(3*c+2*d)-B*(2*c^2+2*c*d+d^2))*arctan((d+
c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d*(A*d*(3*c + 2*d) - B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(A*(
c^2 - 6*c*d - 10*d^2) + B*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/(3*a^2*(c
- d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos
[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - ((
A - B)*Cos[e + f*x])/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int \frac{-c}{}}{}$$

$$= -\frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{}{3(c}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))}$$

$$= \frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+f\right.}{\sqrt{c^2-d^2}}\right.}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f}$$

Mathematica [A]
time = 1.94, size = 313, normalized size = 1.14

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(A(c-7d)+2B(c+2d))\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-\frac{6d(-Ad(3c+2d)+B(2c^2+2cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)\sqrt{c^2-d^2}}+\frac{3d^2(-Bc+Ad)\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{(c+d)(c+d\sin(e+fx))}\right)}{3a^2(c-d)^3f(1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d) +
 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
(6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)
*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1
 + Sin[e + f*x])^2)
```

Maple [A]
time = 0.67, size = 263, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{-2A+2B}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2A-2B)}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(Ac-3Ad+2Bd)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}-\cdots\right)}{\cdots}}{a^2f}$
default	$\frac{-\frac{-2A+2B}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2A-2B)}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(Ac-3Ad+2Bd)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}-\cdots\right)}{\cdots}}{a^2f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(-1/2*(-2*A+2*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A-2*B)/(c-
d)^2/(tan(1/2*f*x+1/2*e)+1)^3-(A*c-3*A*d+2*B*d)/(c-d)^3/(tan(1/2*f*x+1/2*e)
+1)+1/(c-d)^3*d*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)/(c+d
))/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(3*A*c*d+2*A*d^2-2*B*c
^2-2*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(273) = 546$.
time = 0.47, size = 3168, normalized size = 11.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*d
^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - 2*((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c
^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*co
s(f*x + e)^3 + 2*((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (
A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B
*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2
*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*B*c^3*d
 - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^
2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*
(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x +
 e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^
4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 -
 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e)
 - 2*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 +
 (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (
11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)
^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*
c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e))*sin(f*x + e
))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5
- 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^
2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(
f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 -
 a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5
- a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^
2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c
^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^
7)*f)*sin(f*x + e)), 1/3*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 +
 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*
A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*
B)*d^5)*cos(f*x + e)^3 + ((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3
*d^2 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2
- 3*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*
d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2
```

```
*B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f
*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B
)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*
d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A
 - B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x +
 e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^5 - (5*A - 8*B)*c^4
*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A
 - 4*B)*d^5)*cos(f*x + e) - ((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^
2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*
(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A -
 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)
*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*co
s(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2
*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7
 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*
d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 +
3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(
f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d
^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*c
os(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^
2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 425, normalized size = 1.55

$$-\frac{2\left(\frac{3\left(2Bc^2d-3Acd^2+2Bcd^2-2Ad^3+Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(Bcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Bc^2d^2-Acd^3\right)}{\left(a^2c^5-2a^2c^4d+2a^2c^2d^3-a^2cd^4\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}+\frac{3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-9Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-15Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+9Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Ac+Bc-8Ad+5Bd}{\left(a^2c^3-3a^2c^2d+3a^2cd^2-a^2d^3\right)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2))
+ 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*d^2
- A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*
f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x + 1/2*
e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*
tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*f*x + 1/
2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/((a^2*c^3
- 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

<u>Mupad</u> [B]
time = 16.76, size = 844, normalized size = 3.07

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,\left(-2\,a^2\,c^4\,d+4\,a^2\,c^3\,d^2-4\,a^2\,c\,d^4+2\,a^2\,d^5\right)\,\left(2\,B\,c^2-2\,A\,d^2+B\,d^2-3\,A\,c\,d+2\,B\,c\,d\right)}{a^2\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}-\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^2\,c^4-2\,a^2\,c^3\,d+2\,a^2\,c\,d^3-a^2\,d^4\right)\,\left(2\,B\,c^2-2\,A\,d^2+B\,d^2-3\,A\,c\,d+2\,B\,c\,d\right)}{a^2\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}}{2\,B\,d^3-4\,A\,d^3-6\,A\,c\,d^2+4\,B\,c\,d^2+4\,B\,c^2\,d}\right)\,\left(2\,B\,c^2-2\,A\,d^2+B\,d^2-3\,A\,c\,d+2\,B\,c\,d\right)}{a^2\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}-\frac{\frac{2\,\left(2\,A\,c^3-3\,A\,d^3+B\,c^3-8\,A\,c\,d^2-6\,A\,c^2\,d+8\,B\,c\,d^2+6\,B\,c^2\,d\right)}{3\,\left(c+d\right)\,\left(c-d\right)\,\left(c^2-2\,c\,d+d^2\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(5\,A\,c^3-9\,A\,d^3+B\,c^3-30\,A\,c\,d^2-11\,A\,c^2\,d+27\,B\,c\,d^2+17\,B\,c^2\,d\right)}{3\,c\,\left(c-d\right)\,\left(c^2-2\,c\,d+d^2\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(A\,c^4-3\,A\,d^4+B\,c^4-9\,A\,c^2\,d^2+8\,B\,c^2\,d^2-7\,A\,c\,d^3-2\,A\,c^3\,d+7\,B\,c\,d^3+4\,B\,c^3\,d\right)}{c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^2-2\,c\,d+d^2\right)}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,A\,c^4-3\,A\,d^4+3\,B\,c^4-27\,A\,c^2\,d^2+30\,B\,c^2\,d^2-25\,A\,c\,d^3-8\,A\,c^3\,d+13\,B\,c\,d^3+14\,B\,c^3\,d\right)}{3\,c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^2-2\,c\,d+d^2\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(A\,c^4-A\,d^4-3\,A\,c^2\,d^2+2\,B\,c^2\,d^2-2\,A\,c^3\,d+B\,c\,d^3+2\,B\,c^3\,d\right)}{c\,\left(c+d\right)\,\left(c-d\right)\,\left(c^2-2\,c\,d+d^2\right)}}{f\,\left(a^2\,c+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,a^2\,c+2\,a^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(3\,a^2\,c+2\,a^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(4\,a^2\,c+6\,a^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(4\,a^2\,c+6\,a^2\,d\right)+a^2\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d*atan(((d*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2)*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c - d)^(7/2
)) - (2*c*d*tan(e/2 + (f*x)/2)*(a^2*c^4 - a^2*d^4 + 2*a^2*c*d^3 - 2*a^2*c^3
*d)*(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c
- d)^(7/2)))/(2*B*d^3 - 4*A*d^3 - 6*A*c*d^2 + 4*B*c*d^2 + 4*B*c^2*d))*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*f*(c + d)^(3/2)*(c - d)^(7
/2)) - ((2*(2*A*c^3 - 3*A*d^3 + B*c^3 - 8*A*c*d^2 - 6*A*c^2*d + 8*B*c*d^2 +
 6*B*c^2*d))/(3*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)^2*(5*A*c^3 - 9*A*d^3 + B*c^3 - 30*A*c*d^2 - 11*A*c^2*d + 27*B*c*d^2 + 17*
B*c^2*d))/(3*c*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*c^
4 - 3*A*d^4 + B*c^4 - 9*A*c^2*d^2 + 8*B*c^2*d^2 - 7*A*c*d^3 - 2*A*c^3*d + 7
*B*c*d^3 + 4*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2
 + (f*x)/2)*(3*A*c^4 - 3*A*d^4 + 3*B*c^4 - 27*A*c^2*d^2 + 30*B*c^2*d^2 - 25
*A*c*d^3 - 8*A*c^3*d + 13*B*c*d^3 + 14*B*c^3*d))/(3*c*(c + d)*(c - d)*(c^2
- 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^4 - A*d^4 - 3*A*c^2*d^2 + 2*
B*c^2*d^2 - 2*A*c^3*d + B*c*d^3 + 2*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c
*d + d^2)))/(f*(a^2*c + tan(e/2 + (f*x)/2)*(3*a^2*c + 2*a^2*d) + tan(e/2 +
(f*x)/2)^4*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c + 6*a^2*d) +
 tan(e/2 + (f*x)/2)^3*(4*a^2*c + 6*a^2*d) + a^2*c*tan(e/2 + (f*x)/2)^5))
```

3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=386

$$\frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f}-\frac{d(A(2c^2-16cd-21d^2}{6a^2(c-d)^3(}$$

```
[Out] -1/6*d*(A*(2*c^2-16*c*d-21*d^2)+B*(4*c^2+19*c*d+12*d^2))*cos(f*x+e)/a^2/(c-
d)^3/(c+d)/f/(c+d*sin(f*x+e))^2-1/3*(A*c-8*A*d+2*B*c+5*B*d)*cos(f*x+e)/a^2/
(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a
+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(A*(2*c^3-16*c^2*d-59*c*d^2-32*d^
3)+B*(4*c^3+37*c^2*d+44*c*d^2+20*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+
d*sin(f*x+e))+d*(A*d*(12*c^2+16*c*d+7*d^2)-B*(6*c^3+12*c^2*d+13*c*d^2+4*d^3
))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d)^2/f/(
c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$\frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^2}-\frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sin(e+fx))}-\frac{(Ac-8Ad+2Bc+5Bd)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+d\sin(e+fx))^2}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3)
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^
2*Sqrt[c^2 - d^2]*f) - (d*(A*(2*c^2 - 16*c*d - 21*d^2) + B*(4*c^2 + 19*c*d
+ 12*d^2))*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2)
 - ((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(a
 + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) - (d*(A*(2*c^3 - 16*c^2*d - 59
*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f*x])/
(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-a}{}}{}$$

$$= -\frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{}{3(c}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= \frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1257 vs. $2(386) = 772$.
time = 5.77, size = 1257, normalized size = 3.26

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-48*d*(-(A*d*(12*c^2 + 16*c*d + 7*
d^2)) + B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 -
d^2] + ((-(A*d*(96*c^4 + 524*c^3*d + 776*c^2*d^2 + 487*c*d^3 + 112*d^4)) +
B*(48*c^5 + 240*c^4*d + 536*c^3*d^2 + 701*c^2*d^3 + 400*c*d^4 + 70*d^5))*Co
s[(e + f*x)/2] - (A*(16*c^5 - 80*c^4*d - 536*c^3*d^2 - 1028*c^2*d^3 - 695*c
*d^4 - 134*d^5) + B*(32*c^5 + 224*c^4*d + 728*c^3*d^2 + 893*c^2*d^3 + 482*c
*d^4 + 98*d^5))*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x))/2] -
12*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 15*
```

```
A*c*d^4*Cos[(5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] + 6*A*d^5*Co
s[(5*(e + f*x))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*Cos[(7*(e +
 f*x))/2] + 8*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos[(7*(e + f*x
))/2] + 59*B*c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*(e + f*x))/2]
 + 76*B*c*d^4*Cos[(7*(e + f*x))/2] - 52*A*d^5*Cos[(7*(e + f*x))/2] + 34*B*d
^5*Cos[(7*(e + f*x))/2] + 48*A*c^5*Sin[(e + f*x)/2] + 48*B*c^5*Sin[(e + f*x
)/2] - 224*A*c^4*d*Sin[(e + f*x)/2] + 416*B*c^4*d*Sin[(e + f*x)/2] - 872*A*
c^3*d^2*Sin[(e + f*x)/2] + 992*B*c^3*d^2*Sin[(e + f*x)/2] - 1144*A*c^2*d^3*
Sin[(e + f*x)/2] + 967*B*c^2*d^3*Sin[(e + f*x)/2] - 685*A*c*d^4*Sin[(e + f*
x)/2] + 496*B*c*d^4*Sin[(e + f*x)/2] - 168*A*d^5*Sin[(e + f*x)/2] + 126*B*d
^5*Sin[(e + f*x)/2] + 48*B*c^4*d*Sin[(3*(e + f*x))/2] - 132*A*c^3*d^2*Sin[(
3*(e + f*x))/2] + 96*B*c^3*d^2*Sin[(3*(e + f*x))/2] - 204*A*c^2*d^3*Sin[(3*
(e + f*x))/2] + 207*B*c^2*d^3*Sin[(3*(e + f*x))/2] - 165*A*c*d^4*Sin[(3*(e
+ f*x))/2] + 174*B*c*d^4*Sin[(3*(e + f*x))/2] - 66*A*d^5*Sin[(3*(e + f*x))/
2] + 42*B*d^5*Sin[(3*(e + f*x))/2] - 16*A*c^4*d*Sin[(5*(e + f*x))/2] - 32*B
*c^4*d*Sin[(5*(e + f*x))/2] + 116*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 224*B*c^
3*d^2*Sin[(5*(e + f*x))/2] + 412*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 409*B*c^2
*d^3*Sin[(5*(e + f*x))/2] + 403*A*c*d^4*Sin[(5*(e + f*x))/2] - 286*B*c*d^4*
Sin[(5*(e + f*x))/2] + 114*A*d^5*Sin[(5*(e + f*x))/2] - 78*B*d^5*Sin[(5*(e
+ f*x))/2] + 15*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 21*A*c*d^4*Sin[(7*(e + f*x
))/2] + 12*B*c*d^4*Sin[(7*(e + f*x))/2] - 12*A*d^5*Sin[(7*(e + f*x))/2] + 6
*B*d^5*Sin[(7*(e + f*x))/2])/(c + d*Sin[e + f*x])^2))/(48*a^2*(c - d)^4*(c
+ d)^2*f*(1 + Sin[e + f*x])^2)
```

Maple [A]
time = 1.43, size = 547, normalized size = 1.42

method	result
derivativedivides	$-\frac{-2A+2B}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2A-2B)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(Ac-4Ad+3Bd)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{d^2\left(9A c^2 d+4Ac d^2-2A d^3-7B c^3-4\right.}{2c\left(c^2+2cd+d^2\right.}\right.}{}$
default	$-\frac{-2A+2B}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2A-2B)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(Ac-4Ad+3Bd)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{d^2\left(9A c^2 d+4Ac d^2-2A d^3-7B c^3-4\right.}{2c\left(c^2+2cd+d^2\right.}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^2*(-1/2*(-2*A+2*B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A-2*B)/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^3-(A*c-4*A*d+3*B*d)/(c-d)^4/(tan(1/2*f*x+1/2*e)
+1)+d/(c-d)^4*((1/2*d^2*(9*A*c^2*d+4*A*c*d^2-2*A*d^3-7*B*c^3-4*B*c^2*d)/c/(
c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(8*A*c^4*d+4*A*c^3*d^2+15*A*c^2*d
^3+8*A*c*d^4-2*A*d^5-6*B*c^5-4*B*c^4*d-13*B*c^3*d^2-8*B*c^2*d^3-2*B*c*d^4)/
(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(23*A*c^2*d+12*A*c*d^2-2*A
*d^3-17*B*c^3-12*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1/
2*d*(8*A*c^2*d+4*A*c*d^2-A*d^3-6*B*c^3-4*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2))/
(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(12*A*c^2*d+16*A*c*
d^2+7*A*d^3-6*B*c^3-12*B*c^2*d-13*B*c*d^2-4*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2
)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2482 vs. $2(384) = 768$.
time = 0.52, size = 5054, normalized size = 13.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/12*(4*(A - B)*c^7 - 4*(A - B)*c^6*d - 12*(A - B)*c^5*d^2 + 12*(A - B)*c
^4*d^3 + 12*(A - B)*c^3*d^4 - 12*(A - B)*c^2*d^5 - 4*(A - B)*c*d^6 + 4*(A -
 B)*d^7 - 2*(2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^
3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*co
s(f*x + e)^4 - 2*(4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*
c^4*d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c
*d^6 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + 2*(2*(A + 2*B)*c^7 - 6*(2*A - 3*
B)*c^6*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*
c^3*d^4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7
)*cos(f*x + e)^2 + 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c
```

^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 + (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^5*d - 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4 - (76*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A - 3*B)*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 4*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 19*B)*c^5*d^2 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A - 21*B)*c^2*d^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - 2*(2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 - (4*(A + 2*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(3*A + B)*c^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A - 8*B)*d^7)*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A - 18*B)*c^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21*A - 22*B)*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x +

```
e)), -1/6*(2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)
*c^4*d^3 + 6*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A -
B)*d^7 - (2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*
d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(
f*x + e)^4 - (4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*c^4*
d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c*d^6
 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + (2*(A + 2*B)*c^7 - 6*(2*A - 3*B)*c^6
*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*c^3*d^
4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7)*cos(
f*x + e)^2 - 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3
 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 + (6*
B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*...
```

<u>**Sympy**</u> [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

<u>**Giac**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. 2(384) = 768.
time = 0.53, size = 944, normalized size = 2.45

$$-\frac{3\left(6Bc^3d-12Ac^2d^2+12Bc^2d^2-16Acd^3+13Bcd^3-7Ad^4+4Bd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^6-2a^2c^5d-a^2c^4d^2+4a^2c^3d^3-a^2c^2d^4-2a^2cd^5+a^2d^6\right)\sqrt{c^2-d^2}} + \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3
- 7*A*d^4 + 4*B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*t
an(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^
4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(c^2 - d^2
)) + 3*(7*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*tan(1/2*f*x + 1/2*
e)^3 + 4*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*tan(1/2*f*x + 1/2*e
)^3 + 2*A*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*tan(1/2*f*x + 1/2*e)^2
 - 8*A*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^2
- 4*A*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^2
- 15*A*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*tan(1/2*f*x + 1/2*e)^2
- 8*A*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*A
*d^7*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*tan(1/2*f*x + 1/2*e) - 23*A*c^3*
d^4*tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*tan(1/2*f*x + 1/2*e) - 12*A*c^2*d^5
```

```
*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*A*c*d^6*tan(1/
2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4*A*c^3*d^4 + B*
c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3
- a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*
d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 12*A*d*t
an(1/2*f*x + 1/2*e)^2 + 9*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x +
1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 21*A*d*tan(1/2*f*x + 1/2*e) + 15*B*d*
tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 11*A*d + 8*B*d)/((a^2*c^4 - 4*a^2*c^3*
d + 6*a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B]
time = 17.69, size = 1686, normalized size = 4.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (d*atan(((d*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c^2*d^5 - 8*a^2*
c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*
A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(2*a^2*(c + d)^(5/2)*(c -
d)^(9/2)) + (c*d*tan(e/2 + (f*x)/2)*(2*a^2*c*d^5 - a^2*d^6 - a^2*c^6 + 2*a^
2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)*(6*B*c^3 - 7*A*d^3 + 4
*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*(c + d)^(
5/2)*(c - d)^(9/2)))/(4*B*d^4 - 7*A*d^4 - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*
A*c*d^3 + 13*B*c*d^3 + 6*B*c^3*d))*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*A*c*d^
2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*f*(c + d)^(5/2)*(c - d)^(9/
2)) - ((tan(e/2 + (f*x)/2)^5*(2*A*c^6 + 2*A*d^6 + 2*B*c^6 - 23*A*c^2*d^4 -
40*A*c^3*d^3 - 38*A*c^4*d^2 + 6*B*c^2*d^4 + 43*B*c^3*d^3 + 40*B*c^4*d^2 - 4
*A*c*d^5 - 4*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^5 - 3*c^4*d - 3*c*d
^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (4*A*c^5 + 3*A*d^5 + 2*B*c^5 - 46*A*c^
2*d^3 - 40*A*c^3*d^2 + 28*B*c^2*d^3 + 52*B*c^3*d^2 - 12*A*c*d^4 - 14*A*c^4*
d + 3*B*c*d^4 + 20*B*c^4*d)/(3*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3
 - d^3)) + (2*tan(e/2 + (f*x)/2)^3*(6*A*c^6 + 9*A*d^6 + 6*B*c^6 - 177*A*c^2
*d^4 - 212*A*c^3*d^3 - 102*A*c^4*d^2 + 105*B*c^2*d^4 + 215*B*c^3*d^3 + 150*
B*c^4*d^2 - 33*A*c*d^5 - 16*A*c^5*d + 9*B*c*d^5 + 40*B*c^5*d))/(3*c^2*(c^2
- d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)*(6*A*c^5 + 6*
A*d^5 + 6*B*c^5 - 160*A*c^2*d^3 - 114*A*c^3*d^2 + 97*B*c^2*d^3 + 156*B*c^3*
d^2 - 33*A*c*d^4 - 20*A*c^4*d + 12*B*c*d^4 + 44*B*c^4*d))/(3*c*(c^2 - d^2)*
(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^2*(14*A*c^7 + 6*A*d^
7 + 4*B*c^7 - 232*A*c^2*d^5 - 583*A*c^3*d^4 - 532*A*c^4*d^3 - 226*A*c^5*d^2
 + 124*B*c^2*d^5 + 412*B*c^3*d^4 + 595*B*c^4*d^3 + 352*B*c^5*d^2 - 6*A*c*d^
6 - 16*A*c^6*d + 6*B*c*d^6 + 82*B*c^6*d))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d
^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^4*(16*A*c^7 + 18*A*d^7 + 2
*B*c^7 - 303*A*c^2*d^5 - 522*A*c^3*d^4 - 502*A*c^4*d^3 - 220*A*c^5*d^2 + 15
```

```
6*B*c^2*d^5 + 453*B*c^3*d^4 + 538*B*c^4*d^3 + 328*B*c^5*d^2 - 48*A*c*d^6 -
14*A*c^6*d + 18*B*c*d^6 + 80*B*c^6*d))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2
- 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^6*(2*A*c^6 + 2*A*d^6 - 9*A*c^
2*d^4 - 8*A*c^3*d^3 - 14*A*c^4*d^2 + 4*B*c^2*d^4 + 13*B*c^3*d^3 + 12*B*c^4*
d^2 - 4*A*c*d^5 - 4*A*c^5*d + 6*B*c^5*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(3
*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/2 + (f*x)/2)*(3*a^2*c^2 + 4*a^2*c
*d) + tan(e/2 + (f*x)/2)^2*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + tan(e/2 +
 (f*x)/2)^5*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + tan(e/2 + (f*x)/2)^3*(7*
a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^4*(7*a^2*c^2 + 12*a
^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(3*a^2*c^2 + 4*a^2*c*d) + a^2*c
^2 + a^2*c^2*tan(e/2 + (f*x)/2)^7))
```

3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=225

$$\frac{d^2(3B(c-d)+Ad)x}{a^3}+\frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(3B(c^2+6cd-15d^2)+A(2c^2+\right.}{15f\left(a^3+a^3\sin(e+\right.}$$

```
[Out] d^2*(3*B*(c-d)+A*d)*x/a^3+1/15*d^2*(3*B*(c-9*d)+A*(2*c+7*d))*cos(f*x+e)/a^3
/f-1/15*(c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*cos(f*x+e)/f/
(a^3+a^3*sin(f*x+e))-1/15*(3*B*(c-3*d)+2*A*(c+2*d))*cos(f*x+e)*(c+d*sin(f*x
+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+
a*sin(f*x+e))^3
```

Rubi [A]

time = 0.55, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3056, 3047, 3102, 2814, 2727}

$$-\frac{(c-d)\left(A(2c^2+7cd+15d^2)+3B(c^2+6cd-15d^2)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}+\frac{d^2x(Ad+3B(c-d))}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}-\frac{(2A(c+2d)+3B(c-3d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*(3*B*(c - d) + A*d)*x)/a^3 + (d^2*(3*B*(c - 9*d) + A*(2*c + 7*d))*Cos[
e + f*x])/(15*a^3*f) - ((c - d)*(3*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*
c*d + 15*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((3*B*(c - 3
*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a*f*(a + a*Si
n[e + f*x])^2) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(5*f*(a + a*
Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{(c+d\sin(e+fx)}{}}{}$$

$$= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx)}{15af(a+a\sin(e+fx))^2}$$

$$= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx)}{15af(a+a\sin(e+fx))^2}$$

$$= \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(3B(c-3d)+}{}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e}{15a^3f}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e}{15a^3f}$$

Mathematica [A]

time = 2.00, size = 366, normalized size = 1.63

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^3\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(c-d)^2(3B(c-6d)+A(2c+13d))\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(c-d)^2(3B(c-6d)+A(2c+13d))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+2(c-d)\left(3B(c^2+8cd-24d^2)+A(2c^2+11cd+32d^2)\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-15d^2(-3Bc-Ad+3Bd)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5-15Bd^3\cos(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\right)}{15a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2
*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*(2*c^
2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
)/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.45, size = 349, normalized size = 1.55

method	result
derivativedivides	$\frac{-\frac{2\left(A\,c^3-A\,d^3-3Bc\,d^2+3B\,d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^3+6A\,c^2d-2A\,d^3+2B\,c^3-6Bc\,d^2+4B\,d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A\,c^3-18A\,c^2d+12Ac\,d^2-2A\,d^3-6B\,c^3+12B\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{}$
default	$\frac{-\frac{2\left(A\,c^3-A\,d^3-3Bc\,d^2+3B\,d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^3+6A\,c^2d-2A\,d^3+2B\,c^3-6Bc\,d^2+4B\,d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A\,c^3-18A\,c^2d+12Ac\,d^2-2A\,d^3-6B\,c^3+12B\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{}$
risch	$\frac{d^3xA}{a^3}+\frac{3d^2xBc}{a^3}-\frac{3d^3xB}{a^3}-\frac{B\,d^3\mathrm{e}^{i(fx+e)}}{2a^3f}-\frac{B\,d^3\mathrm{e}^{-i(fx+e)}}{2a^3f}-\frac{2\left(3B\,c^3+2A\,c^3+21Ac\,d^2+9A\,c^2d-96Bc\,d^2+21\right.}{}$
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-(A*c^3-A*d^3-3*B*c*d^2+3*B*d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A*
c^3+6*A*c^2*d-2*A*d^3+2*B*c^3-6*B*c*d^2+4*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1
/3*(8*A*c^3-18*A*c^2*d+12*A*c*d^2-2*A*d^3-6*B*c^3+12*B*c^2*d-6*B*c*d^2)/(ta
n(1/2*f*x+1/2*e)+1)^3-1/4*(-8*A*c^3+24*A*c^2*d-24*A*c*d^2+8*A*d^3+8*B*c^3-2
4*B*c^2*d+24*B*c*d^2-8*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A*c^3-12*A*c^
2*d+12*A*c*d^2-4*A*d^3-4*B*c^3+12*B*c^2*d-12*B*c*d^2+4*B*d^3)/(tan(1/2*f*x+
1/2*e)+1)^5+d^2*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+3*B*c-3*B*d)*arctan(tan
(1/2*f*x+1/2*e))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1828 vs. $2(226) = 452$.
time = 0.67, size = 1828, normalized size = 8.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1
1*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*sin(f*x + e)/(cos(f*x + e) +
1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(
f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a^3) - A*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1)
+ 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*
a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) + A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^2*d*(5*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^
3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c*d^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*B*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x +
```

```
 e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*c^2*d*(5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(226) = 452$.
time = 0.41, size = 663, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(15*B*d^3*cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A - B
)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A -
 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos
(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x - (2*(2*A + 3*B)*c^3 + 3*(
6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^2 + (
A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((3*A + 2*B)*c^3 + 3*(2*A + 3*B)*c^2*
d + 9*(A - 6*B)*c*d^2 - 9*(2*A - 7*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*
f*x)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A - B)*c^
2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x
- ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 - 2*(16*A -
 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((2*A +
 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A - 62*B)*d^3
- 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^3*f*co
s(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a
^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 11456 vs. $2(206) = 412$.
time = 17.20, size = 11456, normalized size = 50.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f
*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e/2 +
 f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3
*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 110*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2
+ f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**
5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 100*A
*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 94*A*c**3*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(
e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x
/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a
**3*f) - 40*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c**3/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 9
0*A*c**2*d*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)**
4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/
2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 180*A*c**2*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 108*A*c**2*d
*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f
*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4
+ 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 - 18*A*c**2*d/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
```

```
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 120*A*c*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 60*A*c*d**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 132*A*c*d**2*tan(e/2 + f*x/2
)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a
**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan
(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 60*A*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d**2/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 15*A*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(226) = 452.
time = 0.50, size = 596, normalized size = 2.65

$$-\frac{\frac{30\,Bd^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\,(3\,Bcd^2+Ad^3-3\,Bd^3)(fx+e)}{a^3}+\frac{2\,\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+\cdots\right)}{\cdots}}{15\,f}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(30*B*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3
- 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^2*
tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*tan(1/2
*f*x + 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*tan(1/2*f*x +
1/2*e)^3 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*tan(1/2*f*x + 1/
2*e)^3 - 75*A*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*tan(1/2*f*x + 1/2*e)^3
 + 40*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 45*A
*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*A*c*
d^2*tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*A*d^3
*tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^3*tan(1
/2*f*x + 1/2*e) + 15*B*c^3*tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*tan(1/2*f*x +
1/2*e) + 30*B*c^2*d*tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*tan(1/2*f*x + 1/2*e)
```

```
- 285*B*c*d^2*tan(1/2*f*x + 1/2*e) - 95*A*d^3*tan(1/2*f*x + 1/2*e) + 240*B*
d^3*tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*B*c^2*d + 6*A*
c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5
))/f
```

Mupad [B]
time = 15.70, size = 593, normalized size = 2.64

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d+3\,B\,c-3\,B\,d\right)}{2\,A\,d^3-6\,B\,d^3+6\,B\,c\,d^2}\right)\,\left(A\,d+3\,B\,c-3\,B\,d\right)}{a^3\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(4\,A\,c^3-10\,A\,d^3+2\,B\,c^3+30\,B\,d^3+6\,A\,c^2\,d-30\,B\,c\,d^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,A\,c^3}{3}-\frac{38\,A\,d^3}{3}+2\,B\,c^3+42\,B\,d^3+4\,A\,c\,d^2+6\,A\,c^2\,d-38\,B\,c\,d^2+4\,B\,c^2\,d\right)+\frac{14\,A\,c^3}{15}-\frac{44\,A\,d^3}{15}+\frac{2\,B\,c^3}{5}+\frac{48\,B\,d^3}{5}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(\frac{22\,A\,c^3}{3}-\frac{64\,A\,d^3}{3}+2\,B\,c^3+64\,B\,d^3+8\,A\,c\,d^2+6\,A\,c^2\,d-64\,B\,c\,d^2+8\,B\,c^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{20\,A\,c^3}{3}-\frac{68\,A\,d^3}{3}+4\,B\,c^3+80\,B\,d^3+4\,A\,c\,d^2+12\,A\,c^2\,d-68\,B\,c\,d^2+4\,B\,c^2\,d\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{94\,A\,c^3}{15}-\frac{334\,A\,d^3}{15}+\frac{12\,B\,c^3}{5}+\frac{378\,B\,d^3}{5}+\frac{44\,A\,c\,d^2}{5}+\frac{36\,A\,c^2\,d}{5}-\frac{334\,B\,c\,d^2}{5}+\frac{44\,B\,c^2\,d}{5}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(2\,A\,c^3-2\,A\,d^3+6\,B\,d^3-6\,B\,c\,d^2\right)+\frac{4\,A\,c\,d^2}{5}+\frac{6\,A\,c^2\,d}{5}-\frac{44\,B\,c\,d^2}{5}+\frac{4\,B\,c^2\,d}{5}}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+5\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+11\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+15\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+15\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+11\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(A*d + 3*B*c - 3*B*d))/(2*A*d^3 - 6*B
*d^3 + 6*B*c*d^2))*(A*d + 3*B*c - 3*B*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^5*(
4*A*c^3 - 10*A*d^3 + 2*B*c^3 + 30*B*d^3 + 6*A*c^2*d - 30*B*c*d^2) + tan(e/2
 + (f*x)/2)*((8*A*c^3)/3 - (38*A*d^3)/3 + 2*B*c^3 + 42*B*d^3 + 4*A*c*d^2 +
6*A*c^2*d - 38*B*c*d^2 + 4*B*c^2*d) + (14*A*c^3)/15 - (44*A*d^3)/15 + (2*B*
c^3)/5 + (48*B*d^3)/5 + tan(e/2 + (f*x)/2)^4*((22*A*c^3)/3 - (64*A*d^3)/3 +
 2*B*c^3 + 64*B*d^3 + 8*A*c*d^2 + 6*A*c^2*d - 64*B*c*d^2 + 8*B*c^2*d) + tan
(e/2 + (f*x)/2)^3*((20*A*c^3)/3 - (68*A*d^3)/3 + 4*B*c^3 + 80*B*d^3 + 4*A*c
*d^2 + 12*A*c^2*d - 68*B*c*d^2 + 4*B*c^2*d) + tan(e/2 + (f*x)/2)^2*((94*A*c
^3)/15 - (334*A*d^3)/15 + (12*B*c^3)/5 + (378*B*d^3)/5 + (44*A*c*d^2)/5 + (
36*A*c^2*d)/5 - (334*B*c*d^2)/5 + (44*B*c^2*d)/5) + tan(e/2 + (f*x)/2)^6*(2
*A*c^3 - 2*A*d^3 + 6*B*d^3 - 6*B*c*d^2) + (4*A*c*d^2)/5 + (6*A*c^2*d)/5 - (
44*B*c*d^2)/5 + (4*B*c^2*d)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan
(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)
^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*ta
n(e/2 + (f*x)/2)))
```

3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{Bd^2x}{a^3}-\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2))\cos(e$$

```
[Out] B*d^2*x/a^3-1/15*(c-d)*(B*(3*c-7*d)+2*A*(c+d))*cos(f*x+e)/a/f/(a+a*sin(f*x+
e))^2-1/15*(B*(3*c^2+14*c*d-29*d^2)+2*A*(c^2+3*c*d+2*d^2))*cos(f*x+e)/f/(a^
3+a^3*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e)
)^3
```

Rubi [A]
time = 0.31, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3056, 3047, 3098, 2814, 2727}

$$-\frac{(2A(c^2+3cd+2d^2)+B(3c^2+14cd-29d^2))\cos(e+fx)}{15f(a^3\sin(e+fx)+a^3)}+\frac{Bd^2x}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*d^2*x)/a^3 - ((c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 + 14*c*d - 29*d^2) + 2*A*(c^2 + 3*c*
d + 2*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((A - B)*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{(c+d\sin(e+fx)}{\cdots}}{\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{ac(B(3c-2d)+}{\cdots}}{\cdots}$$

$$= -\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)}{\cdots}$$

$$= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \cdots$$

$$= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 514 vs. 2(164) = 328.
time = 0.62, size = 514, normalized size = 3.13

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(30(2Ad(c+d)+B(c^2+4cd+d^2(-9+5e+5fx)))\cos\left(\frac{1}{2}(e+fx)\right)-5(4A(c^2+3cd+2d^2)+B(6c^2+16cd+d^2(-46+15e+15fx)))\cos\left(\frac{3}{2}(e+fx)\right)-15Bd^2e\cos\left(\frac{5}{2}(e+fx)\right)-15Bd^2fx\cos\left(\frac{5}{2}(e+fx)\right)+40Ac^2\sin\left(\frac{1}{2}(e+fx)\right)+30Bc^2\sin\left(\frac{1}{2}(e+fx)\right)+60Acd\sin\left(\frac{1}{2}(e+fx)\right)+160Bcd\sin\left(\frac{1}{2}(e+fx)\right)+80Ad^2\sin\left(\frac{1}{2}(e+fx)\right)-370Bd^2\sin\left(\frac{1}{2}(e+fx)\right)+150Bd^2e\sin\left(\frac{1}{2}(e+fx)\right)+150Bd^2fx\sin\left(\frac{1}{2}(e+fx)\right)+60Bcd\sin\left(\frac{3}{2}(e+fx)\right)+30Ad^2\sin\left(\frac{3}{2}(e+fx)\right)-90Bd^2\sin\left(\frac{3}{2}(e+fx)\right)+75Bd^2e\sin\left(\frac{3}{2}(e+fx)\right)+75Bd^2fx\sin\left(\frac{3}{2}(e+fx)\right)-4Ac^2\sin\left(\frac{5}{2}(e+fx)\right)-6Bc^2\sin\left(\frac{5}{2}(e+fx)\right)-12Acd\sin\left(\frac{5}{2}(e+fx)\right)-28Bcd\sin\left(\frac{5}{2}(e+fx)\right)-14Ad^2\sin\left(\frac{5}{2}(e+fx)\right)+64Bd^2\sin\left(\frac{5}{2}(e+fx)\right)-15Bd^2e\sin\left(\frac{5}{2}(e+fx)\right)-15Bd^2fx\sin\left(\frac{5}{2}(e+fx)\right)\right)}{60a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d
+ d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2)
+ B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] - 15
*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*A*c^
2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x)/2]
+ 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*Sin[(e
 + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f*x)/2]
+ 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 90*B*d^2*
Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*f*x*Sin[(
3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(e + f*x))/
2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/2] - 14*A*d
^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*d^2*e*Sin[(5
*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f*(1 + Sin[e +
 f*x])^3)
```

Maple [A]
time = 0.41, size = 241, normalized size = 1.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-(A*c^2-B*d^2)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A*c^2+4*A*c*d+2*B*c^
2-2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-8*A*c^2+16*A*c*d-8*A*d^2+8*B*c^2-
16*B*c*d+8*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A*c^2-8*A*c*d+4*A*d^2-4*B
*c^2+8*B*c*d-4*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(8*A*c^2-12*A*c*d+4*A*d^
2-6*B*c^2+8*B*c*d-2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^3+B*d^2*arctan(tan(1/2*f*
x+1/2*e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(165) = 330$.
time = 0.55, size = 1230, normalized size = 7.50

$$\frac{2\left(Bd^{2}\left(\frac{\frac{95 \sin(fx+e)}{\cos(fx+e)+1}+\frac{145 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{75 \sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{15 \sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+22}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}+\frac{15\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^{3}}\right)-\frac{Ac^{2}\left(\frac{20 \sin(fx+e)}{\cos(fx+e)+1}+\frac{40 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{30 \sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{15 \sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+7\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{4Bcd\left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1}+\frac{10 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{2Ad^{2}\left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1}+\frac{10 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{3Bc^{2}\left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1}+\frac{5 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{5 \sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{6Acd\left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1}+\frac{5 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{5 \sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+1\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
```

```
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 4*B*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*d^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 6*A*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(165) = 330$.
time = 0.57, size = 444, normalized size = 2.71

$$-\frac{60\,Bd^2fx-(15\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+7\,B)cd-(7\,A-32\,B)d^2)\cos(fx+e)^3-3\,(A-B)c^2+6\,(A-B)cd-3\,(A-B)d^2-(45\,Bd^2fx+2\,(2\,A+3\,B)c^2+2\,(6\,A-B)cd-(A+19\,B)d^2)\cos(fx+e)^2+3\,(10\,Bd^2fx-(3\,A+2\,B)c^2-2\,(2\,A+3\,B)cd-3\,(A-6\,B)d^2)\cos(fx+e)+(60\,Bd^2fx+3\,(A-B)c^2-6\,(A-B)cd+3\,(A-B)d^2-(15\,Bd^2fx+(2\,A+3\,B)c^2+2\,(3\,A+7\,B)cd+(7\,A-32\,B)d^2)\cos(fx+e)^2+3\,(10\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+2\,B)cd-(2\,A-17\,B)d^2)\cos(fx+e))\sin(fx+e)}{15\,(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f+(a^3f\cos(fx+e)^2-2\,a^3f\cos(fx+e)-4\,a^3f)\sin(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d -
 (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B)*d
^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*c*d
- 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A - B)
*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B)*c*d
+ (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*
(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(
f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3
*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. $2(151) = 302$.

time = 9.64, size = 3468, normalized size = 21.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*
tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c**2*tan(e/2 + f*x/2)**2/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 40*A*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)
**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*
a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A
*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*
a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 60*A*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2
+ f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*A*d**2*t
an(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*A*d**2*tan(e/2 + f*x/2)/(15*a*
*3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e
/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) - 4*A*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/
2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)
**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a*
*3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
```

```
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 30*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*
tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 +
 f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 80*B*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*c*d*ta
n(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*B*c*d/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d
**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*d**2*f*x*tan(e/2 +
f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**3/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 7
5*B*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d**2*f*x/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) + 30*B*d**2*tan(e/2 + f*x/2)**4/(1...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(165) = 330.
time = 0.48, size = 382, normalized size = 2.33

$$\frac{\frac{15\,(fx+e)Bd^2}{a^3}-\frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-75\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+40\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+20\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-145\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+20\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+20\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+10\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-95\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,Ac^2+3\,Bc^2+6\,Acd+4\,Bcd+2\,Ad^2-22\,Bd^2\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^
2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1
/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f*x
```

```
+ 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e
)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 2
0*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^
2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*tan(1/2*f
*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x + 1/2*e)
 - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4*B*c*d +
2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]

time = 16.54, size = 286, normalized size = 1.74

$$\frac{B\,d^2\,x}{a^3}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{16\,A\,c^2}{3}+\frac{8\,A\,d^2}{3}+2\,B\,c^2-\frac{58\,B\,d^2}{3}+4\,A\,c\,d+\frac{16\,B\,c\,d}{3}\right)+\frac{14\,A\,c^2}{15}+\frac{4\,A\,d^2}{15}+\frac{2\,B\,c^2}{5}-\frac{44\,B\,d^2}{15}+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(4\,A\,c^2+2\,B\,c^2-10\,B\,d^2+4\,A\,c\,d\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,A\,c^2-2\,B\,d^2\right)+\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,A\,c^2}{3}+\frac{4\,A\,d^2}{3}+2\,B\,c^2-\frac{38\,B\,d^2}{3}+4\,A\,c\,d+\frac{8\,B\,c\,d}{3}\right)+\frac{4\,A\,c\,d}{5}+\frac{8\,B\,c\,d}{15}}{f\,\left(a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+5\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+10\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,a^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (B*d^2*x)/a^3 - (tan(e/2 + (f*x)/2)^2*((16*A*c^2)/3 + (8*A*d^2)/3 + 2*B*c^2
 - (58*B*d^2)/3 + 4*A*c*d + (16*B*c*d)/3) + (14*A*c^2)/15 + (4*A*d^2)/15 +
(2*B*c^2)/5 - (44*B*d^2)/15 + tan(e/2 + (f*x)/2)^3*(4*A*c^2 + 2*B*c^2 - 10*
B*d^2 + 4*A*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*B*d^2) + tan(e/2 + (f*
x)/2)*((8*A*c^2)/3 + (4*A*d^2)/3 + 2*B*c^2 - (38*B*d^2)/3 + 4*A*c*d + (8*B*
c*d)/3) + (4*A*c*d)/5 + (8*B*c*d)/15)/(f*(10*a^3*tan(e/2 + (f*x)/2)^2 + 10*
a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)
/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2Ac+3Bc+3Ad+7Bd)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -1/5*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c+3*A*d+3*B*c-8*
B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*A*c+3*A*d+3*B*c+7*B*d)*cos(f
*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3047, 3098, 2829, 2727}

$$-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) - ((2*A*c +
3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*
A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc+3Ad-3Bd)-5aB}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bc}{15af(a+a\sin(e+}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bc}{15af(a+a\sin(e+}$$

Mathematica [A]
time = 0.49, size = 176, normalized size = 1.39

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15(Ad+B(c+2d))\cos\left(\frac{1}{2}(e+fx)\right)-5(2Ac+3Bc+3Ad+4Bd)\cos\left(\frac{3}{2}(e+fx)\right)-2(-3(3Ac+2Bc+2Ad+8Bd)+(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)+(2Ac+3Bc+3Ad+7Bd)\cos(2(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)\right)}{30a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*x
)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*A*
c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x] +
 (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*a
^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.36, size = 151, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{-8Ac+8Ad+8Bc-8Bd}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4Ac-4Ad-4Bc+4Bd)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4Ac+2Ad+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-6Ad-6Bc+4Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{f\,a^3}$

default	$\frac{-\frac{-8Ac+8Ad+8Bc-8Bd}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(4Ac-4Ad-4Bc+4Bd)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4Ac+2Ad+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(8Ac-6Ad-6Bc+4Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{f\,a^{3}}$
risch	$-\frac{2\left(2Ac+15Bd\,e^{4i(fx+e)}-10iAc\,e^{i(fx+e)}+15iAd\,e^{3i(fx+e)}-20iBd\,e^{i(fx+e)}-15iBc\,e^{i(fx+e)}+30iBd\,e^{3i(fx+e)}-15id\,e^{i(\ldots}\right.}{15f\,a^{3}\left(e^{i(fx+\ldots}\right.}$
norman	$\frac{-\frac{14Ac+6Ad+6Bc+4Bd}{15fa}-\frac{2Ac\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(2Ac+Ad+Bc)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(34Ac+11Ad+11Bc+14Bd)\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A*c+8*A*d+8*B*c-8*B*d)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A*
c-4*A*d-4*B*c+4*B*d)/(tan(1/2*f*x+1/2*e)+1)^5-A*c/(tan(1/2*f*x+1/2*e)+1)-1/
2*(-4*A*c+2*A*d+2*B*c)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-6*A*d-6*B*c+4*B*
d)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(127) = 254$.
time = 0.31, size = 797, normalized size = 6.28

$$-\frac{2\left(\frac{Ac\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{2Bd\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3Bc\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3Ad\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
```

```
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(127) = 254$.
time = 0.79, size = 283, normalized size = 2.23

$$-\frac{((2A+3B)c+(3A+7B)d)\cos(fx+e)^3-(2(2A+3B)c+(6A-B)d)\cos(fx+e)^2-3(A-B)c+3(A-B)d-3((3A+2B)c+(2A+3B)d)\cos(fx+e)-(((2A+3B)c+(3A+7B)d)\cos(fx+e)^2-3(A-B)c+3(A-B)d+3((2A+3B)c+(3A+2B)d)\cos(fx+e))\sin(fx+e)}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c +
(6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*c
+ (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x +
e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*cos(f*
x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^
3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) -
 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(121) = 242$.
time = 5.37, size = 1819, normalized size = 14.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(
```

```
e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)/(15*a**3*f*t
an(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f
*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15
*a**3*f) - 6*A*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2
)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*
f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*d*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*B*d*tan(e/2 + f*x/2)/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 4*B*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
d*sin(e))/(a*sin(e) + a)**3, True))
```

Giac [A]
time = 0.51, size = 223, normalized size = 1.76

$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+20\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+20\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+10\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,Ac+3\,Bc+3\,Ad+2\,Bd\right)}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/2*
f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/2*e
)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*B*c*
tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*x + 1
/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B]

time = 14.26, size = 245, normalized size = 1.93

$$\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53Ac}{4}+3Ad+3Bc+\frac{13Bd}{4}-4Ac\cos\left(e+fx\right)+\frac{3Ad\cos(e+fx)}{2}+\frac{3Bc\cos(e+fx)}{2}+Bd\cos\left(e+fx\right)+\frac{25Ac\sin(e+fx)}{2}+\frac{15Ad\sin(e+fx)}{2}+\frac{15Bc\sin(e+fx)}{2}+\frac{5Bd\sin(e+fx)}{2}-\frac{9Ac\cos(2e+2fx)}{4}-\frac{3Ad\cos(2e+2fx)}{2}-\frac{3Bc\cos(2e+2fx)}{2}-\frac{9Bd\cos(2e+2fx)}{4}-\frac{5Ac\sin(2e+2fx)}{4}+\frac{5Bd\sin(2e+2fx)}{4}\right)}{15a^3f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}\cos\left(\frac{5e}{2}-\frac{\pi}{4}+\frac{5fx}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A*c)/4 + 3*A*d + 3*B*c + (13*B*d)/4 - 4*A*c*cos(
e + f*x) + (3*A*d*cos(e + f*x))/2 + (3*B*c*cos(e + f*x))/2 + B*d*cos(e + f*
x) + (25*A*c*sin(e + f*x))/2 + (15*A*d*sin(e + f*x))/2 + (15*B*c*sin(e + f*
x))/2 + (5*B*d*sin(e + f*x))/2 - (9*A*c*cos(2*e + 2*f*x))/4 - (3*A*d*cos(2*
e + 2*f*x))/2 - (3*B*c*cos(2*e + 2*f*x))/2 - (9*B*d*cos(2*e + 2*f*x))/4 - (
5*A*c*sin(2*e + 2*f*x))/4 + (5*B*d*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1
/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/
2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

```
[Out] -1/5*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A+3*B)*cos(f*x+e)/a/f/(a
+a*sin(f*x+e))^2-1/15*(2*A+3*B)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2829, 2729, 2727}

$$-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2A+3B)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) - ((2*A + 3*B)*Cos[e
 + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A + 3*B)*Cos[e + f*x])/(15*f
*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx &= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{(2A+3B)\int\frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}\\ &= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}+\frac{(2A+3B)\int\frac{1}{a+a\sin(e+\cdots}}{15a^2}\\ &= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2A+3B)\cos(e+\cdots}{15f\left(a^3+a^3\sin(e+\cdots\right.}\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 63, normalized size = 0.62

$$-\frac{\cos(e+fx)\left(7A+3B+(6A+9B)\sin(e+fx)+(2A+3B)\sin^2(e+fx)\right)}{15a^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Sin
[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [**A**]
time = 0.24, size = 114, normalized size = 1.12

method	result
risch	$-\frac{2i\left(20iA\,e^{2i(fx+e)}+15iB\,e^{2i(fx+e)}+15B\,e^{3i(fx+e)}-2iA-10A\,e^{i(fx+e)}-3iB-15B\,e^{i(fx+e)}\right)}{15f\,a^3\left(e^{i(fx+e)}+i\right)^5}$
derivativedivides	$\frac{-\frac{2(4A-4B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-4A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8A-6B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{f\,a^3}$
default	$\frac{-\frac{2(4A-4B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-4A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8A-6B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{f\,a^3}$
norman	$\frac{-\frac{14A+6B}{15fa}-\frac{2A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{(94A+36B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15fa}-\frac{(20A+12B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{(22A+6B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{(8A-\cdots}{\cdots}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a^3*(-1/5*(4*A-4*B)/(tan(1/2*f*x+1/2*e)+1)^5-A/(tan(1/2*f*x+1/2*e)+1)-1
/4*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-4*A+2*B)/(tan(1/2*f*x+1/2*e)+1
)^2-1/3*(8*A-6*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(102) = 204$.

time = 0.32, size = 421, normalized size = 4.13

$$-\frac{2\left(\frac{A\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3B\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*B*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A]

time = 0.39, size = 202, normalized size = 1.98

$$-\frac{(2A+3B)\cos(fx+e)^3-2(2A+3B)\cos(fx+e)^2-3(3A+2B)\cos(fx+e)-\left((2A+3B)\cos(fx+e)^2+3(2A+3B)\cos(fx+e)-3A+3B\right)\sin(fx+e)-3A+3B}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A +
 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x +
e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(87) = 174$.

time = 2.96, size = 1015, normalized size = 9.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
```

```
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 40*A*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 30*B*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan
(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**3, True))
```

Giac [A]
time = 0.47, size = 130, normalized size = 1.27

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A+3\,B\right)}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*tan
(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/2*
e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B]
time = 13.80, size = 150, normalized size = 1.47

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{53\,A}{4}+3\,B-4\,A\,\cos\left(e+f\,x\right)+\frac{3\,B\,\cos(e+f\,x)}{2}+\frac{25\,A\,\sin(e+f\,x)}{2}+\frac{15\,B\,\sin(e+f\,x)}{2}-\frac{9\,A\,\cos(2\,e+2\,f\,x)}{4}-\frac{3\,B\,\cos(2\,e+2\,f\,x)}{2}-\frac{5\,A\,\sin(2\,e+2\,f\,x)}{4}\right)}{15\,a^{3}\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}-\frac{\pi}{4}+\frac{5\,f\,x}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A)/4 + 3*B - 4*A*cos(e + f*x) + (3*B*cos(e + f*x
))/2 + (25*A*sin(e + f*x))/2 + (15*B*sin(e + f*x))/2 - (9*A*cos(2*e + 2*f*x
))/4 - (3*B*cos(2*e + 2*f*x))/2 - (5*A*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*
2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f
*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=229

$$\frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}\,f}-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3}-\frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

```
[Out] -1/5*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c-7*A*d+3*B*c+2*
B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/15*(B*(3*c^2-16*c*d-2*d^2)
+A*(2*c^2-9*c*d+22*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))+2*d^2*(-
A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^3/f/(c^
2-d^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3057, 12, 2739, 632, 210}

$$\frac{2d^2(Bc-Ad)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{(A(2c^2-9cd+22d^2)+B(3c^2-16cd-2d^2))\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*d^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(
c - d)^3*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Si
n[e + f*x])^3) - ((2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/(15*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 - 16*c*d - 2*d^2) + A*(2*c^2 -
9*c*d + 22*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc-5Ad)-2a(A-B)}{(a+a\sin(e+fx))^2(c+d\sin(}}{5a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2B}{15a(c-d)^2f(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2B}{15a(c-d)^2f(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2B}{15a(c-d)^2f(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2B}{15a(c-d)^2f(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2B}{15a(c-d)^2f(a+a\sin}$$

$$= \frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}\,f} - \frac{(A-B)\cos(e}{5(c-d)f(a+a\sin}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 502 vs. 2(229) = 458.

time = 0.86, size = 502, normalized size = 2.19

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(15Bc^2\cos\left(\tfrac{1}{2}(e+fx)\right)-15Acd\cos\left(\tfrac{1}{2}(e+fx)\right)-75Bcd\cos\left(\tfrac{1}{2}(e+fx)\right)+75Ad^2\cos\left(\tfrac{1}{2}(e+fx)\right)-10Ac^2\cos\left(\tfrac{3}{2}(e+fx)\right)-15Bc^2\cos\left(\tfrac{3}{2}(e+fx)\right)+45Acd\cos\left(\tfrac{3}{2}(e+fx)\right)+65Bcd\cos\left(\tfrac{3}{2}(e+fx)\right)-95Ad^2\cos\left(\tfrac{3}{2}(e+fx)\right)+10Bd^2\cos\left(\tfrac{3}{2}(e+fx)\right)+20Ac^2\sin\left(\tfrac{1}{2}(e+fx)\right)+15Bc^2\sin\left(\tfrac{1}{2}(e+fx)\right)-75Acd\sin\left(\tfrac{1}{2}(e+fx)\right)-85Bcd\sin\left(\tfrac{1}{2}(e+fx)\right)+145Ad^2\sin\left(\tfrac{1}{2}(e+fx)\right)-20Bd^2\sin\left(\tfrac{1}{2}(e+fx)\right)-\frac{60d^2(-Bc+Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{\sqrt{c^2-d^2}}-15Bcd\sin\left(\tfrac{3}{2}(e+fx)\right)+15Ad^2\sin\left(\tfrac{3}{2}(e+fx)\right)-2Ac^2\sin\left(\tfrac{5}{2}(e+fx)\right)-3Bc^2\sin\left(\tfrac{5}{2}(e+fx)\right)+9Acd\sin\left(\tfrac{5}{2}(e+fx)\right)+16Bcd\sin\left(\tfrac{5}{2}(e+fx)\right)-22Ad^2\sin\left(\tfrac{5}{2}(e+fx)\right)+2Bd^2\sin\left(\tfrac{5}{2}(e+fx)\right)\right)}{30a^3(c-d)^3f(1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c*
d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2]
- 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c*d*
Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*(e +
 f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2] + 15*
B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e + f*x)
/2] + 145*A*d^2*Sin[(e + f*x)/2] - 20*B*d^2*Sin[(e + f*x)/2] - (60*d^2*(-(B
*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/Sqrt[c^2 - d^2] - 15*B*c*d*Sin[(3*(e + f*x))/2] +
 15*A*d^2*Sin[(3*(e + f*x))/2] - 2*A*c^2*Sin[(5*(e + f*x))/2] - 3*B*c^2*Sin
[(5*(e + f*x))/2] + 9*A*c*d*Sin[(5*(e + f*x))/2] + 16*B*c*d*Sin[(5*(e + f*x
))/2] - 22*A*d^2*Sin[(5*(e + f*x))/2] + 2*B*d^2*Sin[(5*(e + f*x))/2]))/(30*
a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 0.77, size = 252, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6Ad+2Bc-4Bd}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-10Ad-6Bc+8Bd)}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(Ac^2-3A\cdots\right.}{(c-d)^3\left(\mathrm{ta}\cdots\right.}}{f\,a^3}$
default	$\frac{-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6Ad+2Bc-4Bd}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-10Ad-6Bc+8Bd)}{3(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(Ac^2-3A\cdots\right.}{(c-d)^3\left(\mathrm{ta}\cdots\right.}}{f\,a^3}$
risch	$\frac{6Acd}{5}+\frac{38iA\,d^2\mathrm{e}^{i(fx+e)}}{3}-\frac{2B\,c^2}{5}-\frac{4A\,c^2}{15}+\frac{4B\,d^2}{15}+10iBcd\,\mathrm{e}^{3i(fx+e)}-\frac{4iB\,d^2\mathrm{e}^{i(fx+e)}}{3}+\frac{32Bcd}{15}-\frac{44A\,d^2}{15}+\frac{8A\,c^2\mathrm{e}^{2i(fx+e)}}{3}+58\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A+8*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*B)/(c-d)
/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+6*A*d+2*B*c-4*B*d)/(c-d)^2/(tan(1/2*f
```

```
*x+1/2*e)+1)^2-1/3*(8*A*c-10*A*d-6*B*c+8*B*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1
)^3-(A*c^2-3*A*c*d+3*A*d^2-B*d^2)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)-d^2*(A*d-B
*c)/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(225) = 450$.
time = 0.46, size = 2337, normalized size = 10.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4
- 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16
*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (18*A
 + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A - 4*B)
*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x
+ e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x +
 e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2
- A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
 e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d + 5*(3*A - B)
*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) - 2*(3*(A - B
)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^
4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A
 - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B)*c^3*d + 5*(3*
A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos(f*x + e))*sin(f
*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*
```

```
d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
+ 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*
a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*
x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3
 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a
^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3
*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)
*f)*sin(f*x + e)), 1/15*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3
- 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d
^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + (2*(2*A + 3*B)
*c^4 - (18*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 -
(29*A - 4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d
^3)*cos(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^
3)*cos(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 +
 2*(B*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c
*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^4 - (
11*A + 9*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*
d^4)*cos(f*x + e) - (3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*
(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 +
 (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4
 - (9*A + 11*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A -
2*B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*
a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*
d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d +
2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2
*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3
*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(225) = 450.

time = 0.58, size = 578, normalized size = 2.52

$$\frac{2\left(\frac{15\,(Bcd^2-Ad^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\sqrt{c^2-d^2}}-\frac{15Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45Acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-15Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+30Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+15Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-105Acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-45Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+135Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-30Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+15Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-135Acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-65Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+185Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-40Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+20Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+15Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-75Acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-55Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+115Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-20Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7Ac^2+3Bc^2-24Acd-11Bcd+32Ad^2-7Bd^2}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
- 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*
d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan
(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan(1/2*f
*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*f*x + 1
/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2
 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 185
*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*
tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*tan(1/2*f*x
 + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f*x + 1/2*e)
- 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d - 11*B*c*d +
 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(
1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 17.18, size = 591, normalized size = 2.58

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,(A\,d-B\,c)\,\left(-2\,a^3\,c^3\,d+6\,a^3\,c^2\,d^2-6\,a^3\,c\,d^3+2\,a^3\,d^4\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}-\frac{2\,c\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,\left(a^3\,c^3-3\,a^3\,c^2\,d+3\,a^3\,c\,d^2-a^3\,d^3\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}}{2\,A\,d^3-2\,B\,c\,d^2}\right)\,(A\,d-B\,c)}{a^3\,f\,\sqrt{c+d}\,(c-d)^{7/2}}-\frac{\frac{2\,(7\,A\,c^2+32\,A\,d^2+3\,B\,c^2-7\,B\,d^2-24\,A\,c\,d-11\,B\,c\,d)}{15\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,A\,c^2+23\,A\,d^2+3\,B\,c^2-4\,B\,d^2-15\,A\,c\,d-11\,B\,c\,d)}{3\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,(2\,A\,c^2+9\,A\,d^2+B\,c^2-2\,B\,d^2-7\,A\,c\,d-3\,B\,c\,d)}{(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,(8\,A\,c^2+37\,A\,d^2+3\,B\,c^2-8\,B\,d^2-27\,A\,c\,d-13\,B\,c\,d)}{3\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,(A\,c^2+3\,A\,d^2-B\,d^2-3\,A\,c\,d)}{(c-d)\,(c^2-2\,c\,d+d^2)}}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+5\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+10\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d^2*atan(((d^2*(A*d - B*c)*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a^
3*c^2*d^2))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/2)
*(A*d - B*c)*(a^3*c^3 - a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c + d)^
(1/2)*(c - d)^(7/2)))/(2*A*d^3 - 2*B*c*d^2))*(A*d - B*c))/(a^3*f*(c + d)^(1
/2)*(c - d)^(7/2)) - ((2*(7*A*c^2 + 32*A*d^2 + 3*B*c^2 - 7*B*d^2 - 24*A*c*d
 - 11*B*c*d))/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(4*A
*c^2 + 23*A*d^2 + 3*B*c^2 - 4*B*d^2 - 15*A*c*d - 11*B*c*d))/(3*(c - d)*(c^2
 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2*A*c^2 + 9*A*d^2 + B*c^2 - 2*B
*d^2 - 7*A*c*d - 3*B*c*d))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*
x)/2)^2*(8*A*c^2 + 37*A*d^2 + 3*B*c^2 - 8*B*d^2 - 27*A*c*d - 13*B*c*d))/(3*
(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^2 + 3*A*d^2 - B
*d^2 - 3*A*c*d))/((c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*tan(e/2 + (f*x)/
2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e
/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=381

$$-\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f}-\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2}{15a^3(c-d)^4(c+d)}$$

```
[Out] -1/15*d*(B*(3*c^3-23*c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*d
^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)/(
c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))-1/15*(2*A*c-9*A*d+3*B*c+4*B*d)*c
os(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/15*(B*(3*c^2-23
*c*d-15*d^2)+A*(2*c^2-12*c*d+45*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x
+e))/(c+d*sin(f*x+e))-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^4/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$-\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\,\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{(A(2c^2-12cd+45d^2)+B(3c^2-23cd-15d^2))\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))}-\frac{d(A(2c^3-12c^2d+43cd^2+72d^3)+B(3c^3-23c^2d-63cd^2-22d^3))\cos(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sin(e+fx))}-\frac{(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^2*(A*d*(4*c + 3*d) - B*(3*c^2 + 3*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(
B*(3*c^3 - 23*c^2*d - 63*c*d^2 - 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 +
 72*d^3))*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e +
f*x])) - ((2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(
a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) - ((B*(3*c^2 - 23*c*d - 15*d^2)
 + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Si
n[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\int \frac{-c}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{}{15a}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{}{15a}$$

$$= -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43}{15a^3(c-d)^4(c+d)f(c+d\sin(e+}$$

$$= -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43}{15a^3(c-d)^4(c+d)f(c+d\sin(e+}$$

$$= -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43}{15a^3(c-d)^4(c+d)f(c+d\sin(e+}$$

$$= -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43}{15a^3(c-d)^4(c+d)f(c+d\sin(e+}$$

$$= -\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\tan^{-1}\left(\frac{d+c\tan(\frac{1}{2}(}{\sqrt{c^2-}}\right.}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1253 vs. $2(381) = 762$.

time = 6.29, size = 1253, normalized size = 3.29

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] (2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*x
)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a + a*
Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos[(e
+ f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2] + 54
0*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430*A*c*d
^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(e + f*x
)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] - 60*B*c^
4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*c^3*d*Cos
```

```
[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c^2*d^2*Cos
[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c*d^3*Cos[(3
*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos[(3*(e + f*x
))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*(e + f*x))/2]
 + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x))/2] + 30*B*d
^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6*B*c^3*d*Cos[(7
*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c^2*d^2*Cos[(7*(e
 + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^3*Cos[(7*(e + f*x
))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*(e + f*x))/2] + 80
*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 340*A*c^3*d*Sin[(e +
f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d^2*Sin[(e + f*x)/2] - 1
520*B*c^2*d^2*Sin[(e + f*x)/2] + 2140*A*c*d^3*Sin[(e + f*x)/2] - 1435*B*c*d
^3*Sin[(e + f*x)/2] + 975*A*d^4*Sin[(e + f*x)/2] - 340*B*d^4*Sin[(e + f*x)/
2] - 90*B*c^3*d*Sin[(3*(e + f*x))/2] + 120*A*c^2*d^2*Sin[(3*(e + f*x))/2] -
 390*B*c^2*d^2*Sin[(3*(e + f*x))/2] + 540*A*c*d^3*Sin[(3*(e + f*x))/2] - 31
5*B*c*d^3*Sin[(3*(e + f*x))/2] + 285*A*d^4*Sin[(3*(e + f*x))/2] - 150*B*d^4
*Sin[(3*(e + f*x))/2] - 8*A*c^4*Sin[(5*(e + f*x))/2] - 12*B*c^4*Sin[(5*(e +
 f*x))/2] + 28*A*c^3*d*Sin[(5*(e + f*x))/2] + 62*B*c^3*d*Sin[(5*(e + f*x))/
2] - 52*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 362*B*c^2*d^2*Sin[(5*(e + f*x))/2]
 - 568*A*c*d^3*Sin[(5*(e + f*x))/2] + 553*B*c*d^3*Sin[(5*(e + f*x))/2] - 55
5*A*d^4*Sin[(5*(e + f*x))/2] + 190*B*d^4*Sin[(5*(e + f*x))/2] - 15*B*c*d^3*
Sin[(7*(e + f*x))/2] + 15*A*d^4*Sin[(7*(e + f*x))/2]))/(120*(c - d)^4*(c +
d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]))
```

Maple [A]
time = 0.88, size = 355, normalized size = 0.93

method	result
derivativedivides	$-\frac{-\frac{-8A+8B}{2(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+8Ad+2Bc-6Bd}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-12Ad-6Bc+10Bd)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\cdots}{f}$
default	$-\frac{-\frac{-8A+8B}{2(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+8Ad+2Bc-6Bd}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-12Ad-6Bc+10Bd)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\cdots}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A+8*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*B)/(c-
d)^2/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+8*A*d+2*B*c-6*B*d)/(c-d)^3/(tan(1
/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-12*A*d-6*B*c+10*B*d)/(c-d)^3/(tan(1/2*f*x+1/2
*e)+1)^3-(A*c^2-4*A*c*d+6*A*d^2-3*B*d^2)/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)-d^2
/(c-d)^4*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)/(c+d))/(c*t
an(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(4*A*c*d+3*A*d^2-3*B*c^2-3*B*
c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c
^2-d^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2227 vs. 2(380) = 760.
time = 0.50, size = 4543, normalized size = 11.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/30*(6*(A - B)*c^6 - 12*(A - B)*c^5*d - 6*(A - B)*c^4*d^2 + 24*(A - B)*c
^3*d^3 - 6*(A - B)*c^2*d^4 - 12*(A - B)*c*d^5 + 6*(A - B)*d^6 - 2*((2*A + 3
*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*
d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - 2*((2*A +
 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*B)*c^
3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^
6)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A -
7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A -
 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(
2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 -
 (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A -
9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*
c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos
```

(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^6 - (11*A + 9*B)*c^5*d + (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 + (47*A + 28*B)*c^2*d^4 - 71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e) - 2*(3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 + 5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 + 2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e)), -1/15*(3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 - ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - ((2*A + 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*B)*c^3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^6)*cos(f*x + e)^3 + (2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A - 7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A - 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 -

```
4*(3*A - B)*d^5 + (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x
 + e)^4 - (3*B*c^3*d^2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A
- B)*d^5)*cos(f*x + e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18
*B)*c*d^4 - 5*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B
)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2
 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2
*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4
*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2
+ 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(380) = 760.
time = 0.54, size = 772, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor(
1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3
*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) - A*d
^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*
a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1
/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*B
*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*ta
n(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*d*tan(1/2*
f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*tan(1/2*f*x +
 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)
^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x + 1/2*e)^2 +
420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*
c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 110*A*c*d*tan(1/
2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*tan(1/2*f*x + 1/
```

```
2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 34*A*c*d - 16*B
*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3
*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 17.67, size = 1349, normalized size = 3.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d^2*atan(((d^2*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(2*a^3*d^
6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^3*c^3*d^3 - 6*a^3*c^4*d
^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(3*B*c
^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*d^4
- 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c - d)^
(9/2)))/(2*B*d^4 - 6*A*d^4 + 6*B*c^2*d^2 - 8*A*c*d^3 + 6*B*c*d^3))*(3*B*c^2
 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))/(a^3*f*(c + d)^(3/2)*(c - d)^(9/2)
) - ((2*(7*A*c^4 + 15*A*d^4 + 3*B*c^4 + 38*A*c^2*d^2 - 48*B*c^2*d^2 + 72*A*
c*d^3 - 27*A*c^3*d - 47*B*c*d^3 - 13*B*c^3*d))/(15*(c + d)*(c - d)*(3*c*d^2
 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(5*A*c^4 + 15*A*d^4 + 3*
B*c^4 + 19*A*c^2*d^2 - 45*B*c^2*d^2 + 84*A*c*d^3 - 18*A*c^3*d - 52*B*c*d^3
- 11*B*c^3*d))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 +
 (f*x)/2)*(20*A*c^5 + 15*A*d^5 + 15*B*c^5 + 346*A*c^2*d^3 + 106*A*c^3*d^2 -
 286*B*c^2*d^3 - 221*B*c^3*d^2 + 219*A*c*d^4 - 76*A*c^4*d - 79*B*c*d^4 - 59
*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(
e/2 + (f*x)/2)^5*(2*A*c^5 + 5*A*d^5 + B*c^5 + 24*A*c^2*d^3 + 4*A*c^3*d^2 -
16*B*c^2*d^3 - 13*B*c^3*d^2 + 13*A*c*d^4 - 6*A*c^4*d - 11*B*c*d^4 - 3*B*c^4
*d))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*
x)/2)^4*(11*A*c^5 + 30*A*d^5 + 3*B*c^5 + 162*A*c^2*d^3 + 4*A*c^3*d^2 - 139*
B*c^2*d^3 - 84*B*c^3*d^2 + 135*A*c*d^4 - 27*A*c^4*d - 84*B*c*d^4 - 11*B*c^4
*d))/(3*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (
f*x)/2)^2*(47*A*c^5 + 75*A*d^5 + 18*B*c^5 + 812*A*c^2*d^3 + 88*A*c^3*d^2 -
757*B*c^2*d^3 - 463*B*c^3*d^2 + 690*A*c*d^4 - 137*A*c^4*d - 305*B*c*d^4 - 6
8*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan
(e/2 + (f*x)/2)^6*(A*c^5 + A*d^5 + 6*A*c^2*d^3 + 2*A*c^3*d^2 - 3*B*c^2*d^3
- 3*B*c^3*d^2 - 3*A*c^4*d - B*c*d^4))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d
 + c^3 - d^3)))/(f*(a^3*c + tan(e/2 + (f*x)/2)*(5*a^3*c + 2*a^3*d) + tan(e/
2 + (f*x)/2)^6*(5*a^3*c + 2*a^3*d) + tan(e/2 + (f*x)/2)^2*(11*a^3*c + 10*a^
3*d) + tan(e/2 + (f*x)/2)^5*(11*a^3*c + 10*a^3*d) + tan(e/2 + (f*x)/2)^3*(1
5*a^3*c + 20*a^3*d) + tan(e/2 + (f*x)/2)^4*(15*a^3*c + 20*a^3*d) + a^3*c*ta
n(e/2 + (f*x)/2)^7))
```

3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=508

$$-\frac{d^2\left(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\,f}-\frac{d(3B(2c^3-20c^2d$$

```
[Out] -1/30*d*(3*B*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)+A*(4*c^3-30*c^2*d+146*c*d^2+1
95*d^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))^2-1/5*(A-B)*cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2-1/15*(2*A*c-11*A*d+3*B*c
+6*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/15*(
3*B*(c^2-10*c*d-12*d^2)+A*(2*c^2-15*c*d+76*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+
a^3*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/30*d*(3*B*(2*c^4-20*c^3*d-119*c^2*d^2-
130*c*d^3-48*d^4)+A*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4))*cos(f*x
+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))-d^2*(A*d*(20*c^2+30*c*d+13*d^2)-
3*B*(4*c^3+8*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2
)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A]

time = 0.99, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3057, 2833, 12, 2739, 632, 210}

$$-\frac{d^2\left(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3)\right)\text{ArcTan}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{\left(A(2c^2-15cd+76d^2)+3B(c^2-10cd-12d^2)\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^2}-\frac{d\left(A(4c^3-30c^2d+146cd^2+195d^3)+3B(2c^3-20c^2d-57cd^2-30d^3)\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^2}-\frac{d\left(A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)+3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2(c+d\sin(e+fx))}-\frac{(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^2*(A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2
*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^5*(c
+ d)^2*Sqrt[c^2 - d^2]*f)) - (d*(3*B*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3)
 + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3))*Cos[e + f*x])/(30*a^3*(c - d
)^4*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f
*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2) - ((2*A*c + 3*B*c - 11*A*d
+ 6*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[
e + f*x])^2) - ((3*B*(c^2 - 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))
*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2) - (d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4) + A*(4
*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \dots}{\dots}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\dots}{15a \dots}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\dots}{15a \dots}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+1\dots}{30a^3(c-d)^4(c+d)f(c+d\sin(e+\dots}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+1\dots}{30a^3(c-d)^4(c+d)f(c+d\sin(e+\dots}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+1\dots}{30a^3(c-d)^4(c+d)f(c+d\sin(e+\dots}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+1\dots}{30a^3(c-d)^4(c+d)f(c+d\sin(e+\dots}$$

$$= -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+1\dots}{30a^3(c-d)^4(c+d)f(c+d\sin(e+\dots}$$

$$= -\frac{d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2\dots}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\dots}$$

Mathematica [A]
time = 3.30, size = 548, normalized size = 1.08

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - d)
*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2 - 19*
c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 +
(30*d^2*(-(A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2
```

```
 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d)*d^3*(B*
c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)*(c
+ d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 + 6*c*d + 2*d
^2))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*(c +
d*Sin[e + f*x]))))/(30*a^3*(c - d)^5*f*(1 + Sin[e + f*x])^3)
```

Maple [A]
time = 1.54, size = 639, normalized size = 1.26

method	result
derivativedivides	$-\frac{-8A+8B}{2(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+10Ad+2Bc-8Bd}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-14Ad-6Bc+12Bd)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(A c^2-5A\right.}{(c-d)^5\left(\right.}$
default	$-\frac{-8A+8B}{2(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+10Ad+2Bc-8Bd}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-14Ad-6Bc+12Bd)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(A c^2-5A\right.}{(c-d)^5\left(\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A+8*B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*B)/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+10*A*d+2*B*c-8*B*d)/(c-d)^4/(tan(
1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-14*A*d-6*B*c+12*B*d)/(c-d)^4/(tan(1/2*f*x+1/
2*e)+1)^3-(A*c^2-5*A*c*d+10*A*d^2-6*B*d^2)/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)-d
^2/(c-d)^5*((1/2*d^2*(11*A*c^2*d+6*A*c*d^2-2*A*d^3-9*B*c^3-6*B*c^2*d)/c/(c^
2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(10*A*c^4*d+6*A*c^3*d^2+19*A*c^2*d^
3+12*A*c*d^4-2*A*d^5-8*B*c^5-6*B*c^4*d-17*B*c^3*d^2-12*B*c^2*d^3-2*B*c*d^4)
/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(29*A*c^2*d+18*A*c*d^2-2*
A*d^3-23*B*c^3-18*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1
/2*d*(10*A*c^2*d+6*A*c*d^2-A*d^3-8*B*c^3-6*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2)
)/(c*tan(1/2*f*x+1/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(20*A*c^2*d+30*A*
c*d^2+13*A*d^3-12*B*c^3-24*B*c^2*d-21*B*c*d^2-6*B*d^3)/(c^2+2*c*d+d^2)/(c^2
-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3631 vs. $2(507) = 1014$.
time = 0.57, size = 7352, normalized size = 14.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/60*(12*(A - B)*c^8 - 24*(A - B)*c^7*d - 24*(A - B)*c^6*d^2 + 72*(A - B)
*c^5*d^3 - 72*(A - B)*c^3*d^5 + 24*(A - B)*c^2*d^6 + 24*(A - B)*c*d^7 - 12*
(A - B)*d^8 + 2*(2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 1
21*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35
*A - 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*cos(f*x + e)^5 - 2*(4*(2*A + 3*B)*c
^7*d - 4*(13*A + 27*B)*c^6*d^2 + 18*(12*A - 37*B)*c^5*d^3 + 6*(181*A - 171*
B)*c^4*d^4 + 3*(328*A - 33*B)*c^3*d^5 - 9*(69*A - 104*B)*c^2*d^6 - (1208*A
- 753*B)*c*d^7 - (413*A - 198*B)*d^8)*cos(f*x + e)^4 - 2*(2*(2*A + 3*B)*c^8
 - 6*(A + 4*B)*c^7*d - 20*(A + 21*B)*c^6*d^2 + 6*(128*A - 293*B)*c^5*d^3 +
3*(892*A - 827*B)*c^4*d^4 + 3*(769*A - 49*B)*c^3*d^5 - (1573*A - 2373*B)*c^
2*d^6 - 3*(1023*A - 643*B)*c*d^7 - (1087*A - 522*B)*d^8)*cos(f*x + e)^3 + 4
*(2*(2*A + 3*B)*c^8 - 5*(4*A + 3*B)*c^7*d + (19*A - 174*B)*c^6*d^2 + 15*(22
*A - 35*B)*c^5*d^3 + 3*(233*A - 173*B)*c^4*d^4 + 15*(23*A + 10*B)*c^3*d^5 -
 (526*A - 591*B)*c^2*d^6 - 5*(131*A - 78*B)*c*d^7 - 4*(49*A - 24*B)*d^8)*co
s(f*x + e)^2 - 15*(48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)
*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B
)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*
A - 6*B)*d^7)*cos(f*x + e)^5 + (24*B*c^4*d^3 - 4*(10*A - 21*B)*c^3*d^4 - 6*
(20*A - 19*B)*c^2*d^5 - (116*A - 75*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x
+ e)^4 - (12*B*c^5*d^2 - 4*(5*A - 18*B)*c^4*d^3 - (110*A - 153*B)*c^3*d^4 -
 (193*A - 162*B)*c^2*d^5 - (142*A - 87*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f
*x + e)^3 - (36*B*c^5*d^2 - 12*(5*A - 16*B)*c^4*d^3 - (290*A - 387*B)*c^3*d
^4 - (479*A - 396*B)*c^2*d^5 - (340*A - 207*B)*c*d^6 - 7*(13*A - 6*B)*d^7)*
cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c
^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*
cos(f*x + e) + (48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^
```

```
3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d
^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A -
 6*B)*d^7)*cos(f*x + e)^4 - 2*(12*B*c^4*d^3 - 4*(5*A - 9*B)*c^3*d^4 - 5*(10
*A - 9*B)*c^2*d^5 - (43*A - 27*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^3
- (12*B*c^5*d^2 - 4*(5*A - 24*B)*c^4*d^3 - 75*(2*A - 3*B)*c^3*d^4 - (293*A
- 252*B)*c^2*d^5 - 3*(76*A - 47*B)*c*d^6 - 5*(13*A - 6*B)*d^7)*cos(f*x + e)
^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(
31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)
)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d
*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e)
)*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2))
+ 12*((3*A + 2*B)*c^8 - (11*A + 9*B)*c^7*d + (9*A - 109*B)*c^6*d^2 + (213*A
 - 353*B)*c^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (237*A + 103*B)*c^3*d^5 - (35
9*A - 399*B)*c^2*d^6 - (439*A - 259*B)*c*d^7 - (128*A - 63*B)*d^8)*cos(f*x
+ e) - 2*(6*(A - B)*c^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B
)*c^5*d^3 - 36*(A - B)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*
(A - B)*d^8 + (2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121
*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A
 - 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*cos(f*x + e)^4 + (4*(2*A + 3*B)*c^7*d
 - 6*(8*A + 17*B)*c^6*d^2 + 6*(31*A - 121*B)*c^5*d^3 + 3*(408*A - 463*B)*c^
4*d^4 + 3*(513*A - 143*B)*c^3*d^5 - 3*(153*A - 383*B)*c^2*d^6 - (1733*A - 1
143*B)*c*d^7 - 3*(239*A - 114*B)*d^8)*cos(f*x + e)^3 - 2*((2*A + 3*B)*c^8 -
 (7*A + 18*B)*c^7*d + (14*A - 159*B)*c^6*d^2 + 3*(97*A - 172*B)*c^5*d^3 + 6
*(121*A - 91*B)*c^4*d^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^
6 - (668*A - 393*B)*c*d^7 - 5*(37*A - 18*B)*d^8)*cos(f*x + e)^2 - 6*((2*A +
 3*B)*c^8 - (9*A + 11*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c
^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)
*c^2*d^6 - 9*(49*A - 29*B)*c*d^7 - (127*A - 62*B)*d^8)*cos(f*x + e))*sin(f*
x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a
^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^5 + (2
*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d
^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^
3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^
9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*
a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*
f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^
3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*
a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*cos(f*x + e)^2
 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^
4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^
3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(507) = 1014$.
time = 0.64, size = 1272, normalized size = 2.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(12*B*c^3*d^2 - 20*A*c^2*d^3 + 24*B*c^2*d^3 - 30*A*c*d^4 + 21*B*c*
d^4 - 13*A*d^5 + 6*B*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan
((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a
^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^
3*d^7)*sqrt(c^2 - d^2)) + 15*(9*B*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 11*A*c^3
*d^5*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*A*c^2*
d^6*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*B*c^5*d^3
*tan(1/2*f*x + 1/2*e)^2 - 10*A*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^4*d^4
*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^3*d^5
*tan(1/2*f*x + 1/2*e)^2 - 19*A*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*B*c^2*d^
6*tan(1/2*f*x + 1/2*e)^2 - 12*A*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^7*ta
n(1/2*f*x + 1/2*e)^2 + 2*A*d^8*tan(1/2*f*x + 1/2*e)^2 + 23*B*c^4*d^4*tan(1/
2*f*x + 1/2*e) - 29*A*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*B*c^3*d^5*tan(1/2*f
*x + 1/2*e) - 18*A*c^2*d^6*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^6*tan(1/2*f*x +
 1/2*e) + 2*A*c*d^7*tan(1/2*f*x + 1/2*e) + 8*B*c^5*d^3 - 10*A*c^4*d^4 + 6*B
*c^4*d^4 - 6*A*c^3*d^5 + B*c^3*d^5 + A*c^2*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a
^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 -
a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) -
 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 150
*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 90*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*
tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 195*A*c*d*tan(1/
2*f*x + 1/2*e)^3 - 75*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*A*d^2*tan(1/2*f*x
+ 1/2*e)^3 - 300*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*
e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*A*c*d*tan(1/2*f*x + 1/2*e)^2 -
 135*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 745*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 420*
B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan
(1/2*f*x + 1/2*e) - 145*A*c*d*tan(1/2*f*x + 1/2*e) - 105*B*c*d*tan(1/2*f*x
+ 1/2*e) + 485*A*d^2*tan(1/2*f*x + 1/2*e) - 270*B*d^2*tan(1/2*f*x + 1/2*e)
+ 7*A*c^2 + 3*B*c^2 - 44*A*c*d - 21*B*c*d + 127*A*d^2 - 72*B*d^2)/((a^3*c^5
 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3*d^5)*(
tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B]
time = 19.88, size = 2387, normalized size = 4.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)
[Out] ((15*A*d^6 - 14*A*c^6 - 6*B*c^6 - 404*A*c^2*d^4 - 420*A*c^3*d^3 - 92*A*c^4*
d^2 + 234*B*c^2*d^4 + 450*B*c^3*d^3 + 222*B*c^4*d^2 - 90*A*c*d^5 + 60*A*c^5
*d + 15*B*c*d^5 + 30*B*c^5*d)/(15*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^
3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^7*(2*A*d^8 - 4*A*c^8 - 2*B*c^8
- 49*A*c^2*d^6 - 141*A*c^3*d^5 - 200*A*c^4*d^4 - 122*A*c^5*d^3 + 2*A*c^6*d^
2 + 12*B*c^2*d^6 + 95*B*c^3*d^5 + 187*B*c^4*d^4 + 146*B*c^5*d^3 + 58*B*c^6*
d^2 - 2*A*c*d^7 + 10*A*c^7*d + 2*B*c*d^7 + 6*B*c^7*d))/(c^2*(c - d)*(2*c*d
+ c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x
)/2)^6*(30*A*d^8 - 28*A*c^8 - 6*B*c^8 - 759*A*c^2*d^6 - 1707*A*c^3*d^5 - 19
60*A*c^4*d^4 - 870*A*c^5*d^3 + 62*A*c^6*d^2 + 336*B*c^2*d^6 + 1257*B*c^3*d^
5 + 1893*B*c^4*d^4 + 1350*B*c^5*d^3 + 414*B*c^6*d^2 - 114*A*c*d^7 + 54*A*c^
7*d + 30*B*c*d^7 + 18*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4
*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(60*A*d^8 - 32
*A*c^8 - 18*B*c^8 - 1857*A*c^2*d^6 - 3763*A*c^3*d^5 - 3560*A*c^4*d^4 - 1294
*A*c^5*d^3 + 70*A*c^6*d^2 + 900*B*c^2*d^6 + 2859*B*c^3*d^5 + 3705*B*c^4*d^4
 + 2358*B*c^5*d^3 + 678*B*c^6*d^2 - 270*A*c*d^7 + 62*A*c^7*d + 60*B*c*d^7 +
 42*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 +
 d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(30*A*d^8 - 108*A*c^8 - 42*B*c^8
 - 2501*A*c^2*d^6 - 8725*A*c^3*d^5 - 10616*A*c^4*d^4 - 4810*A*c^5*d^3 + 10*
A*c^6*d^2 + 1056*B*c^2*d^6 + 5235*B*c^3*d^5 + 9891*B*c^4*d^4 + 7770*B*c^5*d
^3 + 2370*B*c^6*d^2 - 30*A*c*d^7 + 290*A*c^7*d + 30*B*c*d^7 + 150*B*c^7*d))
/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (
tan(e/2 + (f*x)/2)^3*(150*A*d^8 - 140*A*c^8 - 90*B*c^8 - 7945*A*c^2*d^6 - 1
9441*A*c^3*d^5 - 18600*A*c^4*d^4 - 6898*A*c^5*d^3 + 210*A*c^6*d^2 + 3660*B*
c^2*d^6 + 13311*B*c^3*d^5 + 19455*B*c^4*d^4 + 12618*B*c^5*d^3 + 3570*B*c^6*
d^2 - 570*A*c*d^7 + 314*A*c^7*d + 150*B*c*d^7 + 246*B*c^7*d))/(15*c^2*(c +
d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x
)/2)*(30*A*d^7 - 40*A*c^7 - 30*B*c^7 - 1901*A*c^2*d^5 - 3400*A*c^3*d^4 - 20
18*A*c^4*d^3 - 190*A*c^5*d^2 + 921*B*c^2*d^5 + 2655*B*c^3*d^4 + 2778*B*c^4*
d^3 + 1050*B*c^5*d^2 - 195*A*c*d^6 + 154*A*c^6*d + 60*B*c*d^6 + 126*B*c^6*d
))/(15*c*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) - (
tan(e/2 + (f*x)/2)^8*(2*A*c^7 - 2*A*d^7 + 11*A*c^2*d^5 + 20*A*c^3*d^4 + 30*
A*c^4*d^3 + 2*A*c^5*d^2 - 6*B*c^2*d^5 - 21*B*c^3*d^4 - 24*B*c^4*d^3 - 12*B*
c^5*d^2 + 6*A*c*d^6 - 6*A*c^6*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c
^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^4*(300*A*d^7 - 204
*A*c^7 - 66*B*c^7 - 10235*A*c^2*d^5 - 14330*A*c^3*d^4 - 7254*A*c^4*d^3 - 31
6*A*c^5*d^2 + 5460*B*c^2*d^5 + 12675*B*c^3*d^4 + 10764*B*c^4*d^3 + 3666*B*c
```

```
^5*d^2 - 1650*A*c*d^6 + 614*A*c^6*d + 300*B*c*d^6 + 276*B*c^6*d))/(15*c^2*(
c + d)*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)))/(f*(tan(e/2 +
(f*x)/2)*(5*a^3*c^2 + 4*a^3*c*d) + tan(e/2 + (f*x)/2)^2*(12*a^3*c^2 + 4*a^3
*d^2 + 20*a^3*c*d) + tan(e/2 + (f*x)/2)^7*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*
c*d) + tan(e/2 + (f*x)/2)^3*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/
2 + (f*x)/2)^6*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (f*x)/2)^
4*(26*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^5*(26*a^3*c^2
 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(5*a^3*c^2 + 4*a^3*c*d)
+ a^3*c^2 + a^3*c^2*tan(e/2 + (f*x)/2)^9)) - (d^2*atan(((d^2*(12*B*c^3 - 13
*A*d^3 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(2*a^
3*d^8 - 6*a^3*c*d^7 - 2*a^3*c^7*d + 2*a^3*c^2*d^6 + 10*a^3*c^3*d^5 - 10*a^3
*c^4*d^4 - 2*a^3*c^5*d^3 + 6*a^3*c^6*d^2))/(2*a^3*(c + d)^(5/2)*(c - d)^(11
/2)) - (c*d^2*tan(e/2 + (f*x)/2)*(12*B*c^3 - 13*A*d^3 + 6*B*d^3 - 30*A*c*d^
2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(a^3*c^7 - a^3*d^7 + 3*a^3*c*d^6
- 3*a^3*c^6*d - a^3*c^2*d^5 - 5*a^3*c^3*d^4 + 5*a^3*c^4*d^3 + a^3*c^5*d^2))
/(a^3*(c + d)^(5/2)*(c - d)^(11/2)))/(6*B*d^5 - 13*A*d^5 - 20*A*c^2*d^3 + 2
4*B*c^2*d^3 + 12*B*c^3*d^2 - 30*A*c*d^4 + 21*B*c*d^4))*(12*B*c^3 - 13*A*d^3
 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d))/(a^3*f*(c
+ d)^(5/2)*(c - d)^(11/2))
```

3.286 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=256

$$\frac{4a(c+d)(Bc-9Ad-8Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}}+\frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+f$$

```
[Out] 4/105*d*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+4/31
5*a*(c+d)*(-9*A*d+B*c-8*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(
f*x+e))^(1/2)+2/63*a*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(
a+a*sin(f*x+e))^(1/2)-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*sin(f*
x+e))^(1/2)+8/315*(5*c-d)*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/f
```

Rubi [A]
time = 0.31, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3060, 2849, 2840, 2830, 2725}

$$\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}+\frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a\sin(e+fx)+a}}+\frac{4d(c+d)(-9Ad+Bc-8Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105af}+\frac{8(5c-d)(c+d)(-9Ad+Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (4*a*(c + d)*(B*c - 9*A*d - 8*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/
(315*d*f*Sqrt[a + a*Sin[e + f*x]]) + (8*(5*c - d)*(c + d)*(B*c - 9*A*d - 8*
B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*d*(c + d)*(B*c - 9
*A*d - 8*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*a*f) + (2*a*(B*
c - 9*A*d - 8*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+}{63df\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+f}{105af}$$

$$= \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos}{315f}$$

$$= \frac{4a(c+d)(Bc-9Ad-8Bd)\,(15c^2+10}{315df\sqrt{a+a\sin(e+}}$$

Mathematica [A]
time = 0.83, size = 305, normalized size = 1.19

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(2520Ac^3+1680Bc^3+5040Ac^2d+4788Bc^2d+4788Acd^2+4104Bcd^2+1368Ad^3+1321Bd^3-4d(27Ad(7c+2d)+B(189c^2+162cd+83d^2))\cos(2(e+fx))+35Bd^3\cos(4(e+fx))+840Bc^3\sin(e+fx)+2520Ac^2d\sin(e+fx)+2016Bc^2d\sin(e+fx)+2016Acd^2\sin(e+fx)+2538Bcd^2\sin(e+fx)+846Ad^3\sin(e+fx)+752Bd^3\sin(e+fx)-270Bcd^2\sin(3(e+fx))-90Ad^3\sin(3(e+fx))-80Bd^3\sin(3(e+fx)))}{1260f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] -1/1260*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
2520*A*c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 4104
*B*c*d^2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^2 +
 162*c*d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*B*c^
3*Sin[e + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x] + 20
16*A*c*d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e + f*x
] + 752*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3*Sin[3*
(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]))
```

Maple [A]
time = 5.87, size = 242, normalized size = 0.95

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(\left(-45A\,d^3-135Bc\,d^2-40B\,d^3\right)\sin(fx+e)\left(\cos^2(fx+e)\right)+\left(315A\,c^2d+252Ac\,d^2+117A\,d^3+105B\,c^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*((-45*A*d^3-135*B*c*d^2-40*B*d^3)*sin
(f*x+e)*cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^3+105*B*c^3+252*B*c^2
*d+351*B*c*d^2+104*B*d^3)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^3+(-189*A*c*d^2-54
*A*d^3-189*B*c^2*d-162*B*c*d^2-118*B*d^3)*cos(f*x+e)^2+315*A*c^3+630*A*c^2*
d+693*A*c*d^2+198*A*d^3+210*B*c^3+693*B*c^2*d+594*B*c*d^2+211*B*d^3)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^3, x)
```

Fricas [A]
time = 0.40, size = 480, normalized size = 1.88

$$-\frac{2\,\left(35\,Bd^{3}\cos\left(fx+e\right)^{5}-5\,\left(27\,Bcd^{2}+\left(9\,A+B\right)d^{3}\right)\cos\left(fx+e\right)^{4}+105\,\left(3\,A+B\right)c^{3}+63\,\left(5\,A+7\,B\right)c^{2}d+9\,\left(49\,A+27\,B\right)cd^{2}+\left(81\,A+107\,B\right)d^{3}-\left(189\,Bc^{2}d+27\,\left(7\,A+6\,B\right)cd^{2}+2\,\left(27\,A+59\,B\right)d^{3}\right)\cos\left(fx+e\right)^{3}+\left(105\,Bc^{3}+63\,\left(5\,A+B\right)c^{2}d+9\,\left(7\,A+36\,B\right)cd^{2}+2\,\left(54\,A+13\,B\right)d^{3}\right)\cos\left(fx+e\right)^{2}+\left(105\,\left(3\,A+2\,B\right)c^{3}+63\,\left(10\,A+11\,B\right)c^{2}d+99\,\left(7\,A+6\,B\right)cd^{2}+\left(198\,A+211\,B\right)d^{3}\right)\cos\left(fx+e\right)-\left(35\,Bd^{3}\cos\left(fx+e\right)^{4}+105\,\left(3\,A+B\right)c^{3}+63\,\left(5\,A+7\,B\right)c^{2}d+9\,\left(49\,A+27\,B\right)cd^{2}+\left(81\,A+107\,B\right)d^{3}+5\,\left(27\,Bcd^{2}+\left(9\,A+8\,B\right)d^{3}\right)\cos\left(fx+e\right)^{3}-3\,\left(63\,Bc^{2}d+9\,\left(7\,A+B\right)cd^{2}+\left(3\,A+26\,B\right)d^{3}\right)\cos\left(fx+e\right)^{2}-\left(105\,Bc^{3}+63\,\left(5\,A+4\,B\right)c^{2}d+9\,\left(28\,A+39\,B\right)cd^{2}+13\,\left(9\,A+8\,B\right)d^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{315\,\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*d^3*cos(f*x + e)^5 - 5*(27*B*c*d^2 + (9*A + B)*d^3)*cos(f*x +
e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A + 27*B)*c*d^2 + (
81*A + 107*B)*d^3 - (189*B*c^2*d + 27*(7*A + 6*B)*c*d^2 + 2*(27*A + 59*B)*d
^3)*cos(f*x + e)^3 + (105*B*c^3 + 63*(5*A + B)*c^2*d + 9*(7*A + 36*B)*c*d^2
 + 2*(54*A + 13*B)*d^3)*cos(f*x + e)^2 + (105*(3*A + 2*B)*c^3 + 63*(10*A +
11*B)*c^2*d + 99*(7*A + 6*B)*c*d^2 + (198*A + 211*B)*d^3)*cos(f*x + e) - (3
5*B*d^3*cos(f*x + e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A
 + 27*B)*c*d^2 + (81*A + 107*B)*d^3 + 5*(27*B*c*d^2 + (9*A + 8*B)*d^3)*cos(
f*x + e)^3 - 3*(63*B*c^2*d + 9*(7*A + B)*c*d^2 + (3*A + 26*B)*d^3)*cos(f*x
+ e)^2 - (105*B*c^3 + 63*(5*A + 4*B)*c^2*d + 9*(28*A + 39*B)*c*d^2 + 13*(9*
A + 8*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f
*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(248) = 496.
time = 0.74, size = 579, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi +
9/2*f*x + 9/2*e) + 630*(8*A*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 12*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c*d^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*B*c^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 6*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(6*B*c^2*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 3*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*
pi + 5/2*f*x + 5/2*e) + 45*(6*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 2*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\sin(e + fx))\ \sqrt{a + a\sin(e + fx)}\ (c + d\sin(e + fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,
 x)
```

3.287 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=192

$$\frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{105df\sqrt{a+a\sin(e+fx)}}+\frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}$$

```
[Out] 2/35*d*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+2/105*a*(-7
*A*d+B*c-6*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/2)
-2/7*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)+4/105*(5*
c-d)*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3060, 2840, 2830, 2725}

$$\frac{2a\left(15c^2+10cd+7d^2\right)(-7Ad+Bc-6Bd)\cos(e+fx)}{105df\sqrt{a\sin(e+fx)+a}}+\frac{2d(-7Ad+Bc-6Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}+\frac{4(5c-d)(-7Ad+Bc-6Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*(B*c - 7*A*d - 6*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(105*d*f
*Sqrt[a + a*Sin[e + f*x]]) + (4*(5*c - d)*(B*c - 7*A*d - 6*B*d)*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(105*f) + (2*d*(B*c - 7*A*d - 6*B*d)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+}{35af}$$

$$= \frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+f}{105f}$$

$$= \frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7\right.}{105df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.50, size = 176, normalized size = 0.92

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(420Ac^2+280Bc^2+560Acd+532Bcd+266Ad^2+228Bd^2-6d(14Bc+7Ad+6Bd)\cos(2(e+fx))+\left(56Ad(5c+2d)+B\left(140c^2+224cd+141d^2\right)\right)\sin(e+fx)-15Bd^2\sin(3(e+fx))\right)}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] -1/210*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
20*A*c^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*d*
(14*B*c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140*c^
```

```
2 + 224*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 6.51, size = 161, normalized size = 0.84

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(-15B\left(\cos^2(fx+e)\right)\sin(fx+e)d^2+\left(70Acd+28A\,d^2+35B\,c^2+56Bcd+39B\,d^2\right)\sin(fx+e)+(-21A}{105\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)*d^2+(7
0*A*c*d+28*A*d^2+35*B*c^2+56*B*c*d+39*B*d^2)*sin(f*x+e)+(-21*A*d^2-42*B*c*d
-18*B*d^2)*cos(f*x+e)^2+105*A*c^2+140*A*c*d+77*A*d^2+70*B*c^2+154*B*c*d+66*
B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^2, x)
```

Fricas [A]
time = 0.38, size = 317, normalized size = 1.65

$$\frac{2\left(15\,Bd^2\cos\left(fx+e\right)^4+3\left(14\,Bcd+\left(7\,A+6\,B\right)d^2\right)\cos\left(fx+e\right)^3-35\left(3\,A+B\right)c^2-14\left(5\,A+7\,B\right)cd-\left(49\,A+27\,B\right)d^2-\left(35\,Bc^2+14\left(5\,A+B\right)cd+\left(7\,A+36\,B\right)d^2\right)\cos\left(fx+e\right)^2-\left(35\left(3\,A+2\,B\right)c^2+14\left(10\,A+11\,B\right)cd+11\left(7\,A+6\,B\right)d^2\right)\cos\left(fx+e\right)+\left(15\,Bd^2\cos\left(fx+e\right)^3+35\left(3\,A+B\right)c^2+14\left(5\,A+7\,B\right)cd+\left(49\,A+27\,B\right)d^2-3\left(14\,Bcd+\left(7\,A+B\right)d^2\right)\cos\left(fx+e\right)^2-\left(35\,Bc^2+14\left(5\,A+4\,B\right)cd+\left(28\,A+39\,B\right)d^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{105\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x + e
)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c^2
 + 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)*c^
2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2*cos
(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^2 - 3
```

```
*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*B)*c*d
 + (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**2, x)
```

Giac [A]
time = 0.58, size = 367, normalized size = 1.91

$$\frac{\sqrt{2}\left(15\,Bd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+105\left(8\,Ac^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4\,Bc^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4\,Ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Bd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+35\left(4\,Bc^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4\,Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Bd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+21\left(4\,Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Ad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Bd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\right)\sqrt{a}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 105*(8*A*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 8*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*si
n(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(4*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*
(4*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi +
5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
 x)
```

3.288 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=118

$$-\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-2B\ldots$$

```
[Out] -2/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*A*c+5*A*d+5*B*c+7
*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/15*(5*A*d+5*B*c-2*B*d)*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3047, 3102, 2830, 2725}

$$-\frac{2(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(15*A*c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(15*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int \sqrt{a+a\sin(e+fx)}\,(Ac+(Bc+Ad$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af}$$

$$= -\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a}}{15f}$$

$$= -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+}{15f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.25, size = 117, normalized size = 0.99

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(30Ac+20Bc+20Ad+19Bd-3Bd\cos(2(e+fx))+2(5Bc+5Ad+4Bd)\sin(e+fx))}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
),x]
```

```
[Out] -1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30
*A*c + 20*B*c + 20*A*d + 19*B*d - 3*B*d*Cos[2*(e + f*x)] + 2*(5*B*c + 5*A*d
 + 4*B*d)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 6.13, size = 102, normalized size = 0.86

method	result

default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(3B\left(\sin^2(fx+e)\right)d+5A\sin(fx+e)d+5B\sin(fx+e)c+4B\sin(fx+e)d+15Ac+10Ad+10Bc+8Bd\right)}{15\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+5
*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*A*d+10*B*c+8*B*d)/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
), x)
```

Fricas [A]
time = 0.39, size = 184, normalized size = 1.56

$$\frac{2\left(3\,Bd\cos\left(fx+e\right)^3-\left(5\,Bc+\left(5\,A+B\right)d\right)\cos\left(fx+e\right)^2-5\left(3\,A+B\right)c-\left(5\,A+7\,B\right)d-\left(5\left(3\,A+2\,B\right)c+\left(10\,A+11\,B\right)d\right)\cos\left(fx+e\right)-\left(3\,Bd\cos\left(fx+e\right)^2-5\left(3\,A+B\right)c-\left(5\,A+7\,B\right)d+\left(5\,Bc+\left(5\,A+4\,B\right)d\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*B*d*cos(f*x + e)^3 - (5*B*c + (5*A + B)*d)*cos(f*x + e)^2 - 5*(3*A
+ B)*c - (5*A + 7*B)*d - (5*(3*A + 2*B)*c + (10*A + 11*B)*d)*cos(f*x + e) -
 (3*B*d*cos(f*x + e)^2 - 5*(3*A + B)*c - (5*A + 7*B)*d + (5*B*c + (5*A + 4*
B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
)), x)
```

Giac [A]
time = 0.55, size = 197, normalized size = 1.67

$$\frac{\sqrt{2}\left(3\,Bd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)+30\left(2\,Ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Bd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\left(2\,Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,Ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Bd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,fx+\frac{3}{2}\,e\right)\right)\sqrt{a}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*f
*x + 5/2*e) + 30*(2*A*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*c*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*
B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x +
 3/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)), x
)
```

3.289 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=62

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

```
[Out] -2/3*a*(3*A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*B*cos(f*x+e)*(a+a*si
n(f*x+e))^(1/2)/f
```

Rubi [A]

time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2830, 2725}

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(3A+B)\int \sqrt{}$$

$$= -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2B\cos(e+fx)\sqrt{a+a\sin(}}{3f}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 1.32

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(3A+2B+B\sin(e+fx))}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*A +
 2*B + B*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.50, size = 58, normalized size = 0.94

method	result	size
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(B\sin(fx+e)+3A+2B)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

Fricas [A]
time = 0.39, size = 92, normalized size = 1.48

$$-\frac{2\left(B\cos\left(fx+e\right)^{2}+\left(3A+2B\right)\cos\left(fx+e\right)+\left(B\cos\left(fx+e\right)-3A-B\right)\sin\left(fx+e\right)+3A+B\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(B*cos(f*x + e)^2 + (3*A + 2*B)*cos(f*x + e) + (B*cos(f*x + e) - 3*A -
 B)*sin(f*x + e) + 3*A + B)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)
```

Giac [**A**]
time = 0.47, size = 90, normalized size = 1.45

$$\frac{\sqrt{2}\left(B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+3\left(2A\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sqrt{a}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x +
3/2*e) + 3*(2*A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e))*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}\,f}-\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e)
)^(1/2))*a^(1/2)/d^(3/2)/f/(c+d)^(1/2)-2*a*B*cos(f*x+e)/d/f/(a+a*sin(f*x+e)
)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {3060, 2852, 214}

$$\frac{2\sqrt{a}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
```

```
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} + \frac{(-aBc+aAd)\int \frac{\sqrt{a+a\sin}}{c+d\sin(}}{ad}$$

$$= -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} + \frac{(2a(Bc-Ad))\mathrm{Subst}\left(\int \frac{}{ac+a}\right.}{}$$

$$= \frac{2\sqrt{a}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.34, size = 903, normalized size = 9.03

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{(2-2i)B\sqrt{d}\cos\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)}{f}+\frac{(-Bc+Ad)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\frac{\mathrm{RootSum}\left[\ldots\right](\cos(e)+i(-1+\sin(e)))\sqrt{\cos(e)-i\sin(e)}}{4f}+(1+i)x\sin(e)\right)}{\sqrt{c+d}\,(\cos(e)+i(-1+\sin(e)))\sqrt{\cos(e)-i\sin(e)}}+\ldots\right)\sqrt{a(1+\sin(e+fx))}}{d^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x]),x]
```

```
[Out] ((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f +
 ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d +
 (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*
f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt
[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I
)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)
/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sq
rt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*
(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e
]))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((-(
B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] +
(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt
```

```
[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + S
qrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #
1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f
*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 +
 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*
e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/(Sqr
t[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((2 - 2*I)*B
*Sqrt[d]*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2])/f)*Sqrt[a*(1 + Sin[e + f*x])])
/(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 8.02, size = 139, normalized size = 1.39

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(A\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)ad-B\operatorname{arctanh}\left(\frac{\sqrt{-a(\mathrm{si}}{\sqrt{}}\right.\right.}{d\sqrt{a(c+d)d}\,\cos(fx+e)\sqrt{a+a\sin(fx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(A*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a*d-B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a*c+B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2))/d/(a*(c+d)*
d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.
time = 0.87, size = 685, normalized size = 6.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*cos(f*x + e) - B*sin(f
*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f*x + e) - B*si
n(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 130, normalized size = 1.30

$$\frac{\sqrt{2}\left(\frac{2\,B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d}+\frac{\sqrt{2}\,\left(Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{\sqrt{-cd-d^2}\,d}\right)\sqrt{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] sqrt(2)*(2*B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/d + sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/s
qrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),
 x)
```

3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{a}\,(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f}+\frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(}$$

```
[Out] -(A*d+B*(c+2*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/
f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {3059, 2852, 214}

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{a}\,(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] -((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c -
A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + -\frac{(-a \cdots}{\cdots}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{(a(Ad \cdots}{\cdots}$$

$$= -\frac{\sqrt{a}\,(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+ \cdots}}\right.}{d^{3/2}(c+d)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 6.19, size = 901, normalized size = 7.15

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] + I
*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((
2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)
*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqr
t[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
 + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sq
rt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)
*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(Cos[e] + I
*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(Cos[e/2] +
```

```
I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E
^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 +
2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*
#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*
#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^
((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*
E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e
] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(3/2)*(Cos[e] + I*
(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqrt[d]*(-(B*c) + A*d)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x]))))/
(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(110) = 220$.
time = 11.34, size = 274, normalized size = 2.17

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)ad(Ad+Bc+2Bd)+A\,\mathrm{ar}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(A*d+B*c+2*B*d)+A*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+B*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a*c^2+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a*c*d+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d-B*(a-a*sin(f*x
+e))^(1/2)*(a*(c+d)*d)^(1/2)*c)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(115) = 230$.
time = 1.01, size = 1054, normalized size = 8.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*cos
(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c*d
 + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*
c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*
cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
+ 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*c
os(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B*c - A*d)*co
s(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 +
d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 +
 d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*
x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d
^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3
*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e)
 - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c - A*d + (B*c -
A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c
*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*
c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)
*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 221, normalized size = 1.75

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\left(Bc\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2Bd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(cd+d^2)\sqrt{-cd-d^2}}-\frac{2\left(Bc\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-Ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(cd+d^2)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/(
(c*d + d^2)*sqrt(-c*d - d^2)) - 2*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*si
n(-1/4*pi + 1/2*f*x + 1/2*e))/((2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c -
d)*(c*d + d^2)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c+d\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
2, x)
```

3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a}\,(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f}+\frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin$$

```
[Out] -1/4*(3*A*d+B*(c+4*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*
sin(f*x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e
)/d/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/4*a*(3*A*d+B*(c+4*d
))*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3059, 2851, 2852, 214}

$$-\frac{\sqrt{a}\,(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a(3Ad+B(c+4d))\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] -1/4*(Sqrt[a]*(3*A*d + B*(c + 4*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(5/2)*f) + (a*(B*
c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e
 + f*x])^2) - (a*(3*A*d + B*(c + 4*d))*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[
a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + -$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{}{4d}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{}{4d}$$

$$= -\frac{\sqrt{a}\,(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx}{\sqrt{c+d}\,\sqrt{a+a\sin(e}}\right.}{4d^{3/2}(c+d)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 7.22, size = 967, normalized size = 5.04

[illegible]

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^3,x]

[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*(((3*A*d + B*(c + 4*d))*(Cos[e/2]
 + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*
E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^(
(-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)
*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d
]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((
I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) &]*(Cos[e] + I*(-1 + Sin[e]))
*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2)*(Cos[e]
 + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4*d))*(Cos[
e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*
I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x
+ (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d
]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*
c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/S
qrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c
 + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) &]*Sqrt
[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(5/2)*(Cos[e
] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*Sqrt[d]*(-(B*c)
+ A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x
])^2) - ((2 - 2*I)*Sqrt[d]*(3*A*d + B*(c + 4*d))*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(d^(3/2)*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(168) = 336$.
time = 12.29, size = 628, normalized size = 3.27

method	result
default	$\left(-2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)a^2cd(3Ad+Bc+4Bd)+\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)a^2d^2(3A\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)

[Out] 1/4/a*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c*d*(3*A*d+B*c+4*B*d)+arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1
/2))*a^2*d^2*(3*A*d+B*c+4*B*d)*cos(f*x+e)^2+3*A*(a-a*sin(f*x+e))^(3/2)*(a*(
c+d)*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^

```
3+B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^(3/2)
*(a*(c+d)*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c
^2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-4*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d
)^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-3*B*(a-a*sin
(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
d)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/
(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(176) = 352$.
time = 1.49, size = 1804, normalized size = 9.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
- (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c
*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d
^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A +
3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d +
d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*
d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x +
e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 +
(c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(
c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
 + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
```

```
^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^
2 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2
*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3
*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*
d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 +
d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*c
os(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
 + 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(
f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))
, 1/8*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 -
 (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c*
d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d^
2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3
*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d +
d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-
a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2
 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2*
A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3*
A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d
^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d
^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*co
s(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
+ 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f
*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(176) = 352.
time = 0.58, size = 443, normalized size = 2.31

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\left(Bc\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3Ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4Bd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(c^2d+2cd^2+d^3)\sqrt{-cd-d^2}}+\frac{2\left(2Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+6Ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+8Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+Bc^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-5Acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-5Ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-4Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{(c^2d+2cd^2+d^3)\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*
A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))
/((c^2*d + 2*c*d^2 + d^3)*sqrt(-c*d - d^2)) + 2*(2*B*c*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*A*d^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + B*c^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*c*d*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*c*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*d^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*d^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c^2*d
 + 2*c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\ \sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
3, x)
```

3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=374

$$\frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}+\frac{8a(5c-d)(c+d)\left(11A\right.}{}$$

```
[Out] 4/1155*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f
*x+e))^(3/2)/f+4/3465*a^2*(c+d)*(15*c^2+10*c*d+7*d^2)*(11*A*(c-17*d)*d-3*B*
(c^2-9*c*d+56*d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/693*a^2*(11*A
*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+
a*sin(f*x+e))^(1/2)+2/99*a^2*(3*B*(c-4*d)-11*A*d)*cos(f*x+e)*(c+d*sin(f*x+e
))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)+8/3465*a*(5*c-d)*(c+d)*(11*A*(c-17*d)*d-3
*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a*B*cos(f
*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.61, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3055, 3060, 2849, 2840, 2830, 2725}

$$\frac{2a^2(11Ad(c-17d)-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}+\frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11Ad(c-17d)-3B(c^2-9cd+56d^2)\right)\cos(e+fx)}{3465d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a\sin(e+fx)+a}}+\frac{4(c+d)\left(11Ad(c-17d)-3B(c^2-9cd+56d^2)\right)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{1155f}+\frac{8a(5c-d)(c+d)\left(11Ad(c-17d)-3B(c^2-9cd+56d^2)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465df}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (4*a^2*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*
c*d + 56*d^2))*Cos[e + f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (8*a*(
5*c - d)*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f) + (4*(c + d)*(11*A*(c - 17*d)*d -
3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*
f) + (2*a^2*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(
c + d*Sin[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(3*B*(
c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[
e + f*x])^4)/(11*d*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{11df}$$

$$= \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(}{99d^2f\sqrt{a+a\sin(e+f}}$$

$$= \frac{2a^2(11A(c-17d)d-3B(c^2-9cd+5}{693d^2f\sqrt{a+}}$$

$$= \frac{4(c+d)\,(11A(c-17d)d-3B(c^2-9c}{}$$

$$= \frac{8a(5c-d)(c+d)\,(11A(c-17d)d-3}{}$$

$$= \frac{4a^2(c+d)\,(15c^2+10cd+7d^2)\,(11A(c}{3465d^2f\sqrt{}}$$

Mathematica [A]
time = 2.94, size = 390, normalized size = 1.04

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(92400Ac^3+72072Bc^3+216216Ac^2d+195624Bc^2d+195624Acd^2+177474Bcd^2+59158Ad^3+55482Bd^3-8(11Ad(189c^2+351cd+137d^2)+3B(231c^3+1287c^2d+1507cd^2+581d^3))\cos(2(e+fx))+70d^2(33Bc+11Ad+21Bd)\cos(4(e+fx))+18480Ac^3\sin(e+fx)+33264Bc^3\sin(e+fx)+99792Ac^2d\sin(e+fx)+100188Bc^2d\sin(e+fx)+100188Acd^2\sin(e+fx)+105468Bcd^2\sin(e+fx)+35156Ad^3\sin(e+fx)+34734Bd^3\sin(e+fx)-5940Bc^2d\sin(3(e+fx))-5940Acd^2\sin(3(e+fx))-11220Bcd^2\sin(3(e+fx))-3740Ad^3\sin(3(e+fx))-4935Bd^3\sin(3(e+fx))+315Bd^3\sin(5(e+fx)))}{27720f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] -1/27720*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(92400*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A*c
*d^2 + 177474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 + 35
1*c*d + 137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*Cos[2
*(e + f*x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 18480*A*
c^3*Sin[e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f*x] +
100188*B*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B*c*d^2*
Sin[e + f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x] - 5940*B
*c^2*d*Sin[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*c*d^2*Sin
[3*(e + f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e + f*x)] +
 315*B*d^3*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 6.08, size = 312, normalized size = 0.83

method	result
default	$2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(315B\sin(fx+e)(\cos^4(fx+e))d^3+(-1485Acd^2-935Ad^3-1485Bc^2d-2805Bcd^2-1470Bd^3)(\mathrm{co}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(315*B*sin(f*x+e)*cos(f*x+e)^4*d^3
+(-1485*A*c*d^2-935*A*d^3-1485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)^
2*sin(f*x+e)+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3+66
33*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+(385*A*d^3+1155*B*c*d^2+735*
B*d^3)*cos(f*x+e)^4+(-2079*A*c^2*d-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*B
*c^2*d-5676*B*c*d^2-2478*B*d^3)*cos(f*x+e)^2+5775*A*c^3+14553*A*c^2*d+14157
*A*c*d^2+4499*A*d^3+4851*B*c^3+14157*B*c^2*d+13497*B*c*d^2+4431*B*d^3)/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^3, x)
```

Fricas [A]
time = 0.42, size = 652, normalized size = 1.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d^
3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 132
*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33*(9
*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*c^3
+ 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1239*B
)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a*c^2*d
```

```
 + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e)^2 + (
231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A + 409*B)*a
*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(f*x + e)^
5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A + 47*B)*a
*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12*B)*a*d^3)*
cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (187*A + 294*
B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c^2*d + 33*(24
*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 + (231*(5*A + 9*
B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c*d^2 + 17*(143*
A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*c
os(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(365) = 730.
time = 0.81, size = 793, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/4*
pi + 11/2*f*x + 11/2*e) + 6930*(24*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 16*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*A*a*c^2*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 42*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 42*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 11*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f
*x + 1/2*e) + 2310*(8*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*B*a*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*A*a*c^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 30*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*A*a
*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a*c*d^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 10*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a
```

```
*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) +
693*(8*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*a*c^2*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 36*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 13*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x
 + 5/2*e) + 495*(12*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*
c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a*c*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 6*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a*d
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e) + 38
5*(6*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*si
n(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(a)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^{3/2}\,(c+d\sin(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,
 x)
```

3.294 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=294

$$\frac{2a^2(15c^2+10cd+7d^2)\left(3A(c-13d)d-B(c^2-7cd+34d^2)\right)\cos(e+fx)}{315d^2f\sqrt{a+a\sin(e+fx)}}+\frac{4a(5c-d)\left(3A(c-13d)d-B(c^2\right.}{}$$

```
[Out] 2/105*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/
2)/f+2/315*a^2*(15*c^2+10*c*d+7*d^2)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*
cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/63*a^2*(-9*A*d+3*B*c-10*B*d)*cos(
f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+4/315*a*(5*c-d)*(3*A
*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/9
*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.47, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3055, 3060, 2840, 2830, 2725}

$$\frac{2a^2(15c^2+10cd+7d^2)\left(3Ad(c-13d)-B(c^2-7cd+34d^2)\right)\cos(e+fx)}{315d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2(3Ad(c-13d)-B(c^2-7cd+34d^2))\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105f}+\frac{4a(5c-d)\left(3Ad(c-13d)-B(c^2-7cd+34d^2)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315df}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^3}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (2*a^2*(15*c^2 + 10*c*d + 7*d^2)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^
2))*Cos[e + f*x])/(315*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (4*a*(5*c - d)*(3*
A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(315*d*f) + (2*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(3*B*c - 9*A*d - 10*B*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d^2*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*
f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9df}$$

$$= \frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(}{63d^2f\sqrt{a+a\sin(e+f}}$$

$$= \frac{2(3A(c-13d)d-B(c^2-7cd+34d^2)}{105}$$

$$= \frac{4a(5c-d)\,(3A(c-13d)d-B(c^2-7c}{}$$

$$= \frac{2a^2(15c^2+10cd+7d^2)\,(3A(c-13d)d}{315d^2f\sqrt{a+a}}$$

Mathematica [A]
time = 1.49, size = 267, normalized size = 0.91

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(4200Ac^2+3276Bc^2+6552Acd+5928Bcd+2964Ad^2+2689Bd^2-4(9Ad(14c+13d)+B(63c^2+234cd+137d^2))\cos(2(e+fx))+35Bd^2\cos(4(e+fx))+840Ac^2\sin(e+fx)+1512Bc^2\sin(e+fx)+3024Acd\sin(e+fx)+3036Bcd\sin(e+fx)+1518Ad^2\sin(e+fx)+1598Bd^2\sin(e+fx)-180Bcd\sin(3(e+fx))-90Ad^2\sin(3(e+fx))-170Bd^2\sin(3(e+fx)))}{1260f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(4200*A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B*d
^2 - 4*(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e + f*
x)] + 35*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*Sin[e
 + f*x] + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d^2*Si
n[e + f*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 90*A*d^
2*Sin[3*(e + f*x)] - 170*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

Maple [A]
time = 5.36, size = 207, normalized size = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)((-45A\,d^2-90Bcd-85B\,d^2)\sin(fx+e)(\cos^2(fx+e))+(105A\,c^2+378Acd+201A\,d^2+189B\,c^2+402B}{315}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*((-45*A*d^2-90*B*c*d-85*B*d^2)*sin(
f*x+e)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2+402*B*c*d+221*
B*d^2)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^2+(-126*A*c*d-117*A*d^2-63*B*c^2-234*
B*c*d-172*B*d^2)*cos(f*x+e)^2+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+858*B
*c*d+409*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [A]
time = 0.40, size = 443, normalized size = 1.51

$$-\frac{2\left(35\,Bad^{2}\cos\left(fx+e\right)^{5}-5\left(18\,Bacd+\left(9\,A+10\,B\right)ad^{2}\right)\cos\left(fx+e\right)^{4}+84\left(5\,A+3\,B\right)ac^{2}+24\left(21\,A+19\,B\right)acd+4\left(57\,A+47\,B\right)ad^{2}-\left(63\,Bac^{2}+18\left(7\,A+13\,B\right)acd+\left(117\,A+172\,B\right)ad^{2}\right)\cos\left(fx+e\right)^{3}+\left(21\left(5\,A+6\,B\right)ac^{2}+6\left(42\,A+43\,B\right)acd+\left(129\,A+134\,B\right)ad^{2}\right)\cos\left(fx+e\right)^{2}+\left(21\left(25\,A+21\,B\right)ac^{2}+6\left(147\,A+143\,B\right)acd+\left(429\,A+409\,B\right)ad^{2}\right)\cos\left(fx+e\right)-\left(35\,Bad^{2}\cos\left(fx+e\right)^{4}+84\left(5\,A+3\,B\right)ac^{2}+24\left(21\,A+19\,B\right)acd+4\left(57\,A+47\,B\right)ad^{2}+5\left(18\,Bacd+\left(9\,A+17\,B\right)ad^{2}\right)\cos\left(fx+e\right)^{3}-3\left(21\,Bac^{2}+6\left(7\,A+8\,B\right)acd+\left(24\,A+29\,B\right)ad^{2}\right)\cos\left(fx+e\right)^{2}-\left(21\left(5\,A+9\,B\right)ac^{2}+6\left(63\,A+67\,B\right)acd+\left(201\,A+221\,B\right)ad^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{315\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a*d^2*cos(f*x + e)^5 - 5*(18*B*a*c*d + (9*A + 10*B)*a*d^2)*cos
(f*x + e)^4 + 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*
B)*a*d^2 - (63*B*a*c^2 + 18*(7*A + 13*B)*a*c*d + (117*A + 172*B)*a*d^2)*cos
(f*x + e)^3 + (21*(5*A + 6*B)*a*c^2 + 6*(42*A + 43*B)*a*c*d + (129*A + 134*
B)*a*d^2)*cos(f*x + e)^2 + (21*(25*A + 21*B)*a*c^2 + 6*(147*A + 143*B)*a*c*
d + (429*A + 409*B)*a*d^2)*cos(f*x + e) - (35*B*a*d^2*cos(f*x + e)^4 + 84*(
5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*B)*a*d^2 + 5*(18*B
*a*c*d + (9*A + 17*B)*a*d^2)*cos(f*x + e)^3 - 3*(21*B*a*c^2 + 6*(7*A + 8*B)
*a*c*d + (24*A + 29*B)*a*d^2)*cos(f*x + e)^2 - (21*(5*A + 9*B)*a*c^2 + 6*(6
3*A + 67*B)*a*c*d + (201*A + 221*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x))**2, x)
```

Giac [A]
time = 0.67, size = 523, normalized size = 1.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
+ 9/2*f*x + 9/2*e) + 630*(12*A*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
8*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 16*A*a*c*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 14*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*A*
a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*A*a*c^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
12*A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 5*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*
a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) +
 126*(2*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*a*c*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(4*B*a*c*d*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 3*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7
/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,
 x)
```

3.295 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=165

$$-\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+a\,s}}{105f}$$

```
[Out] -2/35*(7*A*d+7*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*d*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f-8/105*a^2*(35*A*c+21*A*d+21*B*c+19*B*d)*co
s(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a*(35*A*c+21*A*d+21*B*c+19*B*d)*cos
(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.22, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3102, 2830, 2726, 2725}

$$-\frac{8a^2(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2a(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*a*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(105*f) - (2*(7*B*c + 7*A*d - 2*B*d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(7*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^{3/2}\left(Ac+(Bc+A\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/}}{7af}$$

$$= -\frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a-}{35f}$$

$$= -\frac{2a(35Ac+21Bc+21Ad+19Bd)\cos}{105f}$$

$$= -\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\,\mathrm{c}}{105f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.75, size = 144, normalized size = 0.87

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(700Ac+546Bc+546Ad+494Bd-6(7Bc+7Ad+13Bd)\cos(2(e+fx))+(140Ac+252Bc+252Ad+253Bd)\sin(e+fx)-15Bd\sin(3(e+fx)))}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(700*A*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2*(
e + f*x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*d*S
in[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.82, size = 150, normalized size = 0.91

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(15B(\sin^3(fx+e))d+21A(\sin^2(fx+e))d+21B(\sin^2(fx+e))c+39B(\sin^2(fx+e))d+35A\sin(fx+e}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*B*sin(f*x+e)^3*d+21*A*sin(f*x+e
)^2*d+21*B*sin(f*x+e)^2*c+39*B*sin(f*x+e)^2*d+35*A*sin(f*x+e)*c+63*A*sin(f*
x+e)*d+63*B*sin(f*x+e)*c+52*B*sin(f*x+e)*d+175*A*c+126*A*d+126*B*c+104*B*d)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c), x)
```

Fricas [A]
time = 0.39, size = 268, normalized size = 1.62

$$\frac{2\left(15\,Bad\cos\left(fx+e\right)^{4}+3\left(7\,Bac+\left(7\,A+13\,B\right)ad\right)\cos\left(fx+e\right)^{3}-28\left(5\,A+3\,B\right)ac-4\left(21\,A+19\,B\right)ad-\left(7\left(5\,A+6\,B\right)ac+\left(42\,A+43\,B\right)ad\right)\cos\left(fx+e\right)^{2}-\left(7\left(25\,A+21\,B\right)ac+\left(147\,A+143\,B\right)ad\right)\cos\left(fx+e\right)+\left(15\,Bad\cos\left(fx+e\right)^{3}+28\left(5\,A+3\,B\right)ac+4\left(21\,A+19\,B\right)ad-3\left(7\,Bac+\left(7\,A+8\,B\right)ad\right)\cos\left(fx+e\right)^{2}-\left(7\left(5\,A+9\,B\right)ac+\left(63\,A+67\,B\right)ad\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{105\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x + e
)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42*A
 + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*d)*
```

```
cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A + 19
*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B)*a*c
 + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x)), x)
```

Giac [A]
time = 0.61, size = 301, normalized size = 1.82

$$\frac{\sqrt{2}\left(15\,Bad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+105\left(12\,Aac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Bac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Aad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7\,Bad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+35\left(4\,Aac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\,Bac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6\,Aad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\,Bad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+21\left(2\,Bac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Aad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Bad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\right)\sqrt{a}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 105*(12*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B*a
*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 7*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 35*(4*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 5*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x +
 3/2*e) + 21*(2*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)), x
)
```

3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=101

$$-\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}$$

```
[Out] -2/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-8/15*a^2*(5*A+3*B)*cos(f*x+e)/f/
(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$-\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(5*A + 3*B)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*A + 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*B*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5A+3B)\int($$
$$= -\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2B\cos}{}$$
$$= -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{}}{15f}$$

Mathematica [A]
time = 0.28, size = 101, normalized size = 1.00

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(50A+39B-3B\cos(2(e+fx))+2(5A+9B)\sin(e+fx))}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
50*A + 39*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 9*B)*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.49, size = 77, normalized size = 0.76

method	result	size
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)(5A+9B)-3B(\cos^2(fx+e))+25A+21B)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(sin(f*x+e)*(5*A+9*B)-3*B*cos(f*x+e)
^2+25*A+21*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [A]
time = 0.36, size = 146, normalized size = 1.45

$$\frac{2\left(3\,Ba\cos\left(fx+e\right)^{3}-\left(5\,A+6\,B\right)a\cos\left(fx+e\right)^{2}-\left(25\,A+21\,B\right)a\cos\left(fx+e\right)-4\left(5\,A+3\,B\right)a-\left(3\,Ba\cos\left(fx+e\right)^{2}+\left(5\,A+9\,B\right)a\cos\left(fx+e\right)-4\left(5\,A+3\,B\right)a\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*a*cos(f*x + e)^3 - (5*A + 6*B)*a*cos(f*x + e)^2 - (25*A + 21*B)*a
*cos(f*x + e) - 4*(5*A + 3*B)*a - (3*B*a*cos(f*x + e)^2 + (5*A + 9*B)*a*cos
(f*x + e) - 4*(5*A + 3*B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(
f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)
```

Giac [A]
time = 0.52, size = 147, normalized size = 1.46

$$\frac{\sqrt{2}\left(3\,Ba\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+30\left(3\,Aa\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Ba\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+5\left(2\,Aa\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\right)\sqrt{a}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*f
*x + 5/2*e) + 30*(3*A*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*A*a*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=153

$$-\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}\,f}+\frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-2a$$

```
[Out] -2*a^(3/2)*(c-d)*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/
(a+a*sin(f*x+e))^(1/2))/d^(5/2)/f/(c+d)^(1/2)+2/3*a^2*(-3*A*d+3*B*c-4*B*d)*
cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*B*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/d/f
```

Rubi [A]
time = 0.34, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3055, 3060, 2852, 214}

$$-\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}+\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (-2*a^(3/2)*(c - d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*
c - 3*A*d - 4*B*d)*Cos[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*
B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{2\int \sqrt{a+a\,\mathrm{s}}\cdots}{\cdots}$$

$$= \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\cdots}{\cdots}$$

$$= \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\cdots}{\cdots}$$

$$= -\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+f\cdots}{\sqrt{c+d}\sqrt{a+a\sin(e\cdots}}\right.}{d^{5/2}\sqrt{c+d}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 356 vs. 2(153) = 306.
time = 2.44, size = 356, normalized size = 2.33

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(-6\sqrt{d}(-2Bc+2Ad+3Bd)\cos\left(\frac{1}{2}(e+fx)\right)-2Bd^{3/2}\cos\left(\frac{3}{2}(e+fx)\right)-\frac{3(c-d)(Bc-Ad)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{c+d}}+\frac{3(c-d)(Bc-Ad)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{\sqrt{c+d}}+6\sqrt{d}(-2Bc+2Ad+3Bd)\sin\left(\frac{1}{2}(e+fx)\right)-2Bd^{3/2}\sin\left(\frac{3}{2}(e+fx)\right)\right)}{6d^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e +
 f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(B*c - A*d)*(e + f
*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d
]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/
Sqrt[c + d] + (3*(c - d)*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*
x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[c + d] + 6*Sqrt[d]*(-2*B*c + 2*A*d + 3*
B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*(e + f*x))/2]))/(6*d^(5/2)*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(131) = 262$.
time = 8.83, size = 291, normalized size = 1.90

method	result
default	$\frac{2(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3A\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a\left(c+d\right)d}}\right)a^{2}cd-3A\,\mathrm{arctanh}\left(\frac{\sqrt{-a(}}{\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d-3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^2*d^2-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^2*c^2+3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^2*c*d+B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d-3*A*(-a*(sin(
f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d+3*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a*c-6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d)/d^2/(a
*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(137) = 274$.
time = 1.02, size = 918, normalized size = 6.00

$$\left[-\frac{3\left(Bac^2-(A+B)acd+Aad^2+(Bac^2-(A+B)acd+Aad^2)\cos(fx+e)+(Bac^2-(A+B)acd+Aad^2)\sin(fx+e)\right)\sqrt{\frac{a}{cd+d^2}}\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2acd-ad^2-(6acd+7ad^2)\cos(fx+e)^2-4\left(c^2d+4cd^2+3d^3-(cd^2+d^3)\cos(fx+e)^2+(c^2d+3cd^2+2d^3)\cos(fx+e)-(c^2d+4cd^2+3d^3+(cd^2+d^3)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\sqrt{\frac{a}{cd+d^2}}-(ac^2+8acd+9ad^2)\cos(fx+e)+\left(ad^2\cos(fx+e)^2-ac^2-2acd-ad^2+2(3acd+4ad^2)\cos(fx+e)\right)\sin(fx+e)}{d^2\cos(fx+e)^3+(2cd+d^2)\cos(fx+e)^2-c^2-2cd-d^2-(c^2+d^2)\cos(fx+e)+\left(d^2\cos(fx+e)^2-2cd\cos(fx+e)-c^2-2cd-d^2\right)\sin(fx+e)}\right)+4\left(Bad\cos(fx+e)^2-3Bac+(3A+4B)ad-(3Bac-(3A+5B)ad)\cos(fx+e)+(Bad\cos(fx+e)+3Bac-(3A+4B)ad)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{6\left(d^2f\cos(fx+e)+d^2f\sin(fx+e)+d^2f\right)}, -\frac{3\left(Bac^2-(A+B)acd+Aad^2+(Bac^2-(A+B)acd+Aad^2)\cos(fx+e)+(Bac^2-(A+B)acd+Aad^2)\sin(fx+e)\right)\sqrt{-\frac{a}{cd+d^2}}\arctan\left(\frac{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)-c-2d)\sqrt{-\frac{a}{cd+d^2}}}{2a\cos(fx+e)}\right)+2\left(Bad\cos(fx+e)^2-3Bac+(3A+4B)ad-(3Bac-(3A+5B)ad)\cos(fx+e)+(Bad\cos(fx+e)+3Bac-(3A+4B)ad)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{3\left(d^2f\cos(fx+e)+d^2f\sin(fx+e)+d^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*
a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqr
t(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a
*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)
*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
- 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(
f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*a*d*cos(f*x + e)^2 - 3*
B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d
*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1/3*(3*(B*a*c^2
 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*cos(f*x +
e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt(-a/(c*d + d^2))
*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*
d + d^2))/(a*cos(f*x + e))) + 2*(B*a*d*cos(f*x + e)^2 - 3*B*a*c + (3*A + 4*
B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d*cos(f*x + e) + 3
*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*co
s(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(137) = 274$.
time = 0.61, size = 287, normalized size = 1.88

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\left(Bac^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Aacd\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bacd\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Aad^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\sqrt{-cd-d^2}\,d^2}+\frac{2\left(2Bad^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+3Bacd\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3Aad^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-6Bad^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{d^3}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(a)*(3*sqrt(2)*(B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) - A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*d*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sq
rt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*
d^2) + 2*(2*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 + 3*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) - 6*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e))/d^3)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\sin(e + f\,x))\,\left(a + a\,\sin(e + f\,x)\right)^{3/2}}{c + d\,\sin(e + f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)),
 x)
```

3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=191

$$-\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f}-\frac{a^2(3Bc-Ad+2Bd)\,\mathrm{co}}{d^2(c+d)f\sqrt{a+a\sin}}$$

```
[Out] -a^(3/2)*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^(
1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^2*(-A*d+3*
B*c+2*B*d)*cos(f*x+e)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)+a*(-A*d+B*c)*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))
```

Rubi [A]

time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3054, 3060, 2852, 214}

$$-\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -((a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqr
t[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d
)^(3/2)*f)) - (a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[
a + a*Sin[e + f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \sqrt{a+ \cdots}}{\cdots}$$

$$= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e \cdots}{d(c+d)f(\cdots}$$

$$= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e \cdots}{d(c+d)f(\cdots}$$

$$= -\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\tanh^{-1}\left(\frac{\cdots}{\sqrt{c+\cdots}}\right.}{d^{5/2}(c+d)^{3/2}f}$$

Mathematica [A]
time = 3.46, size = 381, normalized size = 1.99

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(-8B\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\frac{(-Ad(c+3d)+B(3c^2+3cd-2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+\frac{(Ad(c+3d)+B(-3c^2-3cd+2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+8B\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)-\frac{4\sqrt{d}(-c+d)(-Bc+Ad)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))}\right)}{4d^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((-(A*d*(c +
 3*d)) + B*(3*c^2 + 3*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] -
Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(3/2) + ((A*d*(c + 3*d) +
 B*(-3*c^2 - 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(
c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] +
Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 8*B*Sqrt[d]*Sin[(e + f*x)/2] - (4*Sq
rt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c +
d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(173) = 346$.
time = 11.04, size = 590, normalized size = 3.09

method	result
default	$-\frac{a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)d\left(A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)acd+3A\operatorname{arctanh}\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e))^
(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*
c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^
(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^
2+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a
*sin(f*x+e))^(1/2)*d)+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2
))*a*c^2*d+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^
2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^3-3*B*arcta
nh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d+2*B*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2-A*(a-a*sin(f*x+e))^(1/2)*(a
*(c+d)*d)^(1/2)*c*d+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d^2+3*B*(a-a
*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
d)^(1/2)*c*d)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(180) = 360$.
time = 1.18, size = 1474, normalized size = 7.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^
3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a
*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c
^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2 - (A + B)*a*c
*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 -
(A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2
*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x
+ e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d
^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c^3 - (A - 6*B)*a*c^2*d
- (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2
- (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A
 + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*
a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B
)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f
*x + e))) - 2*(3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B
*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e)
```

```
 - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x
 + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f -
((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(180) = 360.
time = 0.59, size = 381, normalized size = 1.99

$$\frac{\sqrt{2}\left(\frac{4\,Ba\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d^2}+\frac{\sqrt{2}\left(3\,Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Aacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Bacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-3\,Aad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,Bad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{(cd^2+d^3)\sqrt{-cd-d^2}}-\frac{2\left(Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Aacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Bacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{(cd^2+d^3)\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}\right)\sqrt{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/d^2 + sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A
*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*c*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^2 + d^3)*sqrt(-c*d - d^2)) - 2*(B*a*c^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a*c*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*a*
c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + A*
a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
(c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
2, x)
```

3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f}+\frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\,s}$$

```
[Out] -1/4*a^(3/2)*(A*d*(c+7*d)+3*B*(c^2+3*c*d+4*d^2))*arctanh(cos(f*x+e)*a^(1/2)
*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+1/4*a^2*
(A*(c-5*d)*d+B*(3*c^2+5*c*d-4*d^2))*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e
))/(a+a*sin(f*x+e))^(1/2)+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2
)/d/(c+d)/f/(c+d*sin(f*x+e))^2
```

Rubi [A]

time = 0.41, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3054, 3059, 2852, 214}

$$-\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/4*(a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*ArcTanh[(Sqrt[a]*
Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c
+ d)^(5/2)*f) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*
(c + d)*f*(c + d*Sin[e + f*x])^2) + (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d
- 4*d^2))*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*
Sin[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \sqrt{a+\cdots}}{\cdots}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(\cdots}{4d^2(c+\cdots}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(\cdots}{4d^2(c+\cdots}$$

$$= -\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\tanh^{-1}\left(\frac{\cdots}{\sqrt{c+\cdots}}\right.}{4d^{5/2}(c+d)^{5/2}f}$$

Mathematica [A]

time = 3.78, size = 416, normalized size = 1.88

$$\frac{(a(1+\sin(e+fx)))^{3/2}\left(-\frac{(Ad(c+7d)+3B(c^2+3cd+4d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\,\sqrt{c+d}\,\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\,\sqrt{c+d}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{5/2}}+\frac{(Ad(c+7d)+3B(c^2+3cd+4d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\,\sqrt{c+d}\,\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{(c+d)^{5/2}}-\frac{8\sqrt{d}\,(-c+d)(-Bc+Ad)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))^2}-\frac{4\sqrt{d}\,\left(Ad(c+7d)+B(-5c^2-7cd+4d^2)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}\right)}{16d^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d
 + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)
/2]))]))/(c + d)^(5/2)) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(e +
 f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[
d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5
/2) - (8*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]*(A*d*(c + 7*d) + B*(-5*c
^2 - 7*c*d + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c +
 d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(197) = 394$.
time = 14.55, size = 895, normalized size = 4.05

method	result
default	$\frac{\left(-2\sin(fx+e)\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{cda+a\,d^{2}}}\right)a^{2}cd\left(Acd+7A\,d^{2}+3B\,c^{2}+9Bcd+12B\,d^{2}\right)+\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx\right.}}{\sqrt{cda+a\,d\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)+arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*
cos(f*x+e)^2+A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+7*A*(a-a*sin(
f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c
*d+a*d^2)^(1/2))*a^2*c^3*d-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a^2*c^2*d^2-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2
))*a^2*c*d^3-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*
d^4-5*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-7*B*(a-a*sin(f*x+e))
^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+4*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*d^3-3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4-9*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-15*B*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-9*B*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-12*B*arctanh((a-a*sin(
f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+A*(a-a*sin(f*x+e))^(1/2)*(a*(c
+d)*d)^(1/2)*a*c^2*d-8*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-9
```

```
*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3+3*B*(a-a*sin(f*x+e))^(1/2
)*(a*(c+d)*d)^(1/2)*a*c^3+12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c
^2*d+5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-4*B*(a-a*sin(f*x+
e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))
/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/cos(f*x+e)/(a+a*sin(f*x+e
))^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(205) = 410$.
time = 1.68, size = 2262, normalized size = 10.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A
 + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3
+ (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2*d^
2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*a*c^
4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A +
 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*
B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2
 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*a*c^3*d
+ (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
```

```
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^3 + (A + 2*B)*a
*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*B*a*c^2*d - (A - 7*
B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 + (A + 7*B)*a*c
^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x + e) - (3*B*a*c^3 + (A + 2*
B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 - (5*B*a*c^2*d - (A
- 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5
*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d
^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c
*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 +
2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*
d^5 + d^6)*f)*sin(f*x + e)), 1/8*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
+ 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^
2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3
*d + (2*A + 21*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*
cos(f*x + e)^2 + (3*B*a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 +
(A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15
*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12
*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*
x + e)^2 + 2*(3*B*a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos
(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e
) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -
2*(3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^
3 + (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 + (A + 7*B)*a*c^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x +
 e) - (3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*
a*d^3 - (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f
*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4
*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4
*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(
f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*
c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(205) = 410$.

time = 0.61, size = 655, normalized size = 2.96

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\left(3\,Bac^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+Aacd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+9\,Bacd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+7\,Aad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+12\,Bad^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^{2}}}\right)}{\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)\sqrt{-cd-d^{2}}}-\frac{2\,\left(10\,Bac^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,Aacd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+14\,Bacd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-14\,Aad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-8\,Bad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-3\,Bac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Aac^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,Bac^{2}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,Aacd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-5\,Bacd^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Aad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,Bad^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)\left(2\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c-d\right)^{2}}\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 7*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*B
*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi +
1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c^2*d^2 + 2*c*d^3 + d^4)*sqrt(-c*d - d
^2)) - 2*(10*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - 2*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4
*pi + 1/2*f*x + 1/2*e)^3 + 14*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 14*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 8*B*a*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a*c^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a*c^2*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 12*B*a*c^2*d*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 8*A*a*c*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*B*a
*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 9*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 4*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e))/((c^2*d^2 + 2*c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
c - d)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
3, x)
```

3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=534

$$-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+}{45045d^3f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -4/15015*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d
^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/13*a*B*cos(f*x+e)*(a+
a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^4/d/f-4/45045*a^3*(c+d)*(15*c^2+10*c*d
+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d
^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/9009*a^3*(13*A*d*(3*c^2-38*c
*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(c+d*sin(f*
x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/1287*a^3*(-39*A*c*d+299*A*d^2+15*B*c
^2-75*B*c*d+280*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^3/f/(a+a*sin(f*x+e))
^(1/2)-8/45045*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-1
50*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/143
*a^2*(-13*A*d+5*B*c-16*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^
(1/2)/d^2/f
```

Rubi [A]
time = 0.81, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3055, 3060, 2849, 2840, 2830, 2725}

$$-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{9009d^3f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)}{45045d^3f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{45045d^2f}+\frac{2a^2(-13Ad+5Bc-16Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{143d^2f}-\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{15015df}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(13*A*d*(3*c^2 - 38*c*d + 355*d^2
) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x])/(45045*d^3
*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(13*A*d*(3*c^2 - 38
*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(45045*d^2*f) - (4*a*(c + d)*(13*A*d*(3*c^2 -
38*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^(3/2))/(15015*d*f) - (2*a^3*(13*A*d*(3*c^2 - 38*c
*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(9009*d^3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*(15
*B*c^2 - 39*A*c*d - 75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(1287*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 13*
A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)
```

```
/(143*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e
+ f*x])^4)/(13*d*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^3}{13df}$$

$$= \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)}{14}$$

$$= -\frac{2a^3(15Bc^2-39Acd-75Bcd+299}{1287d^3f}$$

$$= -\frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(}{}$$

$$= -\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2\right.}{}$$

$$= -\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38c\right.}{}$$

$$= -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)(13A}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.60, size = 1565, normalized size = 2.93

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] (B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78*B
*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*Cos
```

```
[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2
))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90
*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1
/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3
+ 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d
^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos[(3*(e + f
*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240
*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[(3*(e + f*x))/
2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 +
120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3
)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e + f*x))/2] - (1/3
20 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150
*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 + I/32
0)*Cos[(5*(e + f*x))/2] - (1/320 - I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d + 30*B*c^2*d + 3
0*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((1/224 + I/224)*Cos[(7*(e + f*x))/2] + (1/224 - I/224)*Sin[(7*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d +
30*B*c^2*d + 30*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e +
 f*x]))^(5/2)*((1/224 - I/224)*Cos[(7*(e + f*x))/2] + (1/224 + I/224)*Sin[(
7*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 +
 6*A*c*d + 15*B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/
288 - I/288)*d*Cos[(9*(e + f*x))/2] + (1/288 - I/288)*d*Sin[(9*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 + 6*A*c*d + 15*
B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/288 + I/288)*d
*Cos[(9*(e + f*x))/2] + (1/288 + I/288)*d*Sin[(9*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e
+ f*x]))^(5/2)*((-1/704 + I/704)*d^2*Cos[(11*(e + f*x))/2] - (1/704 + I/704
)*d^2*Sin[(11*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) +
 ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/704 - I/704)*d^
2*Cos[(11*(e + f*x))/2] - (1/704 - I/704)*d^2*Sin[(11*(e + f*x))/2]))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - (B*d^3*(a*(1 + Sin[e + f*x]))^(5/2
)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]

time = 6.02, size = 374, normalized size = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(\left(4095A\,d^3+12285Bc\,d^2+11970B\,d^3\right)\sin(fx+e)\left(\cos^4(fx+e)\right)+\left(-19305A\,c^2d-55770Ac\,d^2-312\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/45045*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*((4095*A*d^3+12285*B*c*d^2+11970*
B*d^3)*sin(f*x+e)*cos(f*x+e)^4+(-19305*A*c^2*d-55770*A*c*d^2-31265*A*d^3-64
35*B*c^3-55770*B*c^2*d-93795*B*c*d^2-44860*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(
42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3+181038*B*
c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)-3465*B*d^3*cos(f*x+e)^6+(15015
*A*c*d^2+14560*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4+
(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^2-56810*A*d^3-25740*B*c^3-123981*B*
c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f*x+e)^2+138138*A*c^3+373230*A*c^2*d+
359502*A*c*d^2+116090*A*d^3+124410*B*c^3+359502*B*c^2*d+348270*B*c*d^2+1138
18*B*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^3, x)
```

Fricas [A]
time = 0.61, size = 880, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/45045*(3465*B*a^2*d^3*cos(f*x + e)^7 - 315*(39*B*a^2*c*d^2 + (13*A + 27*B
)*a^2*d^3)*cos(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)
*a^2*c^2*d - 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3
- 35*(429*B*a^2*c^2*d + 39*(11*A + 32*B)*a^2*c*d^2 + 4*(104*A + 205*B)*a^2*
d^3)*cos(f*x + e)^5 + 5*(1287*B*a^2*c^3 + 429*(9*A + 19*B)*a^2*c^2*d + 39*(
209*A + 320*B)*a^2*c*d^2 + 2*(2080*A + 2813*B)*a^2*d^3)*cos(f*x + e)^4 + (1
287*(7*A + 20*B)*a^2*c^3 + 429*(180*A + 289*B)*a^2*c^2*d + 39*(3179*A + 437
0*B)*a^2*c*d^2 + (56810*A + 72109*B)*a^2*d^3)*cos(f*x + e)^3 - (429*(77*A +
 85*B)*a^2*c^3 + 429*(255*A + 263*B)*a^2*c^2*d + 39*(2893*A + 2965*B)*a^2*c
*d^2 + (38545*A + 39113*B)*a^2*d^3)*cos(f*x + e)^2 - 2*(429*(161*A + 145*B)
*a^2*c^3 + 429*(435*A + 419*B)*a^2*c^2*d + 39*(4609*A + 4465*B)*a^2*c*d^2 +
```

```
(58045*A + 56909*B)*a^2*d^3)*cos(f*x + e) - (3465*B*a^2*d^3*cos(f*x + e)^6
 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)*a^2*c^2*d - 1248*(143*A
+ 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3 + 315*(39*B*a^2*c*d^2 + (
13*A + 38*B)*a^2*d^3)*cos(f*x + e)^5 - 35*(429*B*a^2*c^2*d + 39*(11*A + 23*
B)*a^2*c*d^2 + (299*A + 478*B)*a^2*d^3)*cos(f*x + e)^4 - 5*(1287*B*a^2*c^3
+ 429*(9*A + 26*B)*a^2*c^2*d + 507*(22*A + 37*B)*a^2*c*d^2 + (6253*A + 8972
*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(429*(7*A + 15*B)*a^2*c^3 + 429*(45*A + 53*
B)*a^2*c^2*d + 39*(583*A + 655*B)*a^2*c*d^2 + (8515*A + 9083*B)*a^2*d^3)*co
s(f*x + e)^2 + 2*(429*(49*A + 65*B)*a^2*c^3 + 429*(195*A + 211*B)*a^2*c^2*d
 + 39*(2321*A + 2465*B)*a^2*c*d^2 + (32045*A + 33181*B)*a^2*d^3)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e)
 + f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(524) = 1048$.
time = 1.01, size = 1061, normalized size = 1.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/1441440*sqrt(2)*(3465*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
13/4*pi + 13/2*f*x + 13/2*e) + 180180*(40*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 30*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*A*a^2*
c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 78*B*a^2*c^2*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 78*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
69*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*A*a^2*d^3*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 21*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(80*A*a^2*c^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 88*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 264*
A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 240*B*a^2*c^2*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 240*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 228*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 76*A*a^2*d^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 71*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*
```

```
x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 9009*(16*A*a^2*c^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 40*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 120*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 144*B*a^2*c^2*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 144*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 150*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 50*A*
a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 51*B*a^2*d^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 12870*(4*B*a^2*c^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 30*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*A*a^2
*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 39*B*a^2*c*d^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 13*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
5*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/
2*e) + 10010*(6*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a^2*c
*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*B*a^2*c*d^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 5*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*
a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-9/4*pi + 9/2*f*x + 9/2*e)
 + 4095*(6*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*d^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sin(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(a)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\sin(e+fx))\ (a+a\sin(e+fx))^{5/2}\ (c+d\sin(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,
 x)
```

3.301 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=429

$$-\frac{2a^3(15c^2+10cd+7d^2)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)}{3465d^3f\sqrt{a+a\sin(e+fx)}}-\frac{4a^2(\cdots}{}$$

```
[Out] -2/1155*a*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))
*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/11*a*B*cos(f*x+e)*(a+a*sin(f*x+e))
^(3/2)*(c+d*sin(f*x+e))^3/d/f-2/3465*a^3*(15*c^2+10*c*d+7*d^2)*(11*A*d*(c^2
-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)/d^3/f/(a+a
*sin(f*x+e))^(1/2)+2/693*a^3*(11*A*(3*c-19*d)*d-B*(15*c^2-65*c*d+194*d^2))*
cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-4/3465*a^2*(5*c-
d)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/99*a^2*(-11*A*d+5*B*c-14*B*d)*cos(f*x+e
)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A]
time = 0.73, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3055, 3060, 2840, 2830, 2725}

$$\frac{2a^3(11Ad(3c-19d)-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3(15c^2+10cd+7d^2)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)}{3465d^3f\sqrt{a\sin(e+fx)+a}}-\frac{4a^2(5c-d)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3465d^2f}+\frac{2a^2(-11Ad+5Bc-14Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^3}{99d^2f}-\frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{1155df}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (-2*a^3*(15*c^2 + 10*c*d + 7*d^2)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^
3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x])/(3465*d^3*f*Sqrt[a + a*S
in[e + f*x]]) - (4*a^2*(5*c - d)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3
 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(3465*d^2*f) - (2*a*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d +
 169*c*d^2 - 710*d^3))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*d*f)
+ (2*a^3*(11*A*(3*c - 19*d)*d - B*(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(693*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B
*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^3)/(99*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])^3)/(11*d*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2840

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = -\frac{2aB\cos(e + fx)(a + a\sin(e + fx)}{11df}$$

$$= \frac{2a^2(5Bc - 11Ad - 14Bd)\cos(e + fx}{}$$

$$= \frac{2a^3(11A(3c - 19d)d - B(15c^2 - 65c}{693d^3 f\sqrt{a}}$$

$$= -\frac{2a(11Ad(c^2 - 10cd + 73d^2) - B(5c}{}$$

$$= -\frac{4a^2(5c - d)\,(11Ad(c^2 - 10cd + 73d}{}$$

$$= -\frac{2a^3(15c^2 + 10cd + 7d^2)\,(11Ad(c^2 -}{}$$

Mathematica [A]
time = 5.96, size = 328, normalized size = 0.76

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,\left(164472Ac^2+137280Bc^2+274560Acd+248732Bcd+124366Ad^2+114640Bd^2-8(11A(63c^2+360cd+254d^2)+2B(990c^2+2794cd+1625d^2))\cos(2(e+fx))+70d(22Bc+11Ad+32Bd)\cos(4(e+fx))+51744Ac^2\sin(e+fx)+66660Bc^2\sin(e+fx)+133320Acd\sin(e+fx)+137104Bcd\sin(e+fx)+68552Ad^2\sin(e+fx)+69890Bd^2\sin(e+fx)-1980Bc^2\sin(3(e+fx))-3960Acd\sin(3(e+fx))-11440Bcd\sin(3(e+fx))-5720Ad^2\sin(3(e+fx))-8675Bd^2\sin(3(e+fx))+315Bd^2\sin(5(e+fx))\right)}{27720f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -1/27720*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x
])]*(164472*A*c^2 + 137280*B*c^2 + 274560*A*c*d + 248732*B*c*d + 124366*A*d
^2 + 114640*B*d^2 - 8*(11*A*(63*c^2 + 360*c*d + 254*d^2) + 2*B*(990*c^2 + 2
794*c*d + 1625*d^2))*Cos[2*(e + f*x)] + 70*d*(22*B*c + 11*A*d + 32*B*d)*Cos
[4*(e + f*x)] + 51744*A*c^2*Sin[e + f*x] + 66660*B*c^2*Sin[e + f*x] + 13332
0*A*c*d*Sin[e + f*x] + 137104*B*c*d*Sin[e + f*x] + 68552*A*d^2*Sin[e + f*x]
 + 69890*B*d^2*Sin[e + f*x] - 1980*B*c^2*Sin[3*(e + f*x)] - 3960*A*c*d*Sin[
3*(e + f*x)] - 11440*B*c*d*Sin[3*(e + f*x)] - 5720*A*d^2*Sin[3*(e + f*x)] -
 8675*B*d^2*Sin[3*(e + f*x)] + 315*B*d^2*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 5.72, size = 257, normalized size = 0.60

method	result

default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(315B\,d^2\sin(fx+e)\left(\cos^4(fx+e)\right)+\left(-990Acd-1430A\,d^2-495B\,c^2-2860Bcd-2405B\,d^2\right)\left(\cos^2(fx+\right.\right.}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*B*d^2*sin(f*x+e)*cos(f*x+e)^4
+(-990*A*c*d-1430*A*d^2-495*B*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(f
*x+e)+(3234*A*c^2+8580*A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*s
in(f*x+e)+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-693*A*c^2-3960*A*
c*d-3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+10626*A*c^2+1
9140*A*c*d+9218*A*d^2+9570*B*c^2+18436*B*c*d+8930*B*d^2)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [A]
time = 0.53, size = 608, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a^
2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*c*
d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c*d
+ (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 + 22*(
180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (33*(7
7*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*a^2*d^
2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*B)*a^2*
c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(f*x + e)
```

```
^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(143*A + 125
*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x + e)^4 - 5*
(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d^2)*cos(f*x
 + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d + (583*A +
655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 + 22*(195*A +
211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.75, size = 717, normalized size = 1.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/
4*pi + 11/2*f*x + 11/2*e) + 6930*(40*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 60*A*a^2*c*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 52*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 26*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*B*a^2*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 231
0*(20*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 22*B*a^2*c^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 44*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) + 40*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*A*a^2*d^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 19*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 693*(8*A*a^2*c^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 20*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 40*
A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*B*a^2*c*d*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + 24*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
25*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5
/2*e) + 495*(4*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*a^2*c*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 10*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 13*B*a^2*d
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e) + 38
```

```
5*(4*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*sin(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^{5/2}\,(c + d\sin(e + fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
 x)
```

3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=212

$$-\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sqrt{a+}}{315f}$$

```
[Out] -2/105*a*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-
2/63*(9*A*d+9*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*B*d*cos(f*
x+e)*(a+a*sin(f*x+e))^(7/2)/a/f-64/315*a^3*(21*A*c+15*A*d+15*B*c+13*B*d)*co
s(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/315*a^2*(21*A*c+15*A*d+15*B*c+13*B*d)*
cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.25, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3102, 2830, 2726, 2725}

$$-\frac{64a^3(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{2(9Ad+9Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{63f}-\frac{2a(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{105f}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x])/(315*f*Sqrt[a +
a*Sin[e + f*x]]) - (16*a^2*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(315*f) - (2*a*(21*A*c + 15*B*c + 15*A*d + 13*B*
d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) - (2*(9*B*c + 9*A*d - 2
*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(63*f) - (2*B*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^{5/2}\left(Ac+(Bc+A\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/}}{9af}$$

$$= -\frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a-}{63f}$$

$$= -\frac{2a(21Ac+15Bc+15Ad+13Bd)\cos}{105f}$$

$$= -\frac{16a^2(21Ac+15Bc+15Ad+13Bd)\,c}{315f}$$

$$= -\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\,c}{315f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 2.75, size = 202, normalized size = 0.95

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(7476Ac+6240Bc+6240Ad+5653Bd-4(63Ac+180Bc+180Ad+254Bd)\cos(2(e+fx))+35Bd\cos(4(e+fx))+2352Ac\sin(e+fx)+3030Bc\sin(e+fx)+3030Ad\sin(e+fx)+3116Bd\sin(e+fx)-90Bc\sin(3(e+fx))-90Ad\sin(3(e+fx))-260Bd\sin(3(e+fx)))}{1260f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(7476*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180*A
*d + 254*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[e +
 f*x] + 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e + f*
x] - 90*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3*(e +
 f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 6.58, size = 152, normalized size = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)((-45Ad-45Bc-130Bd)\sin(fx+e)(\cos^2(fx+e))+(294Ac+390Ad+390Bc+422Bd)\sin(fx+e)+}{315\cos(fx+e)\sqrt{a+a\sin(fx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*((-45*A*d-45*B*c-130*B*d)*sin(f*x+e
)*cos(f*x+e)^2+(294*A*c+390*A*d+390*B*c+422*B*d)*sin(f*x+e)+35*B*d*cos(f*x+
e)^4+(-63*A*c-180*A*d-180*B*c-289*B*d)*cos(f*x+e)^2+966*A*c+870*A*d+870*B*c
+838*B*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c), x)
```

Fricas [A]
time = 0.40, size = 374, normalized size = 1.76

$$-\frac{2\,\left(35\,Ba^2d\cos\left(fx+e\right)^5-5\,\left(9\,Ba^2c+\left(9\,A+19\,B\right)a^2d\right)\cos\left(fx+e\right)^4+96\,\left(7\,A+5\,B\right)a^2c+32\,\left(15\,A+13\,B\right)a^2d-\left(9\,\left(7\,A+20\,B\right)a^2c+\left(180\,A+289\,B\right)a^2d\right)\cos\left(fx+e\right)^3+\left(3\,\left(77\,A+85\,B\right)a^2c+\left(255\,A+263\,B\right)a^2d\right)\cos\left(fx+e\right)^2+2\,\left(3\,\left(161\,A+145\,B\right)a^2c+\left(435\,A+419\,B\right)a^2d\right)\cos\left(fx+e\right)-\left(35\,Ba^2d\cos\left(fx+e\right)^4+96\,\left(7\,A+5\,B\right)a^2c+32\,\left(15\,A+13\,B\right)a^2d+5\,\left(9\,Ba^2c+\left(9\,A+26\,B\right)a^2d\right)\cos\left(fx+e\right)^3-3\,\left(3\,\left(7\,A+15\,B\right)a^2c+\left(45\,A+53\,B\right)a^2d\right)\cos\left(fx+e\right)^2-2\,\left(3\,\left(49\,A+65\,B\right)a^2c+\left(195\,A+211\,B\right)a^2d\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{315\,\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*d*cos(f*x + e)^5 - 5*(9*B*a^2*c + (9*A + 19*B)*a^2*d)*cos(
f*x + e)^4 + 96*(7*A + 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d - (9*(7*A + 20*B
)*a^2*c + (180*A + 289*B)*a^2*d)*cos(f*x + e)^3 + (3*(77*A + 85*B)*a^2*c +
(255*A + 263*B)*a^2*d)*cos(f*x + e)^2 + 2*(3*(161*A + 145*B)*a^2*c + (435*A
 + 419*B)*a^2*d)*cos(f*x + e) - (35*B*a^2*d*cos(f*x + e)^4 + 96*(7*A + 5*B)
*a^2*c + 32*(15*A + 13*B)*a^2*d + 5*(9*B*a^2*c + (9*A + 26*B)*a^2*d)*cos(f*
x + e)^3 - 3*(3*(7*A + 15*B)*a^2*c + (45*A + 53*B)*a^2*d)*cos(f*x + e)^2 -
2*(3*(49*A + 65*B)*a^2*c + (195*A + 211*B)*a^2*d)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^{\frac{5}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(202) = 404.
time = 0.65, size = 425, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
+ 9/2*f*x + 9/2*e) + 630*(20*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
15*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*A*a^2*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 13*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 210*(10*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 11*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*A*a^2*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(2*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 5*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a^2*d*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*B*a^2*c*sgn(cos(-1/4*pi + 1/
```

```
2*f*x + 1/2*e)) + 2*A*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*d
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(
a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^{5/2}\,(c+d\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)), x
)
```

3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=138

$$-\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{2a(7A+5B)\cos(e+fx)(}{35f}$$

```
[Out] -2/35*a*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/f-64/105*a^3*(7*A+5*B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/
2)-16/105*a^2*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2726, 2725}

$$-\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(7*A + 5*B)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f} + \frac{1}{7}(7A+5B)\int$$
$$= -\frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2B\,c}{}$$
$$= -\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2a(}{}$$
$$= -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(7A+5B)\cos(e+}{10}$$

Mathematica [A]
time = 1.00, size = 119, normalized size = 0.86

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\,(1246A+1040B-6(7A+20B)\cos(2(e+fx))+(392A+505B)\sin(e+fx)-15B\sin(3(e+fx)))}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(1246*A + 1040*B - 6*(7*A + 20*B)*Cos[2*(e + f*x)] + (392*A + 505*B)*Sin[
e + f*x] - 15*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
)
```

Maple [A]
time = 5.74, size = 99, normalized size = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(-15B\left(\cos^2(fx+e)\right)\sin(fx+e)+(98A+130B)\sin(fx+e)+(-21A-60B)\left(\cos^2(fx+e)\right)+322A+29}{105\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(98*
A+130*B)*sin(f*x+e)+(-21*A-60*B)*cos(f*x+e)^2+322*A+290*B)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [A]
time = 0.38, size = 202, normalized size = 1.46

$$\frac{2\,{\left(15\,B a^{2} \cos\left(f x + e\right)^{4} + 3\,{\left(7\,A + 20\,B\right)} a^{2} \cos\left(f x + e\right)^{3} - {\left(77\,A + 85\,B\right)} a^{2} \cos\left(f x + e\right)^{2} - 2\,{\left(161\,A + 145\,B\right)} a^{2} \cos\left(f x + e\right) - 32\,{\left(7\,A + 5\,B\right)} a^{2} + {\left(15\,B a^{2} \cos\left(f x + e\right)^{3} - 3\,{\left(7\,A + 15\,B\right)} a^{2} \cos\left(f x + e\right)^{2} - 2\,{\left(49\,A + 65\,B\right)} a^{2} \cos\left(f x + e\right) + 32\,{\left(7\,A + 5\,B\right)} a^{2}\right)} \sin\left(f x + e\right)\right)} \sqrt{a \sin\left(f x + e\right) + a}}{105\,{\left(f \cos\left(f x + e\right) + f \sin\left(f x + e\right) + f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(f*x + e)^4 + 3*(7*A + 20*B)*a^2*cos(f*x + e)^3 - (77*A
 + 85*B)*a^2*cos(f*x + e)^2 - 2*(161*A + 145*B)*a^2*cos(f*x + e) - 32*(7*A +
 5*B)*a^2 + (15*B*a^2*cos(f*x + e)^3 - 3*(7*A + 15*B)*a^2*cos(f*x + e)^2 -
2*(49*A + 65*B)*a^2*cos(f*x + e) + 32*(7*A + 5*B)*a^2)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{5}{2}} \left(A + B \sin{\left(e + f x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x)), x)
```

Giac [A]
time = 0.51, size = 213, normalized size = 1.54

$$\frac{\sqrt{2} {\left(15\,B a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) \sin\left(-\frac{7}{4}\,\pi + \frac{7}{2}\,f x + \frac{7}{2}\,e\right) + 525\,{\left(4\,A a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) + 3\,B a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right)\right)} \sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right) + 35\,{\left(10\,A a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) + 11\,B a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right)\right)} \sin\left(-\frac{3}{4}\,\pi + \frac{3}{2}\,f x + \frac{3}{2}\,e\right) + 21\,{\left(2\,A a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right) + 5\,B a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,f x + \frac{1}{2}\,e\right)\right)\right)} \sin\left(-\frac{5}{4}\,\pi + \frac{5}{2}\,f x + \frac{5}{2}\,e\right)\right)} \sqrt{a}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7
/2*f*x + 7/2*e) + 525*(4*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*
(10*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=218

$$\frac{2a^{5/2}(c-d)^2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c+d}\,f}+\frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d}{15d^3f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2/5*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f+2*a^(5/2)*(c-d)^2*(-A*d+B*c)
*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(
7/2)/f/(c+d)^(1/2)+2/15*a^3*(5*A*(3*c-7*d)*d-B*(15*c^2-35*c*d+32*d^2))*cos(
f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/15*a^2*(-5*A*d+5*B*c-8*B*d)*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A]

time = 0.60, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3055, 3060, 2852, 214}

$$\frac{2a^{5/2}(c-d)^2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{7/2}f\sqrt{c+d}}+\frac{2a^3(5Ad(3c-7d)-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15d^2f}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (2*a^(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2)*Sqrt[c + d]*f) + (2*a^3*(5*A
*(3*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(15*d^3*f*Sqrt
[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(15*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*d*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df} + \frac{2\int \frac{(a+a\sin(e\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}\cdots}{15d^2f}$$

$$= \frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx\cdots}{15d^3f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx\cdots}{15d^3f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^{5/2}(c-d)^2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx\cdots}{\sqrt{c+d}\sqrt{a+a\sin(e\cdots}}\right.}{d^{7/2}\sqrt{c+d}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 450 vs. 2(218) = 436.
time = 4.30, size = 450, normalized size = 2.06

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(-30\sqrt{d}\,(Ad(-2c+5d)+B(2c^2-5cd+5d^2))\cos\left(\frac{1}{2}(e+fx)\right)-5d^{3/2}(-2Bc+2Ad+5Bd)\cos\left(\frac{3}{2}(e+fx)\right)+3Bd^{5/2}\cos\left(\frac{5}{2}(e+fx)\right)+\frac{15(c-d)^2(Bc-Ad)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{c+d}}-\frac{15(c-d)^2(Bc-Ad)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{\sqrt{c+d}}+30\sqrt{d}\,(Ad(-2c+5d)+B(2c^2-5cd+5d^2))\sin\left(\frac{1}{2}(e+fx)\right)-5d^{3/2}(-2Bc+2Ad+5Bd)\sin\left(\frac{3}{2}(e+fx)\right)-3Bd^{5/2}\sin\left(\frac{5}{2}(e+fx)\right)\right)}{30d^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 - 5
*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[(3
*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] + (15*(c - d)^2*(B*c - A*
d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d
 + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)
/2]))]))/Sqrt[c + d] - (15*(c - d)^2*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e +
f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2
*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[c + d] + 30*Sqrt[d]*(A*d*(-
2*c + 5*d) + B*(2*c^2 - 5*c*d + 5*d^2))*Sin[(e + f*x)/2] - 5*d^(3/2)*(-2*B*
c + 2*A*d + 5*B*d)*Sin[(3*(e + f*x))/2] - 3*B*d^(5/2)*Sin[(5*(e + f*x))/2])
)/(30*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(192) = 384$.
time = 11.74, size = 543, normalized size = 2.49

method	result
default	$\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3B(a-a\sin(fx+e))^{\frac{5}{2}}\sqrt{a(c+d)d}\,d^2+5A(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{a(c+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)*
(a*(c+d)*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2-15
*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+e))^(
3/2)*(a*(c+d)*d)^(1/2)*a*c*d+20*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*
a*d^2+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^3-30
*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+15*B*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2+15*A*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-45*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d
)*d)^(1/2)*a^2*d^2-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c^2+45
*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-60*B*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a^2*d^2)/d^3/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(200) = 400.
time = 1.89, size = 1356, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^
3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*cos
(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*
d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 -
2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 +
 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x +
 e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*
cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d
 + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)
*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f
*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B
*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32
*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 - (15*B*a
^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x + e) - (3
*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A +
32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x + e) +
d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A
*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d
^3)*cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 -
 A*a^2*d^3)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) +
```

```
2*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35
*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 -
(15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x +
e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (
35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x
+ e) + d^3*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(200) = 400.
time = 0.58, size = 548, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/15*sqrt(2)*sqrt(a)*(15*sqrt(2)*(B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^2*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*a^2*d^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d^3) + 2*(12*B*a^2*d^4*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 10*B*a^2*c*d^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*
A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 - 40*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^3 + 15*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e) - 15*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 45*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 45*A*a^2*d^4*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 60*B*a^2*d^4*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^5)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2}}{c + d \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)),
x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)),
 x)
```

3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=265

$$\frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}(c+d)^{3/2}f}-\frac{a^3(3Ad(3c+d)-}{3d^3(c+}$$

```
[Out] a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1
/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(3/2)/f+a*(-A
*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+d)/f/(c+d*sin(f*x+e))-1/3*a^
3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*cos(f*x+e)/d^3/(c+d)/f/(a+a*sin(f
*x+e))^(1/2)-1/3*a^2*(-3*A*d+5*B*c+2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)
/d^2/(c+d)/f
```

Rubi [A]

time = 0.64, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3054, 3055, 3060, 2852, 214}

$$\frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{7/2}f(c+d)^{3/2}}-\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3f(c+d)\sqrt{a\sin(e+fx)+a}}-\frac{a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3d^2f(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2 + 5*c*d - 2*d^2))*ArcTanh[(Sqr
t[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2
)*(c + d)^(3/2)*f) - (a^3*(3*A*d*(3*c + d) - B*(15*c^2 - 5*c*d - 14*d^2))*C
os[e + f*x])/(3*d^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*(5*B*c - 3*A
*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d^2*(c + d)*f) + (a*(
B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c + d*Sin
[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin}{}}{}$$

$$= -\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f}$$

$$= -\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\tanh^{-1}\Bigg(}{d^{7/2}(c+d)^{3/2}f}$$

Mathematica [A]
time = 4.47, size = 460, normalized size = 1.74

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(-12\sqrt{d}(-4Bc+2Ad+5Bd)\cos\left(\frac{1}{2}(e+fx)\right)-4Bd^{3/2}\cos\left(\frac{3}{2}(e+fx)\right)-\frac{3(c-d)(-Ad(3c+5d)+B(5c^2+5cd-2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(-\sec^2\left(\frac{1}{4}(e+fx)\right)\left(c+d+\sqrt{d}\sqrt{c+d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sqrt{c+d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+\frac{3(c-d)(-Ad(3c+5d)+B(5c^2+5cd-2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left((c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)+\sqrt{d}\sqrt{c+d}\left(-1+2\tan\left(\frac{1}{4}(e+fx)\right)+\tan^2\left(\frac{1}{4}(e+fx)\right)\right)\right)\right)}{(c+d)^{3/2}}+12\sqrt{d}(-4Bc+2Ad+5Bd)\sin\left(\frac{1}{2}(e+fx)\right)-\frac{12(c-d)^2\sqrt{d}(-Bc+Ad)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))}-4Bd^{3/2}\sin\left(\frac{3}{2}(e+fx)\right)\right)}{12d^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-12*Sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e
+ f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(-(A*d*(3*c + 5*d
)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Lo
g[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt
[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(3/2) + (3*(c - d)*(-(A*d*(3*
c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]
 + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e +
f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 12*Sqrt[d]*(-4*B*c + 2*A*d
 + 5*B*d)*Sin[(e + f*x)/2] - (12*(c - d)^2*Sqrt[d]*(-(B*c) + A*d)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])) - 4*B*d^(3/2)*
Sin[(3*(e + f*x))/2]))/(12*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(241) = 482$.
time = 13.43, size = 932, normalized size = 3.52

method	result
default	$-\frac{a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)d\left(-9A\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)a^2c^2d-6A\,\mathrm{arct}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(-9*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d-6*A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+15*A*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+
d)*d)^(1/2)*c*d-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2+15*a^2*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3-21*B*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+6*B*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3+6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+
d)*d)^(1/2)*a*c*d+6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2-12*B*(
a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2+6*B*(a-a*sin(f*x+e))^(1/2)*(a
*(c+d)*d)^(1/2)*a*c*d+18*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2)-
9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-6*A*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+15*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-2*B*(a-a*sin(f*x+e
))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/
2)*c*d^2+15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4
-21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+6*B
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+9*A*(a-a*s
in(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*A*(a-a*sin(f*x+e))^(1/2)*(a*(c
+d)*d)^(1/2)*a*d^3-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3+12*B
*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+15*B*(a-a*sin(f*x+e))^(1/
2)*(a*(c+d)*d)^(1/2)*a*c*d^2)/d^3/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(250) = 500$.
time = 1.67, size = 2096, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 +
(3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*
d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^
2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*co
s(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2
 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c
^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*
d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*
d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d
 + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e)
 + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*
cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^3 -
(9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2
*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*
a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10
*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*
a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a
^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*
(A + B)*a^2*c*d^2 - (3*A + 8*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a))/((c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*c
os(f*x + e) - (c^2*d^3 + 2*c*d^4 + d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) +
 (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin(f*x + e)), 1/6*(3*(5*B*a^2*c^4 - (3*A - 5
*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*
B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*
A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*
B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A
- 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A +
 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 +
(5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arcta
n(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^
2))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A -
```

```
 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*
x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*
cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 -
(3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d
+ 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d
^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*(A + B)*a^2*c*d^2 - (3*A + 8*B)*a
^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^4 + d^5
)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^2*d^3 + 2*c*d^4
+ d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) + (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin
(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(250) = 500.
time = 0.64, size = 625, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/6*sqrt(2)*sqrt(a)*(3*sqrt(2)*(5*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*A*a^2*c*d^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 5*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a^2*d^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^3 + d^4)*sqrt(-c*d - d^2)) - 6*(B*a^2*c^
3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a^
2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
- 2*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) - A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e))/((c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 - c - d)) + 4*(2*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 + 6*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
```

```
))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^6)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
2, x)
```

3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=308

$$-\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{7/2}(c+d)^{5/2}f}+$$

```
[Out] -1/4*a^(5/2)*(A*d*(3*c^2+10*c*d+19*d^2)-B*(15*c^3+30*c^2*d+7*c*d^2-20*d^3))
*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(
7/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+
d)/f/(c+d*sin(f*x+e))^2+1/4*a^3*(3*A*d*(c+3*d)-B*(15*c^2+25*c*d+4*d^2))*cos
(f*x+e)/d^3/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*(A*d*(c+7*d)-B*(5*c^2+
7*c*d-4*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/(c+d*sin(f*x+
e))
```

Rubi [A]

time = 0.67, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3054, 3060, 2852, 214}

$$-\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{7/2}f(c+d)^{5/2}}+\frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3f(c+d)^2\sqrt{a\sin(e+fx)+a}}-\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/4*(a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d
^2 - 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a +
a*Sin[e + f*x]])])/(d^(7/2)*(c + d)^(5/2)*f) + (a^3*(3*A*d*(c + 3*d) - B*(1
5*c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(4*d^3*(c + d)^2*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4
*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*d^2*(c + d)^2*f*(c + d*Sin
[e + f*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int (a+a}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(Ad}{}$$

$$= \frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2}{4d^{7/2}(c+}$$

Mathematica [A]
time = 5.67, size = 504, normalized size = 1.64

$$\frac{(a(1+\sin(e+fx)))^{5/2}\left(\frac{(-Ad(3c^2+10cd+19d^2)+B(15c^3+30c^2d+7cd^2-20d^3))\left(e+fx-2\log(\sec^2(\frac{1}{4}(e+fx)))+2\log\left(-\sec^2(\frac{1}{4}(e+fx))\left(c+d+\sqrt{d}\sqrt{c+d}\cos(\frac{1}{2}(e+fx))-\sqrt{d}\sqrt{c+d}\sin(\frac{1}{2}(e+fx))\right)\right)\right)}{(c+d)^{5/2}}+\frac{(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\left(e+fx-2\log(\sec^2(\frac{1}{4}(e+fx)))+2\log\left((c+d)\sec^2(\frac{1}{4}(e+fx))+\sqrt{d}\sqrt{c+d}(-1+2\tan(\frac{1}{4}(e+fx))+\tan^2(\frac{1}{4}(e+fx)))\right)\right)}{(c+d)^{5/2}}-\frac{4\sqrt{d}(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))(15Bc^4-3Ac^3d+20Bc^3d-8Ac^2d^2-Bc^2d^2+9Acd^3+10Bcd^3+2Ad^4+4Bd^4-4Bd^2(c+d)^2\cos(2(e+fx))+d(Ad(-5c^2-6cd+11d^2)+B(25c^3+34c^2d+cd^2+4d^3))\sin(e+fx))}{(c+d)^2(c+d\sin(e+fx))^2}\right)}{16d^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-(A*d*(3*c^2 + 10*c*d + 19*d^2)) + B*(15*c
^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - S
qrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(5/2) + ((A*d*(3*c^2 + 10*
c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[
Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d
]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt
[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^4 - 3*A*c^3*d + 20*B*c^3*
d - 8*A*c^2*d^2 - B*c^2*d^2 + 9*A*c*d^3 + 10*B*c*d^3 + 2*A*d^4 + 4*B*d^4 -
4*B*d^2*(c + d)^2*Cos[2*(e + f*x)] + d*(A*d*(-5*c^2 - 6*c*d + 11*d^2) + B*(
25*c^3 + 34*c^2*d + c*d^2 + 4*d^3))*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[e
+ f*x])^2)))/(16*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(280) = 560$.
time = 15.55, size = 1587, normalized size = 5.15

method	result	size
default	Expression too large to display	1587

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*a*(-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-4*B*(-a*(sin(
f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-15*a^2*arctanh((-a*(sin(f*x+e)-1))^(
1/2)*d/(a*(c+d)*d)^(1/2))*B*c^5+3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a*c*d^3+29*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-3*B*(
-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2-13*B*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3+10*A*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4+40*B*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c*d^4-15*B*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-30*B*ar
ctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d
^3-7*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*
a^2*c*d^4+6*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*
x+e)*a^2*c^3*d^2+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*sin(f*x+e)*a^2*c^2*d^3+38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*
d)^(1/2))*sin(f*x+e)*a^2*c*d^4-3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1
/2)*a*c^3*d-13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2+8*B*
(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d^2+16*B*(-a
*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c*d^3+16*B*(-a*(sin
(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^3*d+32*B*(-a*(sin(f*x+e)
-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^2*d^2+16*B*(-a*(sin(f*x+e)-1))^
(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c*d^3+8*B*(-a*(sin(f*x+e)-1))^(1/2)*(a
*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*d^4-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d
/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^4*d-60*B*arctanh((-a*(sin(f*x+e)-1))^(
1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2-14*B*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3+15*B*(-a*(sin(f*x+
e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^2*c^4*d-7*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^2*c^3*d^2+20*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a^2*c^2*d^3+13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a
*d^4+15*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^4+4*B*(-a*(sin(f*
x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+20*B*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+5*A*(-a*(sin(f*x+e)-1))^
(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(
1/2)*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c
```

```
^4*d+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^
2+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3-9
*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d-2*B*(-a*(sin(f*x+e)-1)
)^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))
/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^3/cos(f*x+e)/(a+a*sin(f*x+e
))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. 2(290) = 580.
time = 2.44, size = 3104, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2
- 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2
*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^
4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B
)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19
*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d
 - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a
^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*
B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(
16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A -
 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*
x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*
c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d
+ d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*
a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x
+ e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3
+ (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a
```

/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2 - 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e)), -1/8*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2 - 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)

```
*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos
(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5
 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(
f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e
))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(290) = 580.
time = 0.72, size = 938, normalized size = 3.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/8*sqrt(2)*(16*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)/d^3 + sqrt(2)*(15*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a^2*c^2*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 19*A*a^2*d
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d -
 d^2))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(-c*d - d^2)) - 2*(18*B*a^2*c^3*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^
2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 + 4*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 - 12*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 - 30*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 22*A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*a^2*d^4*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 7*B*a^2*c^4*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*a^2*c^3*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 13*B
*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 13*A*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
```

```
*f*x + 1/2*e) + 11*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 13*B*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 13*A*a^2*d^4*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*a^2*d^4*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c^2*d^3 + 2*c*d^4
 + d^5)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)^2))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}}{\left(c+d\,\sin(e+f\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3, x)
```

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=284

$$-\frac{\sqrt{2}\,(A-B)(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d\cdots}{105f\sqrt{a+a\sin(e+\cdots}}$$

```
[Out] -(A-B)*(c-d)^3*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
))*2^(1/2)/f/a^(1/2)-4/105*(7*A*d*(21*c^2-12*c*d+7*d^2)+B*(36*c^3-63*c^2*d+
144*c*d^2-37*d^3))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/35*(7*A*d+6*B*c-B*
d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-2/7*B*cos(f*x+e)*
(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)-2/105*d*(7*A*(9*c-d)*d+B*(24*c^
2-15*c*d+31*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.66, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3062, 3047, 3102, 2830, 2728, 212}

$$-\frac{2d(7Ad(9c-d)+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105af}-\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2(7Ad+6Bc-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(A-B)(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(7*A*d*(21*c^2 - 12*c*d + 7*d^2) +
B*(36*c^3 - 63*c^2*d + 144*c*d^2 - 37*d^3))*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*d*(7*A*(9*c - d)*d + B*(24*c^2 - 15*c*d + 31*d^2))*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*a*f) - (2*(6*B*c + 7*A*d - B*d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(35*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*C
os[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))^2(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \cdots$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \cdots$$

$$= -\frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\cdots}{105af}$$

$$= -\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2\cdots}{105f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2\cdots}{105f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 375, normalized size = 1.32

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((840+840i)(-1)^{3/4}(A-B)(c-d)^3\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-105(4Ad(6c^2-3cd+2d^2)+B(8c^3-12c^2d+24cd^2-5d^3))\cos\left(\frac{1}{2}(e+fx)\right)-35d(2A(6c-d)d+B(12c^2-6cd+5d^2))\cos\left(\frac{3}{2}(e+fx)\right)+21d^2(6Bc+2Ad-Bd)\cos\left(\frac{5}{2}(e+fx)\right)+15Bd^3\cos\left(\frac{7}{2}(e+fx)\right)+105(4Ad(6c^2-3cd+2d^2)+B(8c^3-12c^2d+24cd^2-5d^3))\sin\left(\frac{1}{2}(e+fx)\right)-35d(2A(6c-d)d+B(12c^2-6cd+5d^2))\sin\left(\frac{3}{2}(e+fx)\right)+21d^2(-2Ad+B(-6c+d))\sin\left(\frac{5}{2}(e+fx)\right)+15Bd^3\sin\left(\frac{7}{2}(e+fx)\right)\right)}{420f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((840 + 840*I)*(-1)^(3/4)*(A - B)*(c
 - d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 105*(4*A*
d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^2*d + 24*c*d^2 - 5*d^3))*Cos[(e
 + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c^2 - 6*c*d + 5*d^2))*Cos[(3*(e
+ f*x))/2] + 21*d^2*(6*B*c + 2*A*d - B*d)*Cos[(5*(e + f*x))/2] + 15*B*d^3*C
os[(7*(e + f*x))/2] + 105*(4*A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^
2*d + 24*c*d^2 - 5*d^3))*Sin[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c
^2 - 6*c*d + 5*d^2))*Sin[(3*(e + f*x))/2] + 21*d^2*(-2*A*d + B*(-6*c + d))*
```

```
Sin[(5*(e + f*x))/2] + 15*B*d^3*Sin[(7*(e + f*x))/2]))/(420*f*Sqrt[a*(1 + S
in[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(259) = 518$.
time = 8.72, size = 610, normalized size = 2.15

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(105Aa^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)c^3-315Aa^{\frac{7}{2}}\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/105*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(105*A*a^(7/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-315*A*a^(7/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+315*A*a^(7/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-105*A*a^(7/2)
*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3-105*B*a^(7
/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3+315*B*a
^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d-31
5*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d
^2+105*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*d^3-30*B*(a-a*sin(f*x+e))^(7/2)*d^3+42*A*(a-a*sin(f*x+e))^(5/2)*a*d^3+126
*B*(a-a*sin(f*x+e))^(5/2)*a*c*d^2+84*B*d^3*a*(a-a*sin(f*x+e))^(5/2)-210*A*(
a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-70*A*(a-a*sin(f*x+e))^(3/2)*a^2*d^3-210*B*(
a-a*sin(f*x+e))^(3/2)*a^2*c^2*d-210*B*(a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-140*
B*(a-a*sin(f*x+e))^(3/2)*a^2*d^3+630*a^3*A*c^2*d*(a-a*sin(f*x+e))^(1/2)+210
*a^3*A*d^3*(a-a*sin(f*x+e))^(1/2)+210*a^3*c^3*B*(a-a*sin(f*x+e))^(1/2)+630*
B*a^3*c*d^2*(a-a*sin(f*x+e))^(1/2))/a^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) +
 a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(271) = 542$.
time = 0.44, size = 653, normalized size = 2.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 -
 (A - B)*a*d^3 + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 - (
A - B)*a*d^3)*cos(f*x + e) + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)
*a*c*d^2 - (A - B)*a*d^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
n(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(15*B*d^3*cos(f*x + e
)^4 - 105*B*c^3 - 105*(3*A - 2*B)*c^2*d + 21*(10*A - 17*B)*c*d^2 - (119*A -
 92*B)*d^3 + 3*(21*B*c*d^2 + (7*A - B)*d^3)*cos(f*x + e)^3 - (105*B*c^2*d +
 21*(5*A - 4*B)*c*d^2 - 4*(7*A - 16*B)*d^3)*cos(f*x + e)^2 - (105*B*c^3 + 1
05*(3*A - B)*c^2*d - 21*(5*A - 16*B)*c*d^2 + 2*(56*A - 23*B)*d^3)*cos(f*x +
 e) + (15*B*d^3*cos(f*x + e)^3 + 105*B*c^3 + 105*(3*A - 2*B)*c^2*d - 21*(10
*A - 17*B)*c*d^2 + (119*A - 92*B)*d^3 - 3*(21*B*c*d^2 + (7*A - 6*B)*d^3)*co
s(f*x + e)^2 - (105*B*c^2*d + 21*(5*A - B)*c*d^2 - (7*A - 46*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin
(f*x + e) + a*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{\sqrt{a(\sin(e + fx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(271) = 542$.
time = 0.67, size = 573, normalized size = 2.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/210*(105*sqrt(2)*(A*sqrt(a)*c^3 - B*sqrt(a)*c^3 - 3*A*sqrt(a)*c^2*d + 3*B
*sqrt(a)*c^2*d + 3*A*sqrt(a)*c*d^2 - 3*B*sqrt(a)*c*d^2 - A*sqrt(a)*d^3 + B*
sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))) - 105*sqrt(2)*(A*sqrt(a)*c^3 - B*sqrt(a)*c^3 - 3*A*sqrt(a
)*c^2*d + 3*B*sqrt(a)*c^2*d + 3*A*sqrt(a)*c*d^2 - 3*B*sqrt(a)*c*d^2 - A*sqr
t(a)*d^3 + B*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*(120*B*a^(13/2)*d^3*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^7 - 252*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^5 - 84*A*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 168*B*a^(13/2)*d^
3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 210*B*a^(13/2)*c^2*d*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 + 210*A*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 2
10*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 70*A*a^(13/2)*d^3*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 140*B*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^3 - 105*B*a^(13/2)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 315*A*a^(13
/2)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 315*B*a^(13/2)*c*d^2*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) - 105*A*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a
^7*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2
), x)
```

3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=200

$$-\frac{\sqrt{2}\,(A-B)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+}{15f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -(A-B)*(c-d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
))*2^(1/2)/f/a^(1/2)-4/15*(5*A*(3*c-d)*d+B*(6*c^2-7*c*d+7*d^2))*cos(f*x+e)/
f/(a+a*sin(f*x+e))^(1/2)-2/5*B*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x
+e))^(1/2)-2/15*d*(5*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.39, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3062, 3047, 3102, 2830, 2728, 212}

$$-\frac{4(5Ad(3c-d)+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{\sqrt{2}\,(A-B)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(5*A*(3*c - d)*d + B*(6*c^2 - 7*c*d
 + 7*d^2))*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*(4*B*c + 5*
A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*a*f) - (2*B*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}ac(5Ac-Bc+4B\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} \cdots$$

$$= -\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d\cdots}{\cdots}$$

$$= -\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 246, normalized size = 1.23

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((60+60i)(-1)^{3/4}(A-B)(c-d)^2\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-30(A(4c-d)d+2B(c^2-cd+d^2))\cos\left(\frac{1}{2}(e+fx)\right)+5d(-2Ad+B(-4c+d))\cos\left(\frac{3}{2}(e+fx)\right)+3Bd^2\cos\left(\frac{5}{2}(e+fx)\right)+30(A(4c-d)d+2B(c^2-cd+d^2))\sin\left(\frac{1}{2}(e+fx)\right)+5d(-2Ad+B(-4c+d))\sin\left(\frac{3}{2}(e+fx)\right)-3Bd^2\sin\left(\frac{5}{2}(e+fx)\right)\right)}{30f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c -
 d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 30*(A*(4*c
- d)*d + 2*B*(c^2 - c*d + d^2))*Cos[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c +
d))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 30*(A*(4*c - d)*d
 + 2*B*(c^2 - c*d + d^2))*Sin[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c + d))*Si
n[(3*(e + f*x))/2] - 3*B*d^2*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[a*(1 + Sin[e
 + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(179) = 358$.
time = 7.36, size = 396, normalized size = 1.98

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(15Aa^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c^{2}-30Aa^{\frac{5}{2}}\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*A*a^(5/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*a^(5/2)*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*a^(5/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*a^(5/2)*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*a^(5/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a*sin(f*
x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x+e))^(3/
2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*a^2*A*c*d*(a-a*sin(f*x+e))^(1
/2)+30*a^2*c^2*B*(a-a*sin(f*x+e))^(1/2)+30*B*a^2*d^2*(a-a*sin(f*x+e))^(1/2)
)/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) +
 a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(188) = 376.
time = 0.43, size = 470, normalized size = 2.35

$$-\frac{\frac{15\sqrt{2}\left((A-B)ac^{2}-2(A-B)acd+(A-B)ad^{2}+\left((A-B)ac^{2}-2(A-B)acd+(A-B)ad^{2}\right)\cos(fx+e)+\left((A-B)ac^{2}-2(A-B)acd+(A-B)ad^{2}\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^{2}-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}-4\left(3Bd^{2}\cos(fx+e)^{3}-15Bc^{2}-10(3A-2B)cd+(10A-17B)d^{2}-(10Bcd+(5A-4B)d^{2})\cos(fx+e)^{2}-(15Bc^{2}+10(3A-B)cd-(5A-16B)d^{2})\cos(fx+e)-\left(3Bd^{2}\cos(fx+e)^{2}-15Bc^{2}-10(3A-2B)cd+(10A-17B)d^{2}+(10Bcd+(5A-B)d^{2})\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{30\left(af\cos(fx+e)+af\sin(fx+e)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*((A - B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2 + ((A -
B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*cos(f*x + e) + ((A - B)*a*c^2 -
 2*(A - B)*a*c*d + (A - B)*a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos
(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x +
 e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(3*B*d^2*cos(
f*x + e)^3 - 15*B*c^2 - 10*(3*A - 2*B)*c*d + (10*A - 17*B)*d^2 - (10*B*c*d
+ (5*A - 4*B)*d^2)*cos(f*x + e)^2 - (15*B*c^2 + 10*(3*A - B)*c*d - (5*A - 1
6*B)*d^2)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - 15*B*c^2 - 10*(3*A - 2*B
)*c*d + (10*A - 17*B)*d^2 + (10*B*c*d + (5*A - B)*d^2)*cos(f*x + e))*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Giac [A]
time = 0.60, size = 375, normalized size = 1.88

$$\frac{\frac{15\sqrt{2}\left(A\sqrt{a}\,c^2-B\sqrt{a}\,c^2-2A\sqrt{a}\,cd+2B\sqrt{a}\,cd+A\sqrt{a}\,d^2-B\sqrt{a}\,d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{15\sqrt{2}\left(A\sqrt{a}\,c^2-B\sqrt{a}\,c^2-2A\sqrt{a}\,cd+2B\sqrt{a}\,cd+A\sqrt{a}\,d^2-B\sqrt{a}\,d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\sqrt{2}\left(12Ba^{\frac{9}{2}}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5-20Ba^{\frac{9}{2}}cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-10Aa^{\frac{9}{2}}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-10Ba^{\frac{9}{2}}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+15Ba^{\frac{9}{2}}c^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+30Aa^{\frac{9}{2}}cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+15Ba^{\frac{9}{2}}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/30*(15*sqrt(2)*(A*sqrt(a)*c^2 - B*sqrt(a)*c^2 - 2*A*sqrt(a)*c*d + 2*B*sqr
t(a)*c*d + A*sqrt(a)*d^2 - B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(a)*c^2
- B*sqrt(a)*c^2 - 2*A*sqrt(a)*c*d + 2*B*sqrt(a)*c*d + A*sqrt(a)*d^2 - B*sqr
t(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) + 4*sqrt(2)*(12*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^5 - 20*B*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^(9/2)*d^2*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^3 + 15*B*a^(9/2)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 30*A*a^(9/2)*
c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/(a^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2
), x)
```

3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{2}(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2Bd\cos(e+ \ldots$$

```
[Out] -(A-B)*(c-d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))
*2^(1/2)/f/a^(1/2)-2/3*(3*A*d+3*B*c-2*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1
/2)-2/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]

time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3102, 2830, 2728, 212}

$$-\frac{2(3Ad+3Bc-2Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (2*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])
/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx &= \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= -\frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}+\frac{2\int \frac{\frac{1}{2}a(3Ac+Bd)+\frac{1}{2}a(\cdots}{\sqrt{a+\cdots}}}{\cdots} \\
&= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2Bd\cos(e+fx)\sqrt{a\cdots}}{3af} \\
&= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2Bd\cos(e+fx)\sqrt{a\cdots}}{3af} \\
&= -\frac{\sqrt{2}\,(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\cdots
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 135, normalized size = 1.04

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-6-6i)(-1)^{3/4}(A-B)(c-d)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3Bc+3Ad-Bd+Bd\sin(e+fx))\right)}{3f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -1/3*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)*
(c - d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x])))/(
f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(113) = 226.
time = 7.42, size = 232, normalized size = 1.78

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3Aa^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c-3Aa^{\frac{3}{2}}\sqrt{2}\,\text{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*a^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-3*A*a^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-3*B*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c+3*B*a^(3/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-2*B*(a-a*sin(f*x+e))^(3/2)*d+6*A
*a*d*(a-a*sin(f*x+e))^(1/2)+6*B*a*c*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a
), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(119) = 238.
time = 0.53, size = 323, normalized size = 2.48

$$\frac{\frac{3\sqrt{2}\left((A-B)ac-(A-B)ad+((A-B)ac-(A-B)ad)\cos(fx+e)+((A-B)ac-(A-B)ad)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}-4\left(Bd\cos\left(fx+e\right)^2+3\,Bc+(3\,A-2\,B)d+(3\,Bc+(3\,A-B)d)\cos\left(fx+e\right)+(Bd\cos\left(fx+e\right)-3\,Bc-(3\,A-2\,B)d)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{6\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*cos
(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e)^2
- (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos
(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*d*co
s(f*x + e)^2 + 3*B*c + (3*A - 2*B)*d + (3*B*c + (3*A - B)*d)*cos(f*x + e) +
 (B*d*cos(f*x + e) - 3*B*c - (3*A - 2*B)*d)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{\sqrt{a(\sin(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1
)), x)
```

Giac [A]
time = 0.53, size = 229, normalized size = 1.76

$$\frac{\frac{3\sqrt{2}\left(A\sqrt{a}c - B\sqrt{a}c - A\sqrt{a}d + B\sqrt{a}d\right)\log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(A\sqrt{a}c - B\sqrt{a}c - A\sqrt{a}d + B\sqrt{a}d\right)\log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{4\sqrt{2}\left(2Ba^{\frac{5}{2}}d\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^3 - 3Ba^{\frac{5}{2}}c\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 3Aa^{\frac{5}{2}}d\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/6*(3*sqrt(2)*(A*sqrt(a)*c - B*sqrt(a)*c - A*sqrt(a)*d + B*sqrt(a)*d)*log(
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
 - 3*sqrt(2)*(A*sqrt(a)*c - B*sqrt(a)*c - A*sqrt(a)*d + B*sqrt(a)*d)*log(-s
in(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
- 4*sqrt(2)*(2*B*a^(5/2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a^(5/2)*c
*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^(5/2)*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e))/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\sin(e + f\,x))\,(c + d\,\sin(e + f\,x))}{\sqrt{a + a\,\sin(e + f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
 x)
```

3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/f/a^(1/2)-2*B*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2728, 212}

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{(2(A-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+}\right.}{f}$$
$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 106, normalized size = 1.34

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}(A-B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+B\left(-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A]
time = 7.88, size = 128, normalized size = 1.62

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)A-\sqrt{a}\,\sqrt{2}\,\text{a}\right.}{a\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*A-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*B+2*B*(a-a*sin(f*x+e))^(1/2))/a/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(72) = 144$.
time = 0.43, size = 228, normalized size = 2.89

$$-\frac{\frac{\sqrt{2}\left((A-B)a\cos(fx+e)+(A-B)a\sin(fx+e)+(A-B)a\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}+4\left(B\cos\left(fx+e\right)-B\sin\left(fx+e\right)+B\right)\sqrt{a\sin\left(fx+e\right)+a}}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a)
*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*
sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e
) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) +
a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin\left(e + fx\right)}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(72) = 144$.
time = 0.45, size = 149, normalized size = 1.89

$$\frac{\frac{4\sqrt{2}B\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*B*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(sqrt(a)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))) + sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt
(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

Mupad [B]
time = 1.06, size = 151, normalized size = 1.91

$$-\frac{A\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}-\frac{B\,\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{a+a\,\sin\left(e+f\,x\right)}{2\,a}}}{f\,\cos\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] - (A*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))
/(f*(a + a*sin(e + f*x))^(1/2)) - (B*(4*ellipticE(asin((2^(1/2)*(1 - sin(e
+ f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))
/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*cos(
e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}-\frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{d}\,\sqrt{c+d}\,f}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)/f/a^(1/2)-2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1
/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3064, 2728, 212, 2852, 214}

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}-\frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,\sqrt{d}\,f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])]])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)*Sqr
t[d]*Sqrt[c + d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx = \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} + \frac{(Bc-Ad)\int \frac{\sqrt{a+}}{c+}}{a(c-}$$

$$= -\frac{(2(A-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} - \frac{2(B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.26, size = 238, normalized size = 1.75

$$\frac{(-1)^{3/4}\left((2+2i)(A-B)\sqrt{d}\,\sqrt{c+d}\,\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\sqrt[4]{-1}\,(Bc-Ad)\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\,\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\,\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)\sqrt{d}\,\sqrt{c+d}\,f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] ((-1)^(3/4)*((2 + 2*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)
^(3/4)*(-1 + Tan[(e + f*x)/4])] + (-1)^(1/4)*(B*c - A*d)*(Log[Sec[(e + f*x)
/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]
- Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*
```

```
Sin[(e + f*x)/2])]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c - d)*Sqrt[d
]*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A]
time = 9.33, size = 199, normalized size = 1.46

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)\sqrt{a(c+d)d}\ A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*A-2*A*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d-2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*B+2*B*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c)/(c-d)/(a*(c+d)*d)^(1/2)/a^(
1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(117) = 234.
time = 1.04, size = 786, normalized size = 5.78

$$\left[\frac{\sqrt{acd+ad^2}\,(Bc-Ad)\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2acd-ad^2-(6acd+7ad^2)\cos(fx+e)^2-4\sqrt{acd+ad^2}\left(d\cos(fx+e)^2-(c+2d)\cos(fx+e)+(d\cos(fx+e)+c+3d)\sin(fx+e)-c-3d\right)\sqrt{a\sin(fx+e)+a}-(ac^2+8acd+9ad^2)\cos(fx+e)+\left(ad^2\cos(fx+e)^2-ac^2-2acd-ad^2+2(3acd+4ad^2)\cos(fx+e)\right)\sin(fx+e)}{d^2\cos(fx+e)^3+(2cd+d^2)\cos(fx+e)^2-c^2-2cd-d^2-(c^2+d^2)\cos(fx+e)+\left(d^2\cos(fx+e)^2-2cd\cos(fx+e)-c^2-2cd-d^2\right)\sin(fx+e)}\right)+\frac{\sqrt{2}\left((A-B)acd+(A-B)ad^2\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}}{2(ac^2d-ad^3)f},\ -\frac{2\sqrt{-acd-ad^2}\,(Bc-Ad)\arctan\left(\frac{\sqrt{-acd-ad^2}\sqrt{a\sin(fx+e)+a}\,(d\sin(fx+e)-c-2d)}{2(acd+ad^2)\cos(fx+e)}\right)-\frac{\sqrt{2}\left((A-B)acd+(A-B)ad^2\right)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}}{2(ac^2d-ad^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2
*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)
*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*si
n(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2
)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c
*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos
(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt
(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) -
2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) +
 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f), -1/2*
(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))
) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
- sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f
)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(117) = 234.
time = 0.56, size = 261, normalized size = 1.92

$$-\frac{\frac{2\sqrt{2}\left(B\sqrt{a}c-A\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(\sqrt{2}ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}-\frac{\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*sqrt(2)*(B*sqrt(a)*c - A*sqrt(a)*d)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*
f*x + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - sqrt(2)*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2))
- (A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*
```

```
a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) + (A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)/(sqrt(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),
 x)
```

3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2 f}+\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2\sqrt{d}\,(c+d)^{3/2} f}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)^2/f/a^(1/2)+(A*d*(3*c+d)-B*(c^2+c*d+2*d^2))*arctanh(cos(f*x+e)*a^(
1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)^2/(c+d)^(3/2)/f/a^(1
/2)/d^(1/2)-(-A*d+B*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x
+e))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3063, 3064, 2728, 212, 2852, 214}

$$-\frac{(Bc-Ad)\cos(e+fx)}{f\,(c^2-d^2)\,\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}\,f(c-d)^2(c+d)^{3/2}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^2*f)) + ((A*d*(3*c + d) - B*(c^2 + c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(Sqrt[a]*(c - d)^2*Sqrt[d]*(c + d)^(3/2)*f) - ((B*c - A*d)*Cos[e +
f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx = -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \cdots$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \cdots$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \cdots$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2 f} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.01, size = 374, normalized size = 1.81

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((8+8i)(-1)^{3/4}(A-B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-\frac{(-Ad(3c+d)+B(c^2+cd+2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{d}\,(c+d)^{3/2}}+\frac{(-Ad(3c+d)+B(c^2+cd+2d^2))\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{d}\,(c+d)^{3/2}}-\frac{4(c-d)(Bc-Ad)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))}\right)}{4(c-d)^2 f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) + B*
(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e +
f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2
])]))/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*d + 2*d^2))
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(Sqrt[d]*(c +
d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(
(c + d)*(c + d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(182) = 364$.
time = 12.52, size = 899, normalized size = 4.34

method	result	size
default	Expression too large to display	899

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(5/2)*(-sin(f*x+e)*d*(-3*A*a^(5/
2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d-A*a^(5/2)*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2+B*a^(5/2)*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^2+B*a^(5/2)*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d+2*B*a^(5/2)*arctanh((a-a*sin(f*x+e)
)^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+A*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d-B*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-B*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^
2*d)+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*
d+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2-B*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^3-B*arctanh(
(a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d-2*B*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+A*(a*(c+d)*d)^(1/2
)*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*c*d-A*(a*(c+d)*d)^(1/2)*a^(3/2)*(a-a*sin(f
*x+e))^(1/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^2*c^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-B*(a*(c+d)*d)^(1/2)*a^(3/2)*(a-a
*sin(f*x+e))^(1/2)*c^2+B*(a*(c+d)*d)^(1/2)*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*c
*d+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c^2+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a^2*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1
/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(189) = 378$.
time = 2.86, size = 2231, normalized size = 10.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f)*sin(f*x + e)), 1/2*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))) + sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(B*c^3*d - A*c^2*d^2 -

```
 B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (
B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^
3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*
c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x
 + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)
*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(189) = 378.
time = 0.62, size = 510, normalized size = 2.46

$$-\frac{\frac{\sqrt{2}\left(\sqrt{2}B\sqrt{a}c^2-3\sqrt{2}A\sqrt{a}cd+\sqrt{2}B\sqrt{a}cd-\sqrt{2}A\sqrt{a}d^2+2\sqrt{2}B\sqrt{a}d^2\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(ac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-ac^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-acd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+ad^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}-\frac{2\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\left(B\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-A\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sqrt{2}ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/2*(sqrt(2)*(sqrt(2)*B*sqrt(a)*c^2 - 3*sqrt(2)*A*sqrt(a)*c*d + sqrt(2)*B*
sqrt(a)*c*d - sqrt(2)*A*sqrt(a)*d^2 + 2*sqrt(2)*B*sqrt(a)*d^2)*arctan(sqrt(
2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a*c^3*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) - a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a*c*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a*d^3*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sqrt(-c*d - d^2)) - 2*(A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2
*sqrt(2)*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a*d^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))) + 2*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
 2*sqrt(2)*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*(B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2
*e) - A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a*c^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\sin\left(e+f\,x\right)}{\sqrt{a+a\sin\left(e+f\,x\right)}\,{\left(c+d\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
), x)
```

3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^3 f}+\frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3)}{4\sqrt{a}\,(c-d)^3\sqrt{d}\,(c+}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)^3/f/a^(1/2)+1/4*(A*d*(15*c^2+10*c*d+7*d^2)-B*(3*c^3+6*c^2*d+19*c*d
^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^
(1/2))/(c-d)^3/(c+d)^(5/2)/f/a^(1/2)/d^(1/2)-1/2*(-A*d+B*c)*cos(f*x+e)/(c^2
-d^2)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*(A*d*(7*c+d)-B*(3*c^2
+c*d+4*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.73, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3063, 3064, 2728, 212, 2852, 214}

$$\frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4f\,(c^2-d^2)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{(Bc-Ad)\cos(e+fx)}{2f\,(c^2-d^2)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}+\frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{a}\sqrt{d}\,f(c-d)^3(c+d)^{5/2}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^3*f)) + ((A*d*(15*c^2 + 10*c*d + 7*d^2) - B*(3
*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[a]*(c - d)^3*Sqrt[d]*(c +
d)^(5/2)*f) - ((B*c - A*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a + a*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^2) + ((A*d*(7*c + d) - B*(3*c^2 + c*d + 4*d^2)
)*Cos[e + f*x])/(4*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx = -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \cdots$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^3 f} + \frac{(Ad\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.51, size = 490, normalized size = 1.59

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((32+32i)(-1)^{3/4}(A-B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\frac{\left(Ad\left(15c^2+10cd+7d^2\right)-B\left(3c^3+6c^2d+19cd^2+4d^3\right)\right)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{d}\,(c+d)^{5/2}}+\frac{\left(-Ad\left(15c^2+10cd+7d^2\right)+B\left(3c^3+6c^2d+19cd^2+4d^3\right)\right)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{\sqrt{d}\,(c+d)^{5/2}}-\frac{8(c-d)^2(Bc-Ad)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))^2}-\frac{4(c-d)\left(-Ad(7c+d)+B\left(3c^2+cd+4d^2\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}\right)}{16(c-d)^3 f\sqrt{a(1+\sin(e+fx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((32 + 32*I)*(-1)^(3/4)*(A - B)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + ((A*d*(15*c^2 + 10*c*
d + 7*d^2) - B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(
e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f
*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]))/(Sqrt[d]*(c + d)^(5/2)) + ((-(A*d*(15
*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x
- 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[
d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(Sqrt[d]*(c + d)^(5/2))
- (8*(c - d)^2*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*
(c + d*Sin[e + f*x])^2) - (4*(c - d)*(-(A*d*(7*c + d)) + B*(3*c^2 + c*d + 4
*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x
]))))/(16*(c - d)^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. $2(276) = 552$.
time = 17.83, size = 2275, normalized size = 7.36

method	result	size
default	Expression too large to display	2275

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^3*d+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^3*d+3*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d^3-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^5-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^2*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^3*d-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*d^4+4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*d^4+4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^2*d^2-B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3+15*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c^2*d^3+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c*d^4-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c^3*d^2-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4+4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4+30*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3+14*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4-6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^4*d-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2-38*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3-8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4+9*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d-A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^2*d^2-9*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3+B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d+B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^2*d^2-6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c^2*d^3-19*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c*d^4-7*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^2*d^2+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d^3+3*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^3*d-2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^2*d^2+8*B*(a*(c+d)*

```
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin
(f*x+e)*a^4*c*d^3-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c^2*d^2-8*A*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2
*a^4*c*d^3+4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c^2*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c*
d^3-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)*a^4*c^3*d-16*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^2*d^2-8*A*(a
*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)*a^4*c*d^3-4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d
)^(1/2))*a^(9/2)*c^2*d^3+A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7
/2)*d^4-5*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^4+7*A*arc
tanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*d^
5-4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*
x+e)^2*d^5+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4+A*(-
a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*d^4-4*B*(-a*(sin(f*x+e)-1
))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*d^4+15*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^4*d+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^3*d^2+7*A*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^2*d^3-6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^4*d-19*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^3*d^2+16*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^2*d^2)*(-
a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/a^(9/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f
*x+e))^2/(c+d)^2/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. $2(286) = 572$.
time = 4.77, size = 4268, normalized size = 13.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")

[Out] [1/16*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A -
8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*
A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 -
(6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (35*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c
*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2
*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A
 - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B
)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 -
 (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d
^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c
^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)
*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*
cos(f*x + e)^2 + 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*
x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x
+ e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 8*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*
c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 +
 (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d
^5 + (A - B)*a*d^6)*cos(f*x + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3
*d^3 + 9*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e
)^2 + ((A - B)*a*c^5*d + 3*(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A -
 B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*
a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4
 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2
*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4
*d^2 + 3*(A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x
 + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e)
 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqr
t(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e
) - cos(f*x + e) - 2))/sqrt(a) + 4*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*
A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6
+ (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A
 - 4*B)*d^6)*cos(f*x + e)^2 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^
3*d^3 + (11*A - B)*c^2*d^4 + (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B
*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4
- (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A
 - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9
)*f*cos(f*x + e)^3 + (2*a*c^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 +
```

```
 6*a*c^3*d^6 + 3*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d
 - 2*a*c^6*d^3 + 2*a*c^2*d^7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d
^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*
d^9)*f + ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2
- 2*(a*c^7*d^2 - 3*a*c^5*d^4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c
^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f)*sin(f*x + e)), 1/8*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d
- 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (
7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4
 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (3
5*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2
 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)
*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 -
 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*
(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3
- (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*
(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x +
e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt
(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x +
e))) - 4*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*
d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a
*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x
 + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3*d^3 + 9*(A - B)*a*c^2*d^4
+ 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + ((A - B)*a*c^5*d + 3*
(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(286) = 572.
time = 0.74, size = 909, normalized size = 2.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/8*(sqrt(2)*(3*sqrt(2)*B*sqrt(a)*c^3 - 15*sqrt(2)*A*sqrt(a)*c^2*d + 6*sqr
t(2)*B*sqrt(a)*c^2*d - 10*sqrt(2)*A*sqrt(a)*c*d^2 + 19*sqrt(2)*B*sqrt(a)*c*
d^2 - 7*sqrt(2)*A*sqrt(a)*d^3 + 4*sqrt(2)*B*sqrt(a)*d^3)*arctan(sqrt(2)*d*s
in(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a*c^5*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) - a*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a*c^3*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*a*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + a*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a*d^5*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - 8*(A*sqrt(a) - B*sqrt(a))*
log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^3*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) - 3*sqrt(2)*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*sqrt(2)*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 8*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 - 3*sqrt(2)*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a*c*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) + 4*(6*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 1
4*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 2*B*sqrt(a)*c*d^2*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^3 - 2*A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^3 + 8*B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*B*sqrt(a)*c^3*s
in(-1/4*pi + 1/2*f*x + 1/2*e) + 9*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e) - 4*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 8*A*sqrt(a)*c*d^
2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x
+ 1/2*e) - A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*sqrt(a)*d^3*s
in(-1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a*c^4*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - 2*sqrt(2)*a*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)
*a*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^2 - c - d)^2))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
), x)
```

3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{(c-d)^2(3B(c-5d)+A(c+11d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d(15Ac^2-99Bc^2-120Acd+16}{15af\sqrt{a+}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
^2*(3*B*(c-5*d)+A*(c+11*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin
(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/15*d*(15*A*c^2-120*A*c*d+65*A*d^2-99*B*
c^2+168*B*c*d-93*B*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+1/10*(5*A-9*B
)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^(1/2)+1/30*d^2*(15*A
*c-35*A*d-51*B*c+39*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]
time = 0.67, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3056, 3062, 3047, 3102, 2830, 2728, 212}

$$-\frac{(c-d)^2(A(c+11d)+3B(c-5d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d^2(15Ac-35Ad-51Bc+39Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{30a^2f}+\frac{d(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2)\cos(e+fx)}{15af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(5A-9B)\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/2*((c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x
])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + (d*(15*A*c^2
- 99*B*c^2 - 120*A*c*d + 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(15
*a*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*C
os[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(30*a^2*f) + ((5*A - 9*B)*d*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(10*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^3)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx)}{}}{}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos}{2f(}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos}{2f(}$$

$$= \frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e}}{30a^2f}$$

$$= \frac{d(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2)\,c}{15af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{d(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2)\,c}{15af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(c-d)^2(3B(c-5d)+A(c+11d))\tanh^{-1}\left(\frac{\sqrt{a}\,\mathrm{co}}{\sqrt{2}\sqrt{a+a}}\right.}{2\sqrt{2}\,a^{3/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 684, normalized size = 2.42

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*c
^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e + f*
x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 110*A
*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3*(e
+ f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e + f*x
))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2] + 30
*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*d^3*Cos
```

```
[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e + f*x)/2
] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*B*c^2*d*S
in[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[(e + f*x)/
2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (30 + 30*I)*
(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*x))/2] + 210*B
*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2] - 123*B*d^3*Sin
[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*d^3*Sin[(5*(e +
f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(e + f*x))/2]))/(6
0*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(256) = 512$.
time = 9.93, size = 1031, normalized size = 3.64

method	result	size
default	Expression too large to display	1031

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/60*(-sin(f*x+e)*(40*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*d^3-360*a^(5/2)*A*c
*d^2*(a-a*sin(f*x+e))^(1/2)+120*a^(5/2)*A*d^3*(a-a*sin(f*x+e))^(1/2)-24*B*d
^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)+120*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^
2-360*B*a^(5/2)*c^2*d*(a-a*sin(f*x+e))^(1/2)+360*B*a^(5/2)*c*d^2*(a-a*sin(f
*x+e))^(1/2)-240*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)-15*A*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-135*A*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+315*A*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-165*A*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3-45*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+315*B*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-495*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+225*B*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)-40*A*(a-a*sin(f*x+e))^
(3/2)*a^(3/2)*d^3+30*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c^3-90*A*(a-a*sin(f*x
+e))^(1/2)*a^(5/2)*c^2*d+450*a^(5/2)*A*c*d^2*(a-a*sin(f*x+e))^(1/2)-150*a^(
5/2)*A*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-1
20*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^2-30*B*(a-a*sin(f*x+e))^(1/2)*a^(5/
2)*c^3+450*B*a^(5/2)*c^2*d*(a-a*sin(f*x+e))^(1/2)-450*B*a^(5/2)*c*d^2*(a-a*
sin(f*x+e))^(1/2)+270*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*ar
```

```
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)*(-a*(sin(f*x+e)-1
))^(1/2)/a^(9/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(268) = 536$.
time = 0.47, size = 812, normalized size = 2.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/120*(15*sqrt(2)*(2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c
*d^2 + 2*(11*A - 15*B)*d^3 - ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A
- 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 + 3*(3*A
 - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e) + (2
*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c*d^2 + 2*(11*A - 15*
B)*d^3 + ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*
A - 15*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(12*B*d^
3*cos(f*x + e)^4 - 15*(A - B)*c^3 + 45*(A - B)*c^2*d - 45*(A - B)*c*d^2 + 1
5*(A - B)*d^3 + 4*(15*B*c*d^2 + (5*A - 3*B)*d^3)*cos(f*x + e)^3 - 4*(45*B*c
^2*d + 15*(3*A - 4*B)*c*d^2 - 4*(5*A - 9*B)*d^3)*cos(f*x + e)^2 - 15*((A -
B)*c^3 - 3*(A - 5*B)*c^2*d + 15*(A - B)*c*d^2 - (5*A - 9*B)*d^3)*cos(f*x +
e) + (12*B*d^3*cos(f*x + e)^3 + 15*(A - B)*c^3 - 45*(A - B)*c^2*d + 45*(A -
 B)*c*d^2 - 15*(A - B)*d^3 - 4*(15*B*c*d^2 + (5*A - 6*B)*d^3)*cos(f*x + e)^
2 - 60*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x
```

```
+ e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
- 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(268) = 536.
time = 0.62, size = 673, normalized size = 2.38

$$\frac{\frac{15\sqrt{2}\left(A\sqrt{a}c^3+3B\sqrt{a}c^3+9A\sqrt{a}c^2d-21B\sqrt{a}c^2d-21A\sqrt{a}cd^2+33B\sqrt{a}cd^2+11A\sqrt{a}d^3-15B\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{15\sqrt{2}\left(A\sqrt{a}c^3+3B\sqrt{a}c^3+9A\sqrt{a}c^2d-21B\sqrt{a}c^2d-21A\sqrt{a}cd^2+33B\sqrt{a}cd^2+11A\sqrt{a}d^3-15B\sqrt{a}d^3\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{30\sqrt{2}\left(A\sqrt{a}c^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-B\sqrt{a}c^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3A\sqrt{a}c^2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+3B\sqrt{a}c^2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+3A\sqrt{a}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3B\sqrt{a}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-A\sqrt{a}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+B\sqrt{a}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{16\sqrt{2}\left(12Ba^{\frac{17}{2}}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5-30Ba^{\frac{17}{2}}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-10Aa^{\frac{17}{2}}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+45Ba^{\frac{17}{2}}c^2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+45Aa^{\frac{17}{2}}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-45Ba^{\frac{17}{2}}cd^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-15Aa^{\frac{17}{2}}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+30Ba^{\frac{17}{2}}d^3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
[Out] 1/120*(15*sqrt(2)*(A*sqrt(a)*c^3 + 3*B*sqrt(a)*c^3 + 9*A*sqrt(a)*c^2*d - 21
*B*sqrt(a)*c^2*d - 21*A*sqrt(a)*c*d^2 + 33*B*sqrt(a)*c*d^2 + 11*A*sqrt(a)*d
^3 - 15*B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(a)*c^3 + 3*B*sqrt(a)*c^3
 + 9*A*sqrt(a)*c^2*d - 21*B*sqrt(a)*c^2*d - 21*A*sqrt(a)*c*d^2 + 33*B*sqrt(
a)*c*d^2 + 11*A*sqrt(a)*d^3 - 15*B*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 30*sqrt(2)*(A*sqr
t(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f
*x + 1/2*e) - 3*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*B*sqrt(a
)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/
2*f*x + 1/2*e) - 3*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*sqrt(
a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))) + 16*sqrt(2)*(12*B*a^(17/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^5 - 30*B*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^(17/
2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 45*B*a^(17/2)*c^2*d*sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 45*A*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 45
*B*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 15*A*a^(17/2)*d^3*sin(-1
/4*pi + 1/2*f*x + 1/2*e) + 30*B*a^(17/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/(a^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2
), x)
```

3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d(3Ac-15Bc-9Ad+13Bd)\,\mathrm{c}}{3af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
*(A*c+7*A*d+3*B*c-11*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/3*d*(3*A*c-9*A*d-15*B*c+13*B*d)*cos(f*x+e
)/a/f/(a+a*sin(f*x+e))^(1/2)+1/6*(3*A-7*B)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/a^2/f
```

Rubi [A]
time = 0.39, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3056, 3047, 3102, 2830, 2728, 212}

$$-\frac{(c-d)(Ac+7Ad+3Bc-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{3af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/2*((c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])
/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) + (d*(3*A*c - 15*
B*c - 9*A*d + 13*B*d)*Cos[e + f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*
A - 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((A - B)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+f}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{2}ac(Ac+3Bc}{}$$

$$= \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)}{}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)}{}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)}{}$$

$$= -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\,c}{\sqrt{2}\sqrt{a+a}}\right.}{2\sqrt{2}\,a^{3/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 357, normalized size = 1.76

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(3+3i)(-1)^{3/4}(c-d)(Ac+3Bc+7Ad-11Bd)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+6d(-4Bc-2Ad+3Bd)\cos\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2Bd^2\cos\left(\frac{3}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-6d(-4Bc-2Ad+3Bd)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2Bd^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sin\left(\frac{3}{2}(e+fx)\right)\right)}{6f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(
-1)^(3/4)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(
3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 6*d
*(-4*B*c - 2*A*d + 3*B*d)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^2 - 2*B*d^2*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - 6*d*(-4*B*c - 2*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - 2*B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[(3*(
e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(180) = 360$.
time = 7.08, size = 694, normalized size = 3.42

method	result
default	$-\frac{\left(\sin(fx+e)\left(3A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}c^{2}+18A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/12/a^(7/2)*(sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*d^2+24*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)+48*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-24*B*d^2*a^(3/2)*(
a-a*sin(f*x+e))^(1/2))+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^2+6*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-12*A*(a-a*sin(f*x+e)
)^(1/2)*a^(3/2)*c*d+30*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)-6*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2+60*B*c*d*a^(3/2)*(a
-a*sin(f*x+e))^(1/2)-30*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))*(-a*(sin(f*x+
e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(189) = 378$.

time = 0.46, size = 610, normalized size = 3.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2
- ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e)^2 + (
(A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e) + (2*(A
+ 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2 + ((A + 3*B)*c^2 + 2*(3
*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) + 4*(4*B*d^2*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
 B)*d^2 - 4*(6*B*c*d + (3*A - 4*B)*d^2)*cos(f*x + e)^2 - 3*((A - B)*c^2 - 2
*(A - 5*B)*c*d + 5*(A - B)*d^2)*cos(f*x + e) - (4*B*d^2*cos(f*x + e)^2 - 3*
(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 + 12*(2*B*c*d + (A - B)*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 -
 a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1)
)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(189) = 378.
time = 0.61, size = 478, normalized size = 2.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/24*(3*sqrt(2)*(A*sqrt(a)*c^2 + 3*B*sqrt(a)*c^2 + 6*A*sqrt(a)*c*d - 14*B*s
qrt(a)*c*d - 7*A*sqrt(a)*d^2 + 11*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt
(a)*c^2 + 3*B*sqrt(a)*c^2 + 6*A*sqrt(a)*c*d - 14*B*sqrt(a)*c*d - 7*A*sqrt(a
)*d^2 + 11*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 6*sqrt(2)*(A*sqrt(a)*c^2*sin(-1/4*pi +
1/2*f*x + 1/2*e) - B*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*A*sqrt(
a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 2*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f
*x + 1/2*e) + A*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*d^2*
sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 16*sqrt(2)*(2*B*a^(9/2)*d^2*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 - 6*B*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)
- 3*A*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*B*a^(9/2)*d^2*sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2
), x)
```

3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{(Ac+3Bc+3Ad-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}-\frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*c+3*A*d+3*B*c-7
*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2
)/f*2^(1/2)-2*B*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3098, 2830, 2728, 212}

$$-\frac{(Ac+3Ad+3Bc-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}-\frac{2Bd\cos(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -1/2*((A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]
*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((A - B)*(c - d)*Cos[e +
 f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)) - (2*B*d*Cos[e + f*x])/(a*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3B(c-d)+A(c+3d))-2aBd\ldots}{\sqrt{a+a\sin(e+fx\ldots}}}{2a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} + \ldots$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \ldots$$

$$= -\frac{(Ac+3Bc+3Ad-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+f\ldots}}\right.}{2\sqrt{2}\,a^{3/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 246, normalized size = 1.85

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\tfrac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(Ac+3Bc+3Ad-7Bd)\tanh^{-1}\left(\left(\tfrac{1}{2}+\tfrac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\tfrac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2-4Bd\cos\left(\tfrac{1}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2+4Bd\sin\left(\tfrac{1}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2\right)}{2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4
)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f*x)/
2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(116) = 232$.
time = 6.89, size = 389, normalized size = 2.92

method	result
default	$-\frac{\left(\sin(fx+e)\left(A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\ \sqrt{2}}{2\sqrt{a}}\right)ac+3A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\ \sqrt{2}}{2\sqrt{a}}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+8*B*
(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d-2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+10*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3
/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(122) = 244$.
time = 0.42, size = 431, normalized size = 3.24

$$-\frac{\sqrt{2}\left(\left((A+3B)c+(3A-7B)d\right)\cos\left(fx+e\right)^{2}-2\left(A+3B\right)c-2\left(3A-7B\right)d-\left((A+3B)c+(3A-7B)d\right)\cos\left(fx+e\right)-\left(2\left(A+3B\right)c+2\left(3A-7B\right)d+\left((A+3B)c+(3A-7B)d\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^{2}+2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(4Bd\cos\left(fx+e\right)^{2}+(A-B)c-(A-B)d+\left((A-B)c-(A-5B)d\right)\cos\left(fx+e\right)+\left(4Bd\cos\left(fx+e\right)-(A-B)c+(A-B)d\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{8\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-2a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+2a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c
 - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A + 3
*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*sin(f
*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
 + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c - (A - B
)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e) - (A - B
)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\ (c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
 x)
```

3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A+3*B)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2829, 2728, 212}

$$-\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f
*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{(A+3B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A+3B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+}}\right.}{2af}$$

$$= -\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} - \frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.15, size = 150, normalized size = 1.72

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(A+3B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)}{2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A +
B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(72) = 144$.
time = 4.80, size = 176, normalized size = 2.02

method	result
default	$-\frac{\left(\sin(fx+e)\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a(A+3B)+A\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*2^(1/2)*a*(A+3*B)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
```

```
(1/2))*a+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*A-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*B)*(-a*
(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(76) = 152.
time = 0.42, size = 315, normalized size = 3.62

$$\frac{\sqrt{2}\,\left((A+3\,B)\cos\left(fx+e\right)^2-(A+3\,B)\cos\left(fx+e\right)-((A+3\,B)\cos\left(fx+e\right)+2\,A+6\,B)\sin\left(fx+e\right)-2\,A-6\,B\right)\sqrt{a}\,\log\left(-\frac{a\cos(fx+e)^2-2\,\sqrt{2}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3\,a\cos(fx+e)-(a\cos(fx+e)-2\,a)\sin(fx+e)+2\,a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\,((A-B)\cos\left(fx+e\right)-(A-B)\sin\left(fx+e\right)+A-B)\sqrt{a\sin\left(fx+e\right)+a}}{8\,\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-(a^2f\cos\left(fx+e\right)+2\,a^2f)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*B
)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin\left(e + fx\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(76) = 152.
time = 0.49, size = 192, normalized size = 2.21

$$\frac{\frac{\sqrt{2}\left(A\sqrt{a}+3\,B\sqrt{a}\right)\log(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))}-\frac{\sqrt{2}\left(A\sqrt{a}+3\,B\sqrt{a}\right)\log(-\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))}-\frac{2\,\sqrt{2}\left(A\sqrt{a}\,\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)-B\sqrt{a}\,\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)\right)}{\left(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-1\right)a^2\mathrm{sgn}(\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(A*sqrt(a) + 3*B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(a) + 3*B*sq
rt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))) - 2*sqrt(2)*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sq
rt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=187

$$-\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^2f}+\frac{2\sqrt{d}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos}{\sqrt{c+d}\,\sqrt{a+a\,s}}\right.}{a^{3/2}(c-d)^2\sqrt{c+d}\,f}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*(c-5*d)+B*(3*c+
d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/
(c-d)^2/f*2^(1/2)+2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/
2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^2/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3057, 3064, 2728, 212, 2852, 214}

$$-\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^2}+\frac{2\sqrt{d}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -1/2*((A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^2*f) + (2*Sqrt[d]*(B*c
- A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e
 + f*x]])])/(a^(3/2)*(c - d)^2*Sqrt[c + d]*f) - ((A - B)*Cos[e + f*x])/(2*(
c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3Bc+A(c-4d))-\frac{1}{2}a(}{\sqrt{a+a\sin(e+fx)}}}{2a^2(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(d(Bc-Ad))\int \frac{\sqrt{a+}}{c}}{a^2(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(2d(Bc-Ad))\text{Subst}\Big(}{}$$

$$= -\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+f}}\right.}{2\sqrt{2}a^{3/2}(c-d)^2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.22, size = 419, normalized size = 2.24

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(A(c-5d)+B(3c+d))\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{\sqrt{d}(Bc-Ad)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{\sqrt{c+d}}+\frac{\sqrt{d}(-Bc+Ad)\left(e+fx-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{\sqrt{c+d}}\right)}{2(c-d)^2f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/
4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A*d)*
(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d
] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-(B*c) + A*d)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*
Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(158) = 316$.
time = 9.64, size = 624, normalized size = 3.34

method	result

default	$-\frac{\left(\sin(fx+e)\left(8A\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)a^{\frac{3}{2}}d^2-8B\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+a\,d^2}}\right)a^{\frac{3}{2}}cd+A\sqrt{a}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(
c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a*d+2*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a*(
c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d-2*B*(a*(c+d)*d)^(1/2)*(a-a*s
in(f*x+e))^(1/2)*a^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(
1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(164) = 328$.
time = 2.07, size = 1633, normalized size = 8.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c +
 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*c
- 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e))*s
qrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*
(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*
a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x + e)^2
+ (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*d)*cos(f
*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
 + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^
2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
 + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)
*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x +
e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d
^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c
*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*
d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2
*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^
2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*
d^2)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x +
e)^2 - 2*(A + 3*B)*c + 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
+ e) - (2*(A + 3*B)*c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
 + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
- (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(2*B*a*c - 2*A*a*d - (B*a*c -
A*a*d)*cos(f*x + e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d +
 (B*a*c - A*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/
2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/
(d*cos(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*d)*cos(
f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/
((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d +
a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 -
 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)
*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(164) = 328.
time = 0.60, size = 488, normalized size = 2.61

$$\frac{\frac{8\sqrt{2}\left(Bcd-Ad^2\right)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(\sqrt{2}\,a^{\frac{3}{2}}c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}\,a^{\frac{3}{2}}cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^{\frac{3}{2}}d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}+\frac{\left(A\sqrt{a}\,c+3B\sqrt{a}\,c-5A\sqrt{a}\,d+B\sqrt{a}\,d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\,a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\left(A\sqrt{a}\,c+3B\sqrt{a}\,c-5A\sqrt{a}\,d+B\sqrt{a}\,d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\,a^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{2}\,a^2cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^2d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(A\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-B\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sqrt{2}\,a^2c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}\,a^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] 1/4*(8*sqrt(2)*(B*c*d - A*d^2)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^(3/2)*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 2*sqrt(2)*a^(3/2)*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*
a^(3/2)*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (A*sqr
t(a)*c + 3*B*sqrt(a)*c - 5*A*sqrt(a)*d + B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2
*sqrt(2)*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*c + 3*B*sqrt(a)*c - 5*A*sqrt(
a)*d + B*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
- 2*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*sin(-1/4*pi + 1/2
*f*x + 1/2*e))/((sqrt(2)*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2
)*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),
 x)
```

3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=292

$$-\frac{(Ac+3Bc-9Ad+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^3f}-\frac{\sqrt{d}\,(Ad(5c+3d)-B(3c^2+3cd+2d^2))\,\mathrm{t}}{a^{3/2}(c-d)^3(}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/4*(
A*c-9*A*d+3*B*c+5*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+
e))^(1/2))/a^(3/2)/(c-d)^3/f*2^(1/2)-(A*d*(5*c+3*d)-B*(3*c^2+3*c*d+2*d^2))*
arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1
/2)/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f+1/2*d*(B*(3*c+d)-A*(c+3*d))*cos(f*x+e)/a/
(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{\sqrt{d}\,(Ad(5c+3d)-B(3c^2+3cd+2d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(Ac-9Ad+3Bc+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^3}+\frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -1/2*((A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]
*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^3*f) - (Sqrt[d]*(A*d*
(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f
*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^3*(c + d)^(3
/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c
+ d*Sin[e + f*x])) + (d*(B*(3*c + d) - A*(c + 3*d))*Cos[e + f*x])/(2*a*(c -
 d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \ldots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \ldots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \ldots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \ldots$$

$$= -\frac{(Ac+3Bc-9Ad+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+\ldots}}\right.}{2\sqrt{2}\,a^{3/2}(c-d)^3 f}\ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.70, size = 542, normalized size = 1.86

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(4(A-B)(c-d)\sin\left(\tfrac{1}{2}(e+fx)\right)+2(-A+B)(c-d)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)+(2+2i)(-1)^{3/4}(Ac+3Bc-9Ad+5Bd)\tanh^{-1}\left(\left(\tfrac{1}{2}+\tfrac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\tfrac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2+\frac{\sqrt{d}\left(-Ad(5c+3d)+B(3c^2+3cd+2d^2)\right)\left(e+fx-2\log\left(\sec^2\left(\tfrac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\tfrac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\tfrac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2}{(c+d)^{3/2}}+\frac{\sqrt{d}\left(Ad(5c+3d)-B(3c^2+3cd+2d^2)\right)\left(e+fx-2\log\left(\sec^2\left(\tfrac{1}{4}(e+fx)\right)\right)+2\log\left(\sec^2\left(\tfrac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\tfrac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2}{(c+d)^{3/2}}+\frac{4(c-d)d(Bc-Ad)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2}{(c+d)(c+d\sin(e+fx))}\right)}{4(c-d)^3 f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-1)
^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-(A*d
*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - S
qrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)
^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*d)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*
Sin[e + f*x]))))/(4*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. $2(259) = 518$.

time = 14.53, size = 2049, normalized size = 7.02

method	result	size
default	Expression too large to display	2049

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)*(-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*d^3+11*B*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^2*d+13*
B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a
^(1/2))*sin(f*x+e)*a*c*d^2-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(s
in(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2+3*B*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x
+e)^2*a*c^2*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2-7
*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*sin(f*x+e)*a*c^2*d-17*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2-6*A*(-a*(sin(f*x+
e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^3+20*A*arctanh((-a*(sin
(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*c*d^3-12*B*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*c^2
*d^2-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*si
n(f*x+e)^2*c*d^3+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*a^(3/2)*sin(f*x+e)*c^2*d^2+32*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c*d^3-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^3*d-24*B*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^2*d^2-20*B*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c*d^3+2*B*
(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^3-4*A*(-a*
(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^3+12*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3-12*B*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^3*d-12*B*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2-8*B*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+20*A*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2+12*A*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*d^4-2*B*(-a*(sin(f*x+
e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3-8*B*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*d^4+12*A*arctanh((-a*(sin(f*x
```

```
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4-8*B*arctanh((-a
*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4+2*A*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3+3*B*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*
a*c^3+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)*a*d^3+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(1/2)*sin(f*x+e)*c^2*d-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d+4*A*(-a*(sin(f*x+e)-1))^(1
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^2-9*A*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*d^3
-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2
)/a^(1/2))*a*c*d^2-6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*
sin(f*x+e)*c^2*d+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*si
n(f*x+e)*c*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2+5*B*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a
*d^3+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)*a*c^3+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^3+2*A*(-a*(sin(f*x+e)-1))^(1/2)
*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^2+6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(1/2)*c*d^2-4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)
*c^2*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^
(1/2)/a^(1/2))*a*c^3)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(
f*x+e))/(c+d)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(269) = 538$.
time = 4.60, size = 3491, normalized size = 11.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d
^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d
^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c
*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^
2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^
3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A
+ 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*
B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin
(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
 - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(6*B*a*c^3 - 2*(5*A - 6*B)*a*
c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A -
 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B
)*a*c^2*d - (13*A - 8*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3
*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*c
os(f*x + e) + (6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^
3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2
- (3*A - 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d
^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*
((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*c
os(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(
f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(
f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2
- c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*
d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c^3 - (A -
B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2
 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A + 3*B)*c*d
^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 +
 (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 +
 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2
*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d
 - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*
(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)
*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2
- (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^
5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3
+ a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*
(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)
*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3
 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x +
 e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A -
```

```
5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A
- 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9
*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A
 - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) - 4*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(
3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*
cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8
*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A -
 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d
- (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5
*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*
sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*((A - B)*
c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A
 + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A
 + 3*B)*c*d^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A -
 B)*c*d^2 + (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(269) = 538.
time = 0.72, size = 898, normalized size = 3.08

$$\frac{\frac{2\sqrt{2}\left(3\sqrt{2}B\sqrt{a}c^2d-5\sqrt{2}A\sqrt{a}cd^2+3\sqrt{2}B\sqrt{a}cd^2-3\sqrt{2}A\sqrt{a}d^3+2\sqrt{2}B\sqrt{a}d^3\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(a^2c^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2a^2c^3d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2a^2cd^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-a^2d^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}+\frac{\left(A\sqrt{a}c+3B\sqrt{a}c-9A\sqrt{a}d+5B\sqrt{a}d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}a^2c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\sqrt{2}a^2c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\sqrt{2}a^2cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}a^2d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\left(A\sqrt{a}c+3B\sqrt{a}c-9A\sqrt{a}d+5B\sqrt{a}d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}a^2c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\sqrt{2}a^2c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\sqrt{2}a^2cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}a^2d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(2A\sqrt{a}cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-6B\sqrt{a}cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+6A\sqrt{a}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-2B\sqrt{a}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-A\sqrt{a}c^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+B\sqrt{a}c^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-2A\sqrt{a}cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+6B\sqrt{a}cd\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-5A\sqrt{a}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+B\sqrt{a}d^2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sqrt{2}a^2c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}a^2c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}a^2cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}a^2d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-3d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+c+d\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(3*sqrt(2)*B*sqrt(a)*c^2*d - 5*sqrt(2)*A*sqrt(a)*c*d^2 + 3*s
qrt(2)*B*sqrt(a)*c*d^2 - 3*sqrt(2)*A*sqrt(a)*d^3 + 2*sqrt(2)*B*sqrt(a)*d^3)
*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^2*c^
```

```
4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 2*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*d^4*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (A*sqrt(a)*c + 3*B*s
qrt(a)*c - 9*A*sqrt(a)*d + 5*B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^2
*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^2*c*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))) - (A*sqrt(a)*c + 3*B*sqrt(a)*c - 9*A*sqrt(a)*d + 5*B*sqrt(a)*d)*log(-
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) - 3*sqrt(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*sqrt(2)*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(2*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 - 6*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*A*sqrt
(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 2*B*sqrt(a)*d^2*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)
*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x
 + 1/2*e) + 6*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*sqrt(a)*d^
2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/
2*e))/((sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*c*d^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
2 - 3*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + c + d)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\sin\left(e + f\,x\right)}{{\left(a + a\,\sin\left(e + f\,x\right)\right)}^{3/2}\,{\left(c + d\,\sin\left(e + f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
), x)
```

3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{(A(c-13d)+3B(c+3d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4f}-\frac{\sqrt{d}\left(Ad(35c^2+42cd+19d^2)-3B(5c^3\right.}{}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/4
*(A*(c-13*d)+3*B*(c+3*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1/2)-1/4*(A*d*(35*c^2+42*c*d+19*d^2)-3*B
*(5*c^3+10*c^2*d+13*c*d^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^
(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f+1/2*d*(
B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*s
in(f*x+e))^(1/2)+1/4*d*(3*B*(3*c^2+3*c*d+2*d^2)-A*(2*c^2+15*c*d+7*d^2))*cos
(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.07, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{\sqrt{d}\left(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(A(c-13d)+3B(c+3d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^4}+\frac{d(3B(3c^2+3cd+2d^2)-A(2c^2+15cd+7d^2))\cos(e+fx)}{4af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] -1/2*((A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2
]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(3/2)*(c - d)^4*f) - (Sqrt[d]*(A*d
*(35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*Ar
cTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])
])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d
)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + (d*(B*(2*c + d) -
A*(c + 2*d))*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^2) + (d*(3*B*(3*c^2 + 3*c*d + 2*d^2) - A*(2*c^2 + 15
*c*d + 7*d^2))*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e +
f*x]]*(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \cdots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \cdots$$

$$= -\frac{(A(c-13d)+3B(c+3d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e \cdots}}\right.}{2\sqrt{2}\,a^{3/2}(c-d)^4 f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.24, size = 1395, normalized size = 3.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^3),x]
```

```
[Out] ((1 + I)*(A*c + 3*B*c - 13*A*d + 9*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[
(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)/((2*(-1)^(1/4)*c^4 - 8*(-1)^(1/4)*c^3*d + 12*(-1)^(1/4)*c
^2*d^2 - 8*(-1)^(1/4)*c*d^3 + 2*(-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(3
/2)) + (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d
```

```
+ 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e +
f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2
]])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(c + d)^(5/2)*f
*(a*(1 + Sin[e + f*x]))^(3/2)) - (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)
) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*
x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2]
 + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16
*(c - d)^4*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) + ((Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-8*A*c^4*Cos[(e + f*x)/2] + 8*B*c^4*Cos[(e + f*x)/2
] - 8*A*c^3*d*Cos[(e + f*x)/2] + 26*B*c^3*d*Cos[(e + f*x)/2] - 22*A*c^2*d^2
*Cos[(e + f*x)/2] + 6*B*c^2*d^2*Cos[(e + f*x)/2] - 10*A*c*d^3*Cos[(e + f*x)
/2] + 4*B*c*d^3*Cos[(e + f*x)/2] + 4*B*d^4*Cos[(e + f*x)/2] - 8*A*c^3*d*Cos
[(3*(e + f*x))/2] + 26*B*c^3*d*Cos[(3*(e + f*x))/2] - 40*A*c^2*d^2*Cos[(3*(
e + f*x))/2] + 31*B*c^2*d^2*Cos[(3*(e + f*x))/2] - 25*A*c*d^3*Cos[(3*(e + f
*x))/2] + 13*B*c*d^3*Cos[(3*(e + f*x))/2] + A*d^4*Cos[(3*(e + f*x))/2] + 2*
B*d^4*Cos[(3*(e + f*x))/2] + 2*A*c^2*d^2*Cos[(5*(e + f*x))/2] - 9*B*c^2*d^2
*Cos[(5*(e + f*x))/2] + 15*A*c*d^3*Cos[(5*(e + f*x))/2] - 9*B*c*d^3*Cos[(5*
(e + f*x))/2] + 7*A*d^4*Cos[(5*(e + f*x))/2] - 6*B*d^4*Cos[(5*(e + f*x))/2]
 + 8*A*c^4*Sin[(e + f*x)/2] - 8*B*c^4*Sin[(e + f*x)/2] + 8*A*c^3*d*Sin[(e +
 f*x)/2] - 26*B*c^3*d*Sin[(e + f*x)/2] + 22*A*c^2*d^2*Sin[(e + f*x)/2] - 6*
B*c^2*d^2*Sin[(e + f*x)/2] + 10*A*c*d^3*Sin[(e + f*x)/2] - 4*B*c*d^3*Sin[(e
 + f*x)/2] - 4*B*d^4*Sin[(e + f*x)/2] - 8*A*c^3*d*Sin[(3*(e + f*x))/2] + 26
*B*c^3*d*Sin[(3*(e + f*x))/2] - 40*A*c^2*d^2*Sin[(3*(e + f*x))/2] + 31*B*c^
2*d^2*Sin[(3*(e + f*x))/2] - 25*A*c*d^3*Sin[(3*(e + f*x))/2] + 13*B*c*d^3*S
in[(3*(e + f*x))/2] + A*d^4*Sin[(3*(e + f*x))/2] + 2*B*d^4*Sin[(3*(e + f*x)
)/2] - 2*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 9*B*c^2*d^2*Sin[(5*(e + f*x))/2]
- 15*A*c*d^3*Sin[(5*(e + f*x))/2] + 9*B*c*d^3*Sin[(5*(e + f*x))/2] - 7*A*d^
4*Sin[(5*(e + f*x))/2] + 6*B*d^4*Sin[(5*(e + f*x))/2]))/(16*(c - d)^3*(c +
d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c + d*Sin[e + f*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4706 vs. $2(363) = 726$.

time = 21.27, size = 4707, normalized size = 11.71

method	result	size
default	Expression too large to display	4707

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4*(-a*(sin(f*x+e)-1))^(1/2)*(3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^5-24*B*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^5-3*A*(-a*(sin(f*
x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^5+4*B*(-a*(sin(f*x+e)
```

-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^5+2*A*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+
d)*d)^(1/2)*a^(1/2)*c*d^4+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^5+7*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*
(c+d)*d)^(1/2)*a^(1/2)*c^3*d^2-2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1
/2)*a^(1/2)*c^2*d^3-B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c
*d^4-13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^4-11*B*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d-15*B*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^3*d^3-30*B*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3
*c^2*d^4-39*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2
)*sin(f*x+e)^3*c*d^5-99*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
(-a(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^5+9*B*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)^3*a^2*d^5-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5-11*A*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d-2
5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^3*d^2-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3+15*B*(a*(c+d)*d)^(1/2)*2^(1/2)
arctanh(1/2(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d+6*B*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))
*sin(f*x+e)^2*a^2*c^4*d-25*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin
(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4+3*B*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x
+e)^3*a^2*c^3*d^2-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d-47*A*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*
c^3*d^2-63*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2
)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^3-26*A*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4+2
1*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)*a^2*c^4*d+18*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4+57*B*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)^2*a^2*c^2*d^3+39*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin
(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^4+2*A*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x
+e)^2*a^2*c^4*d-21*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-
1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-61*A*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2
*a^2*c^2*d^3-51*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^4+A*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3

```
*d^2-11*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^3+15*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^
3+21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4+51*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^3+51*B*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)*a^2*c^3*d^2+33*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2+35*A*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2+42*A*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3+19*A*arc
tanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^4+5*A*(-a
*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^5-4*B*(-a*(sin(f*x+e)-1)
)^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^5-15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^5*d+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^6+21*B*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d^2+9*B*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2834 vs. 2(375) = 750.
time = 8.88, size = 5968, normalized size = 14.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*(2*sqrt(2)*(2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c
^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d
^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13
*A - 9*B)*d^5)*cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A - 11*B)*c^3*d^2
 - (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15*B)*c^3*d^2 -
```

2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B)*d^5)*cos(f*
x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 -
2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c^3*d^2 -
4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d^5 - ((A
+ 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13*A - 9*B
)*d^5)*cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3*d^2 - 36*(A
- B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^2 + (
(A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 - 2*(37*A - 3
3*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e))*sin(
f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(
56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d
^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*
(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^4 - (30*B*a*c^4*d
- 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80*A - 63*B)*a*
c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^3 - (15*B*a*c^5 - 5*(7*A - 18*B)*
a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2*d^3 - (202*A
- 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5)*cos(f*x + e)^2 + (15*B*a*c^5 - 5
*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A - 20*B)*a*c^2*d
^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e) + (30*B*a*c^
5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*
a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 - (15*B*a*c^3*d
^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^
5)*cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2 - (77*A - 69*
B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^2
+ (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*
A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x
+ e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d +
7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 -
c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d +
3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*
cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 +
d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d
- d^2)*sin(f*x + e))) - 4*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*
d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^
3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^3
+ ((4*A - 13*B)*c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A
- 4*B)*c*d^4 + 2*(2*A - B)*d^5)*cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11
*B)*c^4*d + (13*A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4
- (5*A - 6*B)*d^5)*cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A -

```
 B)*c^3*d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A -
 9*B)*c^3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x
 + e)^2 - ((4*A - 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 -
(17*A - 7*B)*c*d^4 - (3*A - 4*B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^
5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d -
3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d
^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6
*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^
7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4
- 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*
a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 -
a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*
x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a
^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c
^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. 2(375) = 750.
time = 0.94, size = 1309, normalized size = 3.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/8*(sqrt(2)*(15*sqrt(2)*B*sqrt(a)*c^3*d - 35*sqrt(2)*A*sqrt(a)*c^2*d^2 + 3
0*sqrt(2)*B*sqrt(a)*c^2*d^2 - 42*sqrt(2)*A*sqrt(a)*c*d^3 + 39*sqrt(2)*B*sqr
t(a)*c*d^3 - 19*sqrt(2)*A*sqrt(a)*d^4 + 12*sqrt(2)*B*sqrt(a)*d^4)*arctan(sq
rt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^2*c^6*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c^5*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - a^2*c^4*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*a^2*c^3*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*c^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 2*a^2*c*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a^2*d^6*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + 2*(A*sqrt(a)*c + 3*B*sqrt(a)
```

```
*c - 13*A*sqrt(a)*d + 9*B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c^3*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^2*c^2*d^2*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(A*sqrt(a)
*c + 3*B*sqrt(a)*c - 13*A*sqrt(a)*d + 9*B*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4
*sqrt(2)*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^2*c^2*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)) - 4*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*sin(-1/4*pi +
1/2*f*x + 1/2*e))/((sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3
*sqrt(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^2*c*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)) + 4*(14*B*sqrt(a)
*c^2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 22*A*sqrt(a)*c*d^3*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^3 + 10*B*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 10*A*sqrt(a)*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*sqrt(a)*d^4*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^3 - 9*B*sqrt(a)*c^3*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e) + 13*A*sqrt(a)*c^2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 12*B*sqrt(a)*c^
2*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 16*A*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2
*f*x + 1/2*e) - 7*B*sqrt(a)*c*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*sqrt
(a)*d^4*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 4*B*sqrt(a)*d^4*sin(-1/4*pi + 1/2*
f*x + 1/2*e))/((sqrt(2)*a^2*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(
2)*a^2*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c^3*d^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*a^2*c^2*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
- sqrt(2)*a^2*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
), x)
```

3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=308

$$-\frac{(c-d)\left(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{d(A(9c^2+$$

```
[Out] -1/16*(3*A*c+9*A*d+5*B*c-17*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin
(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(
5/2)-1/32*(c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*arctanh(1
/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/2
4*d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*cos(f*x+e)/a^2/f/(
a+a*sin(f*x+e))^(1/2)+1/48*d^2*(9*A*c+39*A*d+15*B*c-95*B*d)*cos(f*x+e)*(a+a
*sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A]
time = 0.72, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3056, 3047, 3102, 2830, 2728, 212}

$$-\frac{(c-d)\left(3A(c^2+6cd+25d^2)+B(5c^2+62cd-163d^2)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{d^2(9Ac+39Ad+15Bc-95Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{48a^3f}+\frac{d\left(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2)\right)\cos(e+fx)}{24a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/16*((c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*
ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2
]*a^(5/2)*f) + (d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*
d^2))*Cos[e + f*x])/(24*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(9*A*c + 15*
B*c + 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(48*a^3*f) -
((3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(16*
a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^3)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+f}{}}{}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+f}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+f}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(}}{48a^3f}$$

$$= \frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(}{24a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(}{24a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(c-d)\left(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)\right)}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.76, size = 523, normalized size = 1.70

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)/
2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c - d)
^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2 + 62
*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 16*B*d
^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*
I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*(Co
```

```
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2*A*d - 5*B
*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[(3*(e + f*x
))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(281) = 562$.
time = 12.45, size = 1438, normalized size = 4.67

method	result	size
default	Expression too large to display	1438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/96*(2*sin(f*x+e)*(9*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^3+45*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c^2*d+171*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c*d^2-225*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*d^3+192*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3+15*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+171*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-675*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+489*B*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+576*B*(a-a*sin(f*
x+e))^(1/2)*a^(3/2)*c*d^2-384*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-64*B*(a-
a*sin(f*x+e))^(3/2)*a^(1/2)*d^3)+(-192*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3
-9*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-45
*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-17
1*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+2
25*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+64
*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3-576*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*
c*d^2+384*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-15*B*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-171*B*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+675*B*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-489*B*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3)*cos(f*x+e)^2+90*A*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+342*A*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+342*B*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-1350*B*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+18*A*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-450*A*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+30*B*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+978*B*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+1836*B*(a-a*sin(f
```

```
*x+e))^(1/2)*a^(3/2)*c*d^2+108*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2*d-396*A
*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2-396*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*
c^2*d+234*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2*d-378*B*(a-a*sin(f*x+e))^(3/
2)*a^(1/2)*c*d^2-90*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2*d+234*A*(a-a*sin(f
*x+e))^(3/2)*a^(1/2)*c*d^2+612*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-1092*B*
(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3+46*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3+
60*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3+36*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)
*c^3-30*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3-18*A*(a-a*sin(f*x+e))^(3/2)*a^
(1/2)*c^3-126*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3)*(-a*(sin(f*x+e)-1))^(1/
2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(294) = 588$.
time = 0.43, size = 1012, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A - 7
5*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d
 + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A + 5
*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3
)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75
*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 12*(5*A
 + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*
B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)
*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*
B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
```

```
2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B)*c^2*d - 36*(
A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*d^3)*cos(f*x +
e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A - 53*B)*c*d^2 +
(53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(A - 9*B)*c^2*d -
27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (32*B*d^3*cos(f*x +
 e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c*d^2 - 12*(A - B)*d
^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*((3*A + 5*B)*c^3 + 3
*(5*A - 13*B)*c^2*d - 3*(13*A - 85*B)*c*d^2 + (85*A - 157*B)*d^3)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(294) = 588.
time = 0.80, size = 778, normalized size = 2.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/192*(3*sqrt(2)*(3*A*sqrt(a)*c^3 + 5*B*sqrt(a)*c^3 + 15*A*sqrt(a)*c^2*d +
57*B*sqrt(a)*c^2*d + 57*A*sqrt(a)*c*d^2 - 225*B*sqrt(a)*c*d^2 - 75*A*sqrt(a
)*d^3 + 163*B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(3*A*sqrt(a)*c^3 + 5*B*sqrt(a
)*c^3 + 15*A*sqrt(a)*c^2*d + 57*B*sqrt(a)*c^2*d + 57*A*sqrt(a)*c*d^2 - 225*
B*sqrt(a)*c*d^2 - 75*A*sqrt(a)*d^3 + 163*B*sqrt(a)*d^3)*log(-sin(-1/4*pi +
1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 6*sqrt(2)
*(3*A*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^3*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 + 15*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^3 - 39*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 39*A*sqrt(a)*
c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 63*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^3 + 21*A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 29*
B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c^3*sin(-1/4*p
i + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*A
```

```
*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 33*B*sqrt(a)*c^2*d*sin(-1/4
*pi + 1/2*f*x + 1/2*e) + 33*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)
- 57*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 19*A*sqrt(a)*d^3*sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 27*B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e
))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))) - 128*sqrt(2)*(2*B*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
 - 9*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^(13/2)*d^3*sin
(-1/4*pi + 1/2*f*x + 1/2*e) + 6*B*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,\sin(e + f\,x))\,(c + d\,\sin(e + f\,x))^3}{(a + a\,\sin(e + f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
), x)
```

3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(c-d)(3Ac+5Bc}{16af(a+}$$

```
[Out] -1/16*(c-d)*(3*A*c+5*A*d+5*B*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2
)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)-1/32*(B*
(5*c^2+38*c*d-75*d^2)+A*(3*c^2+10*c*d+19*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/
2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/4*(A-9*B)*d^2*cos(f*
x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3056, 3047, 3098, 2830, 2728, 212}

$$-\frac{(A(3c^2+10cd+19d^2)+B(5c^2+38cd-75d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{d^2(A-9B)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/16*((B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[
(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2
)*f) - ((c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/(16*a*f*(a +
 a*Sin[e + f*x])^(3/2)) + ((A - 9*B)*d^2*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*
Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(4*f*(a + a*
Sin[e + f*x])^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+fx)}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{1}{2}ac(3Ac+5Bc}{}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-}{}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-}{4a^2f\sqrt{}}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-}{4a^2f\sqrt{}}$$

$$= -\frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\tanh^{-1}\Big(}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.78, size = 544, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*Cos[(e + f*x)/2] + 3*B*c^
2*Cos[(e + f*x)/2] + 6*A*c*d*Cos[(e + f*x)/2] + 10*B*c*d*Cos[(e + f*x)/2] +
 5*A*d^2*Cos[(e + f*x)/2] - 45*B*d^2*Cos[(e + f*x)/2] - 3*A*c^2*Cos[(3*(e +
 f*x))/2] - 5*B*c^2*Cos[(3*(e + f*x))/2] - 10*A*c*d*Cos[(3*(e + f*x))/2] +
26*B*c*d*Cos[(3*(e + f*x))/2] + 13*A*d^2*Cos[(3*(e + f*x))/2] - 69*B*d^2*Co
s[(3*(e + f*x))/2] + 16*B*d^2*Cos[(5*(e + f*x))/2] + 11*A*c^2*Sin[(e + f*x)
/2] - 3*B*c^2*Sin[(e + f*x)/2] - 6*A*c*d*Sin[(e + f*x)/2] - 10*B*c*d*Sin[(e
 + f*x)/2] - 5*A*d^2*Sin[(e + f*x)/2] + 45*B*d^2*Sin[(e + f*x)/2] + (2 + 2*
I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^4 - 3*A*c^2*Sin[(3*(e + f*x))/2] - 5*B*c^2*Sin[(3*(e + f*
x))/2] - 10*A*c*d*Sin[(3*(e + f*x))/2] + 26*B*c*d*Sin[(3*(e + f*x))/2] + 13
*A*d^2*Sin[(3*(e + f*x))/2] - 69*B*d^2*Sin[(3*(e + f*x))/2] - 16*B*d^2*Sin[
(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(196) = 392$.

time = 11.49, size = 982, normalized size = 4.48

method	result	size
default	Expression too large to display	982

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/32*(2*sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^2+10*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c*d+19*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*d^2+5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c^2+38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a^2*c*d-75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*d^2+64*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+(-3*A*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-10*A*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-19*A*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-5*B*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-64*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)
)*cos(f*x+e)^2+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*c^2+20*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c*d+38*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*d^2-6*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-20*A*(a-a*sin(f*x+e))^(3/2)*
a^(1/2)*c*d+26*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+20*A*(a-a*sin(f*x+e))^(
1/2)*a^(3/2)*c^2+24*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d-44*A*d^2*a^(3/2)*(
a-a*sin(f*x+e))^(1/2)+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2+76*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c*d-150*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*d^2-10*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+52*B*(a-a*sin(f*x+
e))^(3/2)*a^(1/2)*c*d-42*B*d^2*(a-a*sin(f*x+e))^(3/2)*a^(1/2)+12*B*(a-a*sin
(f*x+e))^(1/2)*a^(3/2)*c^2-88*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+204*B*d^
2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(
f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(205) = 410$.
time = 0.41, size = 774, normalized size = 3.53

$$-\frac{\sqrt{2}\left(\left(\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos(fx+e)^3-4(3A+5B)c^2-8(5A+19B)cd-4(19A-75B)d^2+3\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos(fx+e)^2-2\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos(fx+e)-\left(4(3A+5B)c^2+8(5A+19B)cd+4(19A-75B)d^2-\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos(fx+e)^2+2\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(32Bd^2\cos(fx+e)^3-4(A-B)c^2+8(A-B)cd-4(A-B)d^2-\left((3A+5B)c^2+2(5A-13B)cd-(13A-53B)d^2\right)\cos(fx+e)^2-\left((7A+B)c^2+2(A-9B)cd-9(A-9B)d^2\right)\cos(fx+e)-\left(32Bd^2\cos(fx+e)^2-4(A-B)c^2+8(A-B)cd-4(A-B)d^2+\left((3A+5B)c^2+2(5A-13B)cd-(13A-85B)d^2\right)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\right)}{64\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*
cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)*d
^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x +
 e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*
x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2 - ((
3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^2 + 2
*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos
(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*s
in(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 - 4*(A - B)*c
^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d
- (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A - 9*B)*c*d - 9*
(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A - B)*c^2 + 8*
(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d - (13*A
 - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(205) = 410$.

time = 0.64, size = 550, normalized size = 2.51

$$\frac{\frac{128 \sqrt{2} B d^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}{a^{\frac{5}{2}} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} + \frac{\sqrt{2} \left(3 A \sqrt{a} c^{2} + 5 B \sqrt{a} c^{2} + 10 A \sqrt{a} c d + 38 B \sqrt{a} c d + 19 A \sqrt{a} d^{2} - 75 B \sqrt{a} d^{2}\right) \log\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)}{a^{3} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{\sqrt{2} \left(3 A \sqrt{a} c^{2} + 5 B \sqrt{a} c^{2} + 10 A \sqrt{a} c d + 38 B \sqrt{a} c d + 19 A \sqrt{a} d^{2} - 75 B \sqrt{a} d^{2}\right) \log\left(-\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)}{a^{3} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{2 \sqrt{2} \left(3 A \sqrt{a} c^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{3} + 5 B \sqrt{a} c^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{3} + 10 A \sqrt{a} c d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{3} - 26 B \sqrt{a} c d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{3} - 13 A \sqrt{a} d^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{3} + 21 B \sqrt{a} d^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{3} - 5 A \sqrt{a} c^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) - 3 B \sqrt{a} c^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) - 6 A \sqrt{a} c d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 22 B \sqrt{a} c d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) + 11 A \sqrt{a} d^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) - 19 B \sqrt{a} d^{2} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{{\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{2} - 1\right)}^{2} a^{3} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}}{64 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/64*(128*sqrt(2)*B*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(a^(5/2)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 + 10
*A*sqrt(a)*c*d + 38*B*sqrt(a)*c*d + 19*A*sqrt(a)*d^2 - 75*B*sqrt(a)*d^2)*lo
g(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))) - sqrt(2)*(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 + 10*A*sqrt(a)*c*d + 38*B
*sqrt(a)*c*d + 19*A*sqrt(a)*d^2 - 75*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt(2)*(3*
A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^2*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 + 10*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 26*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 13*A*sqrt(a)*d^2*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^3 + 21*B*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^3 - 5*A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c^2*si
n(-1/4*pi + 1/2*f*x + 1/2*e) - 6*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 22*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 11*A*sqrt(a)*d^2*sin
(-1/4*pi + 1/2*f*x + 1/2*e) - 19*B*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*
e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
), x)
```

3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=151

$$-\frac{(3Ac+5Bc+5Ad+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3Ac+5B\cdots}{16\cdots}$$

```
[Out] -1/4*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c+5*A*d+5*B*
c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A*c+5*A*d+5*B*c+19*
B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)
/f*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3047, 3098, 2829, 2728, 212}

$$-\frac{(3Ac+5Ad+5Bc+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -1/16*((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)*f) - ((A - B)*(c - d)*Cos
[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*c + 5*B*c + 5*A*d - 13*
B*d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
```

```
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3098

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc+5Ad-5Bd)-4a\cdots}{(a+a\sin(e+fx))^{3/\cdots}}}{4a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13B\cdots}{16af(a+a\sin(e+\cdots}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13B\cdots}{16af(a+a\sin(e+\cdots}$$

$$= -\frac{(3Ac+5Bc+5Ad+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e\cdots}}\cdots\right.}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 267, normalized size = 1.77

$$\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)\sin\left(\tfrac{1}{2}(e+fx)\right)-4(A-B)(c-d)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)+2(3Ac+5Bc+5Ad-13Bd)\sin\left(\tfrac{1}{2}(e+fx)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^2-(3Ac+5Bc+5Ad-13Bd)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3+(1+i)(-1)^{3/4}(3Ac+5Bc+5Ad+19Bd)\tanh^{-1}\left(\left(\tfrac{1}{2}+\tfrac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\tfrac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^4\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2]
- 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*B*
c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(132) = 264$.
time = 9.06, size = 448, normalized size = 2.97

method	result
default	$\frac{\left(-2\sin(fx+e)\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2(3Ac+5Ad+5Bc+19Bd)+\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+}}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/32/a^(9/2)*(-2*sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*2^(1/2)*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)+arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)*cos(f*x+e)^2-6*
A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-10*A*2^
(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+6*A*(a-a*si
n(f*x+e))^(3/2)*a^(1/2)*c+10*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d-20*A*(a-a*s
in(f*x+e))^(1/2)*a^(3/2)*c-12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-10*B*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-38*B*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+10*B*(a-a*sin(f*x+
e))^(3/2)*a^(1/2)*c-26*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d-12*B*(a-a*sin(f*x
+e))^(1/2)*a^(3/2)*c+44*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d)*(-a*(sin(f*x+e)
-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5
/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(138) = 276$.
time = 0.40, size = 564, normalized size = 3.74

$$\frac{\sqrt{2}\left(\left(((3A+5B)c+(5A+19B)d)\cos(fx+e)^3+3((3A+5B)c+(5A+19B)d)\cos(fx+e)^2-4(3A+5B)c-4(5A+19B)d-2((3A+5B)c+(5A+19B)d)\cos(fx+e)+(((3A+5B)c+(5A+19B)d)\cos(fx+e)^2-4(3A+5B)c-4(5A+19B)d-2((3A+5B)c+(5A+19B)d)\cos(fx+e))\sin(fx+e)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(((3A+5B)c+(5A-13B)d)\cos(fx+e)^2+4(A-B)c-4(A-B)d+((7A+B)c+(A-9B)d)\cos(fx+e)-(4(A-B)c-4(A-B)d-((3A+5B)c+(5A-13B)d)\cos(fx+e))\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\right)}{64\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+(a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*(((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*
d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c + (5*
A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((3*A
+ 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(
f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin
(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2
*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
+ 4*(((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c - 4*(A -
 B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c - 4*(A - B)
*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(138) = 276$.
time = 0.54, size = 378, normalized size = 2.50

$$\frac{\frac{\sqrt{2}\left(3A\sqrt{a}c+5B\sqrt{a}c+5A\sqrt{a}d+19B\sqrt{a}d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3A\sqrt{a}c+5B\sqrt{a}c+5A\sqrt{a}d+19B\sqrt{a}d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(3\sqrt{2}A\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+5\sqrt{2}B\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+5\sqrt{2}A\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-13\sqrt{2}B\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\sqrt{2}A\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3\sqrt{2}B\sqrt{a}c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3\sqrt{2}A\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+11\sqrt{2}B\sqrt{a}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*A*sqrt(a)*c + 5*B*sqrt(a)*c + 5*A*sqrt(a)*d + 19*B*sqrt(a)
*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))) - sqrt(2)*(3*A*sqrt(a)*c + 5*B*sqrt(a)*c + 5*A*sqrt(a)*d + 19*B*
sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*A*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^3 + 5*sqrt(2)*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*sqrt(2)*A*
sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 13*sqrt(2)*B*sqrt(a)*d*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*A*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1
/2*e) - 3*sqrt(2)*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(2)*A*
sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 11*sqrt(2)*B*sqrt(a)*d*sin(-1/4*
pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\sin\left(e + f\,x\right))\,\left(c + d\,\sin\left(e + f\,x\right)\right)}{\left(a + a\,\sin\left(e + f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A+5*B)*cos(f*x+e)/a/
f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A+5*B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2829, 2729, 2728, 212}

$$-\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(3A+5B)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/16*((3*A + 5*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[2]*a^(5/2)*f) - ((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e
+ f*x])^(5/2)) - ((3*A + 5*B)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}+\frac{(3A+5B)\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}+\frac{(3A+5B)\int \frac{\ldots}{\sqrt{a\ldots}}}{3\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}-\frac{(3A+5B)\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}\ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 227, normalized size = 1.80

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+4(-A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(3A+5B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+(1+i)(-1)^{3/4}(3A+5B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right)}{16f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A
+ B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A + 5*B)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(107) = 214.
time = 9.95, size = 279, normalized size = 2.21

method	result
default	$-\frac{\left(2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{}}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^3*(3*A+5*B)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^3*(3*A+5*B)*cos(f*x+e)^2-6*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+20*A*(a-a
*sin(f*x+e))^(1/2)*a^(5/2)-10*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*si
n(f*x+e))^(1/2)*a^(5/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(113) = 226$.
time = 0.41, size = 420, normalized size = 3.33

$$\frac{\sqrt{2}\left(\left(3A+5B\right)\cos\left(fx+e\right)^3+3\left(3A+5B\right)\cos\left(fx+e\right)^2-2\left(3A+5B\right)\cos\left(fx+e\right)+\left(\left(3A+5B\right)\cos\left(fx+e\right)^2-2\left(3A+5B\right)\cos\left(fx+e\right)-12A-20B\right)\sin\left(fx+e\right)-12A-20B\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(\left(3A+5B\right)\cos\left(fx+e\right)^2+\left(7A+B\right)\cos\left(fx+e\right)+\left(\left(3A+5B\right)\cos\left(fx+e\right)-4A+4B\right)\sin\left(fx+e\right)+4A-4B\right)\sqrt{a\sin\left(fx+e\right)+a}}{64\left(a^3f\cos\left(fx+e\right)^3+3a^3f\cos\left(fx+e\right)^2-2a^3f\cos\left(fx+e\right)-4a^3f+\left(a^3f\cos\left(fx+e\right)^2-2a^3f\cos\left(fx+e\right)-4a^3f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 -
2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*co
s(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*B)*cos(
```

```
f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin\left(e + fx\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(113) = 226.
time = 0.50, size = 246, normalized size = 1.95

$$\frac{\frac{\sqrt{2}\left(3A\sqrt{a}+5B\sqrt{a}\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(3A\sqrt{a}+5B\sqrt{a}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(3\sqrt{2}A\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+5\sqrt{2}B\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\sqrt{2}A\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3\sqrt{2}B\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2)*(3*A*sqrt(a) + 5*B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*sqrt(a) + 5
*B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^3 + 5*sqrt(2)*B*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*A*sqr
t(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(2)*B*sqrt(a)*sin(-1/4*pi + 1/2
*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\sin\left(e + f\,x\right)}{{\left(a + a\,\sin\left(e + f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=261

$$-\frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^3f}-\frac{2d^{3/2}(Bc-Ad)\tan}{a^5}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c-11*A*d+5*B
*c+3*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)-1/32*(B*(5*c^2-34*c
*d-3*d^2)+A*(3*c^2-14*c*d+43*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(
a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^3/f*2^(1/2)-2*d^(3/2)*(-A*d+B*c)*arcta
nh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(
c-d)^3/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3057, 3064, 2728, 212, 2852, 214}

$$-\frac{(A(3c^2-14cd+43d^2)+B(5c^2-34cd-3d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^3}-\frac{2d^{3/2}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}-\frac{(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -1/16*((B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(
Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/2)
*(c - d)^3*f) - (2*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(5/2)*(c - d)^3*Sqrt[c + d]*
f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*
A*c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e
+ f*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-8Ad)-}{(a+a\sin(e+fx))^{3/2}}}{4a^2(c}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad}{16a(c-d)^2f(a+}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad}{16a(c-d)^2f(a+}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad}{16a(c-d)^2f(a+}$$

$$= -\frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\tanh^{-1}}{16\sqrt{2}\,a^{5/2}(c-d)^3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.90, size = 550, normalized size = 2.11

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*
(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2) + A
*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (8*d^(3/2)*(-(B*c) + A*d
)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
 d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)/Sqrt[c + d] + (8*d^(3/2)*(B*c - A*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4)/Sqrt[c + d]))/(16*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. 2(228) = 456.
time = 16.36, size = 1418, normalized size = 5.43

method	result	size
default	Expression too large to display	1418

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32*(2*sin(f*x+e)*(-64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*a^(5/2)*d^3+64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a^(5/2)*c*d^2+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^
(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-14*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+43*A*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+5*B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
^2-34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a^2*c*d-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)+(64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(
a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3-64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*
d+a*d^2)^(1/2))*a^(5/2)*c*d^2-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+14*A*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-43*A*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^
2-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c^2+34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)*cos(f*x+e)^2-128*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+128*B*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2-6*A*(a*(c+d)*d)^(
1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+28*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x
+e))^(3/2)*a^(1/2)*c*d-22*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2
)*d^2+20*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-72*A*(a*(c+
d)*d)^(1/2)*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*c*d+52*A*(a*(c+d)*d)^(1/2)*a^(3/
2)*(a-a*sin(f*x+e))^(1/2)*d^2+6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-28*A*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+86*A*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^
2-10*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+4*B*(a*(c+d)*d)
^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d+6*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*
x+e))^(3/2)*a^(1/2)*d^2+12*B*(a*(c+d)*d)^(1/2)*a^(3/2)*(a-a*sin(f*x+e))^(1/
2)*c^2+8*B*(a*(c+d)*d)^(1/2)*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*c*d-20*B*(a*(c+
d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^2+10*B*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-68*B*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
```

```
c*d-6*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a^2*d^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+
d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(236) = 472$.
time = 4.04, size = 2665, normalized size = 10.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*(((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*c
os(f*x + e)^3 - 4*(3*A + 5*B)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2
 + 3*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)
^2 - 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x +
e) - (4*(3*A + 5*B)*c^2 - 8*(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A +
 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A
 + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*s
qrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x +
e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) - 32*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)
*cos(f*x + e)^3 - 3*(B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d
^2)*cos(f*x + e) + (4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e
)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))
*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2
*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^
2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^
2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x
```

```
+ e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2
 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^
2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (1
1*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A - 3*B)*c*d + (15*
A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2
 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2
- a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*
d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f
*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3
*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 -
 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c
^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), 1/64*(sqrt(2)*(((3*A + 5*B)
*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^3 - 4*(3*A + 5*B
)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2 + 3*((3*A + 5*B)*c^2 - 2*(7
*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 - 2*((3*A + 5*B)*c^2 - 2*
(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e) - (4*(3*A + 5*B)*c^2 - 8*
(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c
*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)
*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*
x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f
*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a
)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
64*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d
 - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*
d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*
cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e)))
 + 4*(4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*
(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A
 - 3*B)*c*d + (15*A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c
*d + 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^
2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*
c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e
)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. 2(236) = 472.
time = 0.64, size = 787, normalized size = 3.02

$$-\frac{\frac{64\,\sqrt{2}\,(Bcd^2-Ad^3)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(\sqrt{2}\,a^{\frac{5}{2}}c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,\sqrt{2}\,a^{\frac{5}{2}}c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,\sqrt{2}\,a^{\frac{5}{2}}cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}\,a^{\frac{5}{2}}d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}}-\frac{\left(3\,A\sqrt{a}\,c^2+5\,B\sqrt{a}\,c^2-14\,A\sqrt{a}\,cd-34\,B\sqrt{a}\,cd+43\,A\sqrt{a}\,d^2-3\,B\sqrt{a}\,d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\,a^3c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,\sqrt{2}\,a^3c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,\sqrt{2}\,a^3cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}\,a^3d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\left(3\,A\sqrt{a}\,c^2+5\,B\sqrt{a}\,c^2-14\,A\sqrt{a}\,cd-34\,B\sqrt{a}\,cd+43\,A\sqrt{a}\,d^2-3\,B\sqrt{a}\,d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\,a^3c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\,\sqrt{2}\,a^3c^2d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,\sqrt{2}\,a^3cd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\sqrt{2}\,a^3d^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(3\,A\sqrt{a}\,c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+5\,B\sqrt{a}\,c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-11\,A\sqrt{a}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+3\,B\sqrt{a}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-5\,A\sqrt{a}\,c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-3\,B\sqrt{a}\,c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+13\,A\sqrt{a}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-5\,B\sqrt{a}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sqrt{2}\,a^3c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,\sqrt{2}\,a^3cd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}\,a^3d^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2}}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -1/32*(64*sqrt(2)*(B*c*d^2 - A*d^3)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^(5/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 3*sqrt(2)*a^(5/2)*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*sqrt(2)*a^(5/2)*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^(5/2
)*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - (3*A*sqrt(a)
*c^2 + 5*B*sqrt(a)*c^2 - 14*A*sqrt(a)*c*d - 34*B*sqrt(a)*c*d + 43*A*sqrt(a)
*d^2 - 3*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^
3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^2*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + (3*A*sqrt(
a)*c^2 + 5*B*sqrt(a)*c^2 - 14*A*sqrt(a)*c*d - 34*B*sqrt(a)*c*d + 43*A*sqrt(
a)*d^2 - 3*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)
*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^2*d*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*(3*A*
sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - 11*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 3*B*sq
rt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c*sin(-1/4*pi + 1/2*
f*x + 1/2*e) - 3*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 13*A*sqrt(a)*
d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*B*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/
2*e))/((sqrt(2)*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^3
*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),
 x)
```

3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=395

$$-\frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^4f}+\frac{d^{3/2}(Ad(7c+5d)}{}$$

```
[Out] d^(3/2)*(A*d*(7*c+5*d)-B*(5*c^2+5*c*d+2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^
(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1/4
*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))-1/16*(3*A
*c-15*A*d+5*B*c+7*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*s
in(f*x+e))-1/32*(B*(5*c^2-58*c*d-43*d^2)+A*(3*c^2-22*c*d+115*d^2))*arctanh(
1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/f*2^
(1/2)-1/16*d*(A*(3*c^2-16*c*d-35*d^2)+B*(5*c^2+32*c*d+11*d^2))*cos(f*x+e)/a
^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 1.07, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{(A(3c^2-22cd+115d^2)+B(5c^2-58cd-43d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^4}+\frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}}-\frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -1/16*((B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2))*ArcTanh
[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/
2)*(c - d)^4*f) + (d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2))*Ar
cTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])
])/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*
f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - ((3*A*c + 5*B*c - 15*A
*d + 7*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c +
 d*Sin[e + f*x])) - (d*(A*(3*c^2 - 16*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 1
1*d^2))*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - $$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - $$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - $$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - $$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - $$

$$= -\frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\tanh}{16\sqrt{2}\,a^{5/2}(c-d)^4 f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.53, size = 1318, normalized size = 3.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 22*A*c*d - 58*B*c*d + 115*A*d^2 - 43*B*d^2)*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e +
 f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^4 - 6
4*(-1)^(1/4)*c^3*d + 96*(-1)^(1/4)*c^2*d^2 - 64*(-1)^(1/4)*c*d^3 + 16*(-1)^
(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(A*d*(7*c + 5*d) - B*
```

```
(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(
e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*
x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2
)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-(A*d*(7*c + 5*d)) + B*(5*c^2
 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*
x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2)*f*(a
*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-22*A
*c^3*Cos[(e + f*x)/2] + 6*B*c^3*Cos[(e + f*x)/2] + 40*A*c^2*d*Cos[(e + f*x)
/2] - 40*B*c^2*d*Cos[(e + f*x)/2] + 54*A*c*d^2*Cos[(e + f*x)/2] - 70*B*c*d^
2*Cos[(e + f*x)/2] + 24*A*d^3*Cos[(e + f*x)/2] + 8*B*d^3*Cos[(e + f*x)/2] -
 6*A*c^3*Cos[(3*(e + f*x))/2] - 10*B*c^3*Cos[(3*(e + f*x))/2] + 21*A*c^2*d*
Cos[(3*(e + f*x))/2] - 29*B*c^2*d*Cos[(3*(e + f*x))/2] + 54*A*c*d^2*Cos[(3*
(e + f*x))/2] - 86*B*c*d^2*Cos[(3*(e + f*x))/2] + 75*A*d^3*Cos[(3*(e + f*x)
)/2] - 19*B*d^3*Cos[(3*(e + f*x))/2] + 3*A*c^2*d*Cos[(5*(e + f*x))/2] + 5*B
*c^2*d*Cos[(5*(e + f*x))/2] - 16*A*c*d^2*Cos[(5*(e + f*x))/2] + 32*B*c*d^2*
Cos[(5*(e + f*x))/2] - 35*A*d^3*Cos[(5*(e + f*x))/2] + 11*B*d^3*Cos[(5*(e +
 f*x))/2] + 22*A*c^3*Sin[(e + f*x)/2] - 6*B*c^3*Sin[(e + f*x)/2] - 40*A*c^2
*d*Sin[(e + f*x)/2] + 40*B*c^2*d*Sin[(e + f*x)/2] - 54*A*c*d^2*Sin[(e + f*x
)/2] + 70*B*c*d^2*Sin[(e + f*x)/2] - 24*A*d^3*Sin[(e + f*x)/2] - 8*B*d^3*Si
n[(e + f*x)/2] - 6*A*c^3*Sin[(3*(e + f*x))/2] - 10*B*c^3*Sin[(3*(e + f*x))/
2] + 21*A*c^2*d*Sin[(3*(e + f*x))/2] - 29*B*c^2*d*Sin[(3*(e + f*x))/2] + 54
*A*c*d^2*Sin[(3*(e + f*x))/2] - 86*B*c*d^2*Sin[(3*(e + f*x))/2] + 75*A*d^3*
Sin[(3*(e + f*x))/2] - 19*B*d^3*Sin[(3*(e + f*x))/2] - 3*A*c^2*d*Sin[(5*(e
+ f*x))/2] - 5*B*c^2*d*Sin[(5*(e + f*x))/2] + 16*A*c*d^2*Sin[(5*(e + f*x))/
2] - 32*B*c*d^2*Sin[(5*(e + f*x))/2] + 35*A*d^3*Sin[(5*(e + f*x))/2] - 11*B
*d^3*Sin[(5*(e + f*x))/2]))/(64*(c - d)^3*(c + d)*f*(a*(1 + Sin[e + f*x]))^
(5/2)*(c + d*Sin[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4091 vs. $2(358) = 716$.
time = 21.73, size = 4092, normalized size = 10.36

method	result	size
default	Expression too large to display	4092

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/32*(-22*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d-224*
A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)
^2*c^2*d^3-608*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(
5/2)*sin(f*x+e)^2*c*d^4+160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*
d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^2+480*B*arctanh((-a*(sin(f*x+e)-1))^(1
```

/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3+288*B*arctanh((-a*(si
n(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4+148*A*(-a*
(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^4+160*B*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d
^3+160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin
(f*x+e)^3*c*d^4+320*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*a^(5/2)*sin(f*x+e)*c^3*d^2-52*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/
2)*a^(3/2)*sin(f*x+e)*d^4-224*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4+480*B*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^3-84*A*(-a*(sin(f*x+e)-1)
)^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d-20*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(
c+d)*d)^(1/2)*a^(3/2)*c^2*d^2+52*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1
/2)*a^(3/2)*c*d^3+52*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*
c^3*d+20*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2-84*B
(-a(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3+6*A*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2-38*A*(-a*(sin(f*x+e)-1))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-32*A*(-a*(sin(f*x+e)-
1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3-6*A*(-a*(sin(f*x+e)-
1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d+230*A*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)
^2*a^2*d^4+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4-86*B*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^4-1
9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^3*d+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4+93*A*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+1
15*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2
)/a^(1/2))*a^2*c*d^3-53*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d+115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^4-
84*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^2
+38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^
2+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^3+
6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)*a^2*c^4+115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4-10*B*(-a*(sin(f
x+e)-1))^(3/2)(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d-22*B*(-a*(sin(f*
x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^2+10*B*(-a*(sin(f
x+e)-1))^(3/2)(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^3+10*B*(a*(c+d)*d)
^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f
*x+e)*a^2*c^4-43*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4+12*B*(-a*(sin(f*x+e)-1))^(1/2)*
(a*(c+d)*d)^(1/2)*a^(3/2)*c^4-224*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*

```
(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(5/2)*c*d^4-160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5+64*B*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5+323*A*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)
*a^2*c*d^3-101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d-255*B*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*
d^2-187*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3+93*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^3+5*B*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)^3*a^2*c^3*d+167*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2+3*A*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)^3*a^2*c^3*d-19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2478 vs. $2(371) = 742$.
time = 8.62, size = 5255, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*(4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c
^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d
- (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*
x + e)^4 - ((3*A + 5*B)*c^4 - (13*A + 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2
+ 7*(43*A - 35*B)*c*d^3 + 2*(115*A - 43*B)*d^4)*cos(f*x + e)^3 - (3*(3*A +
5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317
*B)*c*d^3 + 5*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*
```

(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e) + (4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e)^3 - ((3*A + 5*B)*c^4 - 2*(5*A + 19*B)*c^3*d + 4*(9*A - 65*B)*c^2*d^2 + 2*(197*A - 173*B)*c*d^3 + 3*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 16*(20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 + (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^4 - (5*B*a*c^3*d - (7*A - 15*B)*a*c^2*d^2 - (19*A - 12*B)*a*c*d^3 - 2*(5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (15*B*a*c^3*d - (21*A - 40*B)*a*c^2*d^2 - (50*A - 31*B)*a*c*d^3 - 5*(5*A - 2*B)*a*d^4)*cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e) + (20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 - (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (5*B*a*c^3*d - (7*A - 20*B)*a*c^2*d^2 - (26*A - 17*B)*a*c*d^3 - 3*(5*A - 2*B)*a*d^4)*cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*cos(f*x + e)^3 + ((3*A + 5*B)*c^4 - (15*A - 7*B)*c^3*d - (7*A - 15*B)*c^2*d^2 - (A + 23*B)*c*d^3 + 4*(5*A - B)*d^4)*cos(f*x + e)^2 + ((7*A + B)*c^4 - 20*(A - B)*c^3*d - 2*(13*A - 21*B)*c^2*d^2 - 4*(3*A + 13*B)*c*d^3 + (51*A - 11*B)*d^4)*cos(f*x + e) - (4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*cos(f*x + e)^2 - ((3*A + 5*B)*c^4 - 12*(A - B)*c^3*d - 2*(13*A - 21*B)*c^2*d^2 - 4*(5*A + 11*B)*c*d^3 + 5*(11*A - 3*B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 +

```
2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a
^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2
 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3
*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos
(f*x + e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*
a^3*c*d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^
4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)
 - 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2
*a^3*c*d^5 + a^3*d^6)*f)*sin(f*x + e)), -1/64*(sqrt(2)*(4*(3*A + 5*B)*c^4 -
 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(
115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 10
1*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e)^4 - ((3*A + 5*B)*c^4 - (13*A
+ 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2 + 7*(43*A - 35*B)*c*d^3 + 2*(115*A -
 43*B)*d^4)*cos(f*x + e)^3 - (3*(3*A + 5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8
*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317*B)*c*d^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1138 vs. 2(371) = 742.
time = 0.91, size = 1138, normalized size = 2.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/32*(16*sqrt(2)*(5*sqrt(2)*B*sqrt(a)*c^2*d^2 - 7*sqrt(2)*A*sqrt(a)*c*d^3
+ 5*sqrt(2)*B*sqrt(a)*c*d^3 - 5*sqrt(2)*A*sqrt(a)*d^4 + 2*sqrt(2)*B*sqrt(a)
*d^4)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a
^3*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c^4*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 2*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*a
^3*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c*d^4*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-
c*d - d^2)) - (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 22*A*sqrt(a)*c*d - 58*B*
sqrt(a)*c*d + 115*A*sqrt(a)*d^2 - 43*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*s
```

```
qrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3*c^2*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
 + (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 22*A*sqrt(a)*c*d - 58*B*sqrt(a)*c*d
 + 115*A*sqrt(a)*d^2 - 43*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e
) + 1)/(sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3
*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3*c^2*d^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 64*(B*s
qrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*sqrt(a)*d^3*sin(-1/4*pi + 1
/2*f*x + 1/2*e))/((sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*
sqrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*a^3*c*d^3
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)) + 2*(3*A*s
qrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 - 19*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 11*B*sq
rt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c*sin(-1/4*pi + 1/2*
f*x + 1/2*e) - 3*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 21*A*sqrt(a)*
d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 13*B*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e))/((sqrt(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^
3*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)))*(sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
), x)
```

3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=519

$$-\frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^5f}+\frac{d^{3/2}(3Ad(21c^2+3}{}$$

```
[Out] 1/4*d^(3/2)*(3*A*d*(21*c^2+30*c*d+13*d^2)-B*(35*c^3+70*c^2*d+67*c*d^2+20*d^
3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/
a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))
^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*A*c-19*A*d+5*B*c+11*B*d)*cos(f*x+e)/a/(c-
d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/32*(B*(5*c^2-82*c*d-115*
d^2)+3*A*(c^2-10*c*d+73*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*s
in(f*x+e))^(1/2))/a^(5/2)/(c-d)^5/f*2^(1/2)-1/16*d*(A*(3*c^2-20*c*d-31*d^2)
+B*(5*c^2+28*c*d+15*d^2))*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^2
/(a+a*sin(f*x+e))^(1/2)-1/16*d*(3*A*(c^3-7*c^2*d-37*c*d^2-21*d^3)+B*(5*c^3+
73*c^2*d+79*c*d^2+35*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e)
)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.46, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3057, 3063, 3064, 2728, 212, 2852, 214}

$$-\frac{(3A(c^2-10cd+73d^2)+B(5c^2-82cd-115d^2))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^5}+\frac{d^{3/2}(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{5/2}f(c-d)^5(c+d)^{5/2}}-\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}-\frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] -1/16*((B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d + 73*d^2))*ArcTanh
[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*a^(5/
2)*(c - d)^5*f) + (d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2) - B*(35*c^3 +
70*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) -
 ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[
e + f*x])^2) - ((3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])/(16*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(A*(3*c^2 -
20*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x])/(16*a^2*(c -
d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (d*(3*A*(
c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^
3))*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \int$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - 1$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - 1$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - 1$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - 1$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - 1$$

$$= -\frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\tanh^{-}}{16\sqrt{2}\,a^{5/2}(c-d)^5f}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 13.05, size = 2103, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]

[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 30*A*c*d - 82*B*c*d + 219*A*d^2 - 115*B*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^5 - 80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 - 160*(-1)^(1/4)*c^2*d^3 + 80*(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 + Sin[e + f*x]))^(5/2)) - (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-44*A*c^5*Cos[(e + f*x)/2] + 12*B*c^5*Cos[(e + f*x)/2] + 84*A*c^4*d*Cos[(e + f*x)/2] - 116*B*c^4*d*Cos[(e + f*x)/2] + 249*A*c^3*d^2*Cos[(e + f*x)/2] - 433*B*c^3*d^2*Cos[(e + f*x)/2] + 385*A*c^2*d^3*Cos[(e + f*x)/2] - 277*B*c^2*d^3*Cos[(e + f*x)/2] + 239*A*c*d^4*Cos[(e + f*x)/2] - 95*B*c*d^4*Cos[(e + f*x)/2] + 47*A*d^5*Cos[(e + f*x)/2] - 51*B*d^5*Cos[(e + f*x)/2] - 12*A*c^5*Cos[(3*(e + f*x))/2] - 20*B*c^5*Cos[(3*(e + f*x))/2] + 40*A*c^4*d*Cos[(3*(e + f*x))/2] - 104*B*c^4*d*Cos[(3*(e + f*x))/2] + 261*A*c^3*d^2*Cos[(3*(e + f*x))/2] - 581*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 781*A*c^2*d^3*Cos[(3*(e + f*x))/2] - 665*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 579*A*c*d^4*Cos[(3*(e + f*x))/2] - 299*B*c*d^4*Cos[(3*(e + f*x))/2] + 79*A*d^5*Cos[(3*(e + f*x))/2] - 59*B*d^5*Cos[(3*(e + f*x))/2] + 12*A*c^4*d*Cos[(5*(e + f*x))/2] + 20*B*c^4*d*Cos[(5*(e + f*x))/2] - 73*A*c^3*d^2*Cos[(5*(e + f*x))/2] + 217*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 353*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 397*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 419*A*c*d^4*Cos[(5*(e + f*x))/2] + 251*B*c*d^4*Cos[(5*(e + f*x))/2] - 127*A*d^5*Cos[(5*(e + f*x))/2] + 75*B*d^5*Cos[(5*(e + f*x))/2] + 3*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 5*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 21*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 73*B*c^2*d^3*Cos[(7*(e + f*x))/2] - 111*A*c*d^4*Cos[(7*(e + f*x))/2] + 79*B*c*d^4*Cos[(7*(e + f*x))/2] - 63*A*d^5*Cos[(7*(e + f*x))/2] + 35*B*d^5*Cos[(7*(e + f*x))/2] + 44*A*c^5*Sin[(e + f*x)/2] - 12*B*c^5*Sin[(e + f*x)/2] - 84*A*c^4*d*Sin[(e + f*x)/2] + 116*B*c^4*d*Sin[(e + f*x)/2] - 249*A*c^3*d^2*Sin[(e + f*x)/2] + 433*B*c^3*d^2*Sin[(e + f*x)/2] - 385*A*c^2*d^3*Sin[(e + f*x)/2] + 277*B*c^2*d^3*Sin[(e + f*x)/2] - 239*A*c*d^4*Sin[(e + f*x)/2] + 95*B*c*d^4*Sin[(e + f*x)/2] - 47*A*d^5*Sin[(e + f*x)/2] + 51*B*d^5*Sin[(e + f*x)/2] - 12*A*c^5*Sin[(3*(e + f*x))/2] - 20*B*c^5*Sin[(3*(e + f*x))/2] + 40*A*c^4*d*Sin[(3*(e + f*x))/2] - 104*B*c^4*d*Sin[(3*(e + f*x))/2] + 261*A*c^3*d^2*Sin[(3*(e + f*x))/2] - 581*B*c^3*d^2*Sin[(3*(e + f*x))/2] + 781*A*c^2*d^3*Sin[(3*(e + f*x))/2] - 665*B*c^2*d^3*Sin[(3*(e + f*x))/2] + 579*A*c*d^4*Sin[(3*(e + f*x))/2]

```
- 299*B*c*d^4*Sin[(3*(e + f*x))/2] + 79*A*d^5*Sin[(3*(e + f*x))/2] - 59*B*
d^5*Sin[(3*(e + f*x))/2] - 12*A*c^4*d*Sin[(5*(e + f*x))/2] - 20*B*c^4*d*Sin
[(5*(e + f*x))/2] + 73*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 217*B*c^3*d^2*Sin[(
5*(e + f*x))/2] + 353*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 397*B*c^2*d^3*Sin[(5
*(e + f*x))/2] + 419*A*c*d^4*Sin[(5*(e + f*x))/2] - 251*B*c*d^4*Sin[(5*(e +
 f*x))/2] + 127*A*d^5*Sin[(5*(e + f*x))/2] - 75*B*d^5*Sin[(5*(e + f*x))/2]
+ 3*A*c^3*d^2*Sin[(7*(e + f*x))/2] + 5*B*c^3*d^2*Sin[(7*(e + f*x))/2] - 21*
A*c^2*d^3*Sin[(7*(e + f*x))/2] + 73*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 111*A*
c*d^4*Sin[(7*(e + f*x))/2] + 79*B*c*d^4*Sin[(7*(e + f*x))/2] - 63*A*d^5*Sin
[(7*(e + f*x))/2] + 35*B*d^5*Sin[(7*(e + f*x))/2]))/(128*(c - d)^4*(c + d)^
2*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7321 vs. $2(476) = 952$.
time = 32.62, size = 7322, normalized size = 14.11

method	result	size
default	Expression too large to display	7322

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4188 vs. $2(492) = 984$.
time = 27.88, size = 8675, normalized size = 16.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

[Out] [1/64*(sqrt(2)*(4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(117*A - 413*B
)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(4
23*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A
+ 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A
 - 115*B)*d^6)*cos(f*x + e)^5 + (2*(3*A + 5*B)*c^5*d - 3*(13*A + 43*B)*c^4*
d^2 + 4*(63*A - 191*B)*c^3*d^3 + 6*(217*A - 241*B)*c^2*d^4 + 2*(831*A - 583
*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*cos(f*x + e)^4 - ((3*A + 5*B)*c^6 - 4*(3
*A + 13*B)*c^5*d + (75*A - 547*B)*c^4*d^2 + 8*(123*A - 203*B)*c^3*d^3 + 19*
(123*A - 115*B)*c^2*d^4 + 4*(525*A - 349*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*
cos(f*x + e)^3 - (3*(3*A + 5*B)*c^6 - 2*(21*A + 83*B)*c^5*d + (267*A - 1507
*B)*c^4*d^2 + 4*(669*A - 1045*B)*c^3*d^3 + (5871*A - 5383*B)*c^2*d^4 + 2*(2
523*A - 1667*B)*c*d^5 + 7*(219*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5
*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A - 413*B)*c^4*d^2 + 4*(177*A - 233*B
)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (219*A -
115*B)*d^6)*cos(f*x + e) + (4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(1
17*A - 413*B)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^
2*d^4 + 8*(423*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*
d^2 - 24*(A + 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*
d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^4 - 2*((3*A + 5*B)*c^5*d - (21*A +
67*B)*c^4*d^2 + 2*(69*A - 173*B)*c^3*d^3 + 2*(285*A - 293*B)*c^2*d^4 + (627
*A - 427*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^3 - ((3*A + 5*B)*c^6
- 6*(A + 7*B)*c^5*d + 3*(11*A - 227*B)*c^4*d^2 + 12*(105*A - 193*B)*c^3*d^3
 + 3*(1159*A - 1119*B)*c^2*d^4 + 6*(559*A - 375*B)*c*d^5 + 5*(219*A - 115*B
)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A
- 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 +
2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e))*sin(f*x + e))*
sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2
*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) - 4*(140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108
*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c
d^5 - 4(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 -
 (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^5 + (70*B*a*c^4*
d^2 - 7*(18*A - 35*B)*a*c^3*d^3 - (369*A - 344*B)*a*c^2*d^4 - (348*A - 241*
B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^4 - (35*B*a*c^5*d - 21*(3*
A - 10*B)*a*c^4*d^2 - 2*(171*A - 226*B)*a*c^3*d^3 - 6*(98*A - 83*B)*a*c^2*d
^4 - (426*A - 281*B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (105
*B*a*c^5*d - 7*(27*A - 80*B)*a*c^4*d^2 - 6*(150*A - 191*B)*a*c^3*d^3 - 2*(7
29*A - 610*B)*a*c^2*d^4 - 3*(340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6
)*cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A -
121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 -
(39*A - 20*B)*a*d^6)*cos(f*x + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*
d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A
- 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)
*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^4 -

```
2*(35*B*a*c^4*d^2 - 21*(3*A - 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 -
3*(43*A - 29*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (35*B*a*c^5
*d - 7*(9*A - 40*B)*a*c^4*d^2 - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 38
6*B)*a*c^2*d^4 - (684*A - 455*B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*cos(f*x +
 e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^
3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*
B)*a*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x
+ e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2
)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e)
 + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*s
in(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x +
 e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(4*(A - B)*c^6 - 8*(A - B)*c^5*
d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*
c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(
15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)
^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38*A - 31*B)*c^3*d^
3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(19*A - 11*B)*d^6)
*cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (31*A - 75*B)*c^4*d
^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
), x)
```

3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=221

$$-\frac{8\sqrt{2}\,a^2BF_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -8*a^2*B*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e)
)*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+s
in(f*x+e))^(1/2)-4*a^2*(A-B)*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d
),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*
x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3066, 2863, 144, 143}

$$-\frac{4\sqrt{2}\,a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}-\frac{8\sqrt{2}\,a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)*Ap
pellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^2(c+d\sin$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(1+}{\sqrt{}}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)(c+d\sin(e+\right.}{}$$

$$= -\frac{8\sqrt{2}\,a^2BF_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin\right.}{}$$

Mathematica [F]
time = 27.86, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F]
time = 0.89, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right)^{2} \left(A+B\sin \left(fx+e\right)\right) \left(c+d\sin \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=217

$$-\frac{4\sqrt{2}\,aBF_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)}{f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -4*a*B*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)-2*a*(A-B)*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/
2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3047, 3096, 2834, 144, 143, 2863}

$$-\frac{2\sqrt{2}\,a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}-\frac{4\sqrt{2}\,aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*Appell
F1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)
]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c + d*Si
n[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3096

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, I
nt[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*S
in[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C,
m}, x] && EqQ[A - B + C, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (c+d\sin(e+fx))^n\,(aA+(aA+aB)$$

$$= (a(A-B))\int (1+\sin(e+fx))(c+d\sin$$

$$= \frac{(a(A-B)\cos(e+fx))\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1}}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+}}$$

$$= \frac{\left(a(A-B)\cos(e+fx)(c+d\sin(e+fx\right.}{f}$$

$$= -\frac{4\sqrt{2}\,aBF_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+\right.}{}$$

Mathematica [F]
time = 6.57, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
[Out] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
 x]
```

Maple [F]
time = 0.72, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))\,(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e) + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\sin(e+fx))\,(a+a\sin(e+fx))\,(c+d\sin(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=221

$$-\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{af\sqrt{1+\sin(e+fx)}}-$$

```
[Out] -1/2*(A-B)*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(
1+sin(f*x+e))^(1/2)-B*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/
2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))/
(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3066, 2863, 144, 143, 2744}

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}\,af\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}\,B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/2,
-n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]
*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx + \frac{B\int(c+d\sin(e+f}{a}$$

$$= \frac{((A-B)\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx, x, \sin(\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right.\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{}}$$

$$= -\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{1+\sin(e}}$$

Mathematica [F]
time = 6.18, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
), x]
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{a+a\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=223

$$-\frac{BF_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{\sqrt{2}\,a^2f\sqrt{1+\sin(e+fx)}}-\frac{(A-$$

```
[Out] -1/2*B*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+
sin(f*x+e))^(1/2)-1/4*(A-B)*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+d),
1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/
(c+d))^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3066, 2863, 144, 143}

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}-\frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]
))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 5/2
, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*
Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,d}{a}$$

$$= \frac{((A-B)\cos(e+fx))\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx, x, \sin(\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right.\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{}}$$

$$= -\frac{BF_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(}{\sqrt{2}\,a^2 f\sqrt{1+\sin(e}}$$

Mathematica [F]
time = 30.61, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2, x]
```

Maple [F]
time = 2.04, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^{n}}{(a+a\sin(fx+e))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x
)
```

3.332 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx$

Optimal. Leaf size=427

$$-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*a^2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))
^(1/2)+2*a^2*B*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(4*
n^2+16*n+15)/(a+a*sin(f*x+e))^(1/2)+2*a^2*(A-B)*(c-d*(5+4*n))*cos(f*x+e)*hy
pergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2
*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*B*(3*c^2-2*c*
d*(7+4*n)+d^2*(16*n^2+56*n+43))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-s
in(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d^2/f/(4*n^2+16*n+15)/(((c+d*sin(f*x+e
))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)
*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A]
time = 0.62, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3066, 2842, 21, 2855, 72, 71, 3060}

$$\frac{2a^2(A-B)(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2B(3c^2-2cd(4n+7)+d^2(16n^2+56n+43))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}+\frac{2a^2B(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{n+1}}{df(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*a^2*(A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) + (2*a^2*B*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^(1 + n))/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]
]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 +
 n))/(d*f*(5 + 2*n)) + (2*a^2*(A - B)*(c - d*(5 + 4*n))*Cos[e + f*x]*Hyperg
eometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d
))^n) - (2*a^2*B*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e
 + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c
+ d*Sin[e + f*x])^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^{3/2}(c+$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e}{df(3+2n)\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e}{df(3+2n)\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e}{df(3+2n)\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e}{df(3+2n)\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e}{df(3+2n)\sqrt{a+a\sin(e+f}}$$

Mathematica [A]
time = 33.12, size = 245, normalized size = 0.57

$$-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left(-30(A+B)(c-d(5+4n))\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(-1+\sin(e+fx))}{c+d}\right)+6Bd(3+2n)\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{d(-1+\sin(e+fx))}{c+d}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+20Bd(3+2n)\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{d(-1+\sin(e+fx))}{c+d}\right)(-1+\sin(e+fx))+30(A+B)(c+d)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{1+n}\right)}{15df(3+2n)\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^n,x]
```

```
[Out] -1/15*(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*n
))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + 6*
B*d*(3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/(c
+ d))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hypergeom
etric2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e + f*
x]) + 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 +
 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(f x +e \right)\right)^{\frac{3}{2}} \left(A +B \sin \left(f x +e \right)\right) \left(c +d \sin \left(f x +e \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,
 x)
```

3.333 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=167

$$-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a(Ad(3+2n)-B(c-2d(1+n)))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(\cdots}{\cdots}\right.}{df(3+2n)\sqrt{a+a\sin(\cdots}}$$

```
[Out] -2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
-2*a*(A*d*(3+2*n)-B*(c-2*d*(1+n)))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(
1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*sin(f*x+e))/(c+d
))^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3060, 2855, 72, 71}

$$\frac{2a\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[
e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/
(c + d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1-}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1-}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1-}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1-}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [F]
time = 5.90, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x]
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin\left(fx+e\right)}\,\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)
^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,
 x)
```

3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=220

$$-\frac{(A-B)F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+f}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -2*B*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(
f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-(A-B)*Appel
lF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)
*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)/f/(1+n)/(1-sin
(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3066, 2867, 142, 141, 2855, 72, 71}

$$-\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -(((A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c
+ d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)
]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[
a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (
d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2855

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

```
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int \sqrt{a+a\sin(e-}}{}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}}{f(a-a\sin(e+fx))\sqrt{}}$$

$$= -\frac{(A-B)F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{(c-d)f(1+n)(1-\sin(e}$$

Mathematica [A]
time = 12.27, size = 244, normalized size = 1.11

$$\frac{\cos(e+fx)\sqrt{a(1+\sin(e+fx))}\,(c+d\sin(e+fx))^n\left(-\left((A+B)F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(1+\sin(e+fx)),\frac{d(1+\sin(e+fx))}{-c+d}\right)\sqrt{2-2\sin(e+fx)}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\right)+\frac{4(A-B)F_1\left(-\frac{1}{2}-n;-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{1+\sin(e+fx)},\frac{-c+d}{d+d\sin(e+fx)}\right)\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}\left(1+\frac{c-d}{d+d\sin(e+fx)}\right)^{-n}}{1+2n}\right)}{4af(-1+\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B)
*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c +
 d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*(A -
B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)
*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{\sqrt{a+a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(e + fx))\,(c + d\sin(e + fx))^n}{\sqrt{a\,(\sin(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2
), x)
```

3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{BF_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}}+$$

```
[Out] -B*AppellF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*co
s(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/(1
+n)/(1-sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+(A-B)*d*AppellF1(1+n,2,1/2,2+n,(c
+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)*(c+d*sin(f*x+e))^(1
+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)^2/f/(1+n)/(a-a*sin(f*x+e))/(a+a*si
n(f*x+e))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {3066, 2867, 142, 141}

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Si
n[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c +
 d*Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e
+ f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Si
n[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a
 - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(c+d\sin(e+\ldots}{\sqrt{a+a\sin(e\ldots}}}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)^2}\,dx, x, \ldots\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\operatorname{Subst}\ldots}{f(a-a\sin(e+fx))\sqrt{\ldots}}$$

$$= -\frac{BF_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+\ldots}{(c-d)f(1+n)(a-a\sin(e+\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 603 vs. 2(269) = 538.
time = 55.98, size = 603, normalized size = 2.24

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(aF_1\left(1;\tfrac{1}{2},-n;2;\tfrac{1}{2}(1+\sin(e+fx)),\tfrac{d(1+\sin(e+fx))}{-c+d}\right)\sqrt{2-2\sin(e+fx)}\,(1+\sin(e+fx))\left(\tfrac{c+d\sin(e+fx)}{c-d}\right)^{-n}-\frac{4\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}\left(-2a(1+2n)F_1\left(\tfrac{1}{2}-n;-\tfrac{1}{2},-n;\tfrac{3}{2}-n;\tfrac{2}{1+\sin(e+fx)},\tfrac{-c+d}{d+d\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\tfrac{1}{2}-n;-\tfrac{1}{2},-n;\tfrac{1}{2}-n;\tfrac{2}{1+\sin(e+fx)},\tfrac{-c+d}{d+d\sin(e+fx)}\right)(1+\sin(e+fx))\right)\left(1+\tfrac{c-d}{d+d\sin(e+fx)}\right)^{-n}}{-1+4n^2}\right)+aA(1+\sin(e+fx))\left(aF_1\left(1;\tfrac{1}{2},-n;2;\tfrac{1}{2}(1+\sin(e+fx)),\tfrac{d(1+\sin(e+fx))}{-c+d}\right)\sqrt{2-2\sin(e+fx)}\,(1+\sin(e+fx))\left(\tfrac{c+d\sin(e+fx)}{c-d}\right)^{-n}-\frac{4\sqrt{\frac{-1+\sin(e+fx)}{1+\sin(e+fx)}}\left(2a(1+2n)F_1\left(\tfrac{1}{2}-n;-\tfrac{1}{2},-n;\tfrac{3}{2}-n;\tfrac{2}{1+\sin(e+fx)},\tfrac{-c+d}{d+d\sin(e+fx)}\right)+a(-1+2n)F_1\left(-\tfrac{1}{2}-n;-\tfrac{1}{2},-n;\tfrac{1}{2}-n;\tfrac{2}{1+\sin(e+fx)},\tfrac{-c+d}{d+d\sin(e+fx)}\right)(1+\sin(e+fx))\right)\left(1+\tfrac{c-d}{d+d\sin(e+fx)}\right)^{-n}}{-1+4n^2}\right)\right)}{8a^3f\sqrt{a(1+\sin(e+fx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2
 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (
4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2
 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]
), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (
c - d)/(d + d*Sin[e + f*x]))^n)) + a*A*(1 + Sin[e + f*x])*((a*AppellF1[1, 1
/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 -
2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (4*S
qrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n
, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] +
 a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (
-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c -
d)/(d + d*Sin[e + f*x]))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^{n}}{(a+a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^{n}}{(a\,(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1)
)**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^{n}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2
), x)
```

3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=351

$$\frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+m)+B(c^2+d^2+cdm)\right))\cos(e+fx)(a+a\sin(e+fx)}{f(1+m)(2+m)(3+m)}$$

```
[Out] (d*(A*d*(3+m)+B*(d*m+2*c))-2*(2+m)*(A*c*d*(3+m)+B*(c*d*m+c^2+d^2)))*cos(f*x
+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)-2^(1/2+m)*(A*(3+m)*(2*c*d*m*(2+m
)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))+B*(d^2*m*(m^2+3*m+5)+c^2*m*(m^2+5*m+6)+2*c
*d*(m^3+4*m^2+4*m+3)))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(3+m)/(m^2+3*m+2)-d*(A
*d*(3+m)+B*(d*m+2*c))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)/(3+m)-B*c
os(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+m)
```

Rubi [A]
time = 0.68, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3062, 3047, 3102, 2830, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2(m^2+3m+2)+2cdm(m+2)+d^2(m^2+m+1)\right)+B\left(c^2m(m^2+5m+6)+2cd(m^3+4m^2+4m+3)+d^2m(m^2+3m+5)\right)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)(m+2)(m+3)}+\frac{\cos(e+fx)\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3)+B(c^2+cdm+d^2)\right)\right)(a\sin(e+fx)+a)^m}{f(m+1)(m+2)(m+3)}-\frac{d\cos(e+fx)(Ad(m+3)+B(2c+dm))(a\sin(e+fx)+a)^{m+1}}{af(m+2)(m+3)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 + m)*(A*c*d*(3 + m) + B*(c^2 + d^2
 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)
) - (2^(1/2 + m)*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 +
 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 + 5*m + m^2) + 2*c*d*(3
+ 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(
f*(1 + m)*(2 + m)*(3 + m)) - (d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*
(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (B*Cos[e + f*x]*(a +
a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(3+m)}$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(3+m)}$$
$$= -\frac{d(Ad(3+m)+B(2c+dm))\cos(e+}{af(2+m)(3+}$$
$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+}{}$$
$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+}{}$$
$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+}{}$$

Mathematica [A]
time = 10.42, size = 300, normalized size = 0.85

$$-\frac{\csc^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^m (a(1+\sin(e+fx)))^m \left(-2(A+B)(c+d)^2\,{}_2F_1\left(\frac{1}{2},4+m;\frac{3}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)-\frac{2}{3}(c+d)(3Ac+Bc-Ad-3Bd)\,{}_2F_1\left(\frac{3}{2},4+m;\frac{5}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan^3\left(\frac{1}{4}(2e-\pi+2fx)\right)-\frac{2}{5}(c-d)(A(3c+d)-B(c+3d))\,{}_2F_1\left(\frac{5}{2},4+m;\frac{7}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan^5\left(\frac{1}{4}(2e-\pi+2fx)\right)-\frac{2}{7}(A-B)(c-d)^2\,{}_2F_1\left(\frac{7}{2},4+m;\frac{9}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan^7\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)^2*Hypergeometric2F1[1/2, 4 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan
[(2*e - Pi + 2*f*x)/4] - (2*(c + d)*(3*A*c + B*c - A*d - 3*B*d)*Hypergeomet
ric2F1[3/2, 4 + m, 5/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x
)/4]^3)/3 - (2*(c - d)*(A*(3*c + d) - B*(c + 3*d))*Hypergeometric2F1[5/2, 4
 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^5)/5 - (
2*(A - B)*(c - d)^2*Hypergeometric2F1[7/2, 4 + m, 9/2, -Tan[(2*e - Pi + 2*f
*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^7)/7))/f)
```

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c+d\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e)^2 - (B*d
^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e)
+ a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin (e + f x)) (a + a \sin (e + f x))^m (c + d \sin (e + f x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=199

$$\frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{2^{\frac{1}{2}+m}(A(2+m)(c+cm+dm)+B(cm(2+\cdots}{\cdots}$$

```
[Out] (B*d-(A*d+B*c)*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m)
*(A*(2+m)*(c*m+d*m+c)+B*(c*m*(2+m)+d*(m^2+m+1)))*cos(f*x+e)*hypergeom([1/2,
 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^
m/f/(m^2+3*m+2)-B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)
```

Rubi [A]
time = 0.26, antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3047, 3102, 2830, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)(m+2)}+\frac{\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)(m+2)}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((B*d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (2^(1/2 + m)*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*
(1 + m + m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^m\,(Ac+(Bc+Ad$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(}{f(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(}{f(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(}{f(1+m)(2+m)}$$

Mathematica [A]
time = 4.25, size = 212, normalized size = 1.07

$$-\frac{\csc^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^m (a(1+\sin(e+fx)))^m \left(-2(A+B)(c+d)\,{}_2F_1\left(\frac{1}{2},3+m;\frac{3}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)-\frac{4}{3}(Ac-Bd)\,{}_2F_1\left(\frac{3}{2},3+m;\frac{5}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan^3\left(\frac{1}{4}(2e-\pi+2fx)\right)-\frac{2}{5}(A-B)(c-d)\,{}_2F_1\left(\frac{5}{2},3+m;\frac{7}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\tan^5\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)*Hypergeometric2F1[1/2, 3 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(
2*e - Pi + 2*f*x)/4] - (4*(A*c - B*d)*Hypergeometric2F1[3/2, 3 + m, 5/2, -T
an[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^3)/3 - (2*(A - B)*(c
- d)*Hypergeometric2F1[5/2, 3 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(
2*e - Pi + 2*f*x)/4]^5)/5))/f)
```

Maple [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int \left(a +a \sin \left(f x +e \right)\right)^{m} \left(A +B \sin \left(f x +e \right)\right) \left(c +d \sin \left(f x +e \right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*si
n(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{m} \left(A + B \sin{\left(e + f x \right)}\right) \left(c + d \sin{\left(e + f x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,(c+d\,\sin\left(e+f\,x\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx\right.}{f(1+m)}$$

```
[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2830, 2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{(A+Am+Bm)\int}{1}$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{((A+Am+Bm)(}{}$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.25, size = 275, normalized size = 2.35

$$-\frac{(a(1+\sin(e+fx)))^m\left(\frac{\sqrt[4]{-1}\,2^{-1-2m}Be^{-\frac{3}{2}i(e+fx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\right)^{1+2m}\left(e^{2i(e+fx)}(-1+m)\,{}_2F_1\left(1,m;-m;-ie^{-i(e+fx)}\right)-(1+m)\,{}_2F_1\left(1,2+m;2-m;-ie^{-i(e+fx)}\right)\right)}{-1+m^2}+\frac{2\sqrt{2}A\cos^{1+2m}\left(\frac{1}{4}(2e-\pi+2fx)\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)}{(1+2m)\sqrt{1-\sin(e+fx)}}\right)\sin^{-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*A*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m (A+B\sin (e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\sin (e+f\,x))\,(a+a\,\sin (e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt{2}\,(Bc-Ad)F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m}{(c-d)df(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] -2^(1/2+m)*B*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1
+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/f-(-A*d+B*c)*AppellF1(1/2+m,1,1/
2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x
+e))^m*2^(1/2)/(c-d)/d/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3065, 2731, 2730, 2867, 142, 141}

$$-\frac{\sqrt{2}\,(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^mF_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}-\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m
)/((c - d)*d*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])) - (2^(1/2 + m)*B*Cos[e +
f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e
+ f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3065

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^m\,dx}{d} - \frac{(Bc-Ad)\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}}{d}$$

$$= -\frac{\left(a^2(Bc-Ad)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac12+m}}{\sqrt{a-ax}\,(c+dx)}\,dx,\right.}{df\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac12+m}B\cos(e+fx)\,{}_2F_1\left(\frac12,\frac12-m;\frac32;\frac12(1-\sin(e+fx))\right)}{df}$$

$$= -\frac{\sqrt{2}\,(Bc-Ad)F_1\left(\frac12+m;\frac12,1;\frac32+m;\frac12(1+\sin(e+fx)\right)}{(c-d)df(1+2m)\sqrt{1}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 473 vs. 2(191) = 382.
time = 5.01, size = 473, normalized size = 2.48

$$\frac{(a(1+\sin(e+fx)))^m\left(\frac{\sqrt{2}B\cos(e+fx)\,{}_2F_1\left(\frac12,\frac12+m;\frac32+m;\frac14\cos^2(e+fx)\csc^2\left(\frac14(2e-\pi+2fx)\right)\right)}{(d+2dm)\sqrt{1-\sin(e+fx)}}+\frac{6(c+d)(Bc-Ad)F_1\left(\frac12;\frac12-m,1;\frac32;\cos^2\left(\frac14(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac14(2e-\pi+2fx)\right)}{c+d}\right)\cos^2\left(\frac14(2e-\pi+2fx)\right)^{\frac12+m}\cot\left(\frac14(2e+\pi+2fx)\right)\sec^2\left(\frac14(2e-\pi+2fx)\right)\sin^2\left(\frac14(2e+\pi+2fx)\right)^{\frac12-m}}{d(c+d\sin(e+fx))\left(3(c+d)F_1\left(\frac12;\frac12-m,1;\frac32;\cos^2\left(\frac14(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac14(2e-\pi+2fx)\right)}{c+d}\right)+\left(4dF_1\left(\frac32;\frac12-m,2;\frac52;\cos^2\left(\frac14(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac14(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac32;\frac32-m,1;\frac52;\cos^2\left(\frac14(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac14(2e-\pi+2fx)\right)}{c+d}\right)\right)\sin^2\left(\frac14(2e-\pi+2fx)\right)\right)}\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*((Sqrt[2]*B*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 + m, 3/2 + m, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4])/((d + 2*d
*m)*Sqrt[1 - Sin[e + f*x]]) + (6*(c + d)*(B*c - A*d)*AppellF1[1/2, 1/2 - m,
 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Sec
[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*(c + d
*Sin[e + f*x])*(3*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*
f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (4*d*AppellF1[3/2,
1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4
]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e
+ Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e -
Pi + 2*f*x)/4]^2))))/f
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m\,(A+B\sin(fx+e))}{c+d\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))\ (a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt{2}\,(Ad(c(1-m)-dm)-B(d^2-c^2m-cdm))\,F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right.}{(c-d)^2d(c+d)f(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2^(1/2+m)*(-A*d+B*c)*m*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)/f-(-A*d+B*c)
*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+e))+(A*d*(c*(1-m)-d
*m)-B*(-c^2*m-c*d*m+d^2))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d
),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^2/d/(c+d)
/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3063, 3065, 2731, 2730, 2867, 142, 141}

$$\frac{\sqrt{2}\cos(e+fx)\left(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2)\right)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}+\frac{2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{df\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*(A*d*(c*(1 - m) - d*m) - B*(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m
, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]
*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*d*(c + d)*f*(1 + 2*m)*Sqrt
[1 - Sin[e + f*x]]) + (2^(1/2 + m)*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)
*(a + a*Sin[e + f*x])^m)/(d*(c^2 - d^2)*f) - ((B*c - A*d)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3065

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
```

```
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{(c^2-d^2)\,f(c+d\sin(e+fx))} - \frac{\int (a+a\sin \cdots}{\cdots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{(c^2-d^2)\,f(c+d\sin(e+fx))} - \frac{((Bc- \cdots}{\cdots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{(c^2-d^2)\,f(c+d\sin(e+fx))} + \frac{(a^2(Ad(\cdots}{\cdots}$$

$$= \frac{2^{\frac{1}{2}+m}(Bc-Ad)m\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin \cdots\right.}{d\,(c^2- \cdots}$$

$$= \frac{\sqrt{2}\,(Ad(c(1-m)-dm)-B(d^2-c^2m-cdm))\,F_1\left(\frac{1}{2}+ \cdots\right.}{(c-d)^2 d \cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 651 vs. 2(293) = 586.

time = 1.63, size = 651, normalized size = 2.22

$$\frac{6(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\left(\frac{(-Bc+Ad)F_1\left(\frac{1}{2};\frac{1}{2}-m,2;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)}{-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,2;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(-8dF_1\left(\frac{3}{2};\frac{1}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}+\frac{BF_1\left(\frac{1}{2};\frac{1}{2}-m,1;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)(c+d\sin(e+fx))}{-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,1;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(-4dF_1\left(\frac{3}{2};\frac{1}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,1;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{df(c+d\sin(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((-(B*c) + A*d)*AppellF1[1/2, 1/2 - m, 2, 3/2,
 Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(
-3*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-8*d*AppellF1[3/2, 1/2 - m, 3, 5/
2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
+ (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]
^2) + (B*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*S
```

```
in[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(-3*(c + d)*Appe
llF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
+ 2*f*x)/4]^2)/(c + d)] + (-4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + P
i + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 +
 2*m)*AppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e
+ Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^2)
```

Maple [F]
time = 2.25, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{(c + d\sin(fx + e))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c+d\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2, x
)
```

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=467

$$\frac{(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-3m+m^2)-d^2}{\sqrt{2}\,(c-d)^3d(c+d)^2f(1}$$

```
[Out] -2^(-1/2+m)*m*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*hype
rgeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*s
in(f*x+e))^m/d/(c^2-d^2)^2/f-1/2*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^m/(
c^2-d^2)/f/(c+d*sin(f*x+e))^2+1/2*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d
*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+1/2*(B*(2
*d^3*m+c^3*(1-m)*m+2*c^2*d*(1-m)*m-c*d^2*(m^2-3*m+3))-A*d*(2*c*d*(2-m)*m-c^
2*(m^2-3*m+2)-d^2*(m^2-m+1)))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/
(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c-d)^3/d/(c+d)^2/f
/(1+2*m)*2^(1/2)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.91, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3063, 3065, 2731, 2730, 2867, 142, 141}

$$\frac{\cos(e+fx)\left(B(c^3(1-m)m+2c^2d(1-m)m-cd^2(m^2-3m+3)+2d^3m)-Ad(-(c^2(m^2-3m+2))+2cd(2-m)m-d^2(m^2-m+1))\right)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{\sqrt{2}\,df(2m+1)(c-d)^3(c+d)^2\sqrt{1-\sin(e+fx)}}-\frac{2^{m-\frac{1}{2}}m\cos(e+fx)\left(Ad(c(3-m)-dm)-B(c^2(1-m)-cdm+2d^2)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{df\,(c^2-d^2)^2}+\frac{\cos(e+fx)\left(Ad(c(3-m)-dm)-B(c^2(1-m)-cdm+2d^2)\right)(a\sin(e+fx)+a)^m}{2f\,(c^2-d^2)^2\,(c+d\sin(e+fx))}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\,(c^2-d^2)\,(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3 - 3*m + m^2)) -
 A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m + m^2)))*AppellF1[
1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
 - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(Sqrt[2]*(c - d)^3*d*(c + d)^2
*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]) - (2^(-1/2 + m)*m*(A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin
[e + f*x])^m)/(d*(c^2 - d^2)^2*f) - ((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e
+ f*x])^m)/(2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/
(2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 3065

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{(a+a\sin(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(Ad(c(3-\ldots}{\ldots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(Ad(c(3-\ldots}{\ldots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(Ad(c(3-\ldots}{\ldots}$$

$$= -\frac{2^{-\frac{1}{2}+m}m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-c\ldots}{\ldots}$$

$$= \frac{(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+\ldots}{\ldots}$$

Mathematica [A]

time = 1.73, size = 651, normalized size = 1.39

$$\frac{6(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\left(\frac{(-Bc+Ad)F_1\left(\frac{1}{2};\frac{1}{2}-m,3;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)}{-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,3;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(-12dF_1\left(\frac{3}{2};\frac{1}{2}-m,4;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}+\frac{BF_1\left(\frac{1}{2};\frac{1}{2}-m,2;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)(c+d\sin(e+fx))}{-3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,2;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(-8dF_1\left(\frac{3}{2};\frac{1}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{df(c+d\sin(e+fx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((-(B*c) + A*d)*AppellF1[1/2, 1/2 - m, 3, 3/2,
 Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(
-3*(c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-12*d*AppellF1[3/2, 1/2 - m, 4, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)
/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4
]^2) + (B*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*
Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(-3*(c + d)*App
ellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d)] + (-8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1
+ 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e
 + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^3)
```

Maple [F]
time = 2.16, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c+d\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)
^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^m}{(c + d \sin(e + f x))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3, x
)
```

3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=284

$$\frac{\sqrt{2}\,(A-B)(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+ }{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (A-B)*(c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*s
in(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(
1+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*(c-d)*AppellF1
(3/2+m,-3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e
)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(
f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3066, 2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}+\frac{\sqrt{2}\,B(c-d)\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{3}{2};\frac{1}{2},-\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (Sqrt[2]*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c
 + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d)*AppellF1[3/2 + m, 1/2, -3
/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*
m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = (A-B)\int (a+a\sin(e+fx))^m(c+$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\text{Subst}\Big(\int \frac{(a}{}}{f\sqrt{a-a\sin(e+fx)}}$$

$$= \frac{\Big(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin}{a}}}{\sqrt{2}\,f(a-}$$

$$= \frac{\Big(a(A-B)(ac-ad)\cos(e+fx)\sqrt{}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}\,(A-B)(c-d)F_1\Big(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3281 vs. $2(284) = 568$.
time = 7.43, size = 3281, normalized size = 11.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -((((-2*B*c*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^
2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 +
 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/2)*Sqrt[c + d -
 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x
)/2]^2)/(c + d)]) - (2*A*d*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e +
Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2
- f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + P
i/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/
```

2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(
-e + Pi/2 - f*x)/2]^2)/(c + d)]) + (B*d*AppellF1[5/2, (1 - 2*m)/2, -1/2, 7/
2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Co
s[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
)*Sin[(-e + Pi/2 - f*x)/2]^5*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
+ (-1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(5*Sqrt[(c +
d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]) - (2*B*d*AppellF1[3/2, (-1 -
2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2
]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^
2)^((-1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^
2)^((-1 - 2*m)/2 + (1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2
])/Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (3*B*d*(c + d)*
AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e
 + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(3 + 2*m)*(Cos[(-e +
 Pi/2 - f*x)/2]^2)^(1/2 + (-4 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(
-e + Pi/2 - f*x)/2]^2)^(3/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^
2])/(-3*(c + d)*AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]
^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -3/2 - m
, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c
 + d)] + (c + d)*(3 + 2*m)*AppellF1[3/2, -1/2 - m, -1/2, 5/2, Sin[(-e + Pi/
2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 -
f*x)/2]^2) - (6*B*c*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi
/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 -
f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 + (-2 - 2*m)/2)*Sin[(-e
 + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(1/2 + m)*Sqrt[c + d - 2
*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3
/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] +
 (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Si
n[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1 + 2*m)*AppellF1[3/2, 1/2 -
m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/
(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) - (6*A*d*(c + d)*AppellF1[1/2, -1/2 -
 m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)
/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(
1/2 + (-2 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^
2)^(1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*Appe
llF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + P
i/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e
+ Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1
+ 2*m)*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*S
in[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) + (6*A*c*(
c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*S
in[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Co
s[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e +
 Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])
/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (

```
2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2, 1/2 - m, 1/2,
 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]
 + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*
x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2
]^2) + (3*B*d*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*
x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]
^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/
2 - f*x)/2]^2])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2
 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2
, 1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)
/2]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[
(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f...
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(A+B\sin\left(fx+e\right)\right) \left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),
 x)
```

3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e$

Optimal. Leaf size=274

$$\frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)
/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*AppellF1(3/2+m,-1/2,
1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f
*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))^(1/2
)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3066, 2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}+\frac{\sqrt{2}\,B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{3}{2};\frac{1}{2},-\frac{1}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -1/2, 5/2 + m, (1
+ Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*S
in[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[
e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = (A-B)\int (a+a\sin(e+fx))^m\sqrt{c+\ldots}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\mathrm{Subst}\Big(\int \frac{(a+\ldots}{\ldots}}{f\sqrt{a-a\sin(e+fx)}\ldots}$$

$$= \frac{\Bigg(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(\ldots}{a}}\ldots}{\sqrt{2}\,f(a-\ldots}$$

$$= \frac{\Bigg(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(\ldots}{a}}\ldots}{\sqrt{2}\,f\ldots}$$

$$= \frac{\sqrt{2}\,(A-B)F_1\Big(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m\ldots}{\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 672 vs. 2(274) = 548.
time = 17.13, size = 672, normalized size = 2.45

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*
x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*(((B*c - A*d)*AppellF1
[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2
*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*A
ppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e
- Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1
/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)])*Cos[(2*e + Pi + 2*f*x)/4]^2) - (B*AppellF1[1/2, 1/2 - m, -3/2, 3/2,
```

```
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c
+ d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-6*d*Appell
F1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -3/2,
5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)
])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(
d*f)
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)
^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e +
f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),
 x)
```

3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{\frac{c}{}}}{f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/f/
(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,1/2,1/
2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x
+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3066, 2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{2}\,B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{3}{2};\frac{1}{2},\frac{1}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1/2, 5/2 + m, (1 +
Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*f*(3 + 2*m)*Sqrt[
1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = (A-B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))}{\sqrt{c+d\sin(e+}}}{a}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,\sqrt{c+dx}}\,dx,\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\mathrm{Subst}\left(\int\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin}{ac+}}\right.}{\sqrt{2}\,f(a-a\sin(e+fx)}$$

$$= \frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\right.}{f(1+2m)\sqrt{1-\sin}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 672 vs. 2(274) = 548.
time = 4.01, size = 672, normalized size = 2.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3
*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*
x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)
/4]^2) - (B*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
```

```
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)
*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c
+ d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]
^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2
))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{\sqrt{c+d\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m (A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e +
f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2
), x)
```

3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{}}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c
-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,
3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*s
in(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/(c-d)/f/(3+2*m)/(
1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3066, 2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{2}\,B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{3}{2};\frac{1}{2},\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 3/2, 5/2 +
m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*(c - d)*f
*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = (A-B)\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))}{(c+d\sin(e+fx))}}{a}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^{3/2}}\,dx,\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\mathrm{Subst}\left(\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+}}$$

$$= \frac{\left(a^3(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d}{ac}}\right.}{\sqrt{2}\,(ac-ad)f(a-a\sin(e+}$$

$$= \frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),\right.}{(c-d)f(1+2m)\sqrt{1}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 672 vs. 2(288) = 576.
time = 4.54, size = 672, normalized size = 2.33

$$\frac{6(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m\left(\frac{(Bc-Ad)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{3}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)}{3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{3}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{5}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}-\frac{BF_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{1}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)(c+d\sin(e+fx))}{3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,\frac{1}{2};\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{df(c+d\sin(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3
*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*
x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)
/4]^2) - (B*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*
```

```
AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e
 - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(
2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d
)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(
Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^(3/2))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{(c + d\sin(fx + e))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(c + d*sin(e + f*x)
)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2
), x)
```

3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e
))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((c+
d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,-n,1/2,5/2+m,
-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+
m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)/(1-s
in(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3066, 2867, 145, 144, 143}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}}+\frac{\sqrt{2}\,B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{3}{2};\frac{1}{2},-n;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2,
-((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c +
 d*Sin[e + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x
])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -n, 5/2 + m, (1 + Sin[e
+ f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*
((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^m(c+d\,\mathrm{s}$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+a}{}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e}{a}}\right.}{\sqrt{2}\,f(a-a\,\mathrm{si}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e}{a}}\right.}{}$$

$$= \frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 682 vs. 2(270) = 540.
time = 4.20, size = 682, normalized size = 2.53

$$\frac{6(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^n\left(\frac{(Bc-Ad)F_1\left(\frac{1}{2};\frac{1}{2}-m,-n;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)}{3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-n;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(-4dnF_1\left(\frac{3}{2};\frac{1}{2}-m,1-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}-\frac{BF_1\left(\frac{1}{2};\frac{1}{2}-m,-1-n;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)(c+d\sin(e+fx))}{3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,-1-n;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\left(-4d(1+n)F_1\left(\frac{3}{2};\frac{1}{2}-m,-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,-1-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{df}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^n*(((B*c - A*d)*AppellF1[1
/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + P
i + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*n*Appel
lF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e -
Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -n,
5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)
])*Cos[(2*e + Pi + 2*f*x)/4]^2) - (B*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c +
d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Cos[(2*e +
```

```
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*(1 + n
)*AppellF1[3/2, 1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*
e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m,
-1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)
/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2
 - m))/(d*f)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c+d\sin(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=277

$$-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx))^{-1+m}\left(\frac{(c+d)(1+\sin(e+fx))}{c+d\sin(e+fx)}\right.}{(c+d)f}$$

```
[Out] -2^(1/2+m)*a*(A-B)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(c-d)*(1-sin
(f*x+e))/(c+d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/(c
+d*sin(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)+B*AppellF1(3/2+m,1+m,1
/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*
x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)/a/(c-d)/f/(3+2*m)/((c+d*sin(
f*x+e))^m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3066, 2867, 134, 145, 144, 143}

$$\frac{\sqrt{2}\,B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{3}{2};\frac{1}{2},m+1;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}}-\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)}{f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] -((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])
^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((
c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1 + m
, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c
- d)*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2867

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a + b*x)^(m - 1/2)*((c + d*x
)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 3066

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx = (A-B)\int (a+a\sin(e+fx))^m(c$$

$$= \frac{(a^2(A-B)\cos(e+fx))\,\mathrm{Subst}\Big(\int}{f\sqrt{a-a\sin(e+}}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\Big(}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\Big(}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\Big(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 573 vs. 2(277) = 554.
time = 4.52, size = 573, normalized size = 2.07

$$\frac{2\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-\frac{1}{2}+m}\cot\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{-m}\left(-\frac{3B(c+d)^2F_1\left(\frac{1}{2};\frac{1}{2}-m,m;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)}{d\left(3(c+d)F_1\left(\frac{1}{2};\frac{1}{2}-m,m;\frac{3}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-\left(-4dmF_1\left(\frac{3}{2};\frac{1}{2}-m,1+m;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+(c+d)(-1+2m)F_1\left(\frac{3}{2};\frac{3}{2}-m,m;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)}-A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)\left(\frac{(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}+\frac{Bc\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)\left(\frac{(c+d)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}}{d}\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{1}{2}-m}}{(c+d)f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1
 + Sin[e + f*x]))^m*((-3*B*(c + d)^2*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*
e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(d*(3*(c
+ d)*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(
2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (-4*d*m*AppellF1[3/2, 1/2 - m, 1 + m, 5/
2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
+ (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, m, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]
^2)) - A*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*
```

```
x)/4]^2)/(c + d*Sin[e + f*x])]*(((c + d)*Cos[(2*e - Pi + 2*f*x)/4]^2)/(c +
d*Sin[e + f*x]))^(1/2 - m) + (B*c*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c
- d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x])]*(((c + d)*Cos[(2*e
- Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]))^(1/2 - m))/d)*(Sin[(2*e + Pi + 2*
f*x)/4]^2)^(1/2 - m))/((c + d)*f*(c + d*Sin[e + f*x])^m)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c+d\sin(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(
-m - 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m +
 1), x)
```

3.348 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=132

$$\frac{2\sqrt{2}\,F_1\left(\frac{1}{2}+m;-\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin$$
$$f(1+2m)$$

[Out] 2*AppellF1(1/2+m,-n,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)*(1-sin(f*x+e))^(1/2)/f/(1+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)

Rubi [A]
time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3087, 145, 144, 143}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Si
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*((c + d*Sin[e + f*
x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx=\frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a-\cdots}\right.}{\cdots}$$

$$=\frac{\left(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{\cdots}\right.}{\cdots}$$

$$=\frac{\left(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{\cdots}\right.}{\cdots}$$

$$=\frac{2\sqrt{2}\,F_1\left(\frac{1}{2}+m;-\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\cdots\right.}{\cdots}$$

Mathematica [F]

time = 12.75, size = 0, normalized size = 0.00

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(a - a\sin\left(fx + e\right)\right)\left(a + a\sin\left(fx + e\right)\right)^{m}\left(c + d\sin\left(fx + e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\sin(e + f x))^m (a - a\sin(e + f x)) (c + d\sin(e + f x))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.349 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2}\,F_1\left(\frac{1}{2}+m;-\frac{1}{2},1+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a}{(c-d)f(1+2m)}$$

```
[Out] 2*AppellF1(1/2+m,1+m,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))
*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)*(1-si
n(f*x+e))^(1/2)/(c-d)/f/(1+2*m)/((c+d*sin(f*x+e))^m)
```

Rubi [A]

time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3087, 145, 144, 143}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{1}{2};-\frac{1}{2},m+1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -(
(d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/((c - d)*f*(1 + 2*
m)*(c + d*Sin[e + f*x])^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx=\frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\right.}{}$$

$$=\frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx\right.}{}$$

$$=\frac{\left(\sqrt{2}\,a\sec(e+fx)(a-a\sin(e+f\right.}{}$$

$$=\frac{2\sqrt{2}\,F_1\left(\frac{1}{2}+m;-\frac{1}{2},1+m;\frac{3}{2}+m\right.}{}$$

Mathematica [F]

time = 5.88, size = 0, normalized size = 0.00

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m(c + d\sin(e + fx))^{-1-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(a - a\sin\left(fx + e\right)\right)\left(a + a\sin\left(fx + e\right)\right)^m\left(c + d\sin\left(fx + e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m\,\left(a-a\,\sin\left(e+f\,x\right)\right)}{(c+d\,\sin\left(e+f\,x\right))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m +
 1), x)
```

3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

```
[Out] -cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Rubi [A]
time = 0.12, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = $\{3053\}$

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)*m + (
c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+}{}$$

Mathematica [A]
time = 0.48, size = 39, normalized size = 1.00

$$-\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)
*m + (c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Maple [**F**]
time = 1.53, size = 0, normalized size = 0.00

$$\int (a+a\sin (fx+e))^{m} (c+d\sin (fx+e))^{-2-m} (d-(c-d)\, m+(c+(c-d)\, m) \sin (fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(((c - d)*m - ((c - d)*m + c)*sin(f*x + e) - d)*(a*sin(f*x + e) +
 a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

Fricas [**A**]
time = 0.46, size = 61, normalized size = 1.56

$$-\frac{(d\cos (fx+e)\sin (fx+e)+c\cos (fx+e))(a\sin (fx+e)+a)^{m}(d\sin (fx+e)+c)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*s
in(f*x + e) + c)^(-m - 2)/f
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d-(c-d)*m+(c+(c-d)*
m)*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 14.77, size = 98, normalized size = 2.51

$$-\frac{\left(a\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(d\,\sin\left(2\,e+2\,f\,x\right)-2\,c\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)\right)}{f\,\left(c+d\,\sin\left(e+f\,x\right)\right)^m\,\left(d^2\,\left(2\sin\left(e+f\,x\right)^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(d - m*(c - d) + sin(e + f*x)*(c + m*(c - d))))
/(c + d*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)^
2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d^
2 + 4*c*d*sin(e + f*x)))
```

3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$

Optimal. Leaf size=40

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

```
[Out] -cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Rubi [A]
time = 0.12, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{3053\}$

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)*m + (
c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 3053

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+}{}$$

Mathematica [A]
time = 0.53, size = 40, normalized size = 1.00

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)
*m + (c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Maple [F]
time = 1.52, size = 0, normalized size = 0.00

$$\int (a-a\sin (fx+e))^{m} (c+d\sin (fx+e))^{-2-m} (d+(c+d)\, m+(c+(c+d)\, m)\sin (fx+e))\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((c + d)*m + ((c + d)*m + c)*sin(f*x + e) + d)*(-a*sin(f*x + e) +
 a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

Fricas [A]
time = 0.46, size = 62, normalized size = 1.55

$$-\frac{(d\cos (fx+e)\sin (fx+e)+c\cos (fx+e))(-a\sin (fx+e)+a)^{m}(d\sin (fx+e)+c)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d*
sin(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d+(c+d)*m+(c+(c+d)*
m)*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 14.62, size = 99, normalized size = 2.48

$$-\frac{\left(-a\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(d\,\sin\left(2\,e+2\,f\,x\right)-2\,c\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)\right)}{f\,\left(c+d\,\sin\left(e+f\,x\right)\right)^m\,\left(d^2\,\left(2\,\sin\left(e+f\,x\right)^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^m*(d + sin(e + f*x)*(c + m*(c + d)) + m*(c + d)))
/(c + d*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)
^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d
^2 + 4*c*d*sin(e + f*x)))
```

3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=199

$$-\frac{b(2bBc-Abd-2aBd)x}{d^3}-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b(2Bc^3-Ac^2d-3Bcd^2+2Ad^3)\right)\tan^{-1}\left(\frac{d+c\tan(\frac{1}{2}(}{\sqrt{c^2-}}\right.}{d^3\left(c^2-d^2\right)^{3/2}f}$$

```
[Out] -b*(-A*b*d-2*B*a*d+2*B*b*c)*x/d^3-2*(-a*d+b*c)*(a*d^2*(A*c-B*d)-b*(-A*c^2*d
+2*A*d^3+2*B*c^3-3*B*c*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2
))/d^3/(c^2-d^2)^(3/2)/f-b^2*B*cos(f*x+e)/d^2/f-(-a*d+b*c)^2*(-A*d+B*c)*cos
(f*x+e)/d^2/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Rubi [A]
time = 0.40, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3067, 3102, 2814, 2739, 632, 210}

$$-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2)\right)\text{ArcTan}\left(\frac{c\tan(\frac{1}{2}(e+fx))+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{bx(-2aBd-Abd+2bBc)}{d^3}-\frac{b^2B\cos(e+fx)}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b*(2*b*B*c - A*b*d - 2*a*B*d)*x)/d^3) - (2*(b*c - a*d)*(a*d^2*(A*c - B*d
) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)*f) - (b^2*B*Cos[e + f*x])/(d
^2*f) - ((b*c - a*d)^2*(B*c - A*d)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*
Sin[e + f*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int -d\left(B(bc-ad)^2-Ad\left(a^2c-\right.\right.}{}$$

$$= -\frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \int$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)}{d^2\left(c^2-d\right.}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)}{d^2\left(c^2-d\right.}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)}{d^2\left(c^2-d\right.}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{2(bc-ad)\left(ad^2(Ac-Bd)-\right.}{}$$

Mathematica [A]
time = 1.10, size = 188, normalized size = 0.94

$$\frac{b(-2bBc+Abd+2aBd)(e+fx)+\frac{2(bc-ad)\left(ad^2(-Ac+Bd)+b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}-b^2Bd\cos(e+fx)+\frac{d(bc-ad)^2(-Bc+Ad)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c) +
 B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e +
 f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (d*(
b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f
*x])))/(d^3*f)
```

Maple [A]
time = 0.72, size = 374, normalized size = 1.88

method	result
derivativedivides	$\frac{2\left(\frac{d^2\left(a^2A\,d^3-2Aabc\,d^2+A\,b^2c^2d-B\,a^2c\,d^2+2Bab\,c^2d-c^3B\,b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(a^2A\,d^3-2Aabc\,d^2+A\,b^2c^2d-B\,a^2c\,d^2+2Bab\,c^2d-\right.}{c^2-d^2}\right.}{c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}$

default	$\frac{2\left(\frac{d^{2}\left(a^{2}A\,d^{3}-2Aabc\,d^{2}+A\,b^{2}c^{2}d-B\,a^{2}c\,d^{2}+2Bab\,c^{2}d-c^{3}B\,b^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)c}+\frac{d\left(a^{2}A\,d^{3}-2Aabc\,d^{2}+A\,b^{2}c^{2}d-B\,a^{2}c\,d^{2}+2Bab\,c^{2}\right.}{c^{2}-d^{2}}\right.}{c\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*(2/d^3*((d^2*(A*a^2*d^3-2*A*a*b*c*d^2+A*b^2*c^2*d-B*a^2*c*d^2+2*B*a*b*c
^2*d-B*b^2*c^3)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)+d*(A*a^2*d^3-2*A*a*b*c*d^2+A
*b^2*c^2*d-B*a^2*c*d^2+2*B*a*b*c^2*d-B*b^2*c^3)/(c^2-d^2))/(c*tan(1/2*f*x+1
/2*e)^2+2*d*tan(1/2*f*x+1/2*e)+c)+(A*a^2*c*d^3-2*A*a*b*d^4-A*b^2*c^3*d+2*A*
b^2*c*d^3-B*a^2*d^4-2*B*a*b*c^3*d+4*B*a*b*c*d^3+2*B*b^2*c^4-3*B*b^2*c^2*d^2
)/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
+2*b/d^3*(-B*b*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*b*d+2*B*a*d-2*B*b*c)*arctan(ta
n(1/2*f*x+1/2*e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(198) = 396.
time = 0.53, size = 1327, normalized size = 6.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^
2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*
```

```
B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b +
2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d
^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2
 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 +
(B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (
B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d - 4*B*b^2*
c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c
^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*
b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x
 + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4*B*b^2*c^4*d^2
+ 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 -
 (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b +
 A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d
^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2
+ 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(c^2
- d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*B*
b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*
b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2
)*c*d^5)*cos(f*x + e) + ((2*B*b^2*c^5*d - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 -
 (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2
)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*
d^5 + c*d^7)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(198) = 396.
time = 0.46, size = 776, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*d
^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x + 1
/2*e)^3 - 2*B*a*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*tan(1/2*f*
x + 1/2*e)^3 + B*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*tan(1/2*f
*x + 1/2*e)^3 - A*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*tan(1/2*f*x
+ 1/2*e)^2 - 2*B*a*b*c^3*d*tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*tan(1/2*f*x
 + 1/2*e)^2 + B*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d^2*tan(1/
2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c*d^3*tan(1
/2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*tan(1/2*f*x + 1/2*e) - 2*B*a*b*c^2*d^2*ta
n(1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^3*tan(1
/2*f*x + 1/2*e) + 2*A*a*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*B*b^2*c*d^3*tan(1/
2*f*x + 1/2*e) - A*a^2*d^4*tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4 - 2*B*a*b*c^3
*d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c^2*d^2 - A*a^2*
c*d^3)/((c^3*d^2 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2
*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*B*
b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f
```

Mupad [B]
time = 27.62, size = 2500, normalized size = 12.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*
A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2*(
A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*a*b
*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2
*d^3 - B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*
d))/(c*d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(A*a^2*d^3 - 3*B*b^2*c^3 + A*
b^2*c^2*d - B*a^2*c*d^2 + 2*B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(
c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2
 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (atan((((b*d*(A*b
+ 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b
^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*
B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B
*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7
+ 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*
a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(
A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^
9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 +
7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1
```

i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))*1i)/d^3 + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*

```
b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*
d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 -
8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*
b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*...
```

3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=840

$$\frac{(c-d)\sqrt{c+d}\,(2Ab^2c-2abBc-2aAbd+3a^2Bd-b^2Bd)\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c-}{(a+b)(c-}\right.}{(a-b)b^2\sqrt{a+b}\,(b}$$

```
[Out] (c-d)*(-2*A*a*b*d+2*A*b^2*c+3*B*a^2*d-2*B*a*b*c-B*b^2*d)*EllipticE((a+b)^(1
/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/
(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1
-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)
/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^(1/2)+(2*A*b*d-3*B*a*
d+3*B*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*s
in(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e
)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f
*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/
f/(a+b)^(1/2)+(2*A*b*(b*(c-2*d)+a*d)-B*(3*a^2*d-6*a*b*d+b^2*(2*c+d)))*Ellip
ticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),
((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((
-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(
f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b^3/f/(c+d)^(1/2)+2*(A*b-B*a)*(
-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^
(1/2)-(2*A*b*(-a*d+b*c)-B*(-3*a^2*d+2*a*b*c+b^2*d))*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 2.14, antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {3068, 3140, 3132, 2890, 3077, 2897, 3075}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((c - d)*Sqrt[c + d]*(2*A*b^2*c - 2*a*b*B*c - 2*a*A*b*d + 3*a^2*B*d - b^2*B
*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sq
rt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]
*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*S
qrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b
*Sin[e + f*x]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (Sqrt[c + d]*(3*b
```

```
*B*c + 2*A*b*d - 3*a*B*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a
 + b]*f) + (2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]]
)/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((2*A*b*(b*c - a*d) - B*(2*a
*b*c - 3*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b
^2)*f*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[a + b]*(2*A*b*(b*(c - 2*d) + a*d) -
 B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/
((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a
 - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^3*Sqrt[c + d
]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
```

```
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
```

```
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} -$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} -$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} -$$

$$= \frac{\sqrt{c+d}\,(3bBc+2Abd-3aBd)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{c+}}\right.\right.}{}$$

$$= \frac{(c-d)\sqrt{c+d}\,(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2042 vs. $2(840) = 1680$.
time = 6.54, size = 2042, normalized size = 2.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*(A*b^2*c*Cos[e + f*x] - a*b*B*c*Cos[e + f*x] - a*A*b*d*Cos[e + f*x] + a
^2*B*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b
*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b^2*
c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*
```

```
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*b^2*d^2 + 2
*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) +
2*(-2*A*b^2*c*d + 2*a*b*B*c*d + 2*a*A*b*d^2 - 3*a^2*B*d^2 + b^2*B*d^2)*((Co
s[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(
a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
- b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]
, (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt
[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a
*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))
/(b*d)))/(2*(a - b)*b*(a + b)*f)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 98.62, size = 6059287, normalized size = 7213.44

method	result	size
default	Expression too large to display	6059287

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt(b
*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin
(f*x + e) - a^2 - b^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin{(e + fx)})\,(c + d\sin{(e + fx)})^{\frac{3}{2}}}{(a + b\sin{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*
x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,\left(c+d\,\sin(e+f\,x)\right)^{3/2}}{\left(a+b\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^
(3/2), x)
```

3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=630

$$\frac{2(Ab-aB)(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(}{(c+d)(a}}}{(a-b)b\sqrt{a+b}\,(bc-ad)f}$$

```
[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1
/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/(-a*d+
b*c)/f/(a+b)^(1/2)+2*(A*b-B*a)*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))
^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c
+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(
1/2)/(a-b)/b/(-a*d+b*c)/f/(c+d)^(1/2)+2*B*EllipticPi((c+d)^(1/2)*(a+b*sin(f
*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(
1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-
b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3071, 2890, 2874, 2897, 3075}

$$\frac{2\sqrt{a+b}\,(c-d)(Ab-aB)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{bf(a-b)\sqrt{c+d}\,(bc-ad)}+\frac{2(c-d)\sqrt{c+d}\,(Ab-aB)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{bf(a-b)\sqrt{a+b}\,(bc-ad)}+\frac{2B\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\Pi\left(\frac{(a+b)d}{b(c+d)};\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{b^2f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*(b*c
- a*d)*f) + (2*Sqrt[a + b]*(A*b - a*B)*(c - d)*EllipticF[ArcSin[(Sqrt[c + d
]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b*Sqrt[
```

```
c + d]*(b*c - a*d)*f) + (2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d))
, ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(b^2*Sqrt[c + d]*f)
```

Rule 2874

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3071

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(\ldots}}{(a+b\sin(e+\ldots}}{b}$$

$$= \frac{2\sqrt{a+b}\,B\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\ldots\right.\right.}{\ldots}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin\ldots}}{\sqrt{c+d}\,\sqrt{a+b\sin\ldots}}\right.\right.}{\ldots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1901 vs. $2(630) = 1260$.
time = 29.54, size = 1901, normalized size = 3.02

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[(
(-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
```

```
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)*((Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
 + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi
[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
 b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b*d) + a*B*d)
*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (S
qrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqr
t[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
 b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d
*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[
e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcS
in[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]])))/(b*d)))/((a - b)*(a + b)*f)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 59.50, size = 3148347, normalized size = 4997.38

method	result	size
default	Expression too large to display	3148347

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin{(e + fx)})\sqrt{c + d\sin{(e + fx)}}}{(a + b\sin{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))
**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^
(3/2), x)
```

3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=417

$$\frac{2(Ab-aB)(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(}{(c+d)(a}}}{(a-b)\sqrt{a+b}\,(bc-ad)^2 f}$$

```
[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1
/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*
c)^2/f/(a+b)^(1/2)+2*(A-B)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+
b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)
*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x
+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)
/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3077, 2897, 3075}

$$\frac{2\sqrt{a+b}\,(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\,\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f(a-b)\sqrt{c+d}\,(bc-ad)}+\frac{2(c-d)\sqrt{c+d}\,(Ab-aB)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\,\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f(a-b)\sqrt{a+b}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c -
a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c -
a*d)*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
```

```
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

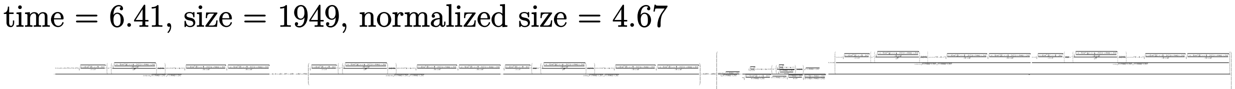
Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b}-$$

$$=\frac{2(Ab-aB)(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin}}{\sqrt{c+d}\sqrt{a+b\sin}}\right.\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1949 vs. $2(417) = 834$.

time = 6.41, size = 1949, normalized size = 4.67



Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
 + f*x]]),x]
```

```
[Out] (-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a
^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*
(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*
Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
+ b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
 b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(A*b^2*d - a*b*B*d)*((Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]
*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*S
in[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) +
a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(
-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*
(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ell
ipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
```

```
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
 + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/((a -
b)*(a + b)*(-(b*c) + a*d)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99489 vs. $2(387) = 774$.
time = 20.68, size = 99490, normalized size = 238.59

method	result	size
default	Expression too large to display	99490

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*c
os(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin{\left(e + fx \right)}}{\left(a + b \sin{\left(e + fx \right)}\right)^{\frac{3}{2}} \sqrt{c + d \sin{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e
 + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
1/2)), x)
```

3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=544

$$\frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{2(A(a^2d^2+b^2(c^2-2d^2))-B(a^2cd-b^2cd+}{}$$

```
[Out] -2*(A*(a^2*d^2+b^2*(c^2-2*d^2))-B*(a^2*c*d-b^2*c*d+a*b*(c^2-d^2)))*Elliptic
E((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-si
n(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2*(-A*a
*d+A*b*c-2*A*b*d+B*a*d+B*b*c)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/
(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x
+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*
c)^2/f/(a+b)^(1/2)/(c+d)^(1/2)+2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c
)/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3079, 3077, 2897, 3075}

$$\frac{2\sec(e+fx)\left(a^2(-A)d^2+a^2Bcd+abB(c^2-d^2)-Ab^2(c^2-2d^2)-b^2Bcd\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}E\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3}+\frac{2b(Ab-aB)\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}+\frac{2\sec(e+fx)(-aAd+aBd+Abc-2Abd+bBc)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*B*c*d - b^2*B*c*d - a^2*A*d^2 -
 A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqr
t[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x])
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/(
(a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqrt[a + b]*(c - d)*
Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(A*b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
```

)

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2266 vs. $2(544) = 1088$.
time = 7.01, size = 2266, normalized size = 4.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
- a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])
) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2 - d
^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*B*c^
3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 + b^3*B
*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2*d - 2*a^2*b*
B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B*c*d^2 + 2*a*b
^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
```

```
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*
d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b^3*c^2*d) + a*b^2*B*c^2*d
 + a^2*b*B*c*d^2 - b^3*B*c*d^2 - a^2*A*b*d^3 + 2*A*b^3*d^3 - a*b^2*B*d^3)*(
(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqr
t[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[
(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b
)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*S
qrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*S
in[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]],
 (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/
2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
 + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 198632 vs. $2(510) = 1020$.
time = 27.28, size = 198633, normalized size = 365.13

method	result	size
default	Expression too large to display	198633

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^
2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 +
 a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x +
e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin{\left(e + fx \right)}}{\left(a + b\sin{\left(e + fx \right)}\right)^{\frac{3}{2}} \left(c + d\sin{\left(e + fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f
*x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
3/2)), x)
```

3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=858

$$\frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}+\frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2c\ldots}{3\,(a^2-b^2)\,(bc-ad)^2\,(c\ldots}$$

```
[Out] 2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(3/2)/(a+b
*sin(f*x+e))^(1/2)+2/3*d*(A*(a^2*d^2+b^2*(3*c^2-4*d^2))-B*(a^2*c*d-b^2*c*d+
3*a*b*(c^2-d^2)))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/
(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(B*(2*a^2*b*c*d*(3*c^2-d^2)-2*b^3*c*
d*(3*c^2-d^2)-a^3*d^2*(c^2+3*d^2)+a*b^2*(3*c^4-5*c^2*d^2+6*d^4))+A*(4*a^3*c
*d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^4)))*E
llipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1
/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c
)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/
(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2
)-2/3*(B*(a^2*d^2*(c+3*d)-b^2*c*(3*c^2+3*c*d-2*d^2)-6*a*b*d*(c^2-d^2))-A*(a
^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d^3)))*Ellipt
icF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-
sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)
/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f/(a+b)^(1/2)
```

Rubi [A]
time = 1.76, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3079, 3134, 3077, 2897, 3075}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(3/2)) + (2*d*(A*(a^2*d^2 + b^2*(3*c^2 - 4*d^2
)) - B*(a^2*c*d - b^2*c*d + 3*a*b*(c^2 - d^2)))*Cos[e + f*x]*Sqrt[a + b*Sin
[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])
^(3/2)) + (2*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^2 - d^2) - a^3*
d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*(4*a^3*c*d^3 - 4
*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4)
))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqr
```

```
t[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*
Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[
-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*
Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) - (2
*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*d^2) - 6*a*b*d*(c^2 - d^2
)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*
c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)^3*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3075

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2837 vs. $2(858) = 1716$.
time = 7.85, size = 2837, normalized size = 3.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x]
 - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x]
)) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c
^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*A*b*
c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e + f*x]
- 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e + f*x
]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c)
 + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4*c^4*d -
 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^4*B*c^3*d
^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 + 10*a^3*b*B
*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^4 - 4*a^4*B
*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*A*b^2*d^5 -
8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2 -
 f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b
```

)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b
)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) +
a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d + 9*a^2*b^2*B*c^4*d -
6*b^4*B*c^4*d - 9*a^2*A*b^2*c^3*d^2 + 15*A*b^4*c^3*d^2 + 5*a^3*b*B*c^3*d^2
- 11*a*b^3*B*c^3*d^2 - 5*a^3*A*b*c^2*d^3 + 11*a*A*b^3*c^2*d^3 - a^4*B*c^2*d
^3 - 7*a^2*b^2*B*c^2*d^3 + 2*b^4*B*c^2*d^3 + 4*a^4*A*c*d^4 + a^2*A*b^2*c*d^
4 - 8*A*b^4*c*d^4 - 5*a^3*b*B*c*d^4 + 8*a*b^3*B*c*d^4 + 5*a^3*A*b*d^5 - 8*a
*A*b^3*d^5 - 3*a^4*B*d^5 + 6*a^2*b^2*B*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2
- f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/(
(a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c +
 d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b
)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
 + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*A*b^4*c^4*d - 3*a*b^3*B*c^4*d - 6*a^2*
b^2*B*c^3*d^2 + 6*b^4*B*c^3*d^2 + 9*a^2*A*b^2*c^2*d^3 - 15*A*b^4*c^2*d^3 +
a^3*b*B*c^2*d^3 + 5*a*b^3*B*c^2*d^3 - 4*a^3*A*b*c*d^4 + 4*a*A*b^3*c*d^4 + 2
*a^2*b^2*B*c*d^4 - 2*b^4*B*c*d^4 - 5*a^2*A*b^2*d^5 + 8*A*b^4*d^5 + 3*a^3*b*
B*d^5 - 6*a*b^3*B*d^5)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a +
 b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]
*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a
+ b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c +
 d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin
[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqr
t[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b
*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-
c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt
[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d
), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x
]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*
Sqrt[c + d*Sin[e + f*x]])))/(b*d))/(3*(a - b)*(a + b)*(c - d)^2*(c + d)^2*

```
(-(b*c) + a*d)^3*f)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 50.63, size = 867629, normalized size = 1011.22

method	result	size
default	Expression too large to display	867629

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 +
(a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3
+ 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c
^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6
*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
5/2)), x)
```

3.358 $\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=38

$$\text{Int}((a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n,x)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^m(A+B\sin(e+$$

Mathematica [A]
time = 16.63, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))^m (A+B\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```